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Computational Tools **ED15013**

Assignment 2

Pseudo Random Number Generation

Linear congruential generator is used to generate random integers. It follows the scheme:

$$X_{n+1} = (aX_n + c)\ mod\ m$$

where a and c are large integers, X_n are the generated random numbers and X_0 is called the seed.

custom_random.py contains the functions that are used for operations involving random numbers. *randint()* generates and returns an array random numbers in range from low to high (low is included but high is not).

choice() takes an array as input and returns randomly selected element from it.

Code

custom_random.py

```
import numpy as np
import time


# Generates random numbers form [low, high)
# Uses linear congruential generator algorithm
def randint(low, high, size):
    a = 2147483629
    c = 2147483587
    m = 2**31 - 1
    seed = int(10 ** 10 * time.process_time())

    rand = np.zeros(size, dtype=int)
    rand[0] = (a * seed + c) % m

    for i in range(1, size):
        rand[i] = (a * rand[i-1] + c) % m

    # Bringing random numbers to the required range
    rand = low + rand % (high-low)

    return rand


# Choose a random number from an array
def choice(arr):
    arr_len = len(arr)
    rand = randint(0, arr_len, 1)
    return arr[rand[0]]
```

1 Question 1

Question 1 requires us to simulate the different cases of the Monte Hall problem. There are 3 different cases involved in this problem which are discussed below. The problem is simulated using 10,000 iterations. The doors are represented as a set of 3 numbers, $S = \{0, 1, 2\}$.

An array of 10,000 correct answers is created by randomly selecting one element from set S. Similarly, another array of initial selections is created. The number of times the contestant wins is counted and the probability of winning is calculated by dividing it by the total number of simulations for each case.

1.1 Never Swap

In this case the contestant never avails the option of swapping his/her selection. This case is easy to simulate, since removing a wrong answer from the set of possible selections does not change the initial selection. It is a redundant step. Hence, a simple check if the initial selection was the correct one is enough to verify if the selection is correct.

```
• correct[] = randint(0, 3, num_sim)
• selection[] = randint(0, 3, num_sim)
• wins = 0
• for i in 0 to num_sim - 1
•     S = {1, 2, 3}
•     remove wrong answer from S which is not selection[i]
•     if (selection[i]= correct[i])
•         wins ← wins + 1
• return wins / number of simulations
```

1.2 Always Swap

In this case, the contestant always swaps his/her selection after a wrong option is removed. On observing, it can be seen the contestant will lose only if his/her initial selection is the correct answer. Now, we have to create set of answers from which the host can remove one wrong answer. It cannot contain the correct answer as well as the initial selection. It can be done by removing the correct answer and selection from the set of possible answers. If the length of the array is 2, it means the contestant has selected the correct answer, therefore he/she will lose. If it is 1, then a wrong answer was selected. This means the contestant will win.

```
• correct[] = randint(0, 3, num_sim)
• selection[] = randint(0, 3, num_sim)
• wins = 0
• for i in 0 to num_sim - 1
•     S = {1, 2, 3}
•     remove correct[i] and selection[i] from S
•     if (length(S) = 1)
•         wins ← wins + 1
• return wins / number of simulations
```

1.3 Coin Toss

In this case, after a wrong answer is removed, the contestant tosses a coin to select between the two remaining doors.

```
correct[] = randint(0, 3, num_sim)
selection[] = randint(0, 3, num_sim)
wins = 0
for i in 0 to num_sim - 1
    S = {1, 2, 3}
    remove wrong answer from S which is not selection[i]
    new_selection = choice(S) # Randomly select one from the remaining 2 elements
    if (new_selection = correct[i])
        wins ← wins + 1
return wins / number of simulations
```

1.4 Results

The program was run 5 times and the results are as follows:

	Never Swap	Always Swap	Coin Toss
	0.3263	0.6737	0.5042
	0.3229	0.6771	0.5058
	0.3372	0.6628	0.5062
	0.3333	0.6667	0.5067
	0.3331	0.6669	0.5065
Mean =	0.33056	0.66944	0.50588

These results agree with the theoretical values.

