Computational and Numerical Methods

Sub Code: CS374, Lab 1
BTech(CS), 5th Semester

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1. Let α be a smallest positive root of $f(x) = 1 - x + \sin x = 0$. Find the interval $[a, b]$ containing α and for which the bisection method will converge to α. Then estimate the number of iterates needed to find α with an error tolerance of 5×10^{-8} . Write *MATLAB* codes to solve this problem.

2. Approximate $f(x) = e^{-x}$ using Taylor polynomials of a given degree n (Where n is input) on the interval $-1 \leq x \leq 1$; $a = 0$. Calculate your approximation at 21 evenly spaced points in the interval $[-1, 1]$.

   The Taylor polynomial of degree n is

   $$p_n(x) = f(a) + (x-a)f^{'}(a) + \frac{(x-a)^2}{2!}f^{''}(a) + \ldots + \frac{(x-a)^n}{n!}f^n(a)$$

   Then find the error approximation $f(x) - p_n(x)$ in $[-1, 1]$ with error tolerance of 10^{-9}. Graph the error approximation using those points in $[-1, 1]$. Find the value of n in $[-1, 1]$ for which $|f(x) - p_n(x)| \leq 10^{-9}$. Write a MATLAB code to solve the problem.

   Check your result whether you get the same value of n from the Taylor's remainder term

   $$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!}f^{(n+1)}(c)$$

   for $-1 \leq x \leq 1$ and c an unknown between 0 and x.