# Relational Model

# Outline

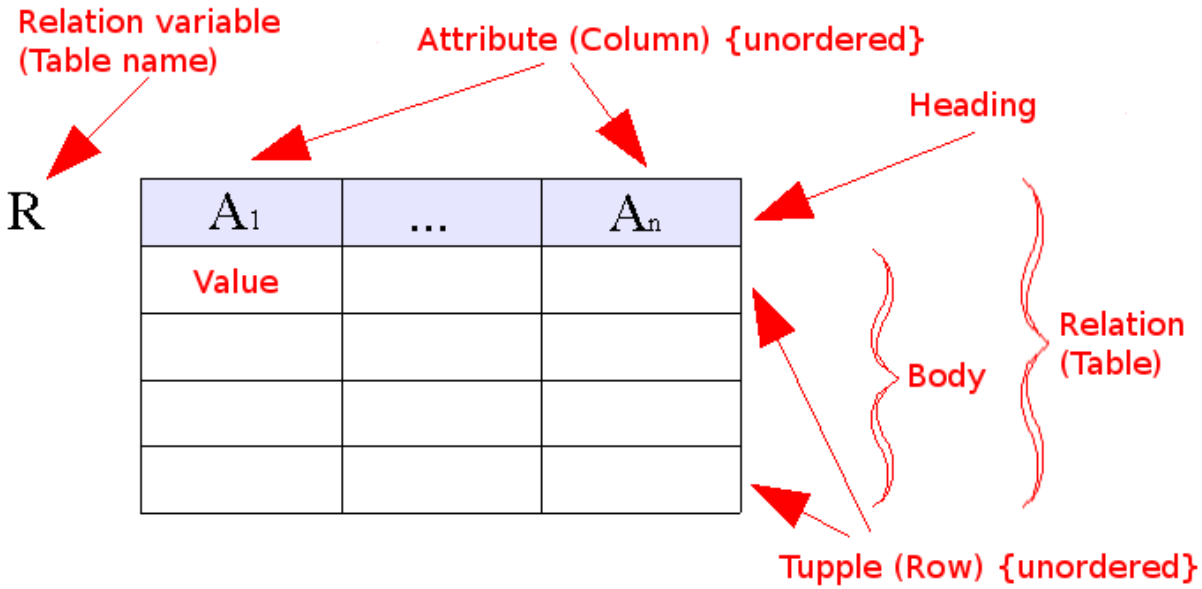
- Structure of Relational Databases
- Database Schema
- Keys
- Schema Diagrams
- Relational Query Languages
- The Relational Algebra

# Relational Model Foundation

- Relational Model of data is based on the concept of RELATION
- A Relation is a Mathematical concept based on idea of SETS
- The strength of the relational approach to data management comes from the formal foundation provided by the theory of relations
- The model was first proposed by Dr. E.F. Codd of IBM in 1970

# Relational Model (1970)

- Data as set of tables
- Having constraints and relationships
- Oracle, DB2,Sybase



# Some Terms

**Informal Name**

- Table
- Row or Record
- Column or Field
- No. of Rows
- No. of Columns
- Unique Identifier
- Legal Values Set

