TABLE 1 Graph Terminology.	*Edges Type*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

1. Draw graph models, stating the type of graph (from Table 1) used, to represent airline routes where every day there are four flights from Bombay to New Delhi, two flights from New Delhi to Bombay, three flights from New Delhi to Chennai, two flights from Chennai to New Delhi, one flight from New Delhi to Bangalore, two flights from Bangalore to New Delhi, three flights from New Delhi to Hyderabad, two flights from Hyderabad to New Delhi, and one flight from Hyderabad toChennai, with

a) an edge between vertices representing cities that have a flight between them (in either direction).
b) an edge between vertices representing cities for each flight that operates between them (in either direction).
c) an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Chennai.
d) an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.
e) an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends.

2. What kind of graph (from Table 1) can be used to model a highway system between major cities where

a) there is an edge between the vertices representing cities if there is an interstate highway between them?
b) there is an edge between the vertices representing cities for each interstate highway between them?
c) there is an edge between the vertices representing cities for each interstate highway between them, and there is a loop at the vertex representing a city if there is an interstate highway that circles this city?

Q3) The intersection graph of a collection of sets $A_1, A_2,...,A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty

intersection. Con- struct the intersection graph of these collections of sets.

a) $A_1 = \{0,2,4,6,8\}$, $A_2 = \{0,1,2,3,4\}$, $A_3 = \{1,3,5,7,9\}$, $A_4 = \{5,6,7,8,9\}$, $A_5 = \{0,1,8,9\}$
b) $A_1 = \{...,-4,-3,-2,-1,0\}$, $A_2 = \{...,-2,-1,0,1,2,...\}$, $A_3 = \{...,-6,-4,-2,0,2,4,6,...\}$,
$A_4 = \{...,-5,-3,-1,1,3,5,...\}$, $A_5 = \{...,-6,-3,0,3,6,...\}$
c) $A_1 = \{x|x<0\}$, $A_2 = \{x|-1<x<0\}$, $A_3 = \{x|0<x<1\}$, $A_4 = \{x|-1<x<1\}$, $A_5 = \{x\ |x > -1\}$, $A_6 = R$

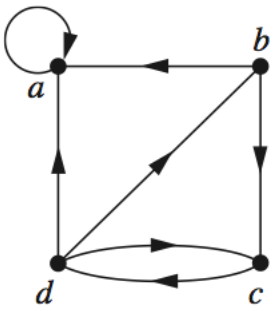
Q4) Construct a niche overlap graph for six species of birds, where the hermit thrush competes with the robin and with the blue jay, the robin also competes with the mockingbird, the mockingbird also competes with the blue jay, and the nuthatch competes with the hairy wood- pecker.

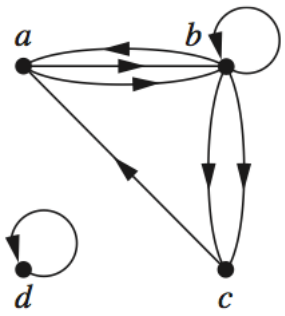
Q5) Draw the acquaintanceship graph that represents that Tarun and Pallavi, Tarun and Harsh, Tarun and Shikhar, Tarun and Archana, Tarun and Mayank, Javed and Pallavi, Javed and Mary, Pallavi and Harsh, Archana and Harsh, and Archana and Mayank know each other, but none of the other pairs of people listed know each other.

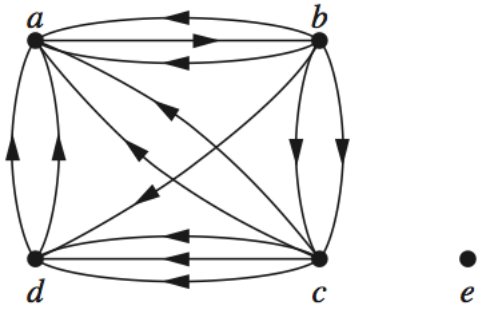
Q6) Describe a graph model that represents a metro system in Delhi as of now. Should edges be directed or undirected? Should multiple edges be allowed? Should loops be al- lowed?

Q7) For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are pre- requisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

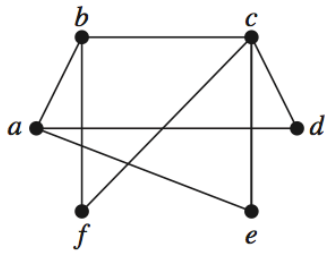
Q8) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

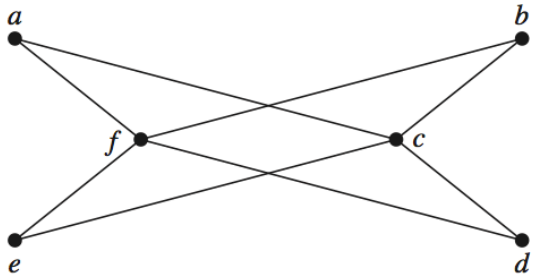


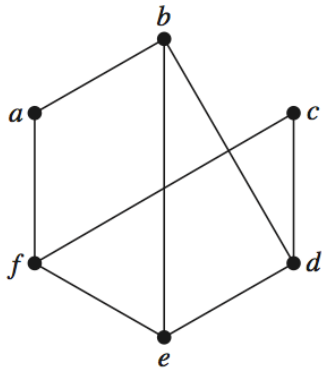




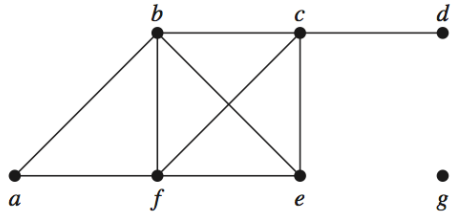
Q9) Determine whether the graphs are bipartite. Is it possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.







The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in nonincreasing order. For example, the degree sequence of the graph G is 4,4,4,3,2,1,0.



Q10) Find the degree sequence of each of the following graphs. a) K_4 b) C_4 c) W_4 d) $K_{2,3}$ e) Q_3

Q11) What is the degree sequence of the bipartite graph $K_{m,n}$ where m and n are positive integers?

Explain your answer.

Q12) What is the degree sequence of K_n, where n is a positive integer? Explain your answer.

Q13) How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2? Draw such a graph.

Q14) How many edges does a graph have if its degree sequence is 5, 2, 2, 2, 2, 1? Draw such a graph.

A sequence $d_1, d_2, \ldots, d_n$ is called graphic if it is the degree sequence of a simple graph.

Q15) Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence. a) 5,4,3,2,1,0 b) 6,5,4,3,2,1

a) 3,3,3,2,2,2 b) 3,3,2,2,2,2 c) 5,3,3,3,3,3 d) 5,5,4,3,2,1

Q16) Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence. a) 3,3,3,3,2 b) 5,4,3,2,1 c) 4,4,3,2,1 d) 4,4,3,3,3 e) 3,2,2,1,0

f) 1,1,1,1,1

Q17) Let G be a graph with v vertices and e edges. Let M be the maximum degree of the vertices of G, and let m be the minimum degree of the vertices of G. Show that a) $2e/v \geq m$. b) $2e/v \leq M$.

Q18) The complementary graph G of a simple graph G has the same vertices as G. Two vertices are adjacent in G if and only if they are not adjacent in G. Describe each of these graphs.

a) $\overline{K_n}$ **b)** $\overline{K_{m,n}}$ **c)** $\overline{C_n}$ **d)** $\overline{Q_n}$