CT303 - DIGITAL COMMUNICATION.

Abhishek Jindal - PhD - communication - wireless

areas of interest - Wireless ", CPS, Information security

- applications of Deep learning to WC & finance.

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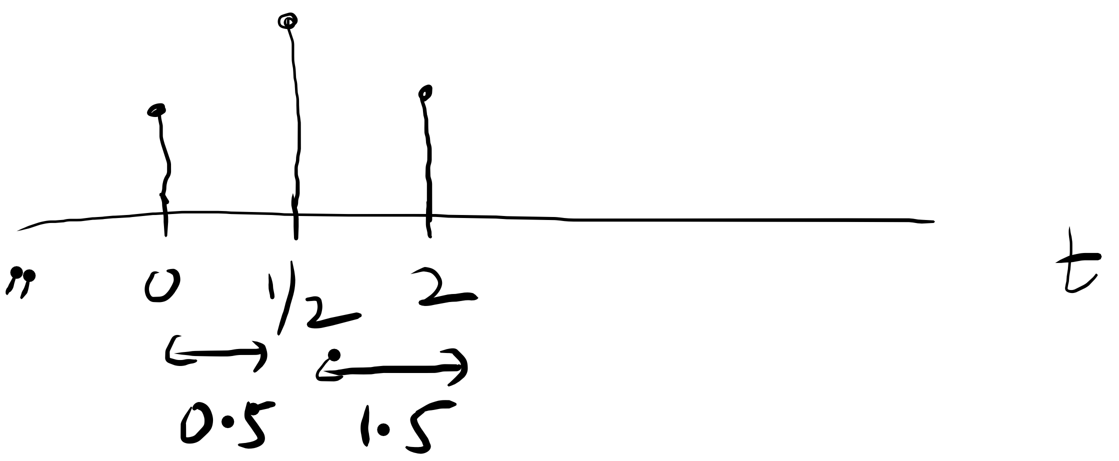
4 different categories of signals - depending upon the characteristic of the time (independent) variable & " values they take.

$[\,x(t)\,]$

a. continuous time signal :- (or analog signal) - defined for every value of time & they take on values in the continuous interval (a, b) where a can be $-\infty$ & b can be ∞. ex- $\cos \pi t$

2. Discrete-Time signal:-

defined only at certain time instants. Note that " need not be equidistant" but in practice they are usually taken at equally spaced intervals for computational convenience & mathematical tractability. - seq. of real or complex nos



ex- $$x(n) = \begin{cases} (0.8)^n, & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

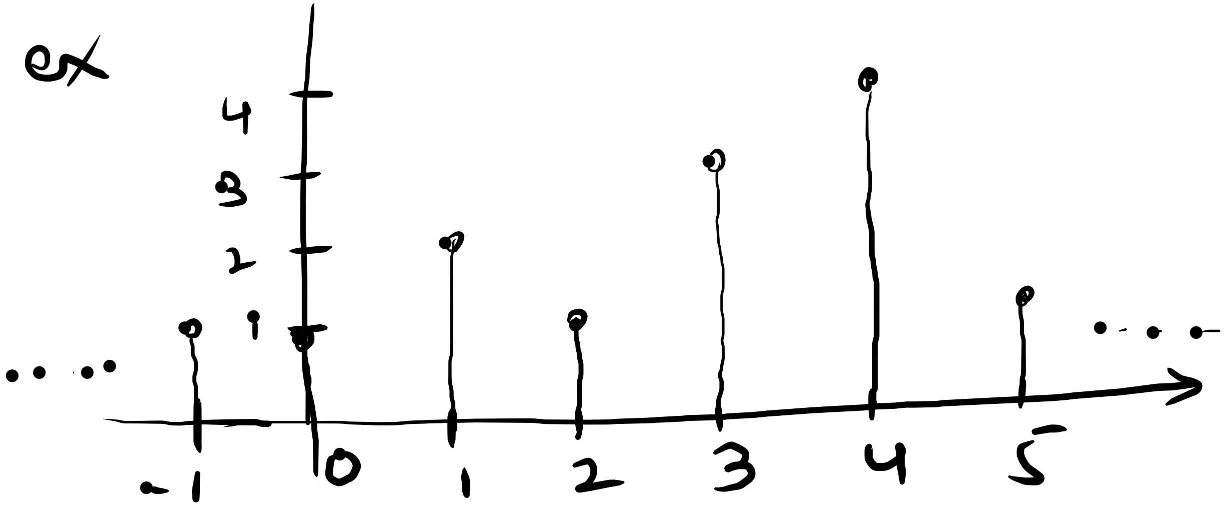
3. Continuous-valued :- Signal takes on all possible values on a finite or infinite range

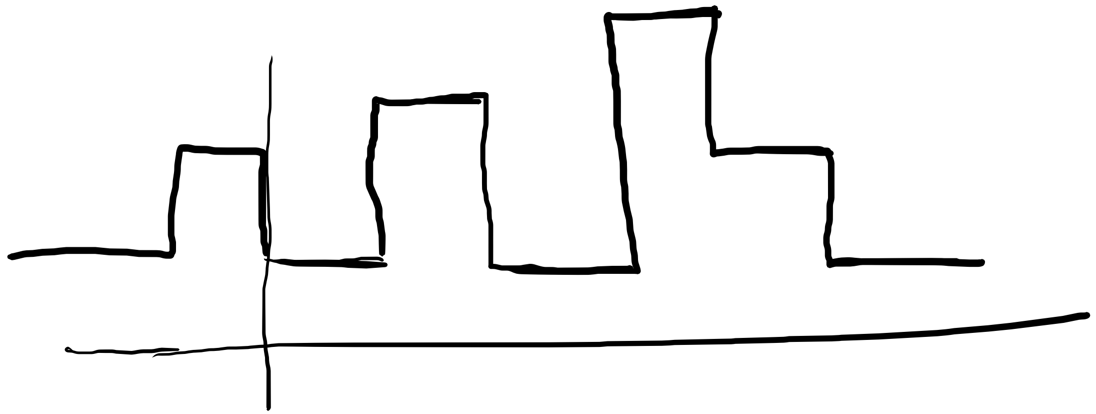
(a, b) (−∞, ∞)

4. Discrete-valued :- values from a finite set of possible values.

→ A discrete-time signal having a set of discrete values is called a digital signal. {1, 2, 3, 4}

ex





1. Analog vs. Digital — Analog or digital?

A. Speech, audio & video, popularly the 'message' signals — they contain information

generation & consumption are analog.

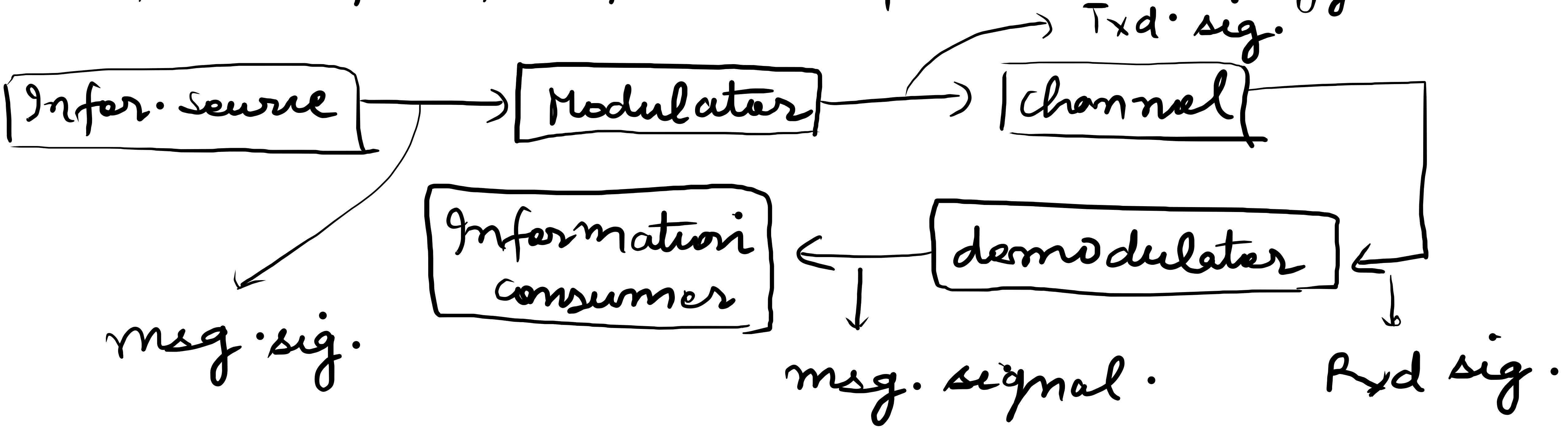
1948, C.E.Shannon

B. transmitted signal corresponding to physical comm. media are also analog ex - wireless & optical comm^n employ EM waves.

Typical choice is AC - analog communication

Given analog nature of both the message & the commu. medium, natural choice is to map analog msg. signal to an analog txd signal that is 'compatible' with over which we wish to communicate.

ex- AM, FM, 1G cellular phone technology.



ex- an audio signal,

Lecture-2, DC

ex- an audio signal, translate from the acoustic to the electrical domain using a microphone. → radio wave which will carry the audio signal → BC audio over the air from an FM/AM radio

→ what we see around is mostly digital?

DC :- communication in terms of bits - foundations were laid by Prof. Claude E. Shannon (1948)

Two main threads :- 1) Source coding & compression

2) Digital information Tx

1) involves compression, or removal of redundancy in a manner

that exploits the prop. of the src sig. ex. heavy correlation among nearby pixels in an image.

2) once source coding is done, task is to "reliably" transfer bit seq. across space or time

→ Notion of channel capacity

$C = 1$ Kbps

$R \leq C$:- error-free Rx

$R > C$:- errors

Thomas Cover's book.

→ Three factors affecting Tx :-

→ signal strength,

→ noise or interference

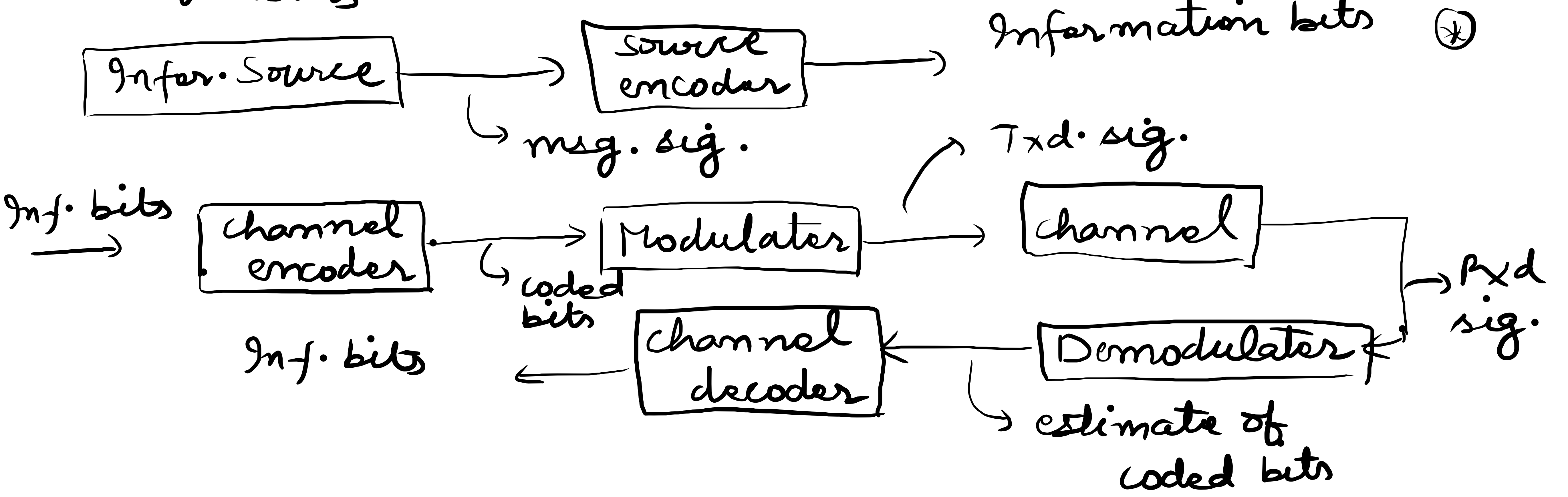
→ distortions imposed by channel

↳ once these three things are fixed for a commⁿ chnl, channel capacity gives the max. possible rate of reliable comm.

let us contrast AC & DC

AC: quality of reproduced source signal typically degrades gradually as the channel condition worsens

DC: Sharp transition b/w reliable & unreliable comm^n.



Inf. bits → Source decoder → Inform. consumer

msg. signal.

Source-chnl separation theorem.

Tx → repeater → repeater → Rx

analog signal

bits

bits

over those links, we tx/Rx bits – hence easy to use for multiple source signal

src. coding – remove redundancy

chnl. coding – "controlled" red. addition

IEEE 802.15a/b/g

Ques:- redundancy removal by src encoder & red. add. by chnl encoder.

Individual descriptions of each of the blocks in the diag. above – you need to study yourself.

As you can see DC involves far more processing than AC then why DC?

→ This is made possible through 'increase in computational power of low-cost silicon integrated circuits".

optimality :- for DC source-channel sep. principle is generally src. indep. & channel optimized.

For AM, waveform TX depends on the msg sig., which is beyond the control of the link designer, hence no freedom to optimize link perf. over all possible commⁿ schemes.

Lecture 3 - DC

→ noise immunity

Scalability :- DC allows ideal regener^n of bits - hence if you can communicate over a link reliably, you are done

① → Inf^n bits are txd without interpretation, the same link can be used for multiple kinds of msgs.

② → Multiple links can be present b/w src encoder & dec. with proper error recovery mechanisms - such as retransmission

SE SD

① & ② have enabled internet

AM :- link perfor^n depends on message prop., successive links means noise accumul^n & this limits the no. of links that can be cascaded.

Why AM still persists? – RF front end is still analog.

– legacy systems – AM/FM are still in use.

– Modulator in a DC system:-

coded bits after channel encoder

→ Txd signals

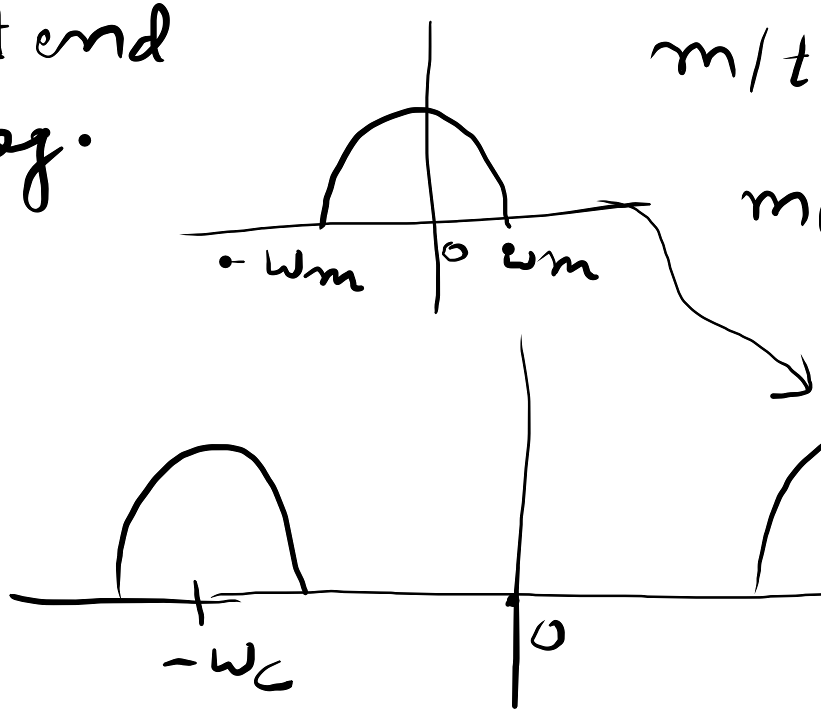
req:- Tx sig. to fit within a given freq. band & adhere to 'stringent power constraint' & 'manage interference'.

bit

0 → $s(t)$ → $s(t)$ must fit into spectral constraint. no interf. to other users i.e. band sep^n.