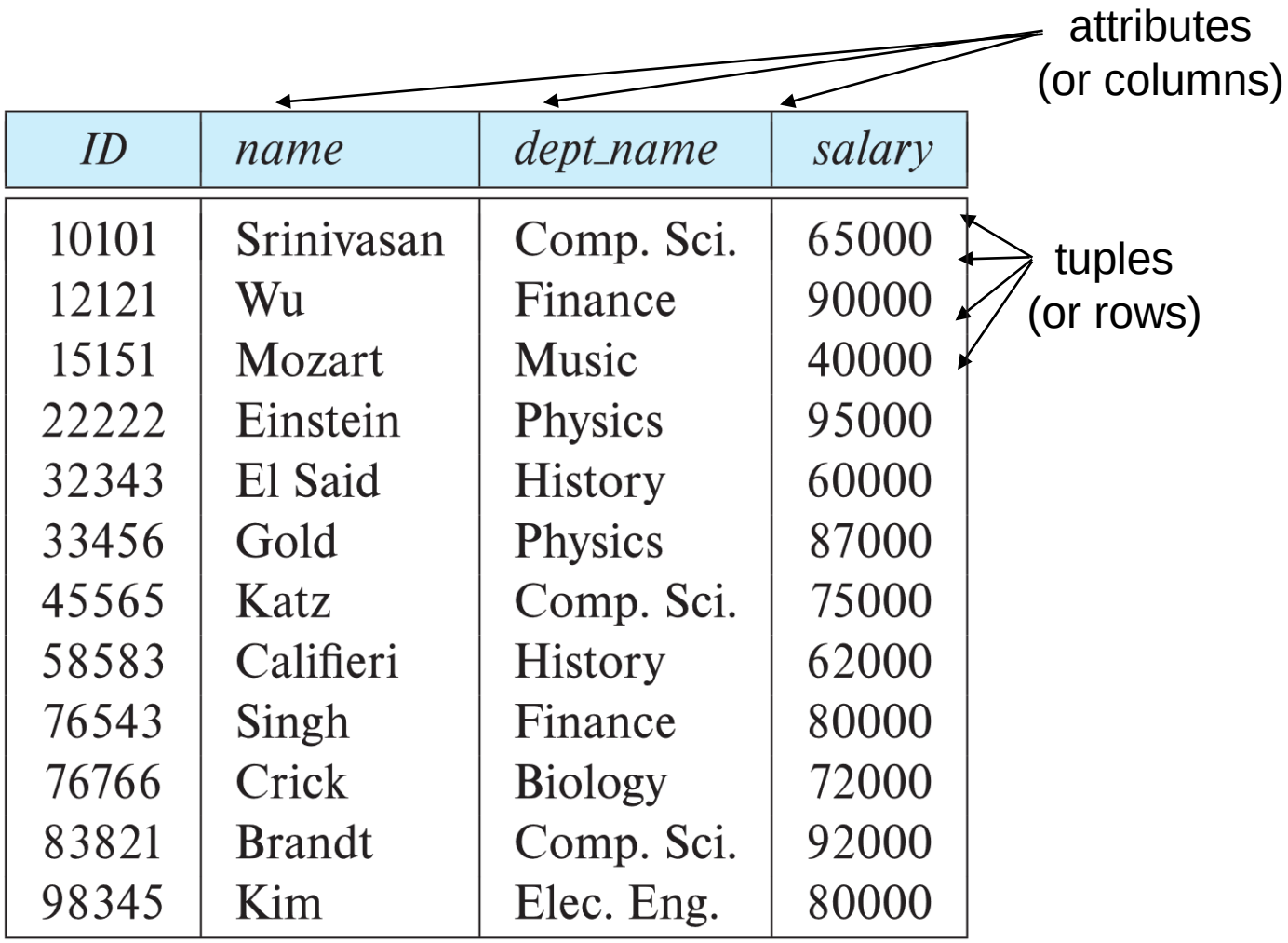
**Formal Name**

- Relation
- Tuple
- Attribute
- Cardinality
- Degree or Arity
- Primary key
- Domain

# Relational Database: Definitions

- Relational database: a set of relations
- Relation: made up of 2 parts:
  - Instance : a table, with rows and columns.
    #Rows = cardinality, #fields = degree / arity.
  - Schema : specifies name of relation, plus name and type of each column.
    - E.G. Students(sid: string, name: string, login: string,
      age: integer, gpa: real).
- Can think of a relation as a set of rows or tuples (i.e., all rows are distinct).

# Example of a *Instructor* Relation




| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 12121 | Wu | Finance | 90000 |
| 15151 | Mozart | Music | 40000 |
| 22222 | Einstein | Physics | 95000 |
| 32343 | El Said | History | 60000 |
| 33456 | Gold | Physics | 87000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 58583 | Califieri | History | 62000 |
| 76543 | Singh | Finance | 80000 |
| 76766 | Crick | Biology | 72000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 98345 | Kim | Elec. Eng. | 80000 |

# Relation Schema and Instance

- $A_1$, $A_2$, …, $A_n$ are *attributes*
- $R = (A_1, A_2, \ldots, A_n)$ is a *relation schema*

  Example:

  *instructor* = (*ID, name, dept_name, salary*)
- A relation instance *r* defined over schema *R* is denoted by *r* (*R*).
- The current values a relation are specified by a table
- An element ***t*** of relation ***r*** is called a *tuple* and is represented by a *row* in a table

# Attributes

- The set of allowed values for each attribute is called the **domain** of the attribute
- Attribute values are (normally) required to be **atomic**; that is, indivisible
- The special value ***null*** is a member of every domain. Indicated that the value is “unknown”
- The null value causes complications in the definition of many operations

# Null Values

- Comparisons with null values return the special truth value *null (unknown)*
- Three-valued logic using the truth value *null*:

  - **OR:** (*null* **OR** *true*) = *true*
    (*null* **OR** *false*) = *null*
    (*null* **OR** *null)* = *null*
  - **AND:** (*true* **AND** *null)* = *null*
    (*false* **AND** null) = *false*
    (*null* **AND** *null)* = *null*
  - **NOT*:*** (**NOT** null) = *null*

# Attributes

- An **attribute** is a characteristic

  Property, data element, field, attribute

  Ex: Student (name, IDNo, course credit)
- **Domain**
- **Attributes can be**
  - Atomic (simple) *vs.* Composite
    - Ex: Student ID , Student name