1.5 Code

monte_hall.py

```python
import numpy as np
import custom_random


# Simulates the condition when the contestant never
# swaps his/her choice
def never_swap(correct, selection):
    n = len(correct)
    # Omitting redundant step of removing wrong answer
    num_wins = np.sum(correct == selection)
    return num_wins / n


# Simulates the condition when the contestant always
# swaps his/her choice
def always_swap(correct, selection):
    n = len(correct)
    num_wins = 0

    for i in range(n):
        # Getting the set of available choices for
        # the host to remove
        # Host cannot remove the correct choice
        available_choices = [0, 1, 2]
        available_choices.remove(correct[i])

        # Host cannot remove the initial selection as well
        # Checking if initial selection was already removed
        # or not, if not then removing it
        if available_choices.__contains__(selection[i]):
            available_choices.remove(selection[i])

        # If initial selection was not correct answer
        # then contestant will win after swapping
        # Since list of available choices for the host
        # to remove will have only one element
        if len(available_choices) == 1:
            num_wins += 1

    return num_wins / n


# Simulates the condition when the user tosses a coin
# To select the new door
def coin_toss(correct, selection):
    n = len(correct)
    num_wins = 0

    for i in range(n):
        # Getting the set of available choices for
        # the host to remove
        # Host cannot remove the correct choice
        available_choices = [0, 1, 2]
        available_choices.remove(correct[i])

        # Host cannot remove the initial selection as well
        # Checking if initial selection was already removed
        # or not, if not then removing it
        if available_choices.__contains__(selection[i]):
            available_choices.remove(selection[i])

        # Now the list only contains wrong answers
        # Randomly removing one from the list
        available_choices.remove(custom_random.choice(available_choices))

        # Adding the correct choice back
        available_choices.append(correct[i])

        # Adding the selected choice back
        # If the initial selection was the correct choice
        # then no need to add it again
        if not available_choices.__contains__(selection[i]):
            available_choices.append(selection[i])

        # Randomly selecting one door from the remaining two
        new_selection = custom_random.choice(available_choices)

        if new_selection == correct[i]:
            num_wins += 1

    return num_wins / n


num_simulations = int(input('Enter the number of simulations: '))
correct = custom_random.randint(low=0, high=3, size=num_simulations)
selection = custom_random.randint(low=0, high=3, size=num_simulations)

print('Never Swap:', never_swap(correct, selection))
print('Always Swap:', always_swap(correct, selection))
print('Coin Toss:', coin_toss(correct, selection))
```

2 Question 2

Simple moving average is taken using 50 consecutive data points. Data for each day is randomly taken from set S.

2.1 Pseudo Code for Moving Average

```
• moving_average(data, points_per_avg)
•     n = length(a)
•     for i in 0 to n - points_per_avg
•         avg_data[i] = mean(data[i to (i + points_per_avg)])
•     return avg_data
```

2.2 Plot for the Trajectory

All the trajectories are plotted. One of them is presented below.