1 → $-s(t)$

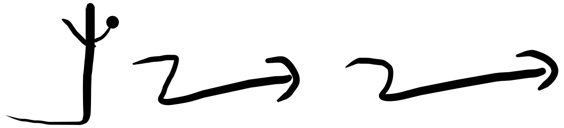
$m(t)$

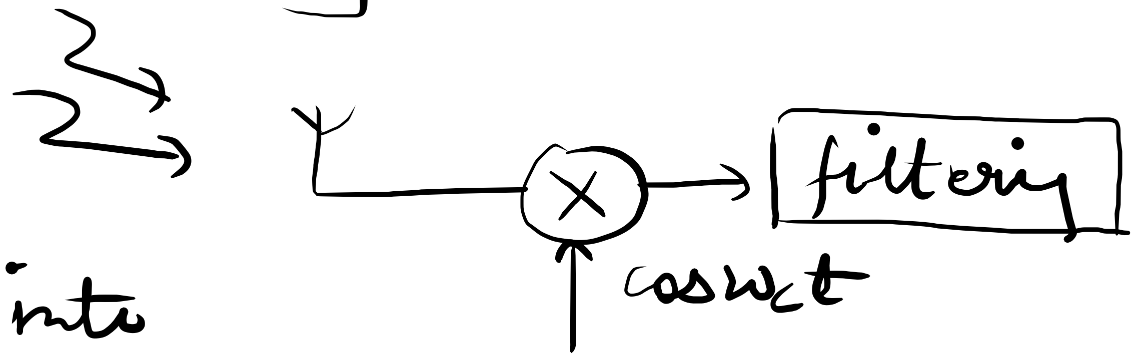


$m(t) \cos w_c t$

→ translation

$w_c >> w_m$





$\cos w_c t$

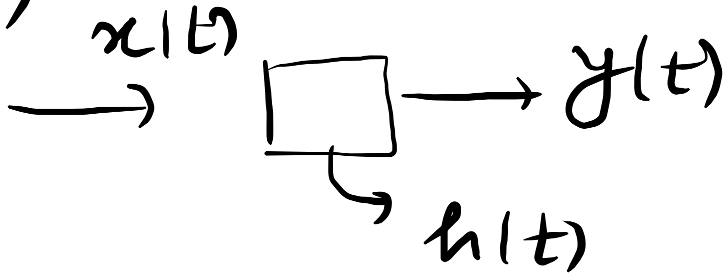
successive bits should not interfere with each other.

→ LTI ? $f(ax+by) = af(x) + bf(y)$

$x(t) \rightarrow y(t)$

TI

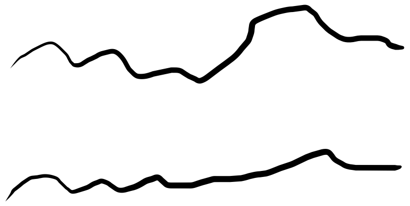
$\rightarrow x(t-t_0) \rightarrow y(t-t_0)$



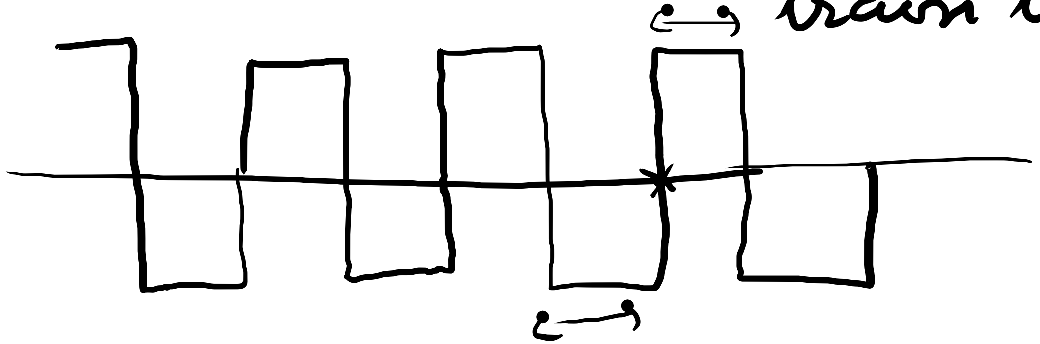
channel:- primarily is LTI but yes others models are also studied.

5/8 9 → 6

6/8



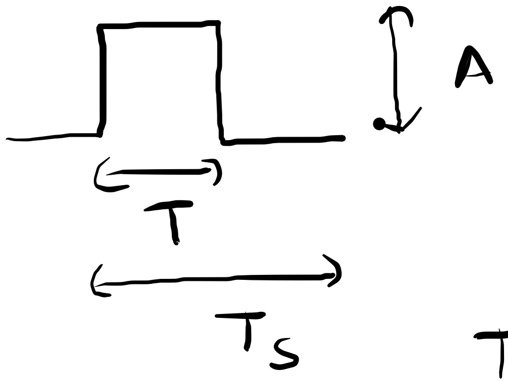
Pulse Modulation (PM) - some parameter of 'periodic' pulse train is varied in accordance with the msg. sig.

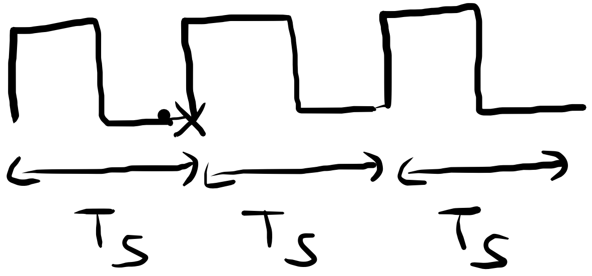


→ amplitude - PAM

→ duration — PWM

→ position — PPM.





$T < T_s$

Two types of PM < APM - analog
DPM - Digital

APM - infor^n is Txd in analog form but at discrete times

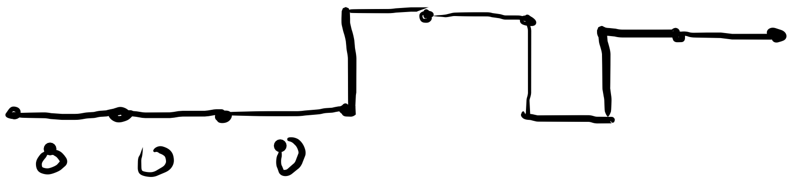
DPM - Message sig. is discrete in time & amplitude both, hence can be txd as seq. of coded pulses.

CW (continuous wave) modulation - some parameter of sinusoidal carrier wave is varied acc. to msg. signal.

{1, 2, 3, 4}

00, 01, 10, 11

1 2, 3, 4



{1, 2, 3, 4}   1 1 2 2 3 4 1 2 3

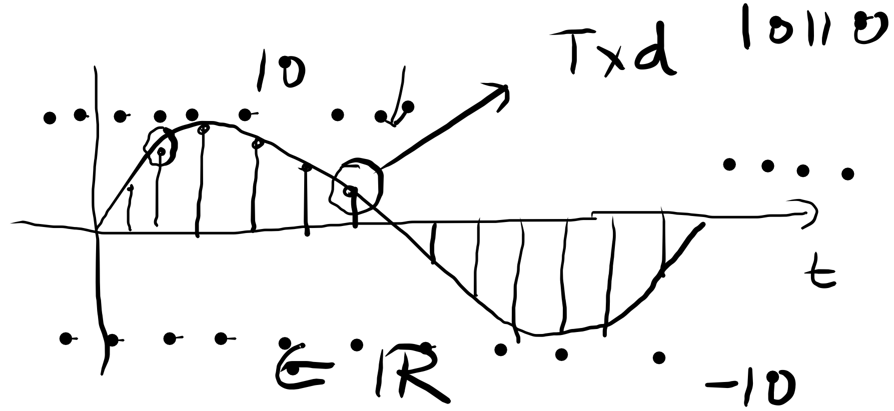
0

For DC, the base req. is use of coded pulses for Tx of analog infor^n bearing signals.

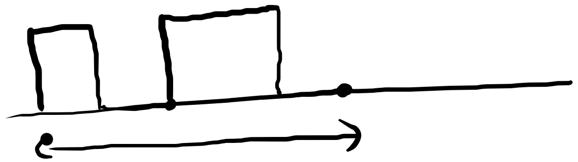
Sampling process:- heart of DSP & DC

Analog sig → seq. of samples that are 'usually' spaced uniformly in time.

Lec-4 DC



coded symbols:-

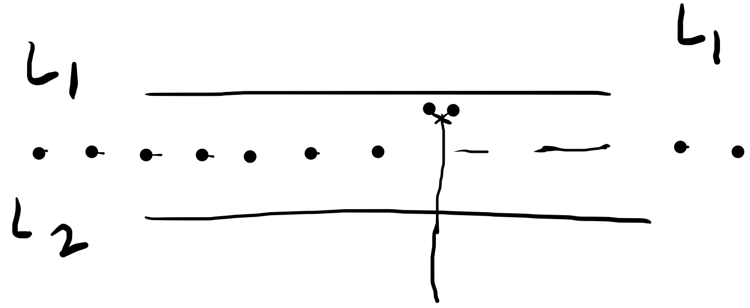


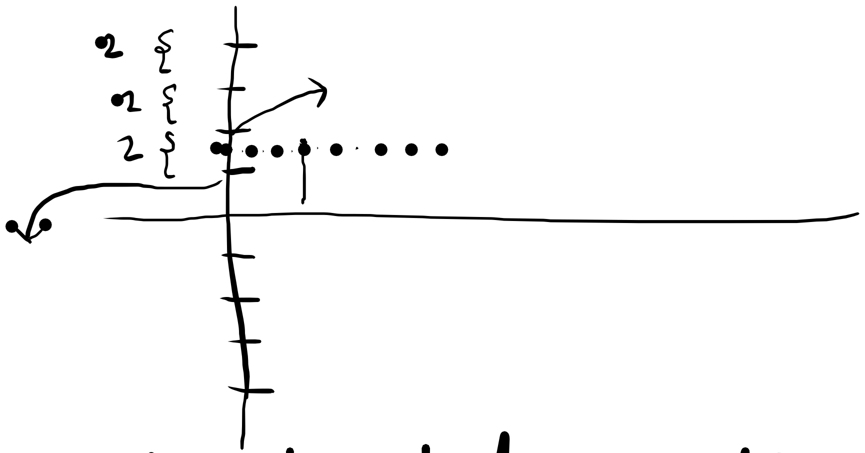
20 fixed points

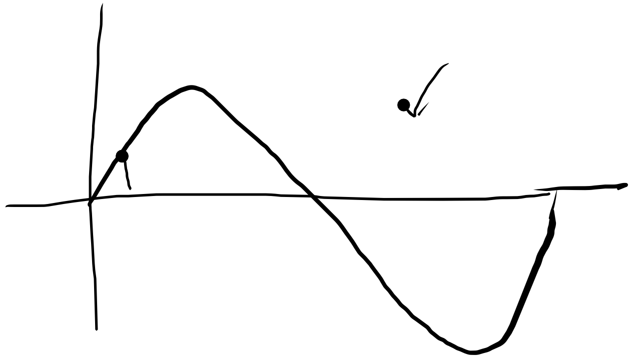
5 bits

$2^4 = 16$

$2^5 = 32$

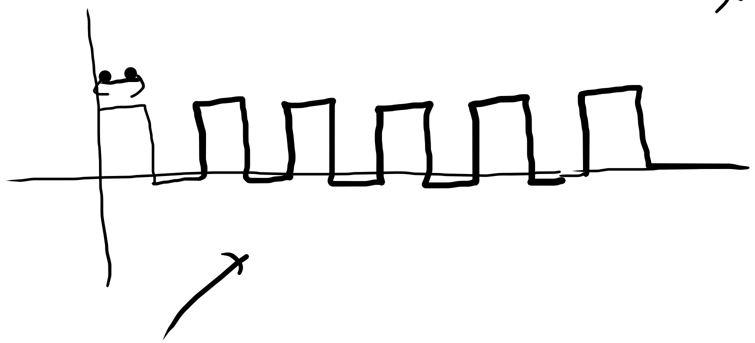


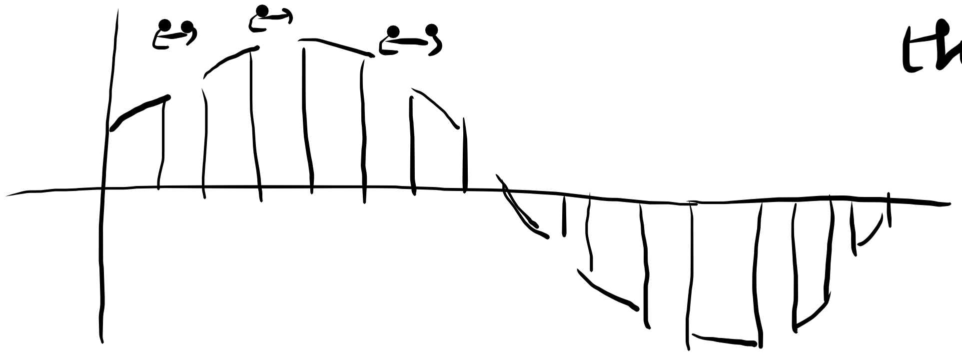




at receiver

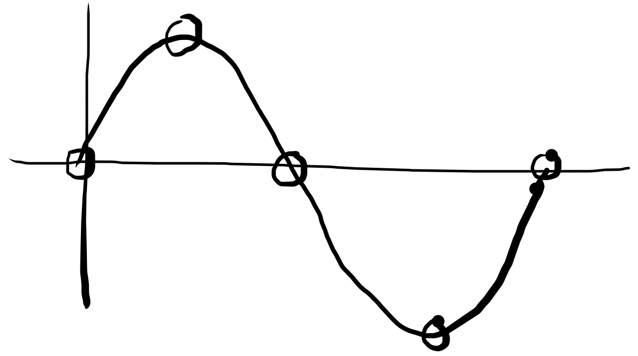
If L_1 is correctly decoded, can you tell the exact value?





agenda for any sampling process / way :- rate or procedure such that seq. of samples "uniquely" define the original analog signal.

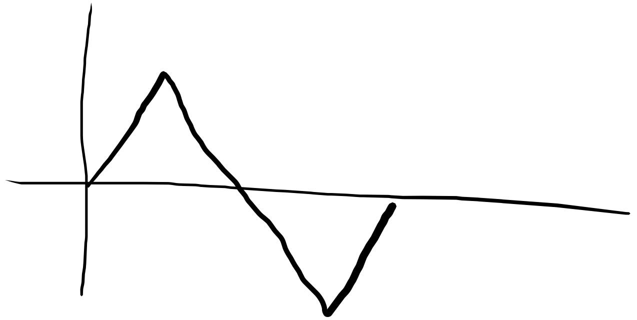
one-many mapping should not be there.



→ Let us take an arbitrary signal $g(t)$ of finite energy', 'specified for all time'

|||

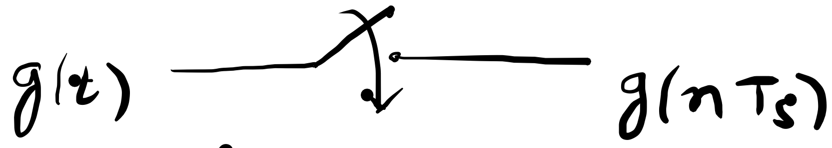
physically realizable



→ sample the signal $g(t)$ instantaneously & at a uniform rate, once every T_s seconds — $\{g(nT_s)\}$, $n \in$ set of integers ($\mathbb{Z}$)

T_s : sampling period — why

as you get infinite seq. of samples spaced T_s sec. apart



$f_s = 1/T_s$:- sampling rate - no. of samples you get every second.

$g_\delta(t) \triangleq$ instantaneously sampled signal

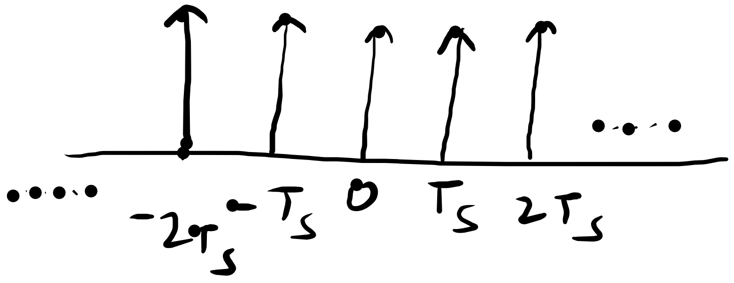
↳ denotes

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$ ✓

$$g(t) \times \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$ ✓

$$\overset{?}{=} g_\delta(t)$$

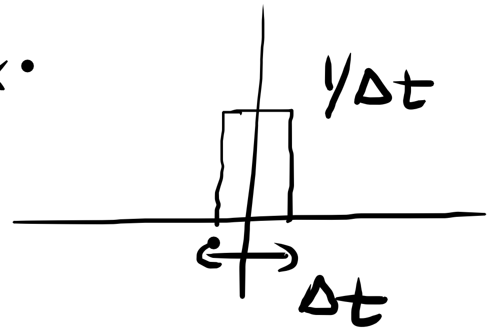
draw $\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$



(1) $\delta(t) = 0, \quad t \neq 0$

(2) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

Closest approx.



as $\Delta t \to 0$

$\longrightarrow \delta(t)$

$$x(t)\,\delta(t-t_0) \overset{?}{=} x(t_0)$$

$$\overset{?}{=} x(t_0)\,\delta(t-t_0)$$ ✓

$g_\delta(t)$:= this is like weighting (individually) the elements of a periodic seq of delta functions spaced ⊛

⊛ T_s sec. apart by seq. of numbers $\{g(nT_s)\}$.