      Where student name may consists of (first, middle, last name)
  - Single-valued *vs.* Multi-valued
    - Ex. number of courses registered, names of registered courses
  - Derived
    - Ex: age of a student

# Relations are Unordered

- Order of tuples is irrelevant (tuples may be stored in an arbitrary order)
- Example: *instructor* relation with unordered tuples

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 76766 | Crick | Biology | 72000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 58583 | Califieri | History | 62000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 76543 | Singh | Finance | 80000 |

# Database Schema

- Database schema -- is the logical structure of the database.
- Database instance -- is a snapshot of the data in the database at a given instant in time.
- Example:
  - schema: i*nstructor* (*ID, name, dept_name, salary*)
  - Instance:

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
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| 15151 | Mozart | Music | 40000 |
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| 76543 | Singh | Finance | 80000 |

# Keys

- **Superkey (SK)**
  - Set of one or more attributes which taken collectively identifies uniquely an entity in entity set
  - Ex: Student ( student name, IDNo, CPI, program, address)
  - Student entity set superkey can be (student name, IDNo)
  - Superkey may contain extraneous attributes
  - There can be more than one superkeys

- **Candidate Key (CK)**
  - A superkey for which no subset is a superkey
  - Ex: IDNo is a Candidate Key as it is minimal and uniquely identifies a student from Student Entity Set
  - There can be more than one candidate keys

# Keys

- **Primary Key (PK)**
  - Is a candidate key
  - One of the candidate key is chosen by database designer as a primary key

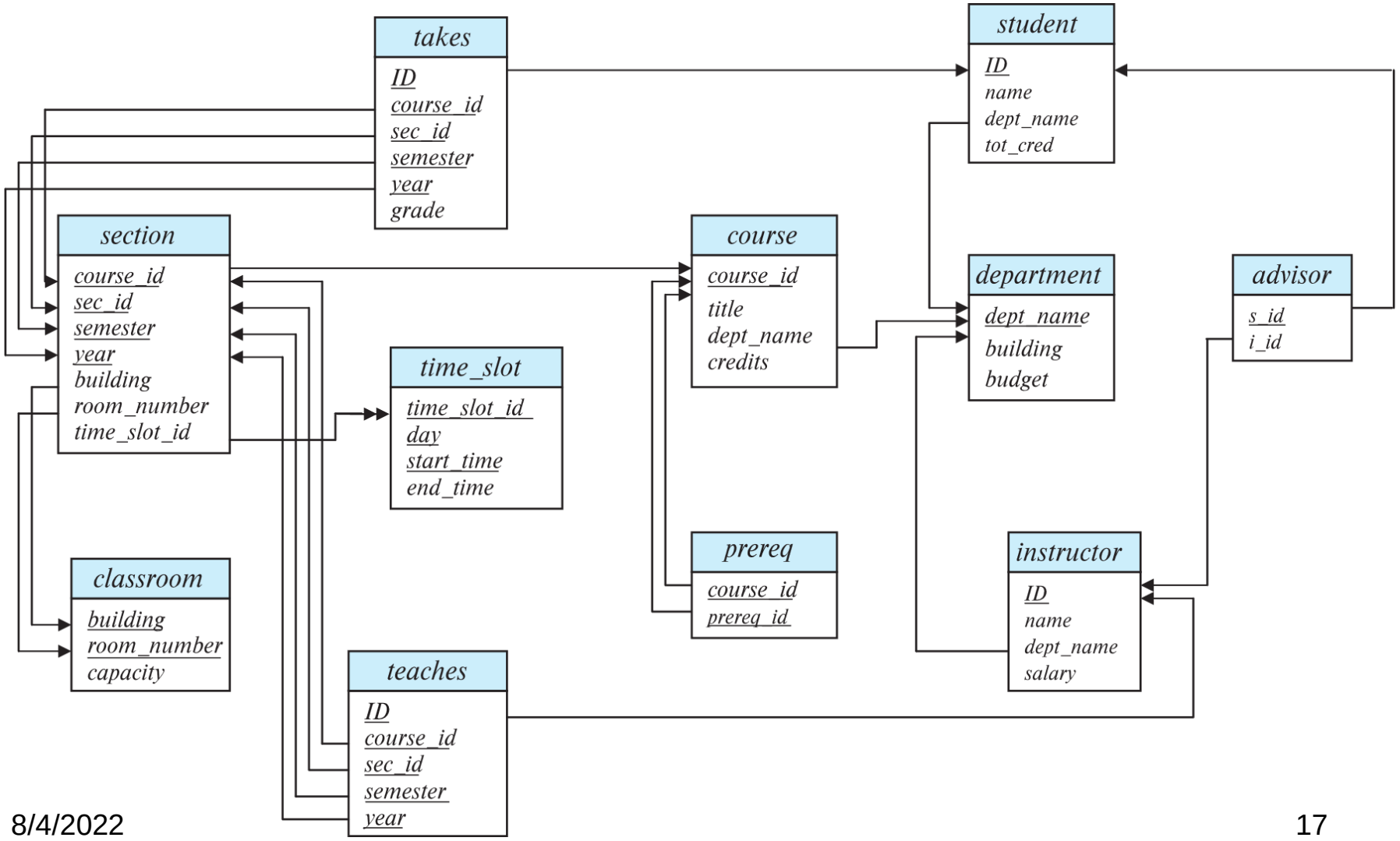
- **Example**
  - Telephone Book( STD Code, Telephone number, Name, Address)
  - Composite Key