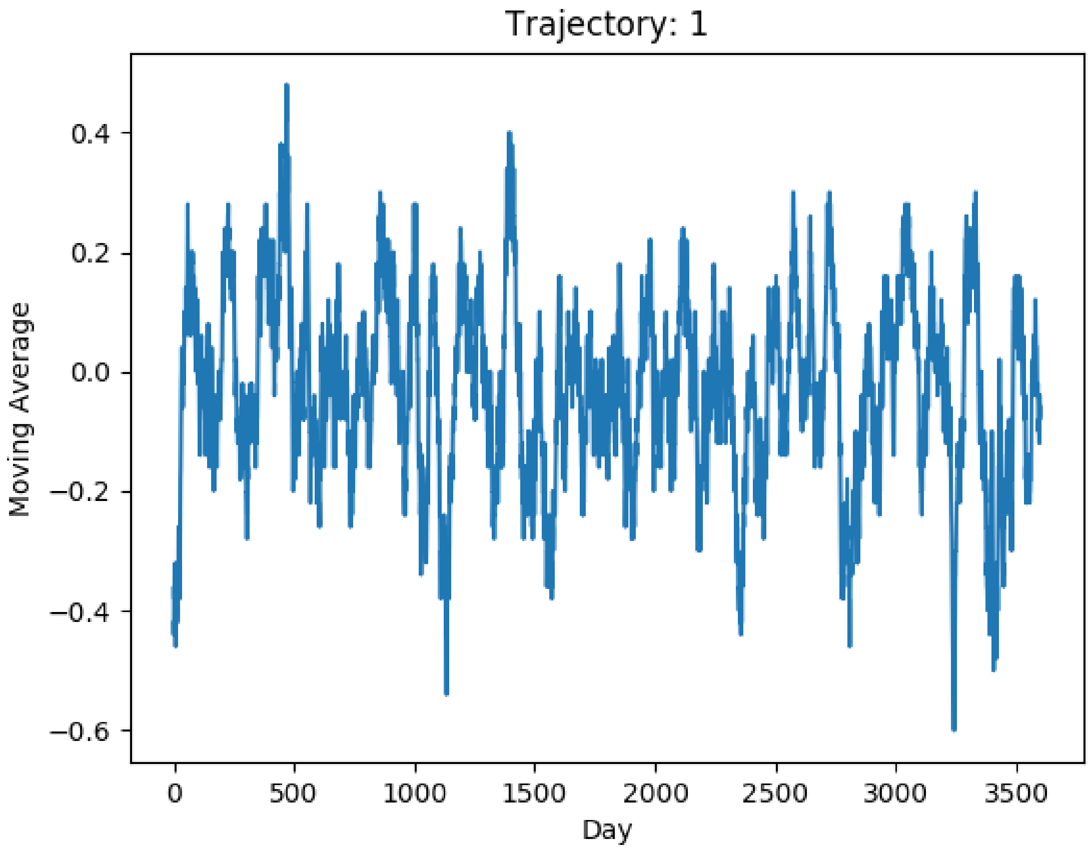
2.3 Code

stock_price.py

```
import numpy as np
import matplotlib.pyplot as plt
import custom_random


# Returns array which stores the moving average of the data
def simple_moving_avg(data, points_per_avg):
    n = len(data)
    avg_data = np.zeros(n-points_per_avg)

    # Taking the moving average
```



```
    for i in range(n-points_per_avg):
        avg_data[i] = np.mean(data[i:i+points_per_avg])

    return avg_data


DAYS_PER_YEAR = 365
NUM_YEARS = 10
NUM_DATA_POINTS = DAYS_PER_YEAR * NUM_YEARS
NUM_TRAJECTORIES = 10
S = np.array([-2, -1, 0, 1, 2])

points_per_avg = int(input('Enter number of data points per average: '))

for i in range(NUM_TRAJECTORIES):
    points_per_avg = 50

    # Randomly choosing points from S
    data = S[custom_random.randint(low=0, high=5, size=NUM_DATA_POINTS)]
    avg_data = simple_moving_avg(data, points_per_avg)

    # Plotting each trajectory
    plt.figure()
    plt.title('Trajectory: ' + str(i+1))
    plt.xlabel('Day')
    plt.ylabel('Moving Average')
    plt.plot(avg_data)
plt.show()
```

3 Question 3

The program is divided into 3 main parts:

3.1 Main

It is the entry point to the program. It initializes the simulation by creating an instance of GridDrawer.

3.2 GridDrawer

Initializes the OpenGL window and handles the drawing as the game iterates.

3.3 GameOfLife

This is used to create the Game of Life instance. An $N \times N$ grid is created. It is a boolean grid whose cells store *true* if an organism is present in it, and *false* otherwise. A certian number of seeds are randomly arranged in the grid that act as the initial configuration. *iterate()* function determines the new configuration of the grid based on the rules, and return the new grid which is used in GridDrawer.