$g_\delta(t) \doteq$ 'ideal sampled signal'

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$

Now, from S&S

$$\sum_{i=-\infty}^{\infty} \delta(t - iT_0) \xrightarrow{FT} \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$$

$$g_\delta(T_s) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(T_s - nT_s)$$

$$g_\delta(t) = g(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$\ldots\ldots g(-T_s)\,\underbrace{\delta(T_s+T_s)}_{0} + g(0)\,\delta(T_s)^{\nearrow 0} + g(T_s)\,\delta(T_s-T_s) + g(2T_s)\,\underbrace{\delta(T_s-2T_s)}_{0} \ldots$$

$$\delta(0) = \infty$$

$$g(T_s)\,\delta(0) = g(T_s)\,.$$

↳ we will discuss

$$FT\{g_\delta(t)\} = FT(g(t)) * FT \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

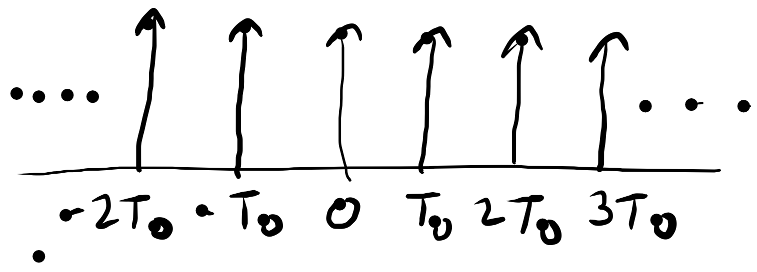
$$= G(\omega) * \frac{1}{T_s} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_s}\right)$$

$\because f_s = 1/T_s$

$$f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

Lecture 5 - DC

$\sum_{i=-\infty}^{\infty} \delta(t - iT_0) \stackrel{?}{=}$ periodic signal



using its FS coeff, we can obtain its FT. # oppenh.. S&S TB

FS coeff are $1/T_0$

$a_k = 1/T_0 \;\forall k$

$$FT\left(\sum_{i=-\infty}^{\infty} \delta(t - iT_0)\right) = \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_0}\right)$$

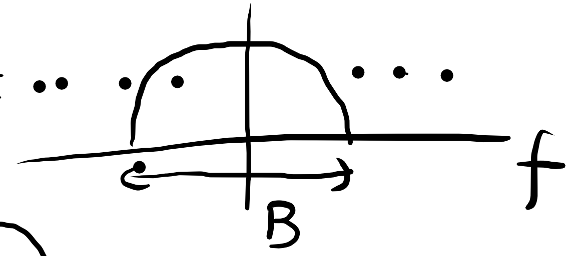
From last lecture

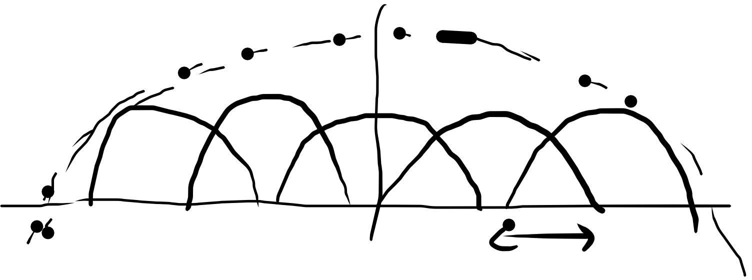
$$G_S(f) \rightarrow FT(g_S(t)) = G(f) * \frac{1}{T_S} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_S}\right) = \frac{1}{T_S} \sum_{m=-\infty}^{\infty} G\left(f - \frac{m}{T_S}\right)$$

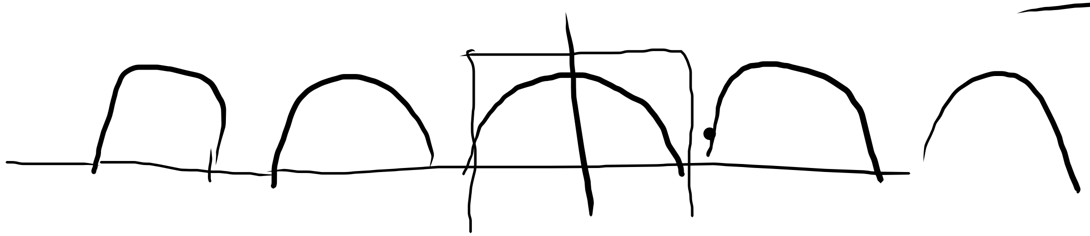
$\frac{1}{T_S} = f_S$;

$$f_S \sum_{m=-\infty}^{\infty} G(f - m f_S)$$

$G(f)$:







$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s) \; ; \qquad FT(g_\delta(t)) = FT \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, FT\left[\delta(t-nT_s)\right]$$

- This is DTFT of $\{g(nT_s)\}$ $\leftarrow$ exp1 :- $\underline{G_\delta(f)} = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n f T_s}$ #
- It may be viewed as a complex FS represen^n of the periodic freq. func^n $G_\delta(f)$ with seq. of samples $\{g(nT_s)\}$ def. the coeff. of expansion.

exp2 :- $G_\delta(f) = \underline{f_s \sum_{m=-\infty}^{\infty} G(f-mf_s)}$

process of uniform sampling a CT signal of finite energy results in a periodic spectrum with period = sampling rate

For periodic signal $x(t)$ with period T,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \underbrace{e^{jk\frac{2\pi}{T}t}}_{\text{or}} \rightarrow e^{jk\omega_0 t} \rightarrow \frac{2\pi}{T}$$

verify.

$$G_\delta(f+\underbrace{f_s}_{1/T_s}) = G_\delta(f)$$

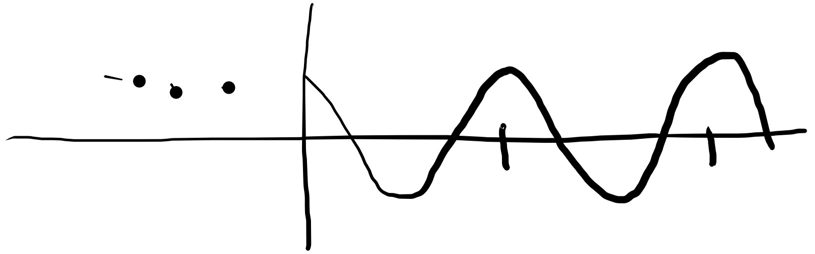
$$e^{-j2\pi n(f+f_s)T_s} = e^{-j2\pi n f T_s} \underbrace{e^{-j2\pi n}}_{1}$$

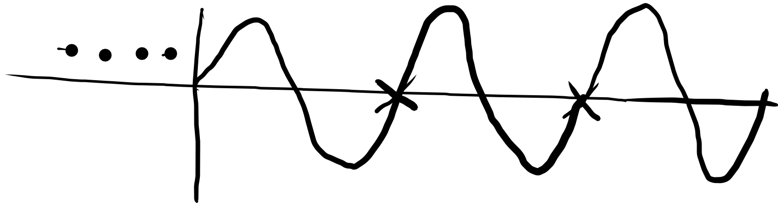
$$\cos(2\pi n) - j\sin 2\pi n$$

$n \in \mathbb{Z}$

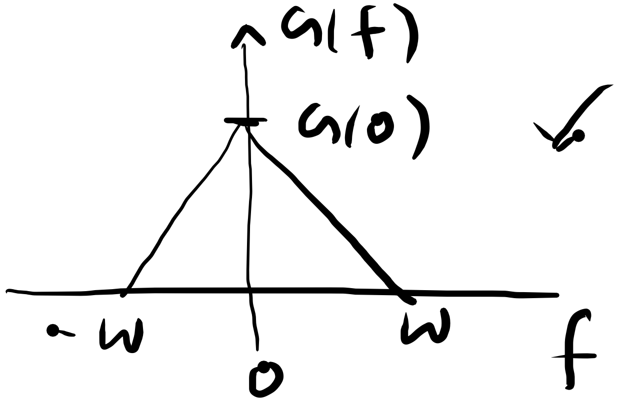
$\cos(2\pi n) \to 1$, $\sin 2\pi n \to 0$

$$G_\delta(f+f_s) = G_\delta(f) \quad \forall f$$

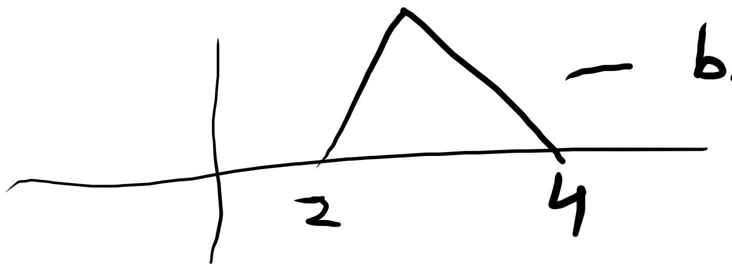




→ we assumed that $g(t)$ is of finite energy & infinite duration. Now, suppose it is also band-limited (strictly) to W Hz



✓ a sample



— bandpass

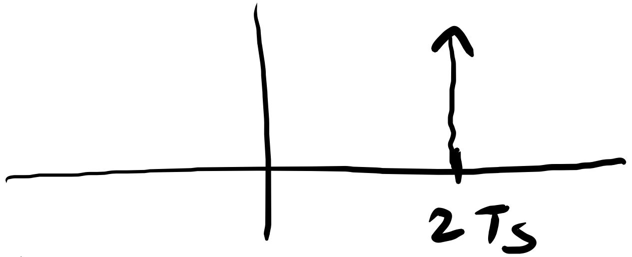
for such a signal, choose the sampling rate as 2W samples/sec or $T_s = 1/2W$, then

① $$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\frac{\pi n f}{W}} \quad \text{— (5a)}$$

also, ② $$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f-mf_s) \quad \text{— (5)}$$

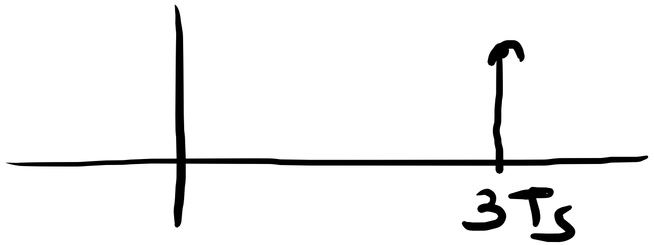
Lec - 6 - DC

$\delta(t-nT_s)$ for $n=2$

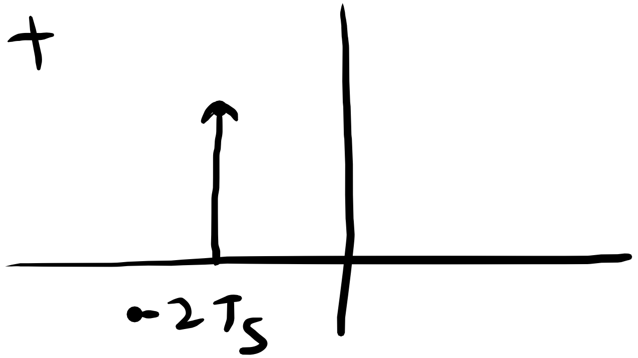


$n=3$

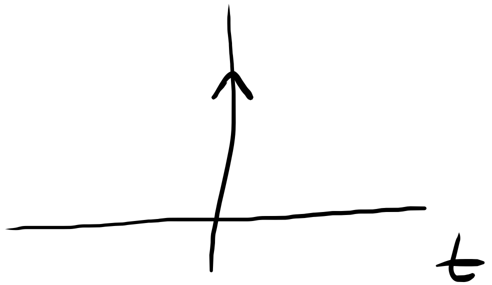
$+$



$+$ $n=-2$



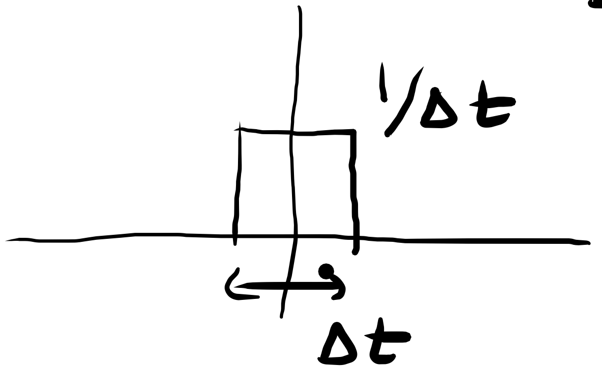
$\delta(t) =$



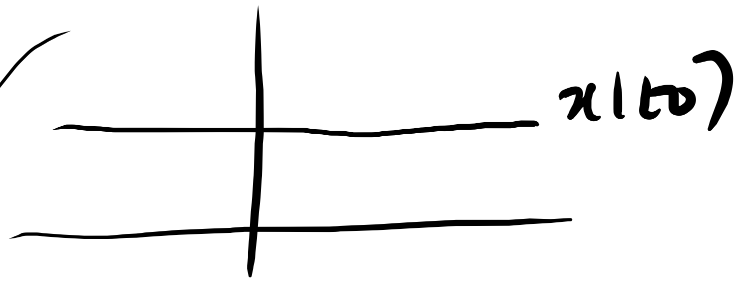
$\delta(t) = 0 \,, \; t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$\Delta t \to 0$



$x(t)\delta(t-t_0) = x(t_0)$



$\delta(t-t_0) = 0 \,, \; t \neq t_0$

$x(t_0)\,\delta(t-t_0)$

$x(t) * \delta(t-t_0) \overset{?}{=} x(t-t_0)$

$$G(f) * \sum_m \delta(f-mf_s) = \sum_m G(f) * \delta(f-mf_s) = x(t_0)$$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$

$$g_\delta(2T_s) \overset{?}{=} g(2T_s)$$

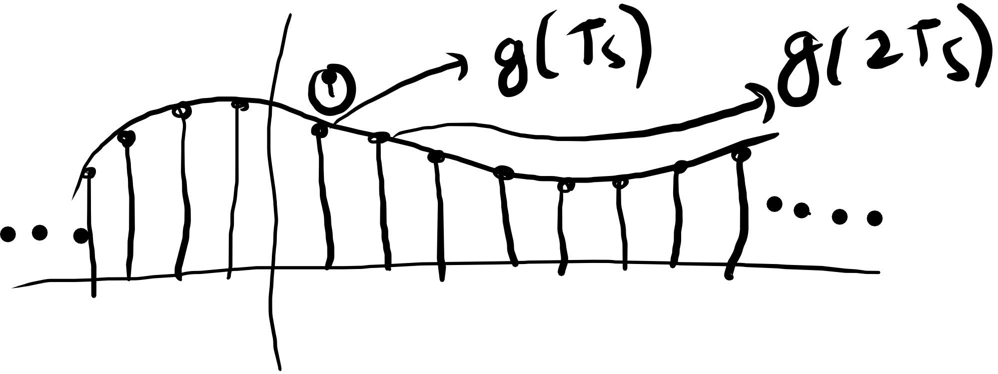
$$g_\delta(2T_s) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(2T_s-nT_s)$$

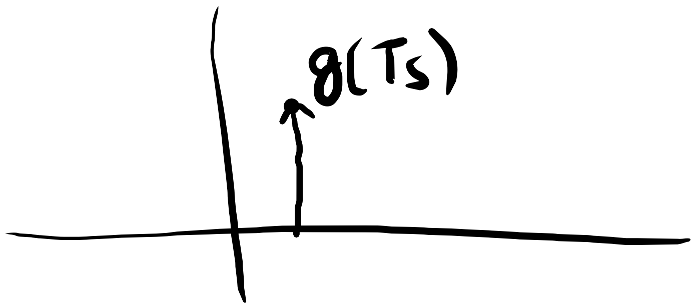
$$g(2T_s)\,\delta(\underbrace{2T_s-2T_s}_{0})$$

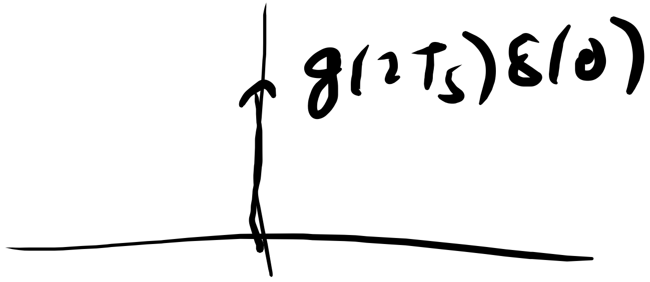
$$\cdots\; g(-T_s)\,\delta(2T_s+T_s) + g(0)\,\delta(2T_s-0) + g(T_s)\,\delta(2T_s-T_s) + \cdots$$

$$= \cdots\; \underset{0}{g(-T_s)\,\delta(3T_s)} + \underset{0}{g(0)\,\delta(2T_s)} + \underset{0}{g(T_s)\,\delta(T_s)} + \cdots$$

$$g(2T_s)\,\delta(0)$$







Lec-7, DC

FS, $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$; $\omega_0 = \frac{2\pi}{T}$ → time period of the periodic sig. $x(t)$

$-\infty < t < \infty$

Ques, will it work with a negative exponent in the exponential func^n? — class assign./HW