# Keys

- Let K $\subseteq$ R
- *K* is a **superkey** of *R* if values for *K* are sufficient to identify a unique tuple of each possible relation *r(R)*
  - Example: {*ID*} and {ID,name} are both superkeys of *instructor.*
- Superkey *K* is a **candidate key** if *K* is minimal

  Example: {*ID*} is a candidate key for *Instructor*
- One of the candidate keys is selected to be the **primary key**.
  - Which one?
- **Foreign key** constraint: Value in one relation must appear in another
  - **Referencing** relation
  - **Referenced** relation
  - Example: *dept_name* in i*nstructor* is a foreign key from *instructor* referencing *department*

# Schema Diagram for University Database



# Schema of University Database

- *classroom*(*building*, *room number*, *capacity*)
- *department*(*dept name*, *building*, *budget*)
- *course*(*course id*, *title*, *dept name*, *credits*)
- *instructor*(*ID*, *name*, *dept name*, *salary*)
- *section*(*course id*, *sec id*, *semester*, *year*, *building*, *room number*, *time slot id*)
- *teaches*(*ID*, *course id*, *sec id*, *semester*, *year*)
- *student*(*ID*, *name*, *dept name*, *tot cred*)
- *takes*(*ID*, *course id*, *sec id*, *semester*, *year*, *grade*)
- *advisor*(*s ID*, *i ID*)
- *time slot*(*time slot id*, *day*, *start time*, *end time*)
- *Prereq* (*course id*, *prereq id*)

# Relational Query Languages

- Procedural versus non-procedural, or declarative
- “Pure” languages:
  - Relational algebra
  - Tuple relational calculus
  - Domain relational calculus
- The above 3 pure languages are equivalent in computing power
- We will concentrate in this chapter on relational algebra
  - Consists of 6 basic operations

# Relational Algebra

- A procedural language consisting of a set of operations that take one or two relations as input and produce a new relation as their result.
- Six basic operators
  - **select: $\sigma$**
  - **project: $\Pi$**
  - **union: $\cup$**
  - **set difference: –**
  - **Cartesian product: x**
  - **rename: $\rho$**

# Relational Algebra Operators

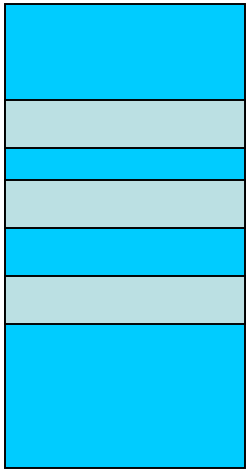
Relational Algebra consists of several groups of operations

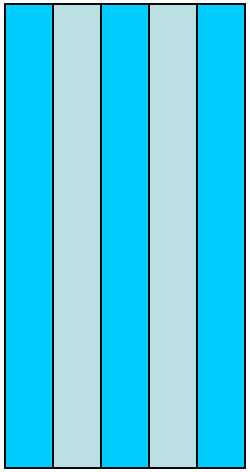
- Unary Relational Operations
  - SELECT (symbol: $\sigma$ (sigma))
  - PROJECT (symbol: $\pi$ (pi))
  - RENAME (symbol: $\rho$ (rho))
- Relational Algebra Operations From Set Theory
  - UNION ( $\cup$ ), INTERSECTION ( $\cap$ ), DIFFERENCE (or MINUS, – )
  - CARTESIAN PRODUCT ( **x** )
- Binary Relational Operations
  - JOIN (several variations of JOIN exist)
  - DIVISION
- Additional Relational Operations
  - OUTER JOINS, OUTER UNION
  - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