3.3.1 Algorithm

```
• iterate(grid, N)
•     newGrid[][] = boolean[N][N]
•     for (i, j) in (0, 0) to (N-1, N-1)
•         nCount = getNCount(grid, i, j) # Neighbour count of grid[i][j]
•         if (nCount = 3)
•             newGrid[i][j] = true # Organism is born
•         else if(nCount = 2)
•             newGrid[i][j] = grid[i][j] # State does not change
•         else
•             newGrid[i][j] = false # Organism dies
•     return newGrid
```

3.4 Code

Main.java

```
/*
* This is the starting point to the program
* Takes the user inputs, validates them and
* initializes the Game of Life
*
* */

import java.io.BufferedReader;
import java.io.IOException;
import java.io.InputStreamReader;
```

```
public class Main {

    private static final int MIN_GRID_SIZE = 10;
    private static final int MAX_GRID_SIZE = 1000;

    private static BufferedReader bufferedReader;


    public static void main(String[] args) throws IOException {

        bufferedReader = new BufferedReader(new InputStreamReader(System.in));


        int gridSize = takeInput(
                ”grid␣size”,
                MIN_GRID_SIZE,
                MAX_GRID_SIZE
        );

        // Number of seeds cannot exceed the number of cells
        int numSeeds = takeInput(
                ”number␣of␣seeds”,
                1,
                gridSize*gridSize
        );

        // Initializing the Game of Life
        GameOfLife game = new GameOfLife(gridSize, numSeeds);
        GridDrawer gridDrawer = new GridDrawer(game);
        gridDrawer.draw();
    }


    private static int takeInput(String varName, int minVal, int maxVal) throws I
        int input;

        while (true) {
            System.out.printf(”Enter␣the␣%s␣(Range␣[%d,␣%d]):␣”, varName, minVal,
            input = Integer.parseInt(bufferedReader.readLine());

            // Validating the input
            if (input >= minVal && input <= maxVal)
                break;
            else
                System.out.println(”Input␣is␣out␣of␣range”);
        }

        return input;
    }
}
```

GridDrawer.java

```java
import com.jogamp.opengl.*;
import com.jogamp.opengl.awt.GLCanvas;
import com.jogamp.opengl.util.FPSAnimator;

import javax.swing.*;
import java.awt.event.MouseAdapter;
import java.awt.event.MouseEvent;
import java.awt.event.MouseWheelEvent;

/*
* This class is uses OpenGL to draw the grid.
* This takes an instance of GameOfLife and iterates
* through it.
*
* */

public class GridDrawer implements GLEventListener {

    // Parameters for dragging window functionality
    // And controlling zoom
    private float DRAG_FACTOR = 500.0f;
    private float ZOOM_FACTOR = 1.1f;
    private int MAX_DRAG = 50;

    private float zoom = 2.0f;
    private int prevScreenX = 0;
    private int prevScreenY = 0;
    private int curScreenX = 0;
    private int curScreenY = 0;

    private float windowX = 0.0f;
    private float windowY = 0.0f;

    private final GameOfLife game;
    private final int gridSize;
    private final float cellLength;
    private int frameIndex;


    public GridDrawer(GameOfLife game) {
        this.game = game;
        this.gridSize = game.getGridSize();
        this.cellLength = game.getCellLength();
        frameIndex = 0;
    }


    // Initializes the window and starts the drawing process
    public void draw() {
        // Setting up the OpenGL window
        final GLProfile profile = GLProfile.get(GLProfile.GL2);
        GLCapabilities capabilities = new GLCapabilities(profile);
        final GLCanvas glcanvas = new GLCanvas(capabilities);

        glcanvas.addGLEventListener(this);
        glcanvas.setSize(800, 800);

        final JFrame frame = new JFrame ("Game of Life");
        frame.getContentPane().add(glcanvas);
        frame.setSize(frame.getContentPane().getPreferredSize());
        frame.setDefaultCloseOperation(WindowConstants.EXIT_ON_CLOSE);
        frame.setVisible(true);

        // Adding zooming functionality
        glcanvas.addMouseWheelListener(new MouseAdapter() {
            @Override
            public void mouseWheelMoved(MouseWheelEvent e) {
                // Updating the zoom depending upon the mouse wheel rotation
                zoom *= e.getWheelRotation() < 0 ? ZOOM_FACTOR : 1 / ZOOM_FACTOR;
            }
        });

        // Adding dragging functionality
        glcanvas.addMouseMotionListener(new MouseAdapter() {
            @Override
            public void mouseDragged(MouseEvent e) {
                curScreenX = e.getX();
                curScreenY = e.getY();

                // Checking for false drags
                // If drag is too much, then ignoring it
                if (Math.abs(curScreenX - prevScreenX) > MAX_DRAG
                        || Math.abs(curScreenY - prevScreenY) > MAX_DRAG) {
                    prevScreenX = curScreenX;
                    prevScreenY = curScreenY;
                    return;
                }

                // Updating the window position
                windowX += (curScreenX - prevScreenX) / DRAG_FACTOR / zoom;
                windowY += -(curScreenY - prevScreenY) / DRAG_FACTOR / zoom;
                prevScreenX = curScreenX;
                prevScreenY = curScreenY;
            }
        });

        // Frequency of the simulation is taken to be 10
        FPSAnimator animator = new FPSAnimator(glcanvas, 10, true);
        animator.start();
    }


    @Override
    public void init(GLAutoDrawable glAutoDrawable) {
    }
    @Override
    public void dispose(GLAutoDrawable glAutoDrawable) {
    }


    // OpenGL function used to draw each frame
    @Override
    public void display(GLAutoDrawable glAutoDrawable) {
        final GL2 gl = glAutoDrawable.getGL().getGL2();
        gl.glClear (GL2.GL_COLOR_BUFFER_BIT);

        gl.glLoadIdentity();
        gl.glScalef(zoom, zoom, 1.0f);
        gl.glTranslatef(-0.5f + windowX, -0.5f + windowY, 0);

        boolean grid[][] = game.iterate(); // Getting the next grid data

        // Drawing the grid
        for (int i = 0; i < gridSize; ++i) {
            for (int j = 0; j < gridSize; ++j) {
                if (grid[i][j]) {
                    gl.glBegin(GL2.GL_QUADS);

                    gl.glVertex2f(j* cellLength, i* cellLength);
                    gl.glVertex2f((j+1)* cellLength, i* cellLength);
                    gl.glVertex2f((j+1)* cellLength, (i+1)* cellLength);
                    gl.glVertex2f(j* cellLength, (i+1)* cellLength);

                    gl.glEnd();
                }
            }
        }

        gl.glFlush();

        System.out.println("Frame Index: " + ++frameIndex);
    }


    // Setting window properties from buffer
    // in case the window size is changed by the user
    @Override
    public void reshape(GLAutoDrawable glAutoDrawable,
                        int i, int i1, int i2, int i3) {
        final GL2 gl = glAutoDrawable.getGL().getGL2();

        gl.glLoadIdentity();
        gl.glScalef(zoom, zoom, 1.0f);
        gl.glTranslatef(-0.5f + windowX, -0.5f + windowX, 0);
    }
}
```