For both exp. of $G_\delta(f)$, find $G_\delta(f+f_s)$ $\forall f$

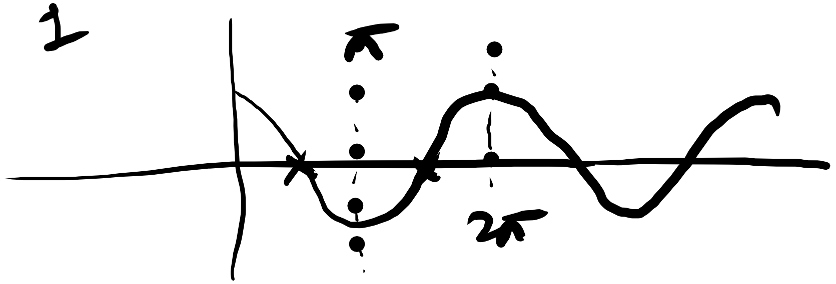
$e^{j2\pi n}$ / $e^{-j2\pi n}$

$\downarrow$ $\downarrow$

1 1

$\cos 2\pi n + j\sin 2\pi n$ → 0

1



$$G_\delta(f+f_s) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n(f+f_s)T_s}$$

$\parallel$ ✓

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n f T_s} \; e^{-j2\pi n}$$

$\because f_s = 1/T_s$

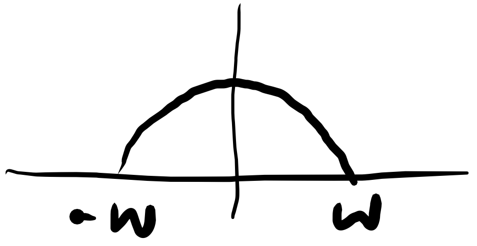
for II expression,

$$G_\delta(f+f_s) = f_s \sum_{m=-\infty}^{\infty} G(f+f_s-mf_s)$$

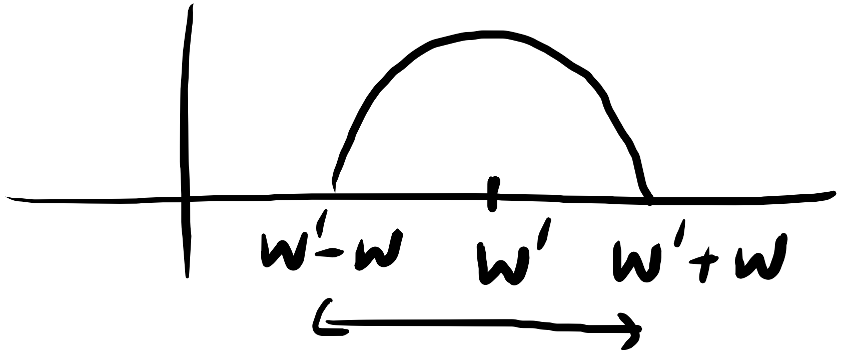
$$= f_s \sum_m G(f-(m-1)f_s)$$

$$f_s \sum_m G(f - m f_s) = f_s \sum_m G(f-(m-1)f_s) \quad -(?) \checkmark$$

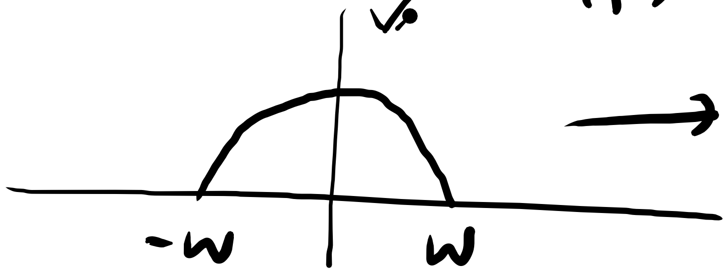
signal is BL to W Hz $\Rightarrow$ around $f = 0$, spectrum is non-zero in $[-W, W]$



translate to higher freq $\longrightarrow$



$G(f)$

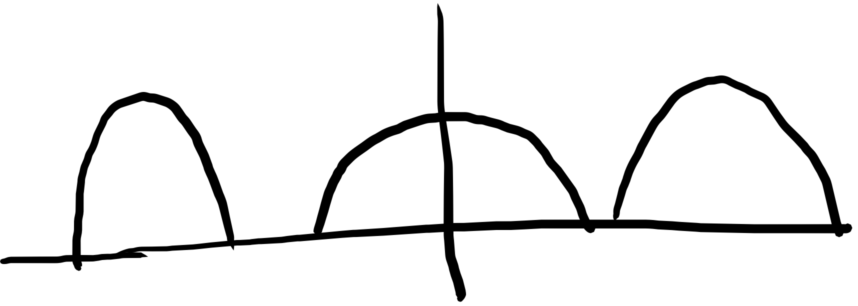


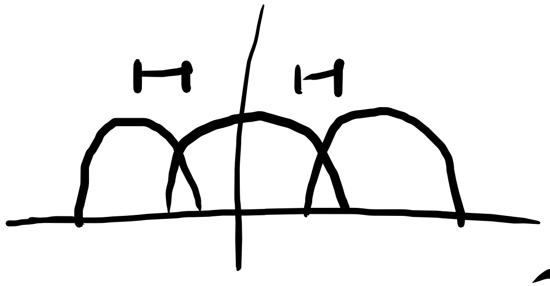
$\longrightarrow$ $G(f+f_s)$

$G(f-f_s)$

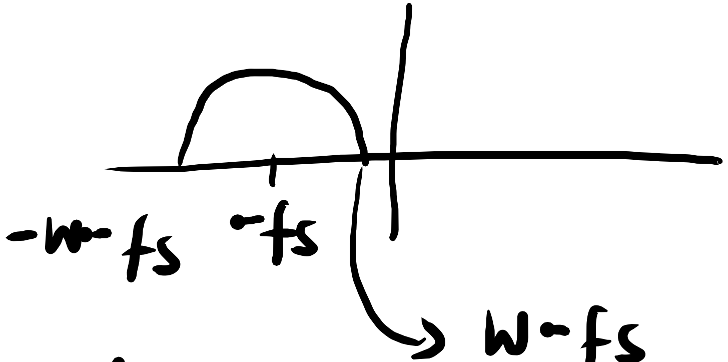
$G(f) + G(f-f_s)$

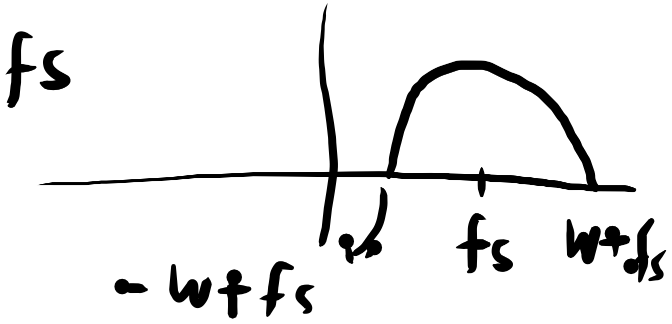
$+ G(f+f_s)$



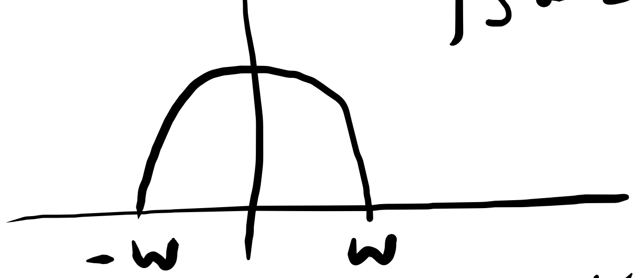


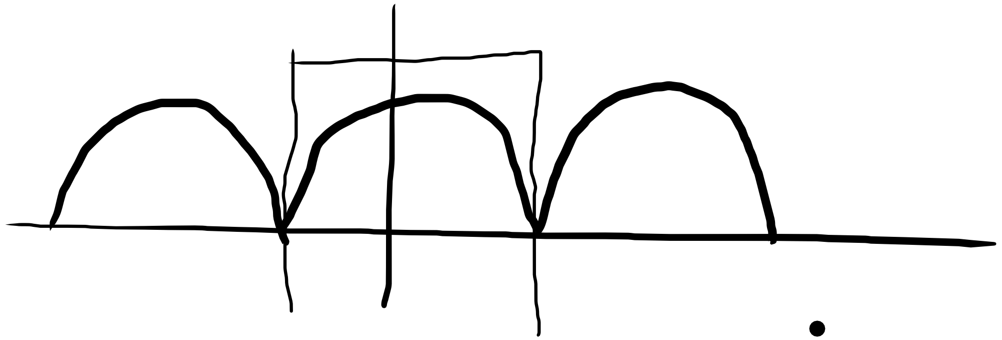
$f_s = 2W$



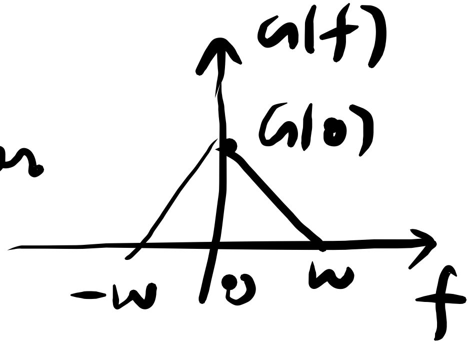


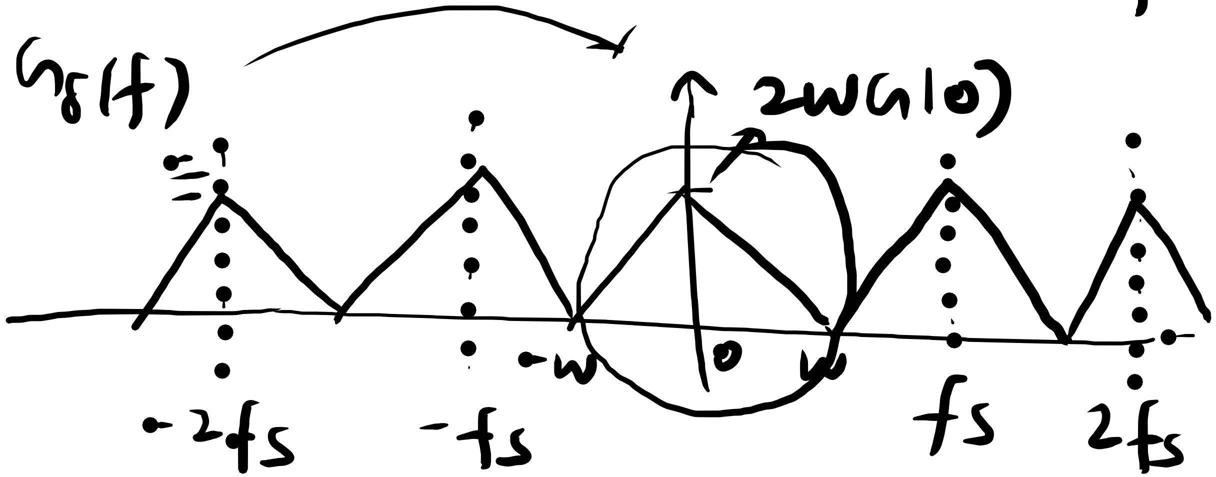
$f_s = 2W$





From lec 5 onwards, for





Hence, under 2 conditions, (1) $G(f)=0$, $|f|>W$ & (2) $f_s=2W$ from eq (5) in lec 5,

$$G(f) = \frac{1}{2W} G_\delta(f), \quad -W<f<W$$

Put (5a) from lec 5 in the above equation

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f-mf_s)$$

exists for all f

$$G(f) = \begin{cases} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\frac{\pi n f}{W}} & -W < f < W \\ 0 \quad , & \text{elsewhere} \end{cases} \quad \text{—(?)}$$

(6) →

for linked G(f) with the sample values

Therefore, if the sample values $g(n/2W)$ of a signal $g(t)$ are specified for 'all n', then FT $G(f)$ of the signal is 'uniquely' determined by using the DTFT of (6)

$g(t)$ & $G(f)$ are related through Inverse FT, so $g(t)$ is itself uniquely det. by the sample values $g(n/2W)$ for $-\infty < n < \infty$.

$\Rightarrow$ seq. $g(n/2W)$ has all the information contained in $g(t)$.

$$g(n/2W)\ \forall n \longrightarrow G(f) \longrightarrow g(t)$$

Recovery / Reconstruction:- $\quad g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$

use the exp. of $G(f)$ in terms of $\{g(n/2W)\}$

$$g(t) = \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\pi n f/W}\, e^{j2\pi f t}\, df$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} e^{j2\pi f\left(t-\frac{n}{2W}\right)}\, df \qquad = \left.\frac{e^{j2\pi f(t-n/2W)}}{j2\pi(t-n/2W)}\right|_{-W}^{W}$$

$$\frac{e^{j2\pi(Wt-n/2)} - e^{-j2\pi(Wt-n/2)}}{j2\pi(t-n/2W)} \qquad = \frac{\cancel{2}j \sin 2\pi(Wt-n/2)}{\cancel{2}j\,\pi(t-n/2W)}$$

$$= \frac{\sin \pi(2Wt-n) . 2W}{\pi(2Wt-n)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n), \quad -\infty < t < \infty$$

this was interpolation formula for reconstructing the original signal $g(t)$ from 'seq of sample values' $\{g(n/2W)\}$ with sinc func^n sinc$(2Wt)$ as the interpolating function

Lecture - 9, DC

delay sinc → multiply → add

Sampling theorem:- For strictly BL (no component higher than W Hz) signal of finite energy,

(1) Such a fun^n/signal is completely described by values separated in time by $1/2W$ seconds.

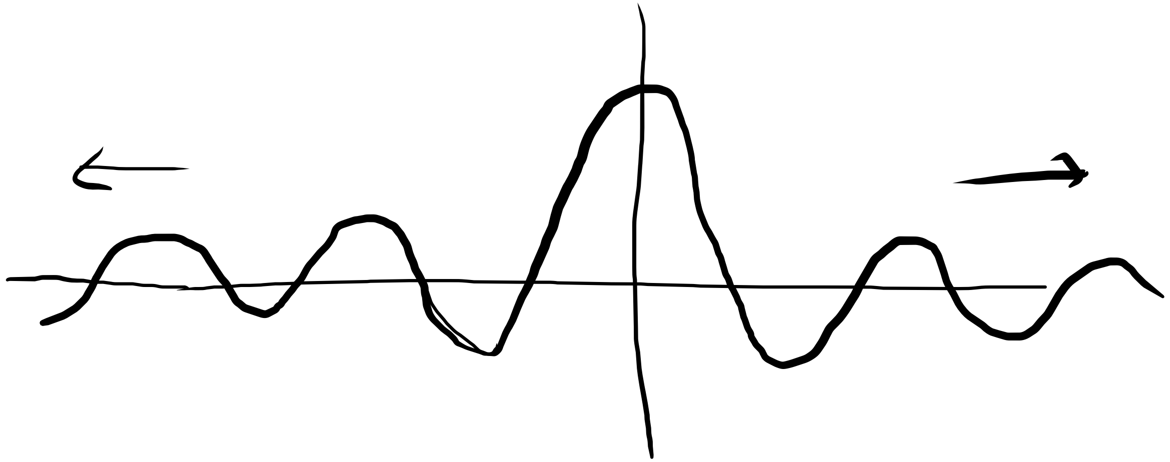
or (2) (equivalent to (1)) It can be recovered from knowledge of its samples taken at the rate of $2W$ samples/sec.

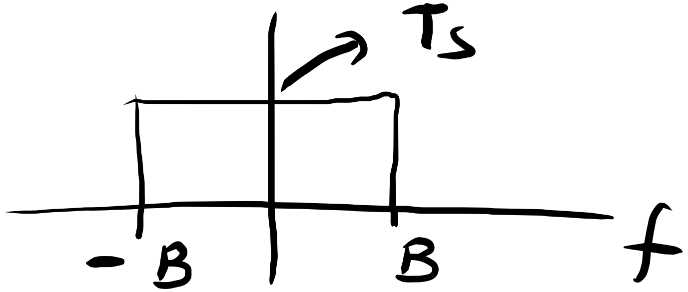
Nyquist rate : $2W$ samples/sec ; Nyquist interval : $1/2W$ sec

→ for BL (W Hz) signal

Another way of seeing the interpolation formula.

