Lec-14, DC

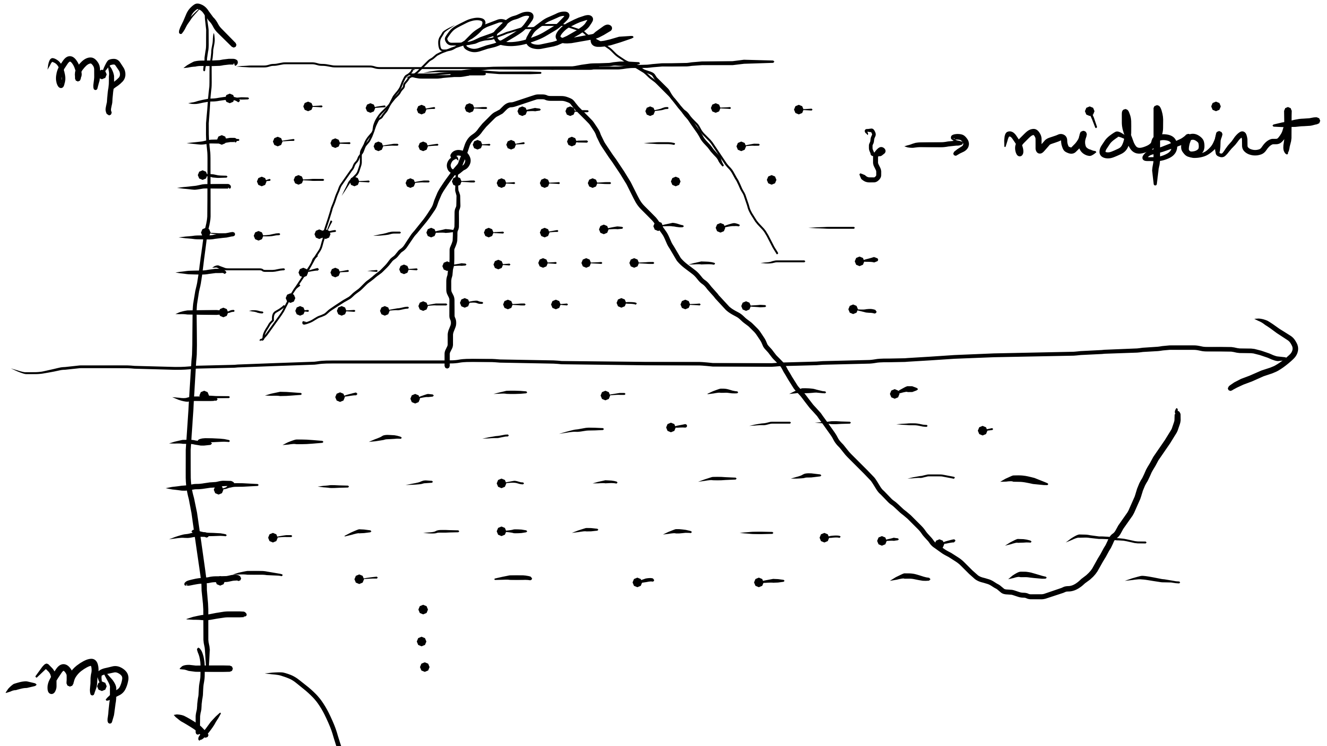
Amplitude quantiⁿ :- process of transforming the sample amplitude $m(nT_s)$ of a msg. sig. $m(t)$ at $t = nT_s$ into a discrete amplitude $v(nT_s)$ taken from a finite set of possible amplitudes.

→ process is assumed to be "memoryless & instantaneous"

Called as scalar quantiⁿ

* Vector quantiⁿ is also a well-studied concept

↳ for sample at $t = nT_s$, quantiⁿ not affected by earlier or later samples



Allowed quantization levels be L $\leftarrow 2^{\text{no. of bits}}$

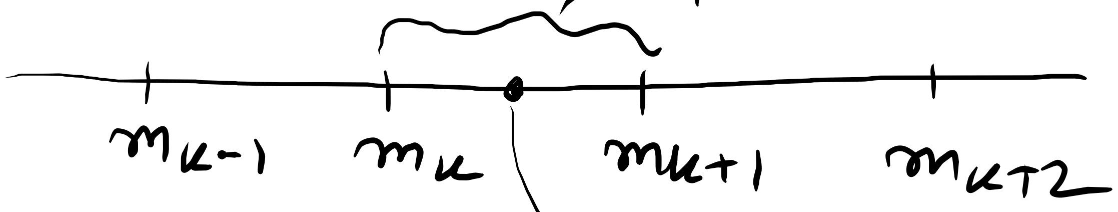
gap b/w levels :- $\dfrac{2m_p}{L}$ (If uniformly spaced)

they can be equally spaced / uniformly spaced
or non-uniformly spaced

anything above m_p or below $-m_p$ is chopped off.

