Q1) If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have?

Q2) If the simple graph G has v vertices and e edges, how many edges does $\overline{G}$ have?

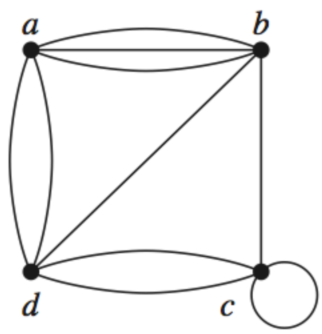
Q3) If the degree sequence of the simple graph G is 4, 3, 3, 2, 2, what is the degree sequence of $\overline{G}$?

Q4) If the degree sequence of the simple graph G is $d_1, d_2, \ldots, d_n$, what is the degree sequence of $\overline{G}$?

Q5) Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq v^2/4$.

Q6) Show that if G is a simple graph with n vertices, then the union of G and $\overline{G}$ is K_n

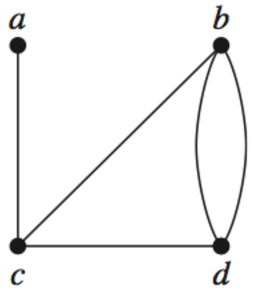
Adjacency Matrix of Undirected Multigraph: When multiple edges connecting the same pair of vertices v_i and v_j , or multiple loops at the same vertex, are present, the adjacency matrix is no longer a zero–one matrix, because the (i, j)th entry of this matrix equals the number of edges that are associated to $\{v_i,v_j\}$. All undirected graphs, including multigraphs and pseudographs, have symmetric adjacency matrices. For example, the adjacency matrix for

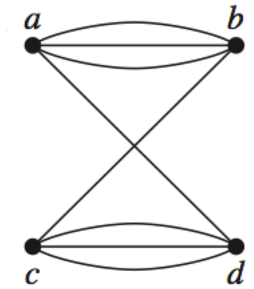


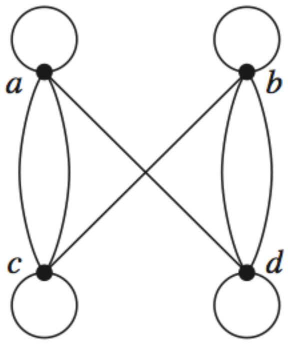
is

$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}.$$

Q7) Represent the given graph using an adjacency matrix.







Q8) Draw an undirected graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

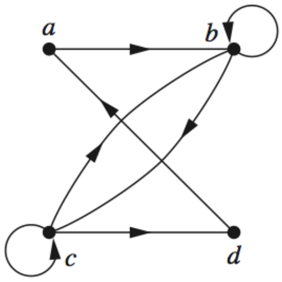
Q9) Draw the graph represented by the given adjacency matrix.

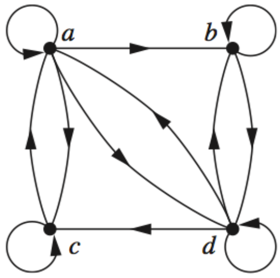
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

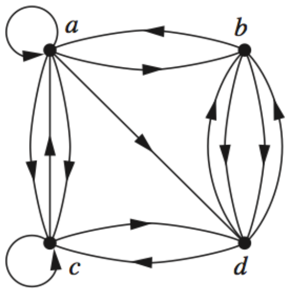
Q10) Find an adjacency matrix and incidence for each of these graphs.

a) K_n b) C_n c) W_n d) $K_{m,n}$ e) Q_n

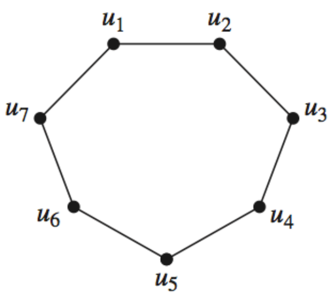
Q11) Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.

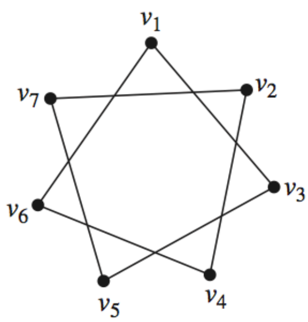


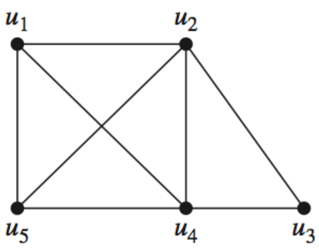


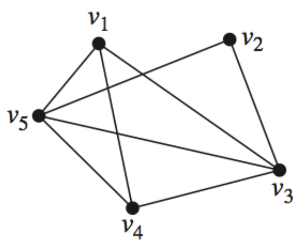


Q12) Determine whether the following pair of graphs is isomorphic. Exhibit an Isomorphism or provide an argument that none exist.









Q13) Are the simple graphs with the following adjacency matrices isomorphic?

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Q14) Show that isomorphism of simple graphs is an equivalence relation.

Q15) Suppose that G and H are isomorphic simple graphs. Show that their complementary graphs $\overline{G}$ and $\overline{H}$ are also isomorphic.