**Step 2: Using the following rules to design your own AND Gate, OR Gate, and NAND Gate**

Design **AND** Gate to calculate the values of **W1,W2 and Y**

Z := ( W1 * X + W2 * Y >= T )

where T := 1.0.

Desired "**And**" Function

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Loop 1:**

W1=W2=0

Z := ( 0 * X + 0 * Y >= T )

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

**Loop 2:**

W1=W2=0.5

Z := ( 0.5 * X + 0.5 * Y >= T )

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Design **OR** Gate to calculate the values of **W1,W2 and Y**

Z := ( W1 * X + W2 * Y >= T )

where T := 1.0.

Desired "**OR**" Function

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**Loop 1:**

W1=W2=0

Z := ( 0* X + 0 * Y >= T )

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

**Loop 2:**

W1=W2=0.5

Z := ( 0.5* X + 0.5 * Y >= T )

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Loop 3:**

W1=W2=1.0

Z := ( 1.0* X + 1.0 * Y >= T )

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Design **NAND** Gate to calculate the values of **W1,W2 and Y**

Z := ( W0 * C + W1 * X + W2 * Y >= T )

where T := 1.0.

The bias C for NAND is 1.0

Desired **"NAND"** Function

| **X** | **Y** | **Z** |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**Loop 1 :**

W0=0.0

W1=W2=0.5

Z := ( 0 * 1.0 + 0.5 * X + 0.5 * Y >= T )

| C | X | Y | Z |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

**Loop 2:**

W0=0.5

W1=W2=0.5

Z := ( 0.5* 1.0 + 0.5 * X + 0.5 * Y >= T )

| C | X | Y | Z |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

**Loop 3:**

W0=1.0

W1=W2=0.5

Z := ( 1.0* 1.0 + 0.5 * X + 0.5 * Y >= T )

| C | X | Y | Z |
|---|---|---|---|
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

**Loop 4:**

W0=1.0

W1=W2=0.0

Z := ( 1.0* 1.0 + 0.0 * X + 0.0 * Y >= T )

| C | X | Y | Z |
|---|---|---|---|
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

**Loop 5:**

W0=1.0

W1=W2=-0.5

Z := ( 1.0* 1.0 + -0.5 * X + -0.5 * Y >= T )

| C | X | Y | Z |
|---|---|---|---|
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

**Loop 6:**

W0=1.5

W1=W2=-0.5

Z := ( 1.5* 1.0 + -0.5 * X + -0.5 * Y >= T )

| C | X | Y | Z |
|---|---|---|---|
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

**Step 3: Please answer**

• What is the formula for

Z1 := X "AND" Y

**Z1 := ( 0.5 * X + 0.5 * Y >= 1.0 )**

**• What is the formula for**

Z1 := X "OR" Y

**Z1 := ( 1.0 * X + 1.0 * Y >= 1.0 )**

**• What is the formula for**

**Z1 := X "NAND" Y**

Bias is +1.5 , C = 1; W0 = 1.5; W1=W2 = -0.5

Z2 := ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0 )

Z2 := ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0)

Z2 := (- 0.5 * Y >= 0.5 * X + -1.5 * 1.0 + 1.0 )

Z2 := ( -0.5 * Y >= 0.5 * X – 0.5 )

**Z2 := ( Y <= -X + 1.0 )**

**• What is the formula for**

Z1 := X "Or" Y

Z2 := X "NAND" Y

Z := Z3 := Z1 "AND" Z2

Z := ( X "OR" Y ) "AND" ( X "NAND" Y )

Z := ( 1.0 * X + 1.0 * Y >= 1.0 ) "AND" ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0)

Z := ( 0.5 * ( 1.0 * X + 1.0 * Y >= 1.0 ) + 0.5 * ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0) >= 1.0 )

**Z := ( 0.5 * ( 1.0 * X + 1.0 * Y >= 1.0 ) + 0.5 * ( 1.5 + -0.5 * X + -0.5 * Y >= 1.0) >= 1.0 )**

- **Step 4: Please prove that your designed XOR Gate work**
  - **X=1, Y=1**
  - **X=1, Y=0**
  - **X=0, Y=1**
  - **X=0, Y=0**