# Relational Algebra

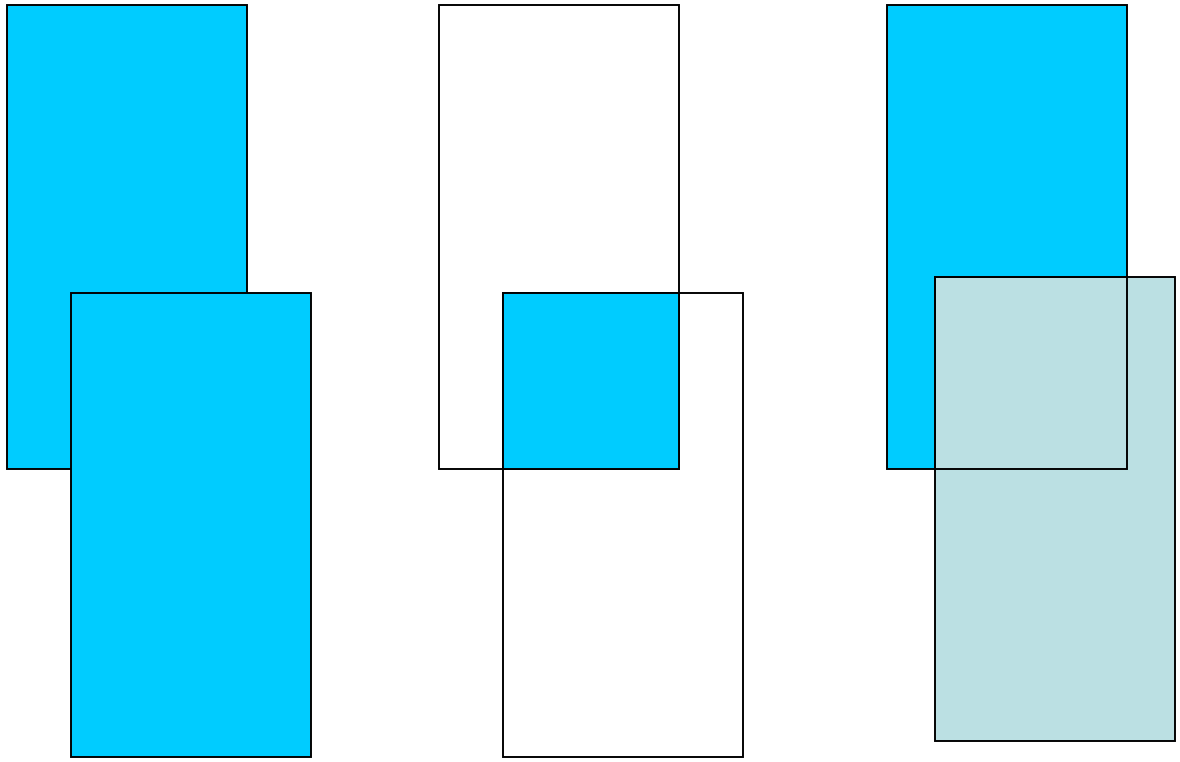
- Collection of operations on relations
- 8 operators + Rename operator
- Codd’s 8 operators do not form a minimal set
- Some of them are not primitive
- Join, Intersect, & Divide can be defined in terms of other 5
- None of these 5 can be defined in terms of the remaining 4

# Restrict & Project





# Union, Intersection & Difference



# Instructor

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 12121 | Wu | Finance | 90000 |
| 32343 | El Said | History | 60000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 76766 | Crick | Biology | 72000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 58583 | Califieri | History | 62000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 76543 | Singh | Finance | 80000 |

# Select Operation

- The **selec**t operation selects tuples that satisfy a given predicate.
- Notation: $\sigma_p(r)$
- *p* is called the **selection predicate**
- Example: select those tuples of the *instructor* relation where the instructor is in the "Physics" department.
  - Query

$$\sigma_{dept\_name="Physics"}(instructor)$$

  - **Result**

| *ID* | *name* | *dept_name* | *salary* |
|---|---|---|---|
| 22222 | Einstein | Physics | 95000 |
| 33456 | Gold | Physics | 87000 |

# Select Operation

- We allow comparisons using

  **=, ≠, >, ≥. <. ≤**

  in the selection predicate.
- We can combine several predicates into a larger predicate by using the connectives:

  **∧ (and), ∨ (or), ¬ (not)**

- Example: Find the instructors in Physics with a salary greater 90,000, we write:

  $\sigma_{dept\_name=\text{“}Physics\text{”} \wedge salary > 90{,}000}(instructor)$

- The select predicate may include comparisons between two attributes.
  - Example, find all departments whose name is the same as their building name:
  - $\sigma_{dept\_name=building}(department)$

# Project Operation

- A unary operation that returns its argument relation, with certain attributes left out.
- Notation:

$$\Pi_{A_1,A_2,A_3 \ldots A_k}(r)$$

where $A_1$, $A_2$, …, $A_k$ are attribute names and $r$ is a relation name.