GameOfLife.java

```java
import java.util.Random;

/*
* Objects of this class can bbe used to simulate individual
* Game of Life instances, each having its own size and
* initial number of seeds
*
* Grid is processed as a boolean array, value of any cell
* being 'true' indicates the presence of an organism in the
* cell, and 'false' being the opposite
*
* */

public class GameOfLife {

    // Setting up the parameters of the Game of Life
    private final int gridSize;
    private final int numSeeds;
    private final float cellLength;

    private boolean[][] grid;


    public GameOfLife(int gridSize, int numSeeds) {
        this.gridSize = gridSize;
        this.numSeeds = numSeeds;
        this.cellLength = 1.0f / gridSize;

        // Initializing the grid
        grid = new boolean[gridSize][gridSize];
        generateSeeds();
    }


    // Puts seeds on random locations in the grid
    private void generateSeeds() {
        Random random = new Random();

        for (int i = 0; i < numSeeds; i++) {
            int xi = random.nextInt(gridSize);
            int yi = random.nextInt(gridSize);
            grid[yi][xi] = true;
        }
    }


    // Returns the next iteration of the grid
    public boolean[][] iterate() {

        boolean newGrid[][] = new boolean[gridSize][gridSize];

        // Copying old grid to new one
    for (int i = 0; i < gridSize; ++i) {
        System.arraycopy(grid[i], 0, newGrid[i], 0, gridSize);
    }

    // Applying the rules to each element of the grid
    for (int i = 0; i < gridSize; ++i) {
        for (int j = 0; j < gridSize; ++j) {
            int neighbourCount = getNeighbourCount(i, j);

            // Organism is born
            if (neighbourCount == 3) {
                newGrid[i][j] = true;
                continue;
            }

            // Cell does not change its state
            if (neighbourCount == 2)
                continue;

            // For the remaining conditions
            // (neighbourCount > 3 || neighbourCount < 2)
            // The organism dies
            newGrid[i][j] = false;
        }
    }

    grid = newGrid; // Updating the grid

    return newGrid;
}


// Returns the number of neighbours for a cell location
private int getNeighbourCount(int i, int j) {
    int neighbourCount = 0;

    for (int k = -1; k <= 1; ++k) {
        for (int l = -1; l <= 1; l++) {

            // Periodic boundary conditions
            int ii = (i + k + gridSize) % gridSize;
            int jj = (j + l + gridSize) % gridSize;

            // Leaving the cell itself
            if (k == 0 && l == 0)
                continue;

            if (grid[ii][jj])
                ++neighbourCount;
        }
    }

    return neighbourCount;
}
```

```
    public int getGridSize() {
        return gridSize;
    }


    public float getCellLength() {
        return cellLength;
    }
}
```