Z1 := X "Or" Y

Z2 := X "NAND" Y

Z := Z3 := Z1 "AND" Z2

Z := ( X "OR" Y ) "AND" ( X "NAND" Y )

Z := ( 1.0 * X + 1.0 * Y >= 1.0 ) "AND" ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0)

Z := ( 0.5 * ( 1.0 * X + 1.0 * Y >= 1.0 ) + 0.5 * ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0) >= 1.0 )

**Z := ( 0.5 * ( 1.0 * X + 1.0 * Y >= 1.0 ) + 0.5 * ( 1.5 + -0.5 * X + -0.5 * Y >= 1.0) >= 1.0 )**

**Take X=1 ,Y=1**

Z := ( 0.5 * ( 1.0 * 1.0 + 1.0 * 1.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.5 * 1.0 + -0.5 * 1.0 >= 1.0) >= 1.0 )

Z := ( 0.5 * ( 1.0 + 1.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.5 + -0.5 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( 2.0 >= 1.0 ) + 0.5 * ( 0.5 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( true ) + 0.5 * ( false ) >= 1.0 )

Z := ( 0.5 * 1 + 0.5 * 0 >= 1.0 )

Z := ( 0.5 + 0.0 >= 1.0 )

Z := ( false )

**Z := 0**

**Take X=1 ,Y=0**

Z := ( 0.5 * ( 1.0 * 1.0 + 1.0 * 0.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.5 * 1.0 + -0.5 * 0.0 >= 1.0) >= 1.0 )

Z := ( 0.5 * ( 1.0 + 0.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.5 + -0.0 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( 1.0 >= 1.0 ) + 0.5 * ( 1.0 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( true ) + 0.5 * ( true) >= 1.0 )

Z := ( 0.5 * 1 + 0.5 * 1 >= 1.0 )

Z := ( 0.5 + 0.5 >= 1.0 )

Z := ( true)

**Z = 1**

**Take X=0 ,Y=1**

Z := ( 0.5 * ( 1.0 * 0.0 + 1.0 * 1.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.5 * 0.0 + -0.5 * 1.0 >= 1.0) >= 1.0 )

Z := ( 0.5 * ( 0.0 + 1.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.0 + -0.5 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( 1.0 >= 1.0 ) + 0.5 * ( 1.0 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( true ) + 0.5 * ( true) >= 1.0 )

Z := ( 0.5 * 1 + 0.5 * 1 >= 1.0 )

Z := ( 0.5 + 0.5 >= 1.0 )

Z := ( true)

**Z = 1**

**Take X=0 ,Y=0**

Z := ( 0.5 * ( 1.0 * 0.0 + 1.0 * 0.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.5 * 0.0 + -0.5 * 0.0 >= 1.0) >= 1.0 )

Z := ( 0.5 * ( 0.0 + 0.0 >= 1.0 ) + 0.5 * ( 1.5 + -0.0 + -0.0 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( 0.0 >= 1.0 ) + 0.5 * ( 1.5 >= 1.0 ) >= 1.0 )

Z := ( 0.5 * ( false) + 0.5 * ( true) >= 1.0 )

Z := ( 0.5 * 0 + 0.5 * 1 >= 1.0 ) Z := ( 0.0 + 0.5 >= 1.0 )

Z := ( false )

**Z = 0**

| **OR** | | **NAND** | | **XOR** |
|---|---|---|---|---|
| X Y \| Z1 | | X Y \| Z2 | | X Y \| Z3 |
| 0 0 \| 0 | | 0 0 \| 1 | | 0 0 \| 0 |
| 0 1 \| 1 | **AND** | 0 1 \| 1 ==> | | 0 1 \| 1 |
| 1 0 \| 1 | | 1 0 \| 1 | | 1 0 \| 1 |
| 1 1 \| 1 | | 1 1 \| 0 | | 1 1 \| 0 |

From Above Calculations, Hence **"OR**" AND "**NAND**" GATE Operations Output is **XOR** GATE.