Because of a term $f_s G(f)$ in the spectrum of sampled sig we can recover $g(t)$ by sending it through an ideal LP filter of BW B Hz & gain T_s





IFT

$$H(f) = T_s \, \Pi\left(f/_{2B}\right)$$

$$\rightarrow h(t) = 2BT_s \, \mathrm{sinc}(2Bt)$$

due to Nyquist sampling rate $2BT_s = 1$, $\Rightarrow$

$$h(t) = \mathrm{sinc}(2Bt)$$

Observe that $h(t) = 0$ at all Nyquist sampling instants $(t = \pm n/2B)$ except $t = 0$

$0 \le n < \infty$ is integer

$\left(\because h(t) = \text{sinc}(2Bt)\right)$

$= \dfrac{\sin 2\pi Bt}{2\pi Bt}$

$h(t)\Big|_{t = \pm n/2B}$

$= \pm \sin \dfrac{\cancel{2}\pi \cdot \cancel{B} \cdot n}{\cancel{2B}}$

$= \pm \sin \pi n = 0$

→ $h(t)$ is the impulse response of the ideal filter

→ Sampled signal (SS) $\longrightarrow \boxed{h(t)} \longrightarrow$ signal

↳ $SS * h(t)$

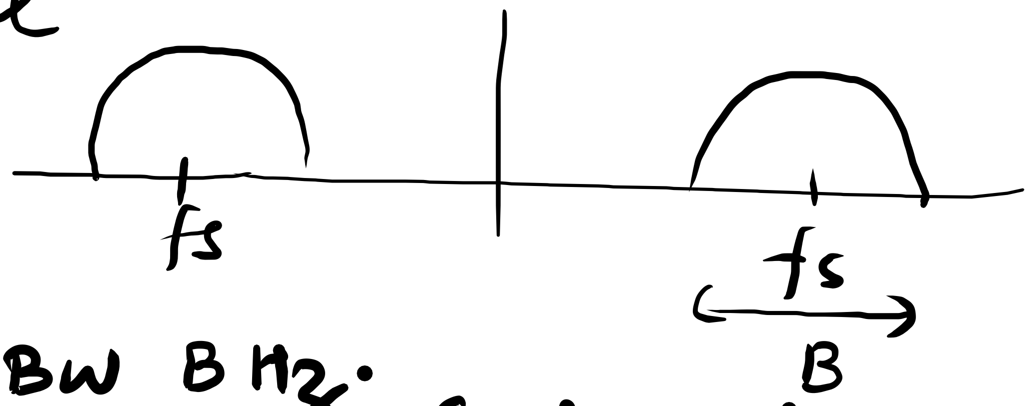
→ Each sample in sampled $g(t)$ being an impulse generates a sinc pulse of height = strength of the sample.
Addition of the sinc pulses generated by all the samples results in $g(t)$

$$\underbrace{\sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)}_{g_\delta(t)} * \text{sinc}(2Bt) = \sum_{n=-\infty}^{\infty} g(nT_s)\left[\delta(t-nT_s) * \text{sinc}(2Bt)\right]$$

$$= \sum_{n} g(nT_s)\,\text{sinc}(2B(t-nT_s))$$

$$= \sum_{n} g(nT_s)\,\text{sinc}(2Bt-n) \quad \because 2BT_s = 1$$

Possibility of bandpass signal



A BP signal whose spectrum exists over a freq. band $f_c - B/2 < |f| < f_c + B/2$ has a BW B Hz. Such a signal is also uniquely determined by samples taken at above the Nyquist freq. 2B.

verify.

→ Bring it to baseband in Inphase & Quadrature phase components, sample at 2B as BL to B Hz, regenerate BP signal
(very loose way of understanding this)

→ Even directly follows from
"A discussion of sampling theorem" D. A. Linden, IRE, 1959

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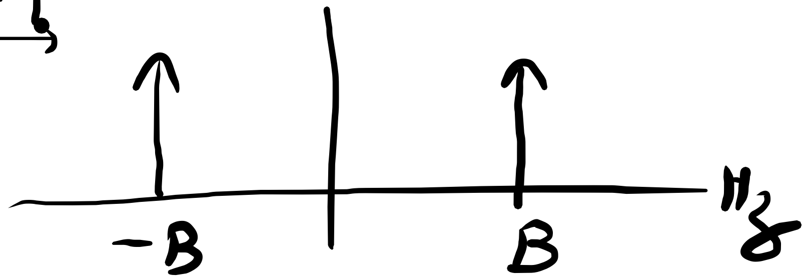
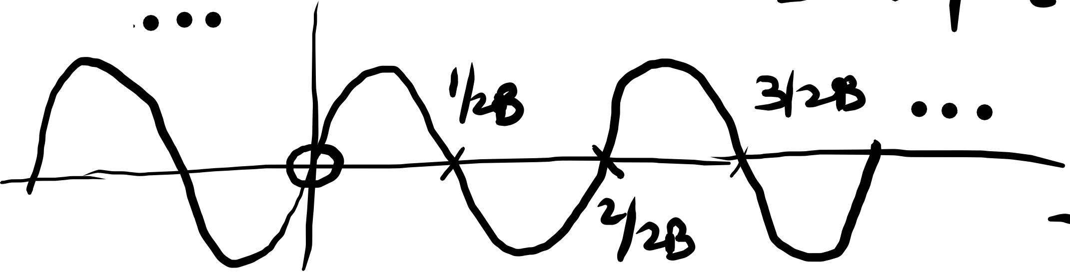
possibility of $f_s = 2B$

$\sin(2\pi B t)$ $\xrightarrow{FT}$

B Hz

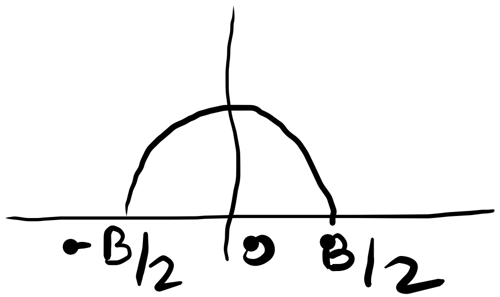


Start at $t = 0$, collect every sample with interval $1/2B$

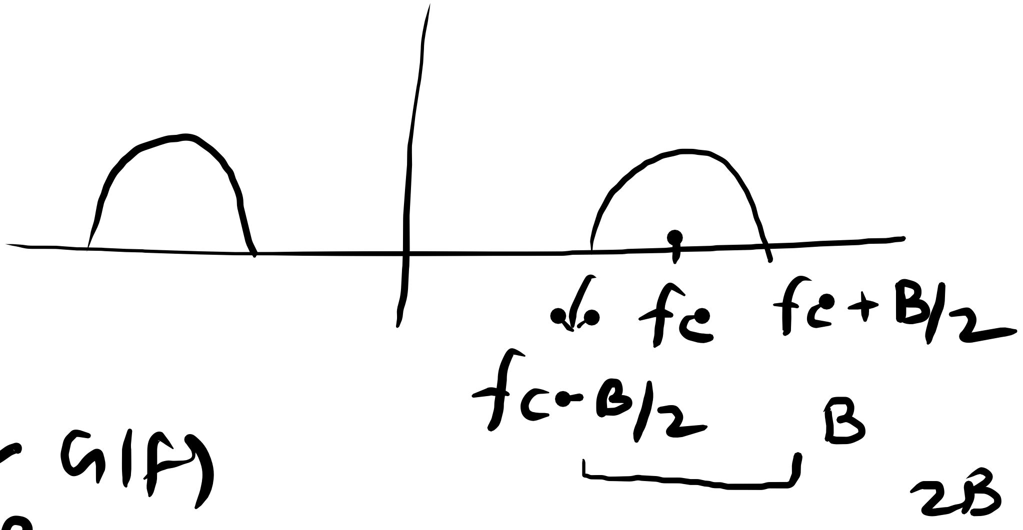


→ leads to seq. of zeros.

Lec-10, DC



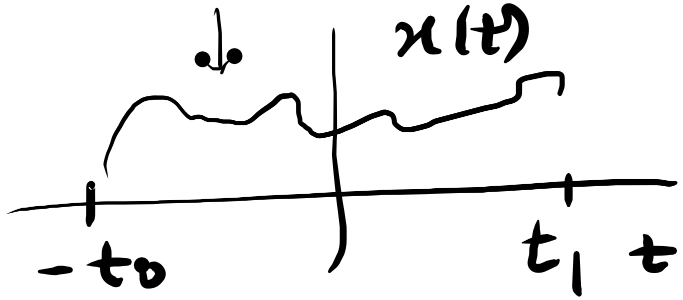
$f_s = 2 \cdot B/2 = B$

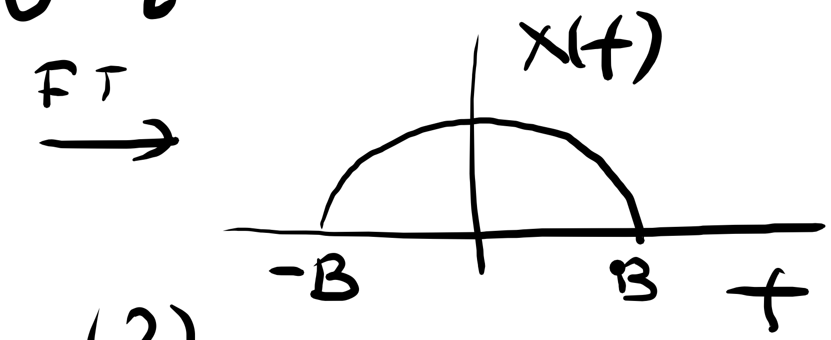


Possibility of $f_s = 2B$:- If the spectrum $G(f)$ has no impulse at the highest freq. B, then the overlap is still zero as long as the sampling rate $\geq$ Nyquist rate

→ On the other hand, if $G(f)$ contains an impulse at the highest freq ($\pm B$), then the equality must be removed or else overlap will occur & the signal cannot be recovered from its Nyquist samples.

explore :- A signal both time-limited & freq. limited.



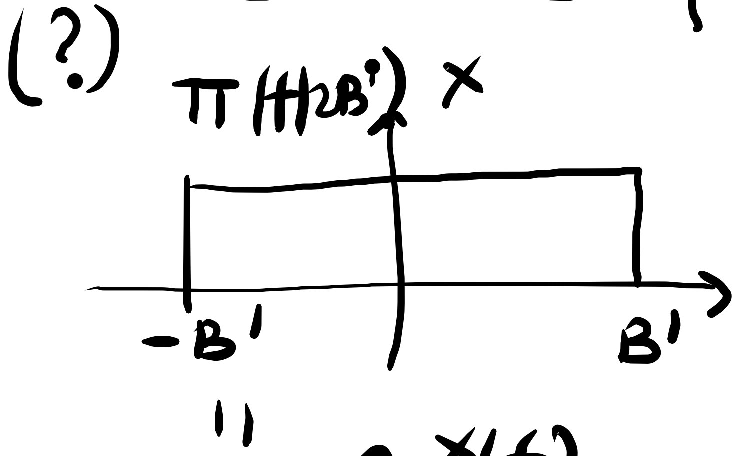


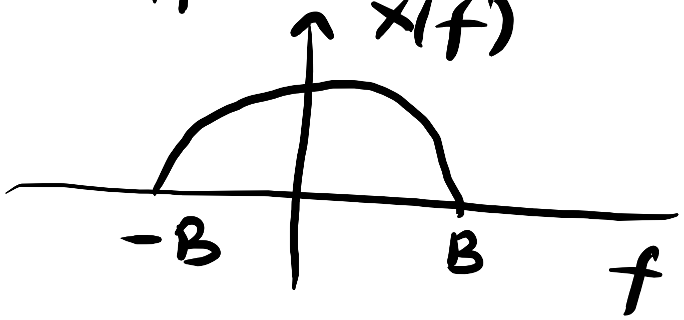
$X(f) = 0$ for $|f| > B$

In this case I can write

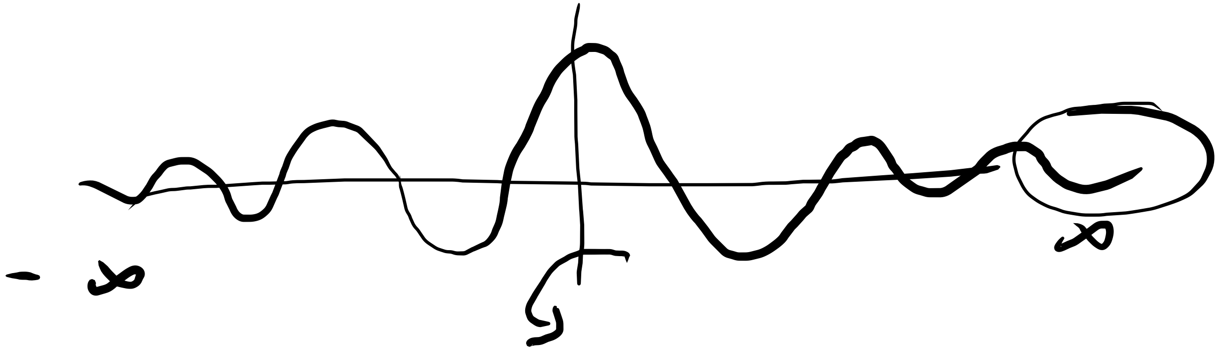
$$X(f) = X(f) \times \Pi(f/2B') \text{ for } B' > B$$

$$x(t) = x(t) * 2B' \,\text{sinc}(2\pi B' t)$$





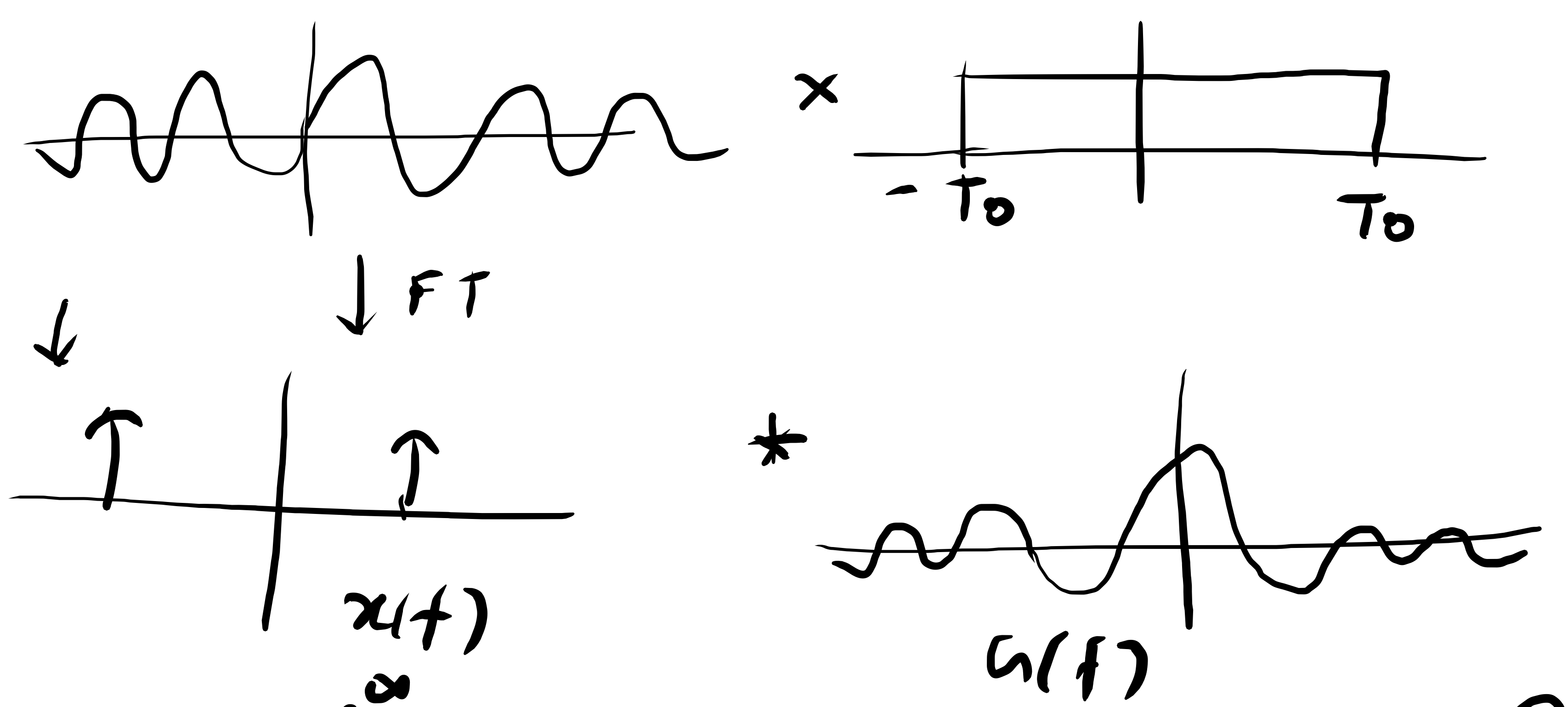
prob 6.1-8 from Lathi's TB.