continuous sample m $\longrightarrow$ $\boxed{\text{quantizer } g(\cdot)}$ $\longrightarrow$ Discrete sample v

Few terms:-



Partition cell, denoted as J_k, $k = 1, 2, 3 \ldots L$

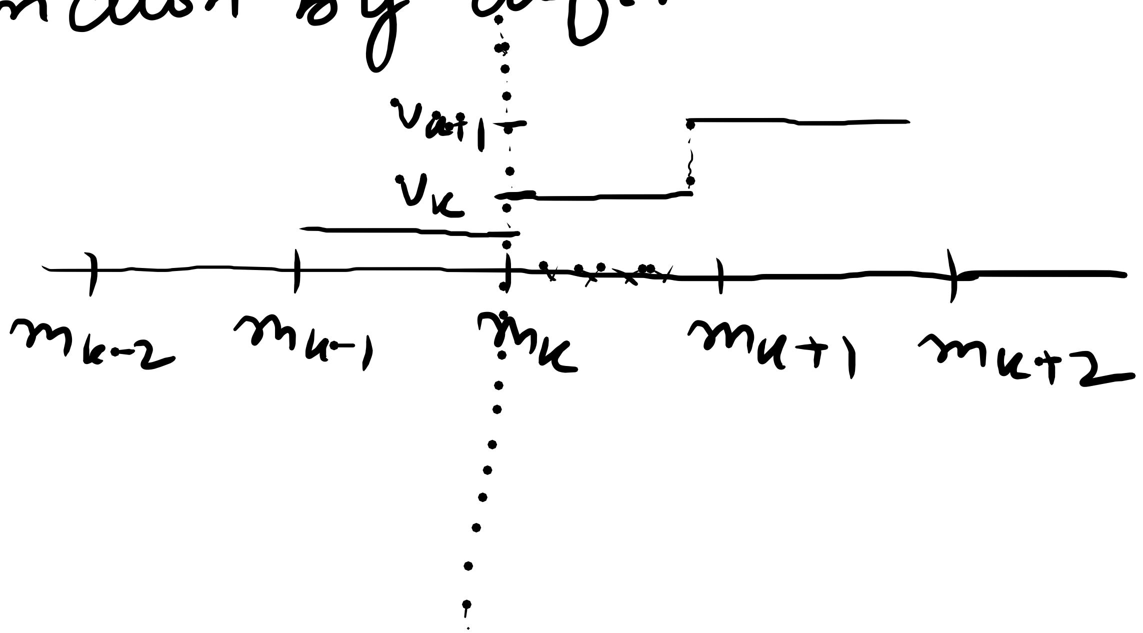
L → total no. of amplitude levels used in quantizer

m_ks:- decision levels or thresholds.

v_k

all amplitudes in $[m_k, m_{k+1})$ are represented by v_k

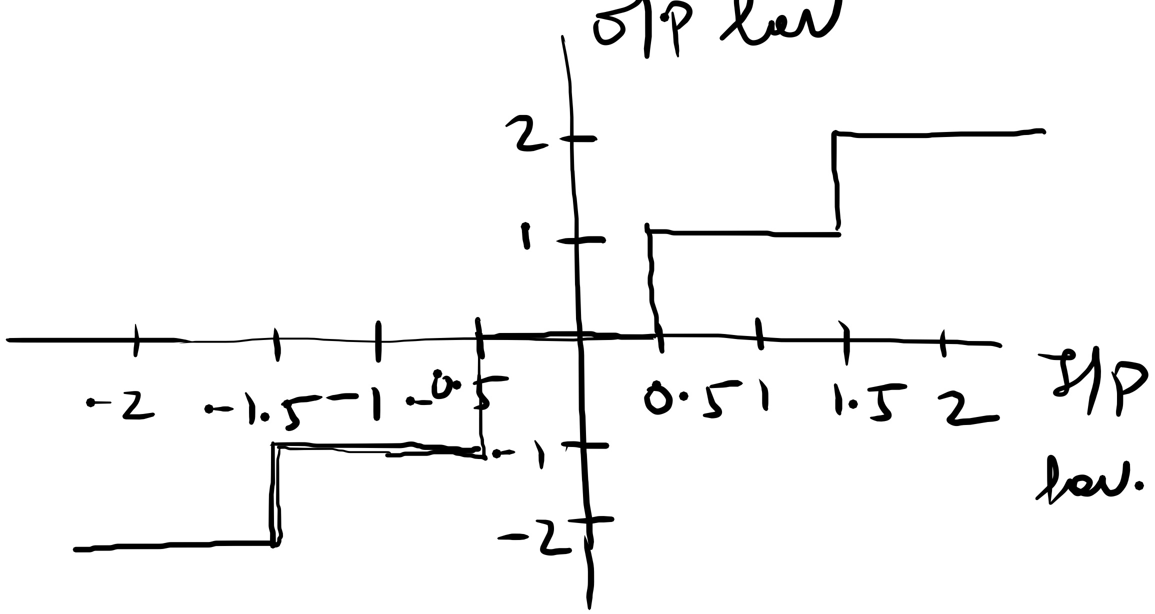
mapping $V = g(m)$ is the quantizer characteristic, which is a staircase function by definition