4 Question 4

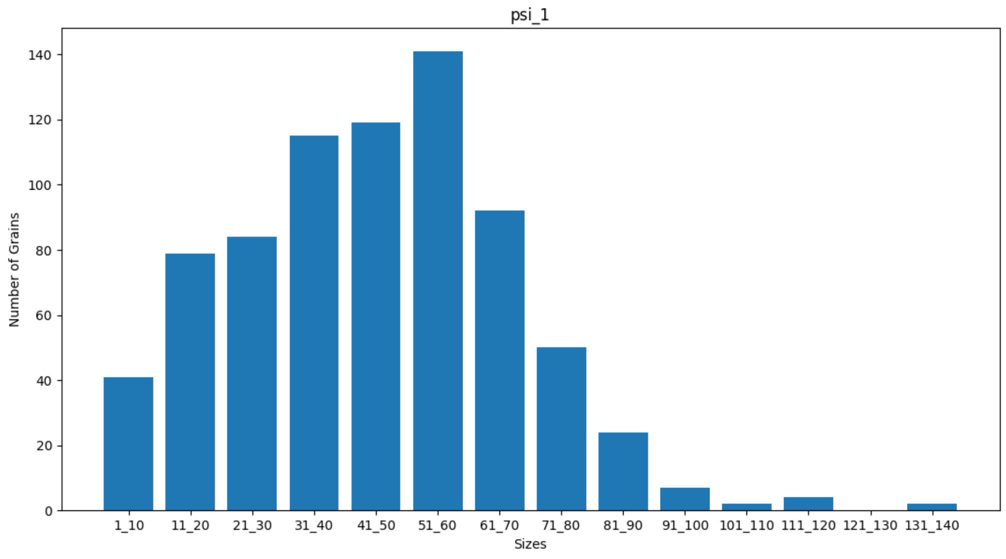
Instead of iterating over all the 512 columns, only 50 columns are randomly chosen and the grain sizes are determined by traversing through each of them. The distribution is plotted and average size is taken as the size of the grains.