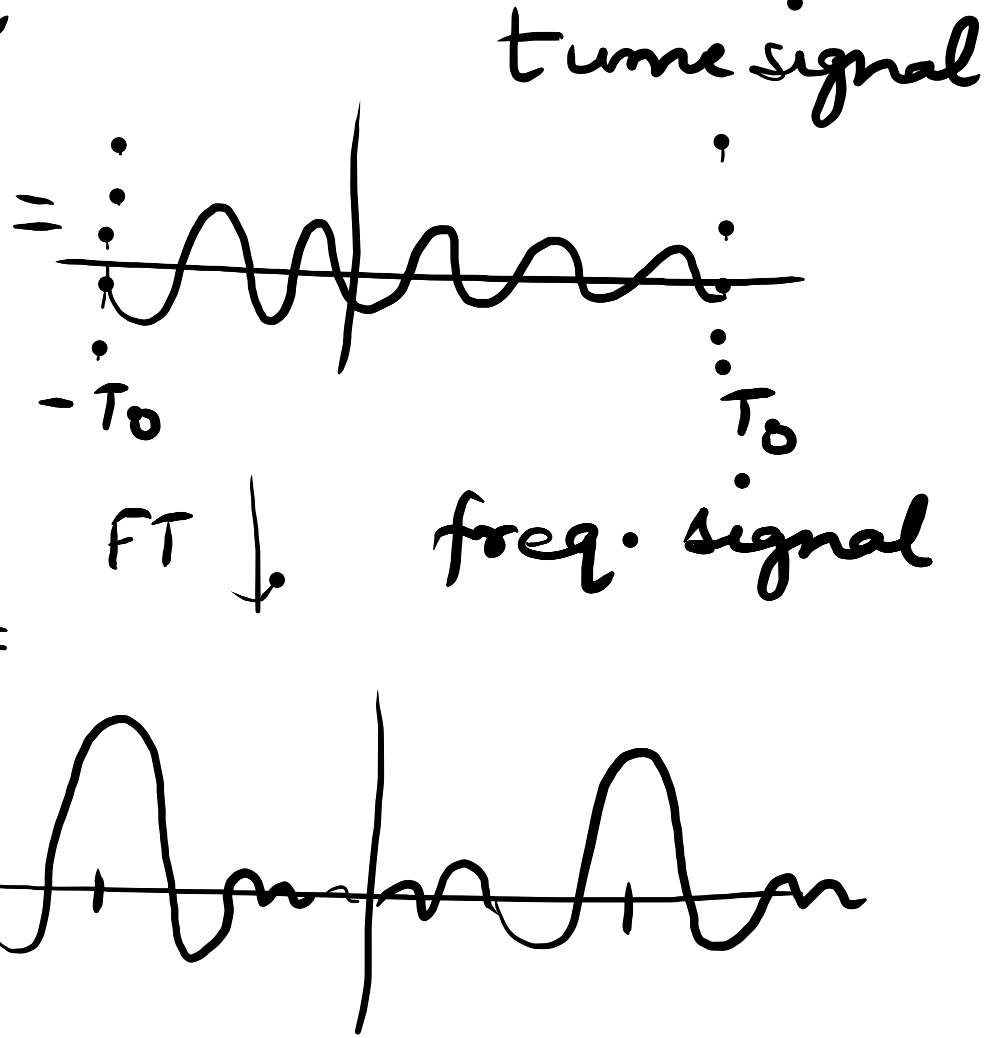


It is possible that a signal can be both time unlimited & freq. "

freq. limited $\longrightarrow$ unlimited in time

time limited $\longrightarrow$ " in freq.





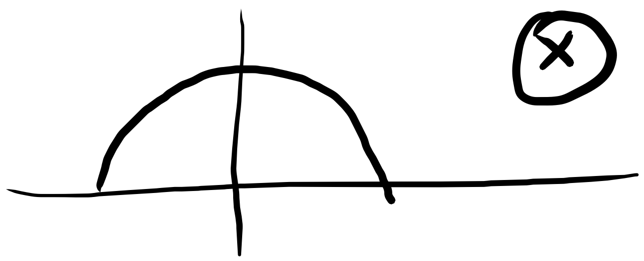
$$Y(f) = \int_{-\infty}^{\infty} x(\tau)\, G(f-\tau)\, d\tau$$

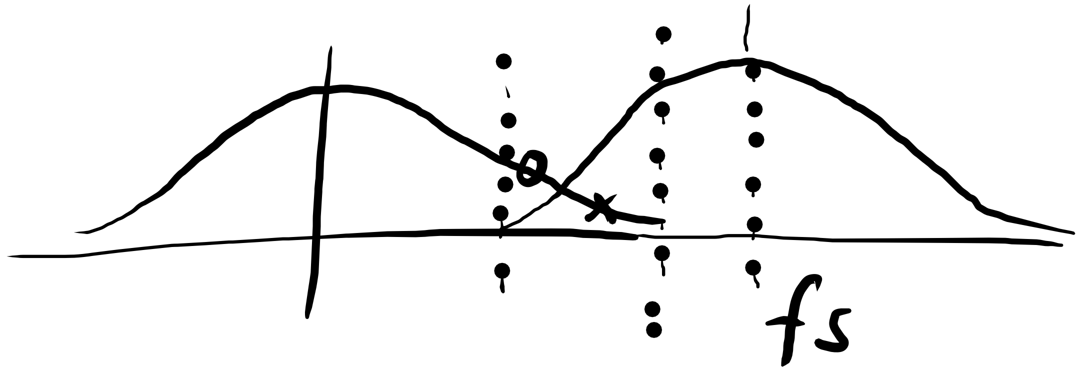
$$\mathrm{sinc}(f-f') + \mathrm{sinc}(f+f') = \left[\delta(f-f') + \delta(f+f')\right] * \mathrm{sinc}(f)$$

Sampling for time-limited signals → unlimited in freq.

→ "whatever sampling rate" you choose, the spectrum of sampled signal consists of overlapping cycles of G(f) repeating every fs Hz.

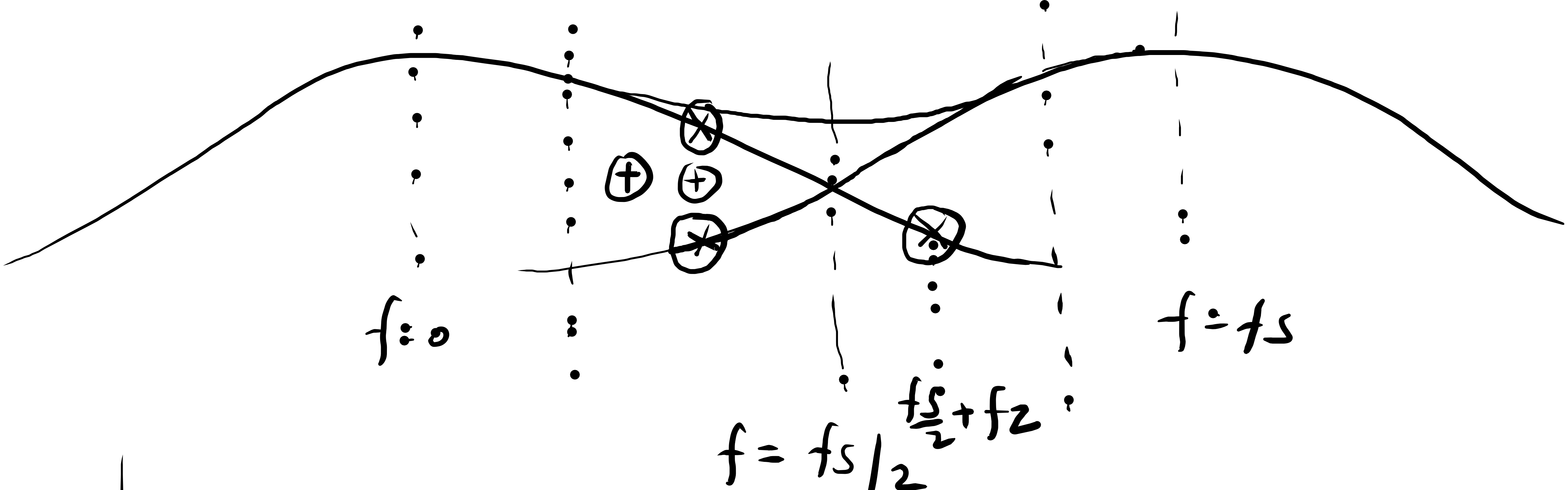
→ Sampled signal / its spectrum no longer has complete information of spectrum / signal (original).



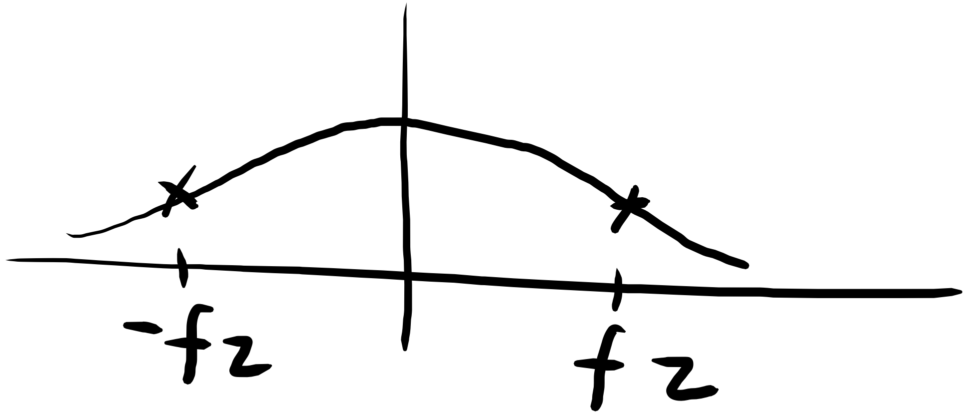


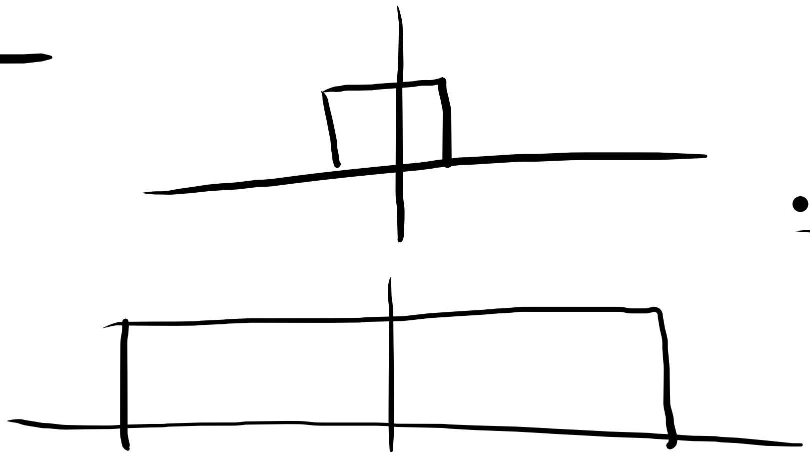
alias (?)

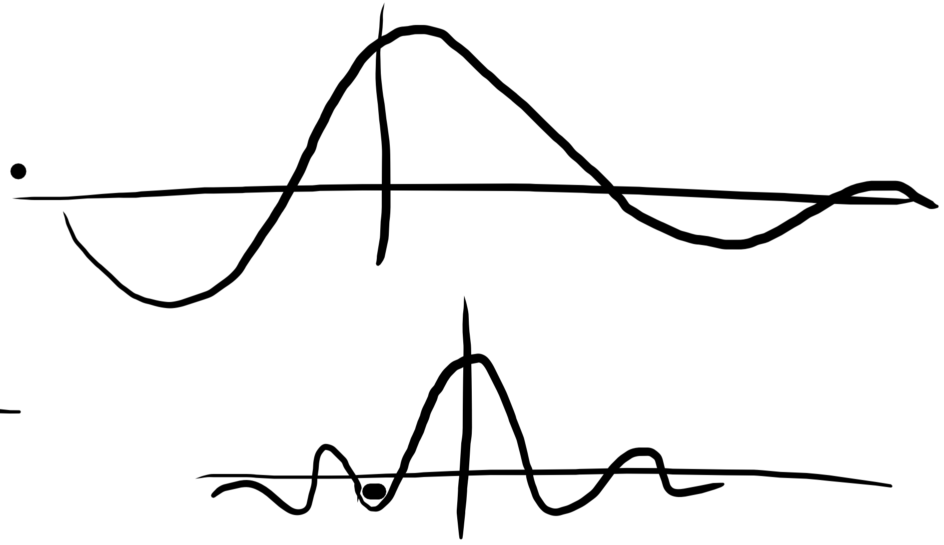
R N



folding freq.







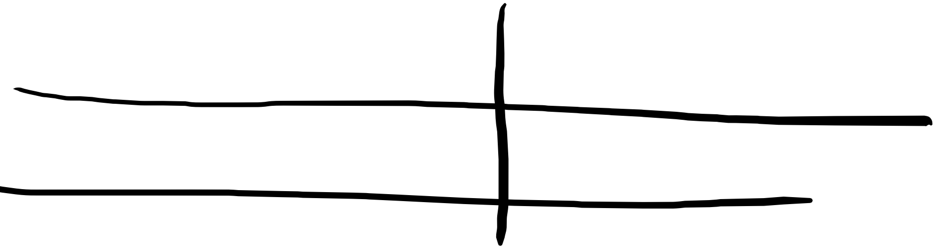
folding freq :- $f_s/2 = \frac{1}{2T_s}$ Hz

↳ spectrum may be viewed as if the lost tail is folding back into itself at the folding freq.

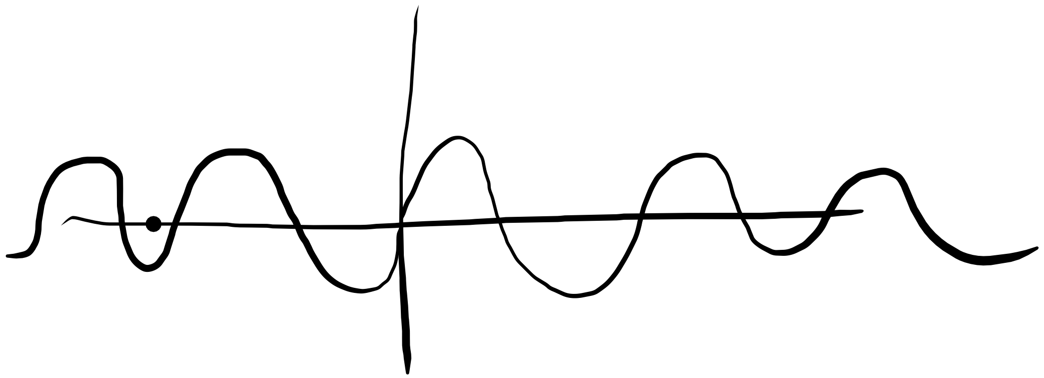
Component of freq. $\frac{f_s}{2} + f_z$ shows up as, or impersonates a component of low. freq. $\frac{f_s}{2} - f_z$ in the reconstructed signal.

→ Solution:- the antialiasing filter → to eliminate the component above $f_s/2$ (folding freq.) from $g(t)$ before sampling.

→ one more benefit:- $y(t) = x(t) + n(t)$

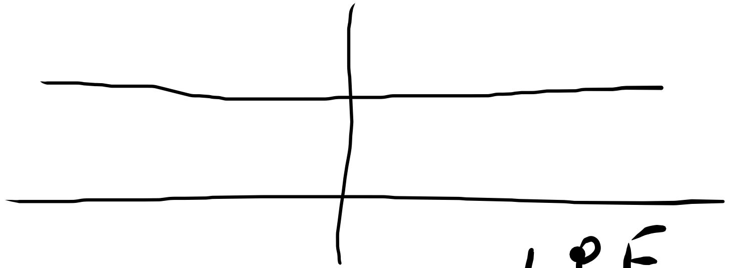


Since noise is wideband, the aliasing phenomenon itself cause the noise components outside the desired signal band to appear in the signal band

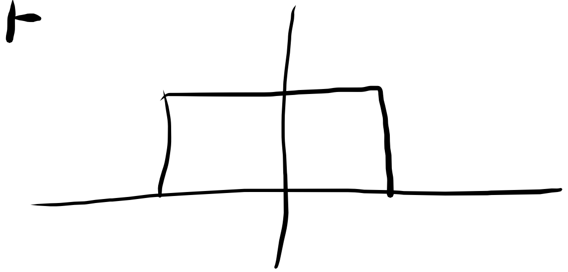


take a snapshot from $-T_0$ to T_0

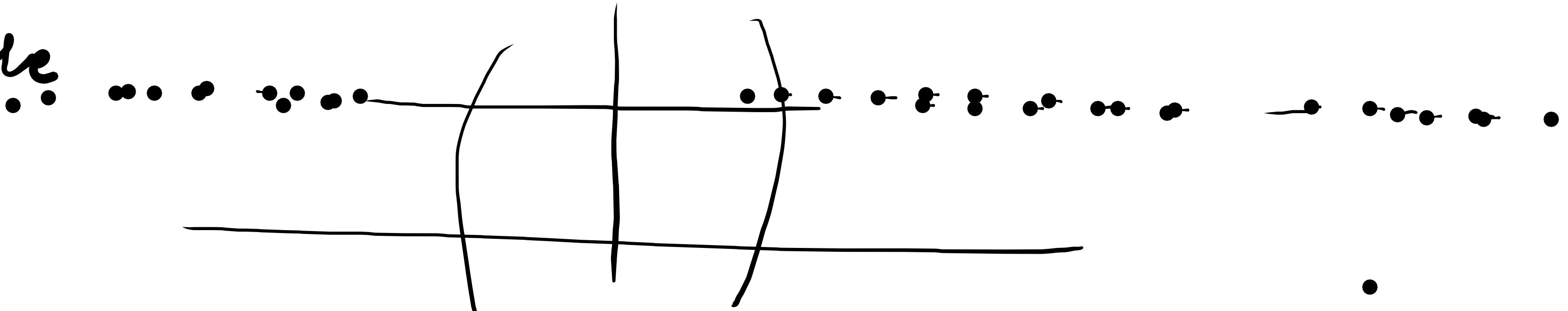
Lec - 11, DC



LPF



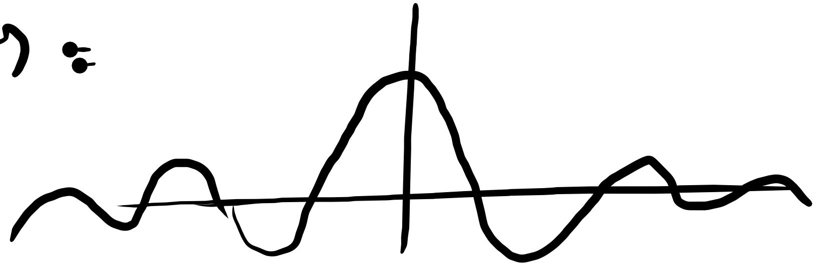
sample



From the last lecture, using an anti-aliasing filter avoids the noise outside signal band to affect sampled signal.