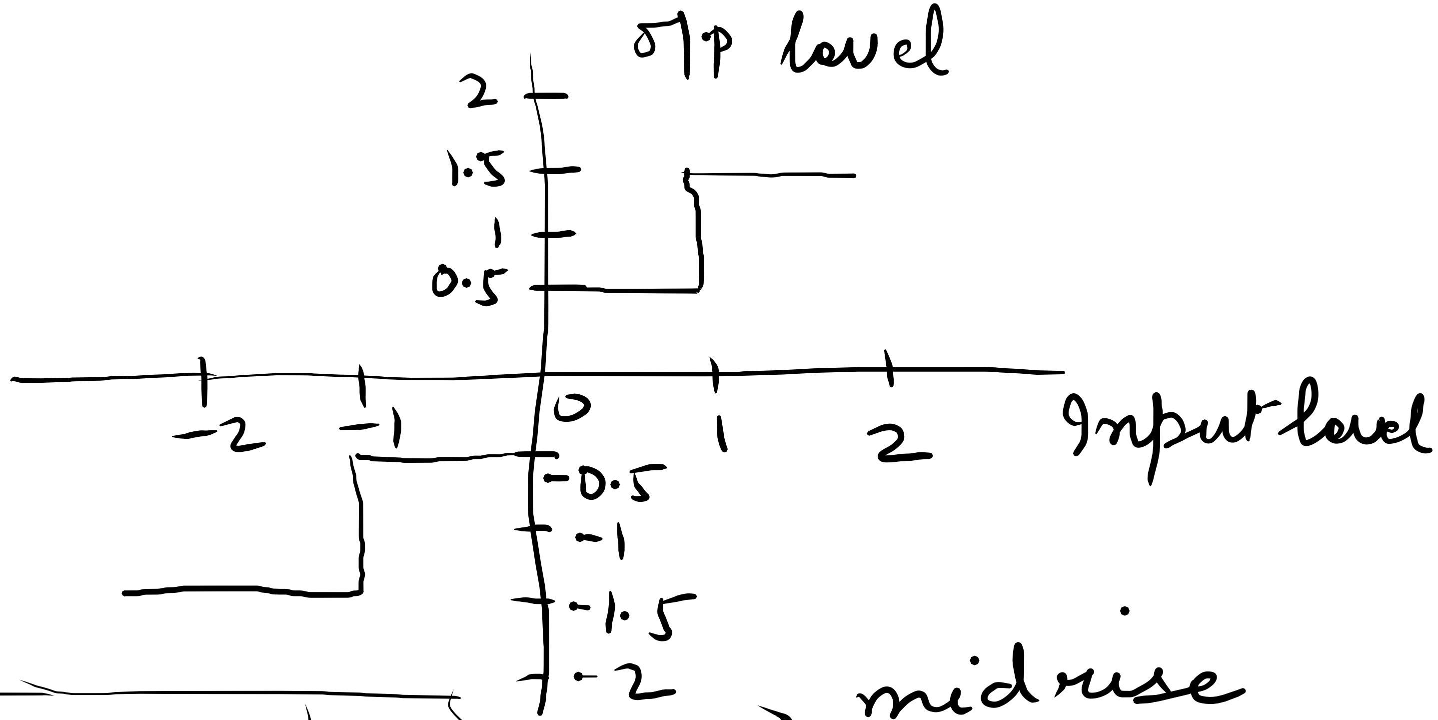


V_ks :- representation levels or reconstruction levels.

midtread
midrise.

} quantizer charac. can be of two types.





midtread

↳ origin lies in the middle of a tread of the staircase like graph

midrise

↳ origin lies in the middle of the rising part.

both are symmetric about the origin

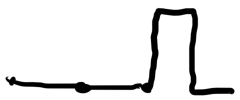
Quantizers
- uniform
- non-uniform.

→ The quantized samples are coded and Txd as binary pulses.

At the rx, some pulse may be detected incorrectly.

