DESCRETE MATHEMATICS

ASSIGNMENT [21-09-2014]

QUESTION 1

We already know that for a given graph G, and its adjacency matrix A, element (i, j) of A^n for n ≥ 0 gives the number of paths of length n from i^{th} vertex to j^{th} vertex.

Now, consider a graph G' with m vertices $\{p_1,p_2,\ldots p_m\}$ whose adjacency matrix is A. Suppose that the minimum distance between some two vertices is 5. Also, let that path with minimum distance be unique. Then, the element $[A^n]_{i,j}$ will be equal to 1 for n = 5.

Therefore, to find the minimum distance from i^{th} vertex to j^{th} vertex, the adjacency matrix can be multiplied with itself recursively until a non-zero element occurs at element $[A^n]_{i,j}$. Then, the value would be the number of paths of shortest distance from i^{th} vertex to j^{th} vertex, each of length n, which should be minimum.

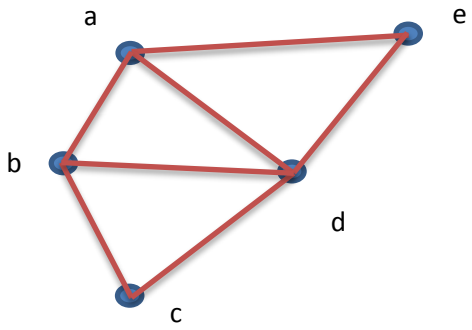
QUESTION 2

If a graph is somehow connected, there should be a minimum distance for any two pair of vertices. Hence, there should be non-zero term in the matrices achieved by constantly raising the power of the adjacency matrix, for the corresponding row and column.

Also, for a simple connected graph, any two vertices can have at most a path of length k-1, k being the number of vertices. This will happen when a path involves all other vertices.

Therefore, we can say that if we keep on raising the powers of the adjacency matrix up to k -1, we should get a non-zero value at every position in the matrix. If this does not happen, this directly means that there is no path present between those corresponding vertices, or in other words, the graph is somehow disconnected!

QUESTION 3



a:	Rajasthan
b:	Gujrat
c:	Madhya Pradesh
d:	Maharashtra
e:	Uttar Pradesh

Adjacency matrix, A=

	a	b	c	d	E
a	0	1	0	1	1
b	1	0	1	1	0
c	0	1	0	1	0
d	1	1	1	0	1
e	1	0	0	1	0

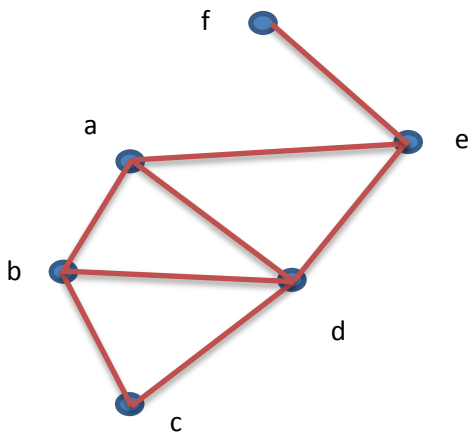
Now, A^5 =

$$\begin{bmatrix} 44 & 52 & 33 & 58 & 38 \\ 52 & 44 & 38 & 58 & 33 \\ 33 & 38 & 24 & 45 & 27 \\ 58 & 58 & 45 & 64 & 45 \\ 38 & 33 & 27 & 45 & 24 \end{bmatrix}$$

As A^5 contains no non-zero term, hence there is a possible path between any two vertices. Or, in other words, the graph in connected.

QUESTION 4

If Delhi is added in the graph, the graph changes to the following



a:	Rajasthan
b:	Gujrat
c:	Madhya Pradesh
d:	Maharashtra
e:	Uttar Pradesh
f:	Delhi

Therefore, the adjacency matrix changes to

	a	b	c	d	e	f
a	0	1	0	1	1	0
b	1	0	1	1	0	0
c	0	1	0	1	0	0
d	1	1	1	0	1	0
e	1	0	0	1	0	1
f	0	0	0	0	1	0

Now, if we find A^6 using MATLAB, it comes out to be

$$\begin{bmatrix} 160 & 140 & 114 & 180 & 118 & 46 \\ 140 & 152 & 104 & 172 & 130 & 36 \\ 114 & 104 & 84 & 126 & 90 & 30 \\ 180 & 172 & 126 & 220 & 140 & 54 \\ 118 & 130 & 90 & 140 & 116 & 28 \\ 46 & 36 & 30 & 54 & 28 & 16 \end{bmatrix}$$

Again, we find that there is no non-zero element. Hence the graph is connected.