- The result is defined as the relation of $k$ columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets

# Project Operation Example

- Example: eliminate the *dept_name* attribute of *instructor*

$$\Pi_{ID,\ name,\ salary}\ (instructor)$$

- **Result:**

| *ID* | *name* | *salary* |
|---|---|---|
| 10101 | Srinivasan | 65000 |
| 12121 | Wu | 90000 |
| 15151 | Mozart | 40000 |
| 22222 | Einstein | 95000 |
| 32343 | El Said | 60000 |
| 33456 | Gold | 87000 |
| 45565 | Katz | 75000 |
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| 76766 | Crick | 72000 |
| 83821 | Brandt | 92000 |
| 98345 | Kim | 80000 |

# Composition of Relational Operations

- The result of a relational-algebra operation is a relation and therefore relational-algebra operations can be composed together into a **relational-algebra expression**.

- *Find the names of all instructors in the Physics department.*

$$\Pi_{name}(\sigma_{dept\_name = \text{“Physics”}} (instructor))$$

- Instead of giving the name of a relation as the argument of the projection operation, we give an expression that evaluates to a relation.

# Cartesian-Product Operation

- The Cartesian-product operation (denoted by **X**) allows us to combine information from any two relations.
- Example: the Cartesian product of the relations *instructor* and t*eaches* is written as:

  ***instructor* X *teaches***

- We construct a tuple of the result out of each possible pair of tuples: one from the *instructor* relation and one from the *teaches* relation

- Since the instructor *ID* appears in both relations we distinguish between these attributes by attaching to the attribute the name of the relation from which the attribute originally came.
  - *instructor.ID*
  - *teaches.ID*

# The *instructor* x *teaches table*

| *instructor.ID* | *name* | *dept_name* | *salary* | *teaches.ID* | *course_id* | *sec_id* | *semester* | *year* |
|---|---|---|---|---|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 12121 | Wu | Finance | 90000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 12121 | Wu | Finance | 90000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 12121 | Wu | Finance | 90000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 12121 | Wu | Finance | 90000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 12121 | Wu | Finance | 90000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 12121 | Wu | Finance | 90000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 15151 | Mozart | Music | 40000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 15151 | Mozart | Music | 40000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 15151 | Mozart | Music | 40000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 22222 | Einstein | Physics | 95000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 22222 | Einstein | Physics | 95000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 22222 | Einstein | Physics | 95000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |

# Join Operation

- The Cartesian-Product

  *instructor* X *teaches*

  associates every tuple of instructor with every tuple of teaches.
  - Most of the resulting rows have information about instructors who did NOT teach a particular course.
- To get only those tuples of "*instructor* X *teaches* " that pertain to instructors and the courses that they taught, we write:

$$\sigma_{instructor.id = teaches.id} (instructor \times teaches))$$

  - We get only those tuples of "*instructor* X *teaches"* that pertain to instructors and the courses that they taught.

# Join Operation (Cont.)

- $\sigma_{instructor.id = teaches.id}$ (*instructor* x *teaches*))

| *instructor.ID* | *name* | *dept_name* | *salary* | *teaches.ID* | *course_id* | *sec_id* | *semester* | *year* |
|---|---|---|---|---|---|---|---|---|
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-101 | 1 | Fall | 2017 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-315 | 1 | Spring | 2018 |
| 10101 | Srinivasan | Comp. Sci. | 65000 | 10101 | CS-347 | 1 | Fall | 2017 |
| 12121 | Wu | Finance | 90000 | 12121 | FIN-201 | 1 | Spring | 2018 |
| 15151 | Mozart | Music | 40000 | 15151 | MU-199 | 1 | Spring | 2018 |
| 22222 | Einstein | Physics | 95000 | 22222 | PHY-101 | 1 | Fall | 2017 |
| 32343 | El Said | History | 60000 | 32343 | HIS-351 | 1 | Spring | 2018 |
| 45565 | Katz | Comp. Sci. | 75000 | 45565 | CS-101 | 1 | Spring | 2018 |
| 45565 | Katz | Comp. Sci. | 75000 | 45565 | CS-319 | 1 | Spring | 2018 |
| 76766 | Crick | Biology | 72000 | 76766 | BIO-101 | 1 | Summer | 2017 |
| 76766 | Crick | Biology | 72000 | 76766 | BIO-301 | 1 | Summer | 2018 |
| 83821 | Brandt | Comp. Sci. | 92000 | 83821 | CS-190 | 1 | Spring | 2017 |
| 83821 | Brandt | Comp. Sci. | 92000 | 83821 | CS-190 | 2 | Spring | 2017 |
| 83821 | Brandt | Comp. Sci. | 92000 | 83821 | CS-319 | 2 | Spring | 2018 |
| 98345 | Kim | Elec. Eng. | 80000 | 98345 | EE-181 | 1 | Spring | 2017 |