$L = 3$

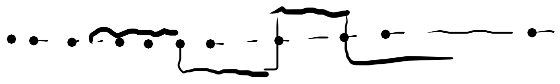
$v_k \rightarrow 001$



Two sources of error → Quantizⁿ process
→ Pulse detecⁿ

$\oplus$ noise

$m \longrightarrow g(m) \quad |m - g(m)|$ — irreducible



1 0 1

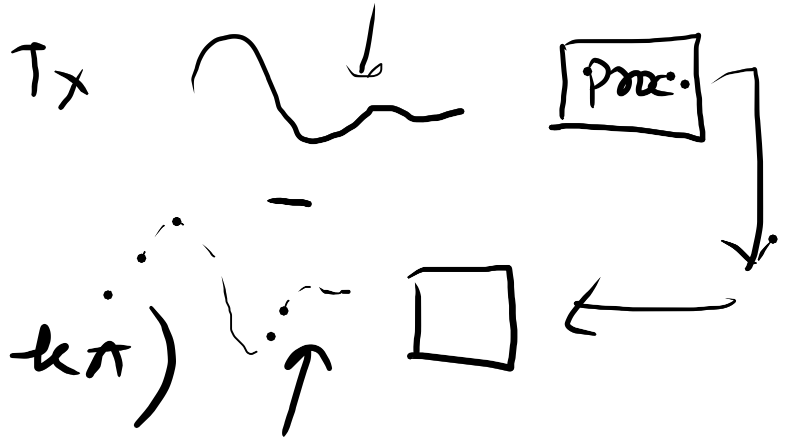
In most cases, pulse detecⁿ error is quite small compared to quantizⁿ error. Basically, for the derivⁿ ahead, we assume that the error in the rxd sig. is caused exclusively by quantizⁿ.

$$m(t) = \sum_k m(kT_s)\, \text{sinc}(2\pi Bt - k\pi) \rightarrow \text{standard interpolation formula.}$$

$m(kT_s)$ ↳ sample values of $m(t)$

$$\hat{m}(t) = \sum_k \hat{m}(kT_s)\, \text{sinc}(2\pi Bt - k\pi)$$

$\hat{m}(kT_s)$ ↳ quantized sample.



distortion component

$$q(t) = \sum_k \left[\hat{m}(kT_s) - m(kT_s)\right] \text{sinc}(2\pi Bt - k\pi)$$

$$\hat{m}(t) - m(t) = \sum_k q(kT_s)\, \text{sinc}(2\pi Bt - k\pi)$$

$q(t)$:- undesired signal called as quantization noise

↳ let us obtain its power or the mean square value.

$$\hat{m}(t) = m(t) + q(t) \quad (?)$$

$$\overline{q^2(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} q^2(t)\,dt \longrightarrow \text{mod 1 of S\&S}$$

mean-square value of $q(t)$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[\sum_k q(kT_s)\, \text{sinc}(2\pi B t - k\pi) \right]^2 dt \quad -(1)$$

using, $$\int_{-\infty}^{\infty} \text{sinc}(2\pi B t - m\pi)\, \text{sinc}(2\pi B t - n\pi)\, dt = \begin{cases} 0, & m \neq n \\ 1/2B, & m = n \end{cases}$$

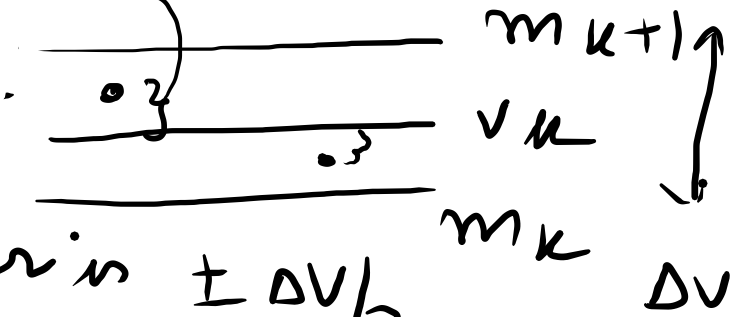
$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \sum_k q^2(kT_s)\, \text{sinc}^2(2\pi B t - k\pi)\, dt \quad -(2)$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_k q^2(kT_s) \int_{-T/2}^{T/2} \text{sinc}^2(2\pi B t - k\pi)\, dt$$

$$\widetilde{q^2(t)} = \lim_{T\to\infty} \frac{1}{2BT} \sum_k q^2(kT_s)$$

→ quantizn levels are separated by $\Delta V = \frac{2m_p}{L}$. Since a sample value is approx. by the midpoint of the subin-terval (of height ΔV) in which the sample falls, the max. quantizn error is $\pm \Delta V/2$

Thus, the quantizn error lies in the range $\left[-\frac{\Delta V}{2}, \frac{\Delta V}{2}\right]$ where max poss. error is $\pm \Delta V/2$



$$\Delta V = \frac{2m_p}{L}$$