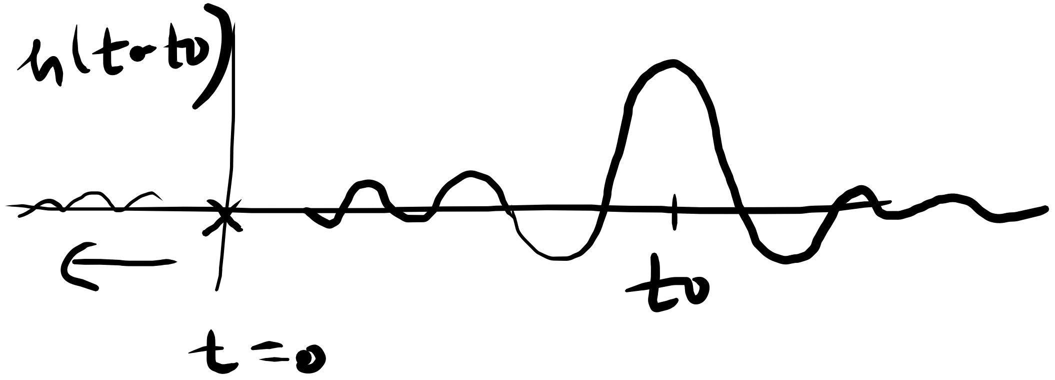
One more practical issue with sampling & recovery:- Ideal filter is unrealizable in practice.

h(t) :

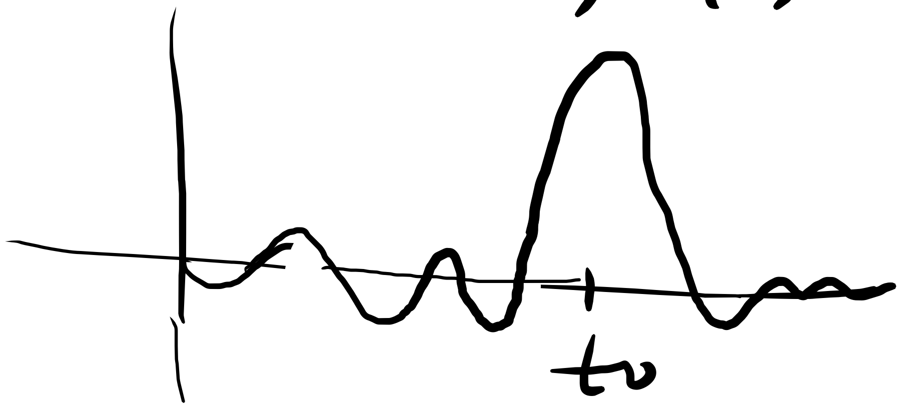


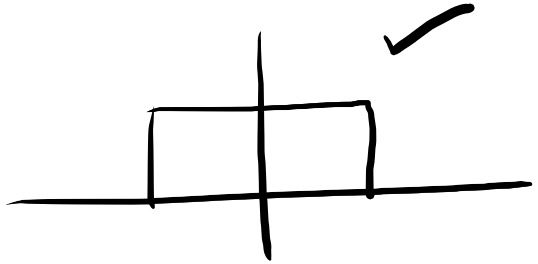
$$\delta(t) \longrightarrow \boxed{} \longrightarrow h(t)$$

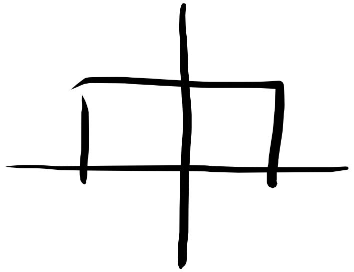
why not delay $h(t)$ sufficiently



$\times \quad u(t) \quad =$







$h(t) \longrightarrow H(\omega)$

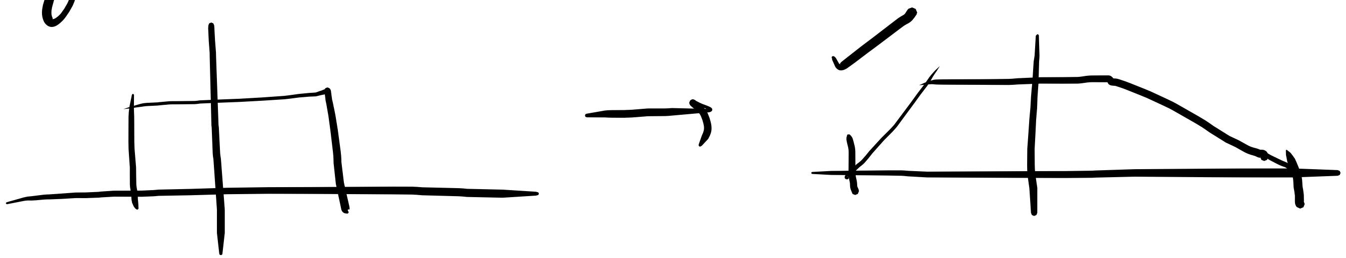
$h(t-t_0) \longrightarrow \left| e^{-j2\pi f t_0} H(f) \right|$

$h(t)u(t) \longrightarrow H(\omega) * FT\{u(t)\}$ ↑ you can substitute & see

another issue:- $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$x(t) \longrightarrow \boxed{h(t-t_0)} \longrightarrow y'(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-t_0-\tau)\, d\tau = y(t-t_0)\,(?)$

why not use those filters which die down slowly



but the issue is:-

Paley-wiener criteria :- for a physically realizable system, $H(f)$ may be zero at some discrete freq. but it cannot be zero over any finite band.

freq. domain filter

side-lobes

With all these issues:- At a higher sampling rate (obviously > Nyquist rate) the recovered signal approaches the desired signal more closely.

Self-study:- 3.3.4 for linear phase from Lathi

3.5 for ideal vs practical filters.

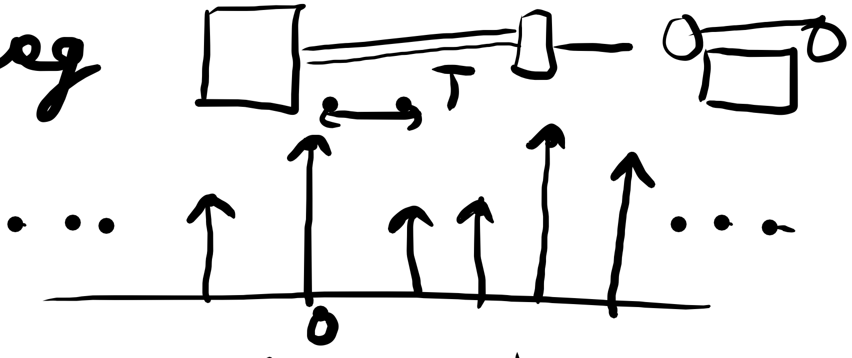
Pulse amplitude mod. or natural and instantaneous sampling.

Sampling theorem provides a way to reproduce an analog WF by using sample values of WF & sinc(x) interpolating func

PAM provides another WF that looks like pulses but contains inf. present in analog WF.

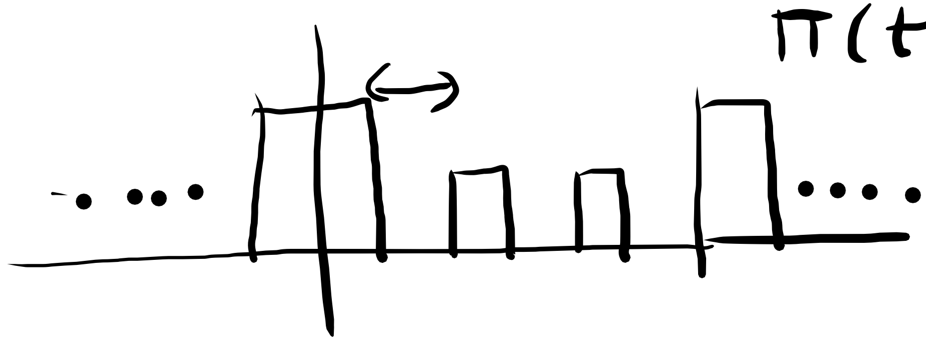
Baseband Tx

Passband Tx

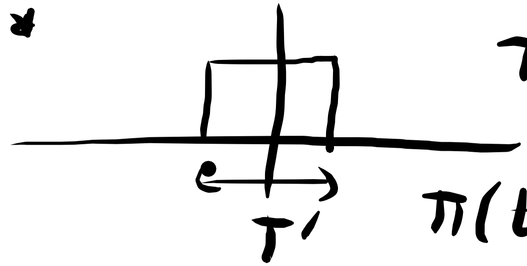


$$\Pi(t) * \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} \Pi(t-nT)$$



$*$



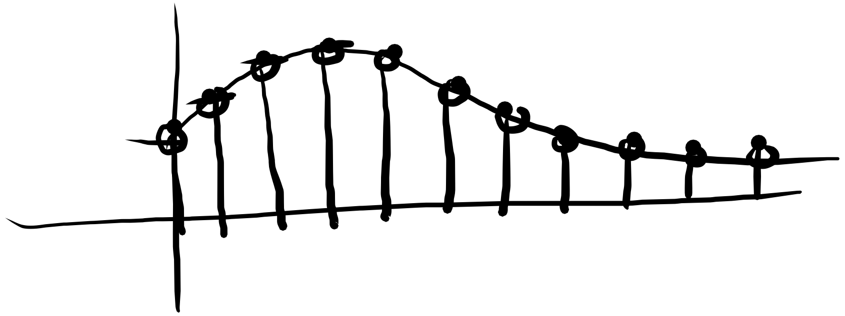
$T' < T$

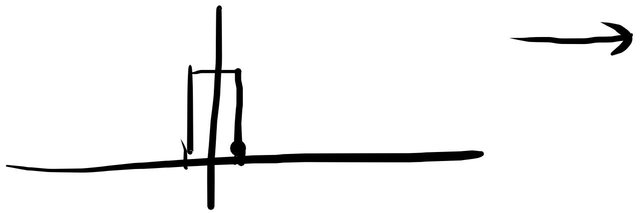
$\Pi(t)$

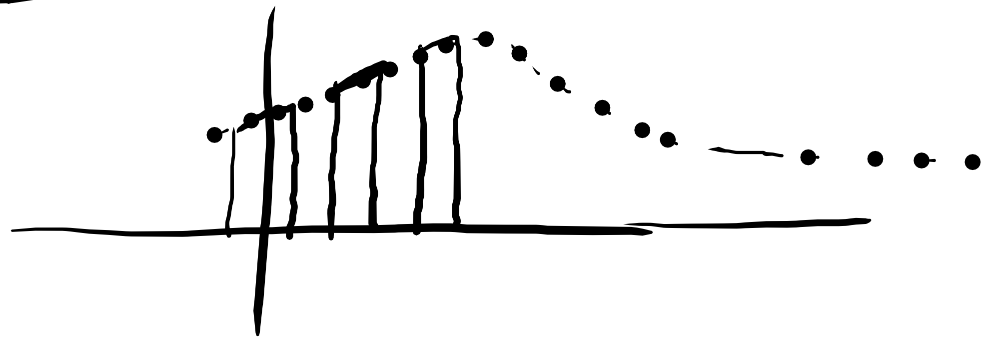
PAM:- conversion of the analog signal to a pulse-type signal in which the amplitude of the pulse denotes the analog information.

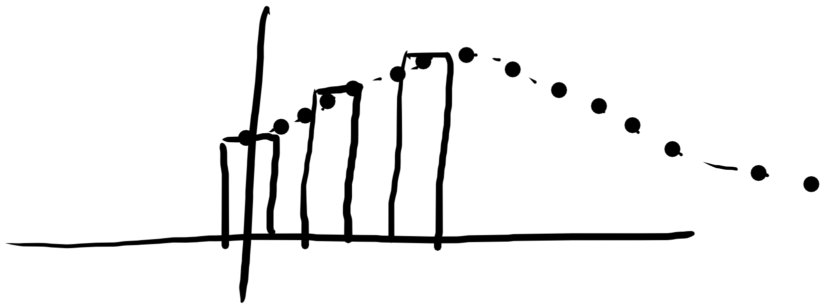
→ Pulses are more practical to use in digital systems.

Two classes
- → natural sampling (gating)
- → Instantaneous sampling to produce flat-top pulse









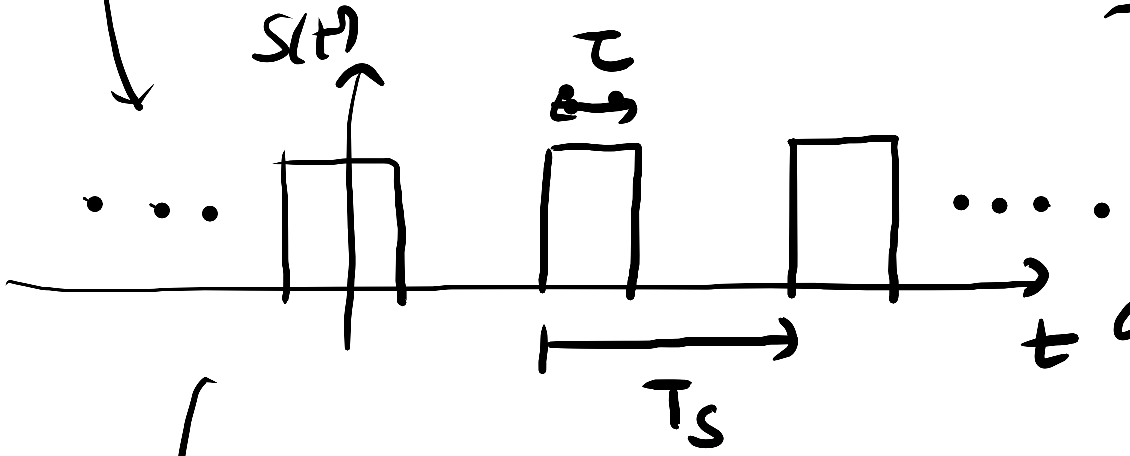
Natural Sampling (Gating) — NS

We consider an analog WF BL to B Hz :- $w(t)$

$$s(t) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_s}{\tau}\right)$$

→ pulse rate

$$f_s = \frac{1}{T_s} \geq 2B$$



$\tau < T_s$

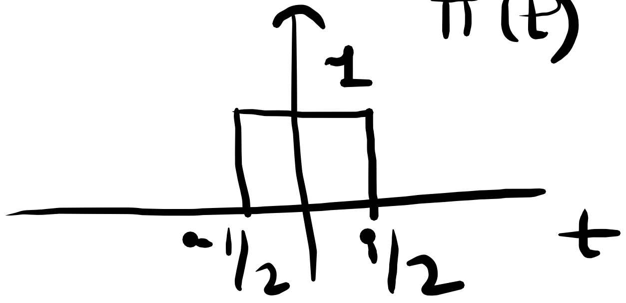
duty cycle : τ/T_s

rectangular wave switching waveform

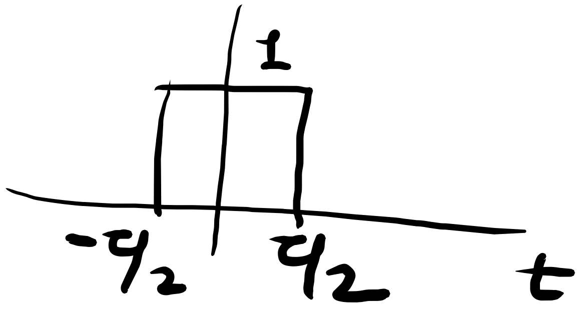
$$w_s(t) = w(t)\, s(t) \Rightarrow W_s(f) = W(f) * S(f)$$