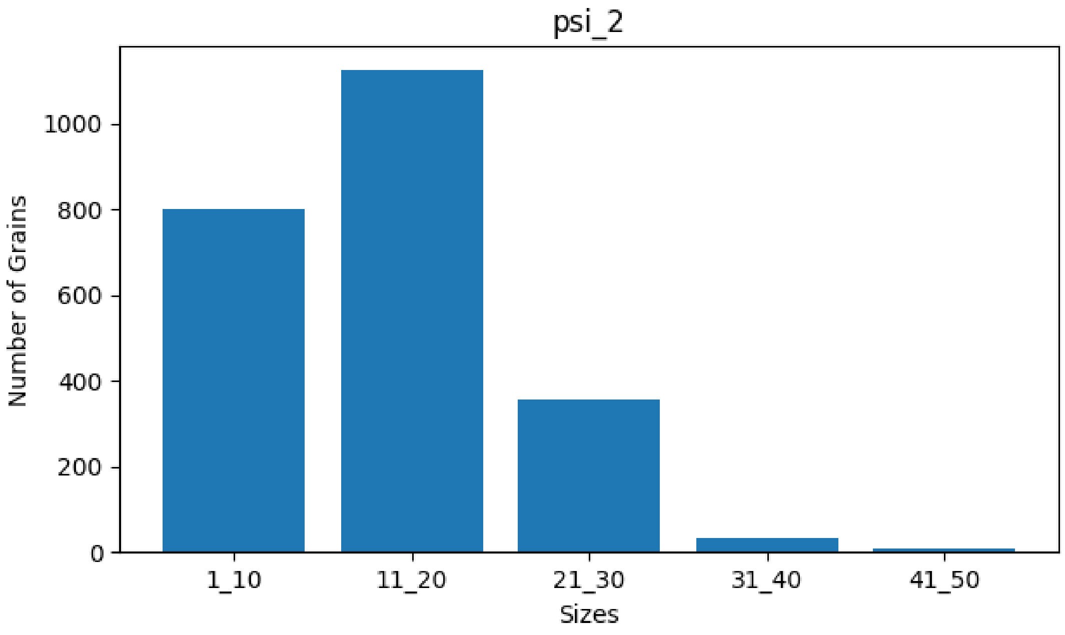
4.1 Pseudo Code

```
• column_indices = randint(0, 512, 50)
• grain_sizes = []
• for i in column_indices
•       append determined sizes in column[i] to grain_sizes list
• mean_size = mean(grain_sizes)
• frequency = 1 / (mean_size × mean_size) # grains per unit area
• divide grain_sizes into ranges
• plot the distribution
```

4.2 Results

	Mean Size (units)	Frequency (grains per unit area)
psi_1.dat	45.67	4.7 e-4
psi_2.dat	14.09	5.0 e-3





4.3 Code

grain.py

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import custom_random


# Returns an array which contains the sizes of different grains
def get_grain_sizes(psi):
    sizes = []

    # Iterating through 50 columns chosen randomly
    for i in custom_random.randint(low=0, high=512, size=50):
        column = psi[i]
        start_j = 0  # Stores the index of start of a grain
        end_j = 0    # Stores the index of end of a grain

        for j in range(1, 512):

            # Grain starts
            if column[j] - column[j - 1] > 0.1:
                start_j = j

            # Grain ends
            elif column[j] - column[j - 1] < -0.1:
                end_j = j
                sizes.append(end_j - start_j)

    return sizes


# Divides the grain sizes according to the range they lie in
def get_distribution(sizes, bin_size):
    dist = init_distribution_dict(bin_size, max(sizes))

    for s in sizes:
        rs = get_elem_range_str(s, bin_size)
        dist[rs] += 1

    return dist


# Initializes all entries of distribution dictionary to 0
# The size range string is used as the key to get the
# number of grains in that range
def init_distribution_dict(bin_size, max_element):
    dist = {}

    for r in range(1, max_element, bin_size):
        dist[get_elem_range_str(r, bin_size)] = 0

    return dist


# Returns range string for an element
# Eg. if bin_size = 10, elem = 15, return value is '11_20'
def get_elem_range_str(elem, bin_size):
    i = 1
    # Finding range in which the element lies
    while bin_size*i < elem:
        i += 1

    rs = str(bin_size*(i-1) + 1) + '_' + str(bin_size*i)

    return rs


def process_psi(file_name, bin_size=10):
    psi = pd.read_csv(filepath_or_buffer=file_name,
                      delimiter='  ', engine='python',
                      header=None)

    grain_sizes = get_grain_sizes(psi)
    dist = get_distribution(grain_sizes, bin_size)

    mean_size = np.mean(grain_sizes)
    frequency = 1 / (mean_size**2)
    print(file_name, 'Mean Size: ', mean_size)
    print(file_name, 'Frequency: ', frequency)
    plt.figure()
    plt.bar(list(dist.keys()), list(dist.values()))
    plt.title(file_name[0:file_name.rindex('.')])
    plt.xlabel('Sizes')
    plt.ylabel('Number of Grains')


bin_size = int(input('Enter the bin size: '))
process_psi('psi_1.dat', bin_size)
process_psi('psi_2.dat', bin_size)
plt.show()
```