# Join Operation (Cont.)

- The **join** operation allows us to combine a select operation and a Cartesian-Product operation into a single operation.
- Consider relations *r* (*R*) and *s* (*S*)
- Let "theta" be a predicate on attributes in the schema R "union" S. The join operation r $\bowtie_\theta$ s is defined as follows:

$$r \bowtie_\theta s = \sigma_\theta (r \times s)$$

- Thus

$$\sigma_{instructor.id = teaches.id} (instructor \times teaches))$$

- Can equivalently be written as

$$instructor \bowtie_{Instructor.id = teaches.id} teaches.$$

# Union Operation

- The union operation allows us to combine two relations
- Notation: $r \cup s$
- For $r \cup s$ to be valid.
  1. *r, s* must have the *same* **arity** (same number of attributes)
  2. The attribute domains must be **compatible** (example: 2nd column of *r* deals with the same type of values as does the 2nd column of *s*)

- Example: to find all courses taught in the Fall 2017 semester, or in the Spring 2018 semester, or in both

$$\Pi_{course\_id} (\sigma_{semester=\text{“Fall”} \wedge year=2017} (section))$$

$$\cup$$

$$\Pi_{course\_id} (\sigma_{semester=\text{“Spring”} \wedge year=2018} (section))$$

# Union Operation

- Result of:

$$\Pi_{course\_id}\,(\sigma_{semester=\text{“Fall”} \wedge year=2017}(section))$$

$$\cup$$

$$\Pi_{course\_id}\,(\sigma_{semester=\text{“Spring”} \wedge year=2018}(section))$$

| *course_id* |
|---|
| CS-101 |
| CS-315 |
| CS-319 |
| CS-347 |
| FIN-201 |
| HIS-351 |
| MU-199 |
| PHY-101 |

# Set-Intersection Operation

- The set-intersection operation allows us to find tuples that are in both the input relations.
- Notation: $r \cap s$
- Assume:
  - $r$, $s$ have the *same arity*
  - attributes of $r$ and $s$ are compatible
- Example: Find the set of all courses taught in both the Fall 2017 and the Spring 2018 semesters.

$$\Pi_{course\_id} (\sigma_{semester=\text{“Fall”} \wedge year=2017} (section)) \cap \Pi_{course\_id} (\sigma_{semester=\text{“Spring”} \wedge year=2018} (section))$$

**Result**

| *course_id* |
| --- |
| CS-101 |

# Set Difference Operation

- The set-difference operation allows us to find tuples that are in one relation but are not in another.
- Notation $r - s$
- Set differences must be taken between **compatible** relations.
  - $r$ and $s$ must have the same arity
  - attribute domains of $r$ and $s$ must be compatible
- Example: to find all courses taught in the Fall 2017 semester, but not in the Spring 2018 semester

$$\Pi_{course\_id}(\sigma_{semester=\text{“Fall”} \wedge year=2017}(section)) - \Pi_{course\_id}(\sigma_{semester=\text{“Spring”} \wedge year=2018}(section))$$

| *course_id* |
| --- |
| CS-347 |
| PHY-101 |

# The Assignment Operation

- It is convenient at times to write a relational-algebra expression by assigning parts of it to temporary relation variables.
- The assignment operation is denoted by $\leftarrow$ and works like assignment in a programming language.
- Example: Find all instructor in the “Physics” and Music department.

$$Physics \leftarrow \sigma_{dept\_name=\text{“Physics”}}(instructor)$$
$$Music \leftarrow \sigma_{dept\_name=\text{“Music”}}(instructor)$$
$$Physics \cup Music$$

- With the assignment operation, a query can be written as a sequential program consisting of a series of assignments followed by an expression whose value is displayed as the result of the query.

# The Rename Operation

- The results of relational-algebra expressions do not have a name that we can use to refer to them. The rename operator, $\rho$, is provided for that purpose
- The expression:

$$\rho_x (E)$$

returns the result of expression *E* under the name *x*

- Another form of the rename operation:

$$\rho_{x(A1,A2, .. An)} (E)$$

# Equivalent Queries

- There is more than one way to write a query in relational algebra.
- Example: Find information about courses taught by instructors in the Physics department with salary greater than 90,000
- Query 1

$$\sigma_{dept\_name = \text{“Physics”} \wedge salary > 90{,}000}(instructor)$$

- Query 2

$$\sigma_{dept\_name = \text{“Physics”}}(\sigma_{salary > 90.000}(instructor))$$

- The two queries are not identical; they are, however, equivalent -- they give the same result on any database.

# Equivalent Queries

- There is more than one way to write a query in relational algebra.
- Example: Find information about courses taught by instructors in the Physics department
- Query 1

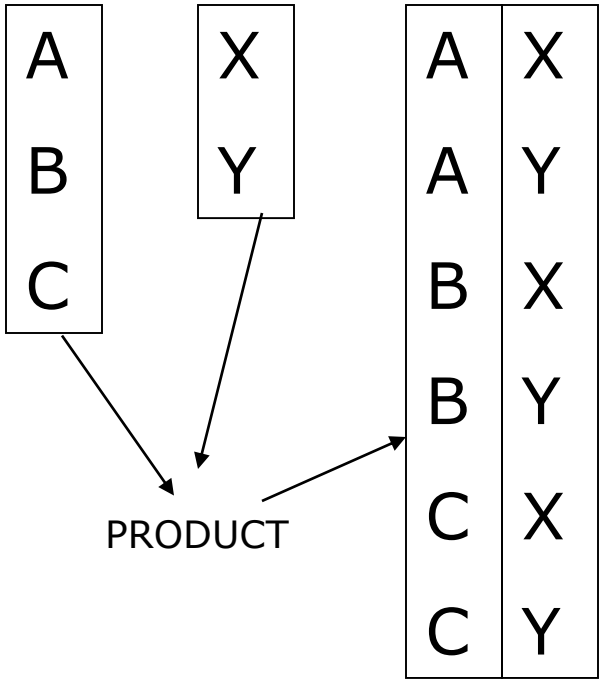
$$\sigma_{dept\_name=\text{“Physics”}} (instructor \bowtie instructor.ID = teaches.ID\ teaches)$$

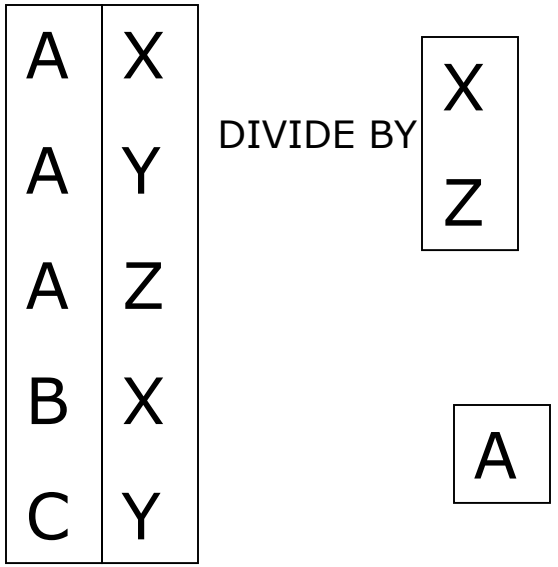
- Query 2

$$(\sigma_{dept\_name=\text{“Physics”}} (instructor)) \bowtie instructor.ID = teaches.ID\ teaches$$

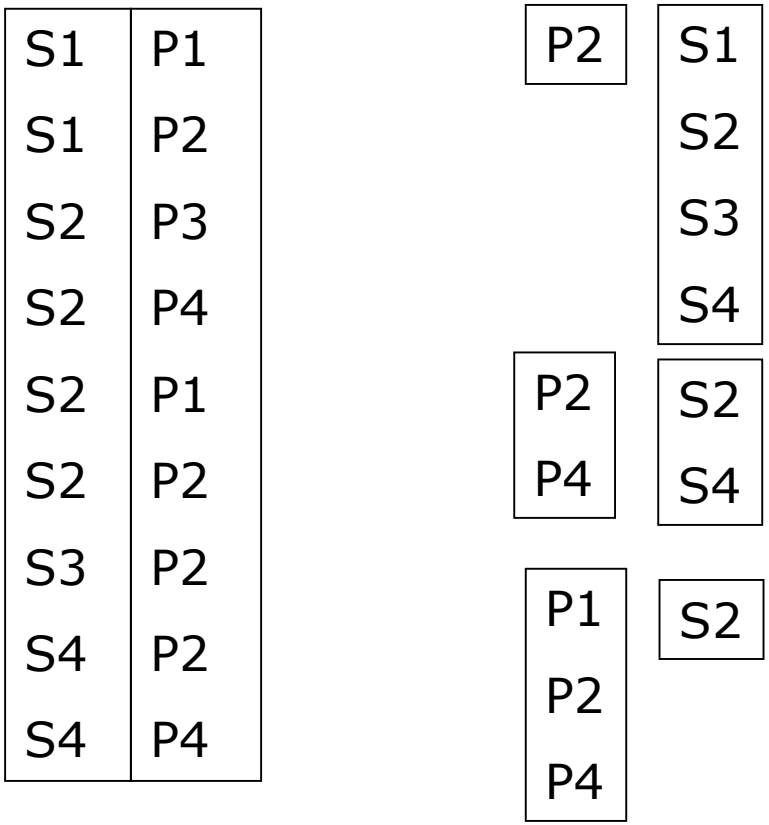
- The two queries are not identical; they are, however, equivalent -- they give the same result on any database.

# Product & Divide



# Division


S1 P1
S1 P2
S2 P3
S2 P4
S2 P1
S2 P2
S3 P2
S4 P2
S4 P4
P2 S1 S2 S3 S4
P2 P4 S2 S4
P1 P2 P4 S2