↳ NS

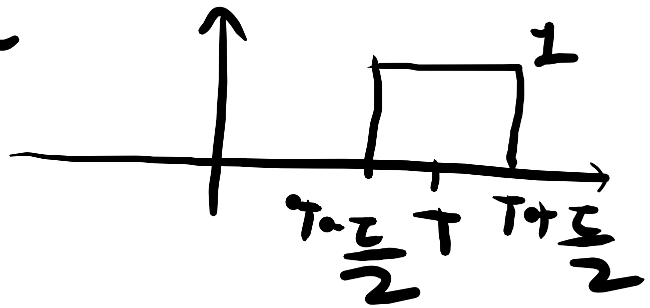
$\Pi(t)$



$\Pi(t/\tau)$



$\Pi\left(\frac{t-T}{\tau}\right)$



s(t) may be represented by the Fourier series

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_s t}$$

where $c_n = d\,\dfrac{\sin n\pi d}{n\pi d}$

$d = \tau / T_s$

Since s(t) is periodic, its spectrum can be written as

$$S(f) = \sum_{n=-\infty}^{\infty} c_n\, \delta(f - nf_s)$$

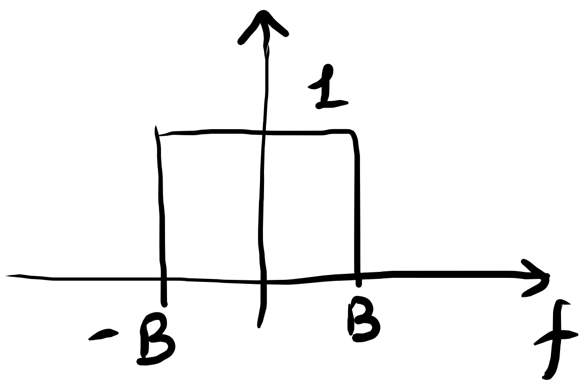
See ex. on Pg 76 from Couch's Book

So, $$W_s(f) = \sum_{n=-\infty}^{\infty} c_n\, W(f) * \delta(f - nf_s)$$

$$= \sum_{n} c_n\, W(f - nf_s).$$

Lecture-12, DC

Suppose



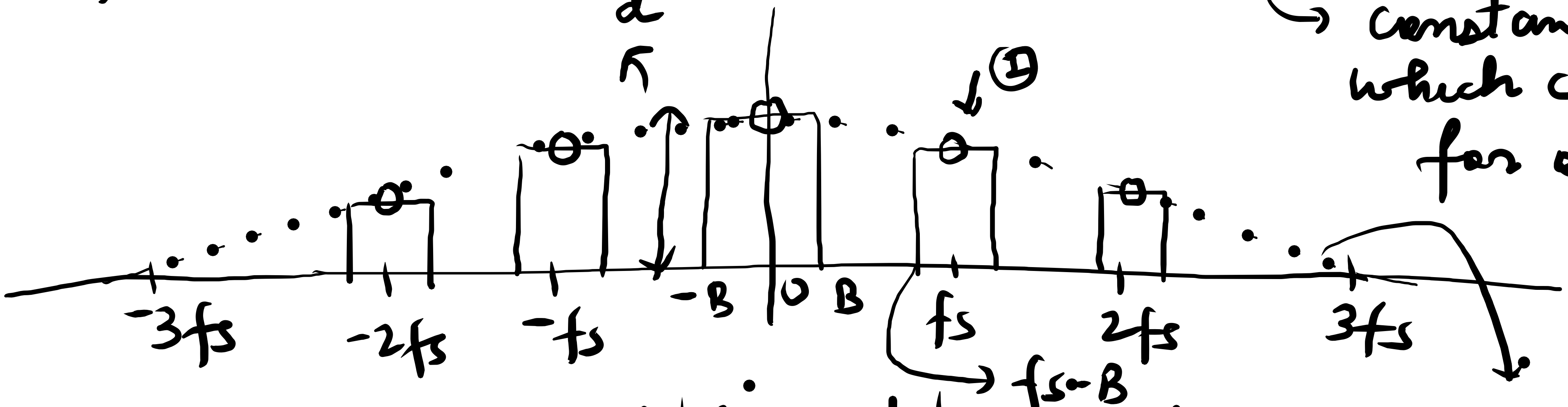
$|W(f)|$ mag. spectrum of LP analog WF

$$|W_s(f)| = \sum_{n=-\infty}^{\infty} \left\{ \underbrace{d \left| \frac{\sin n\pi d}{n\pi d} \right|}_{\text{constant value which changes for each } n} \cdot |W(f-nf_s)| \right\}$$

assuming $d = 1/3$

or $\frac{\tau}{T_s} = \frac{1}{3}$

or $T_s = 3\tau$



dotted envelop is of the sinc function

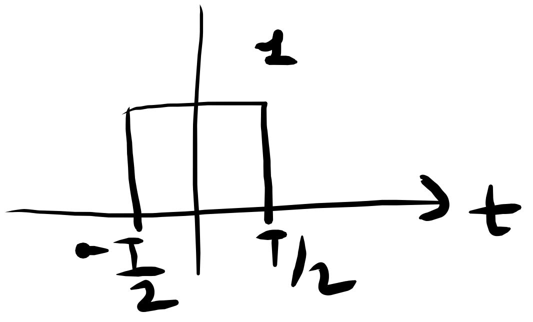
option 1: $2\sqrt{3}$

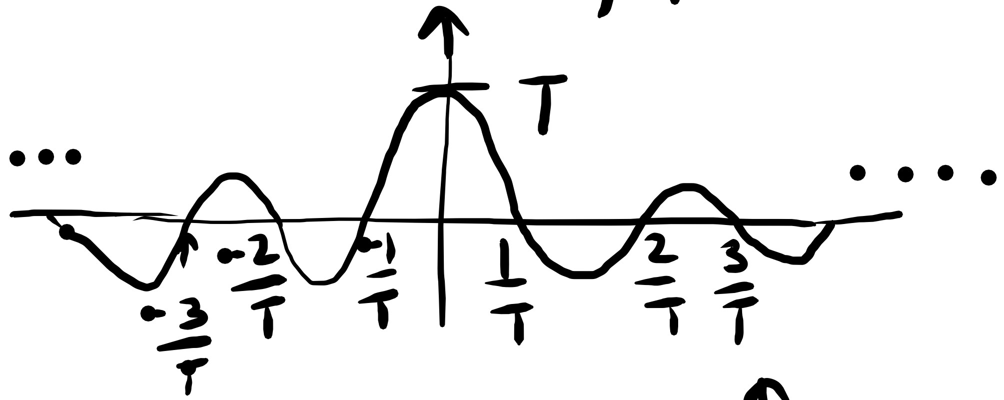
$$d\left|\frac{\sin \pi d}{\pi d}\right| = \frac{1}{3} \cdot \left|\frac{\sin \pi/3}{\pi/3}\right| = \frac{\sqrt{3}}{2\pi}$$

$\dfrac{\sin \frac{n\pi}{3}}{n\pi/3}$ at what values of n, will this be zero?

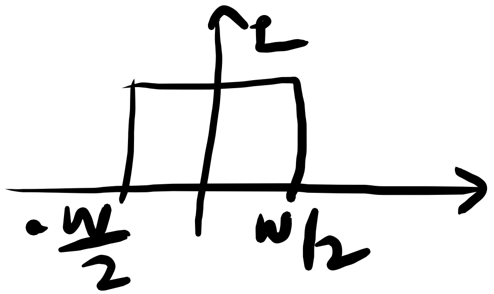
$n = 3k, \; k \in \mathbb{Z}, \; k \neq 0$

$\Pi(t/T) \xrightarrow{FT} T\dfrac{\sin(\pi f T)}{\pi f T}$ or $T\,\mathrm{sinc}(fT)$ if $\mathrm{sinc}(x) = \dfrac{\sin \pi x}{\pi x}$

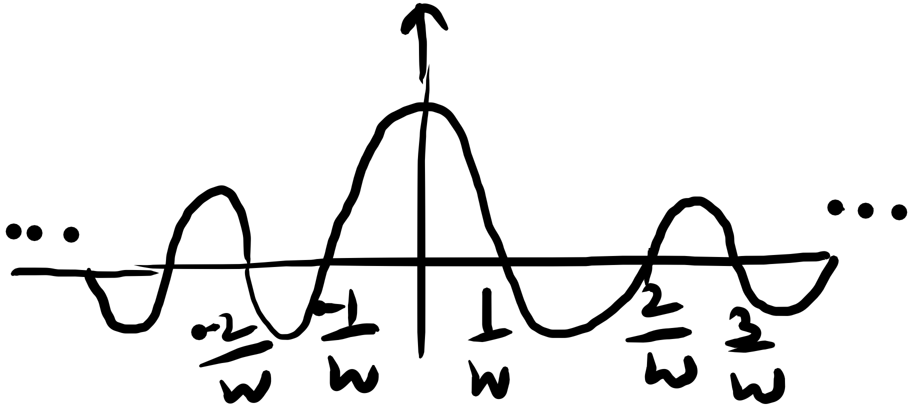




$\Pi(f/W)$



$\xrightarrow{IFT}$



Recovery from a naturally sampled (PAM) $w(t)$

→ $w(t)$ can be recovered from $w_s(t)$ by passing the PAM signal through a LP filter where the cutoff freq. is

$$B < f_{cutoff} < f_s - B$$

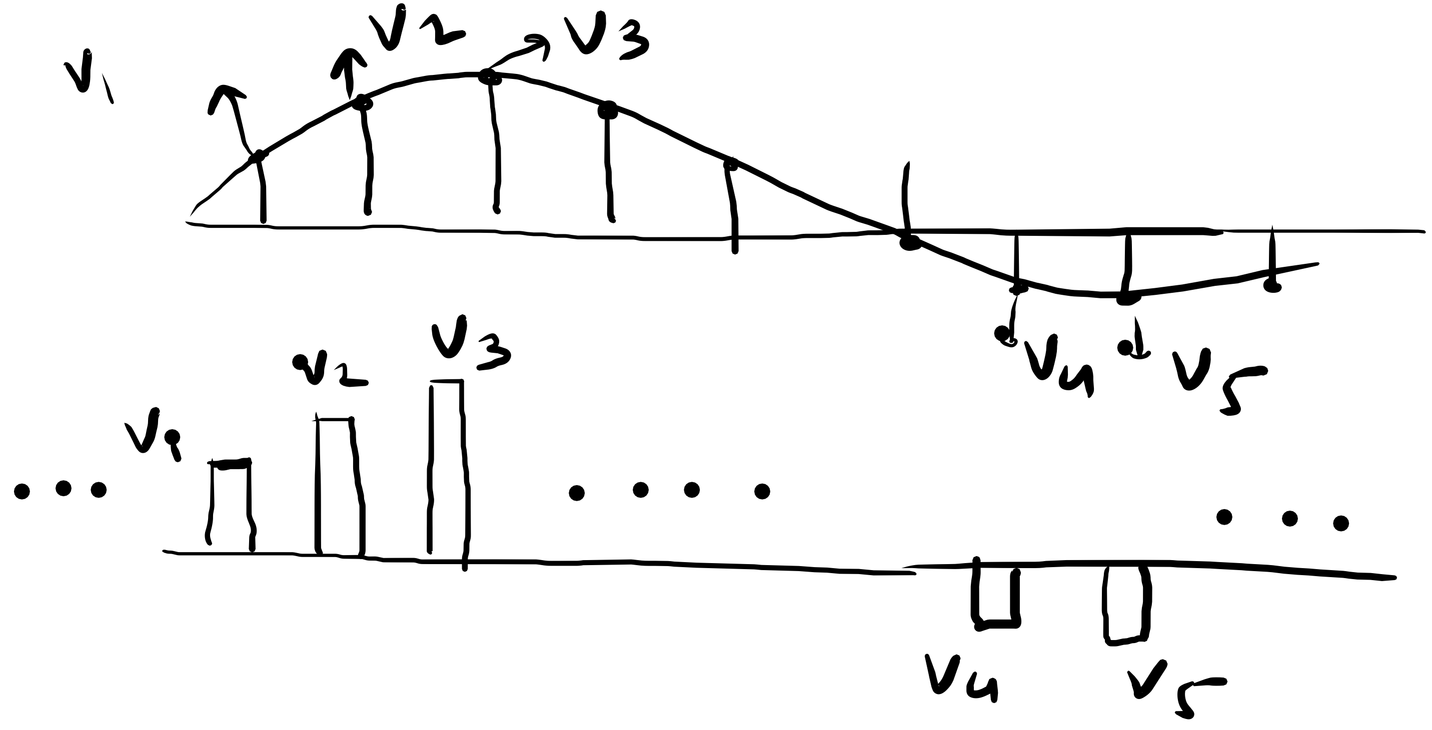
Also, $f_s \geq 2B$ (Nyquist rate) is reqd. to avoid aliasing

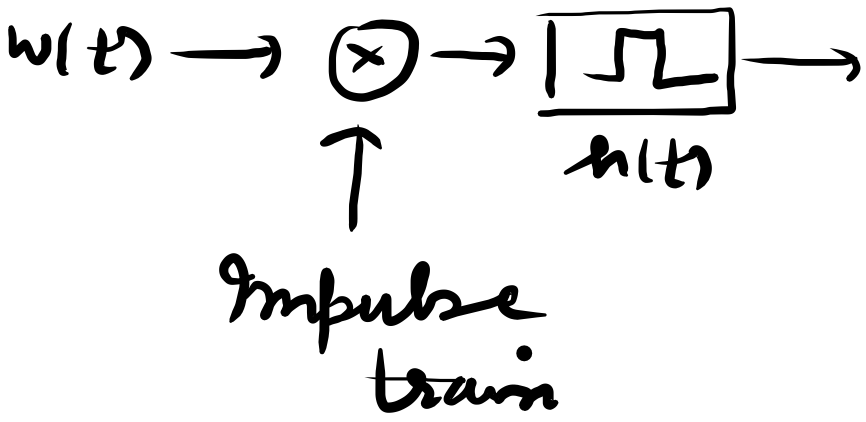
→ Prefiltering $w(t)$ before is reqd. because of TL- BL paradox.

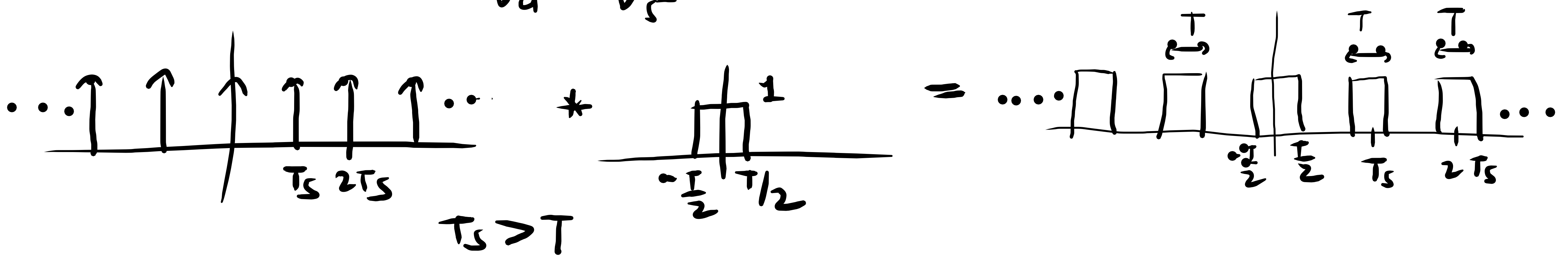
You need to compensate the gain factor of d by using an amplifier.

Instantaneous sampling (flat-top PAM)

→ generalization of the impulse train sampling technique.



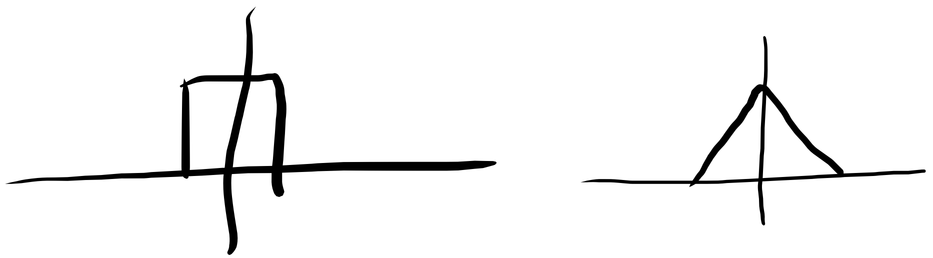




For BL WF $w(t)$, (to B Hz)

$$w_s(t) = \sum_{k=-\infty}^{\infty} w(kT_s)\, h(t - kT_s) \quad -① $$

$h(t)$ = sampling pulse shape



for "flat-top" sampling, it is

$$h(t) = \Pi(t/\tau) = \begin{cases} 1, & |t| < \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

where $\tau \leq T_s = 1/f_s$ & $f_s \geq 2B$

Spectrum of $w_s(t)$ is

$$W_s(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} W(f - kf_s) \; ; \; H(f) = FT[h(t)] = \frac{\tau \sin(\pi \tau f)}{\pi \tau f}$$

Derivation :- from ①

$$w_s(t) = \sum_k w(kT_s)\left[h(t) * \delta(t - kT_s)\right]$$

$$= h(t) * \left[\sum_k w(kT_s)\,\delta(t - kT_s)\right]$$

Hence, $w_s(t) = h(t) * \left[w(t) \times \sum_k \delta(t-kT_s) \right]$

Spectrum

$$W_s(f) = H(f) \left[W(f) * \left(\underbrace{\sum_k e^{-j2\pi f k T_s}}_{} \right) \right]$$

↳ But the sum of the exponential func^ns is equivalent to a FS expansion (in freq. domain) where the periodic func^n is an impulse train

⊛ That is, $\frac{1}{T_s} \sum_k \delta(f-kf_s) = \frac{1}{T_s} \sum_n c_n e^{j2\pi n T_s f}$

$\frac{c_n}{T_s} = 1$

⊛ $c_n = 1/f_s$

$$W_s(f) = H(f) \left[w(t) * \frac{1}{T_s} \sum_k \delta(f-kf_s) \right]$$

$$= \frac{H(f)}{T_s} \sum_k \left[W(f) * \delta(f-kf_s) \right]$$

$$= \frac{H(f)}{T_s} \sum_k W(f-kf_s) \quad \#$$

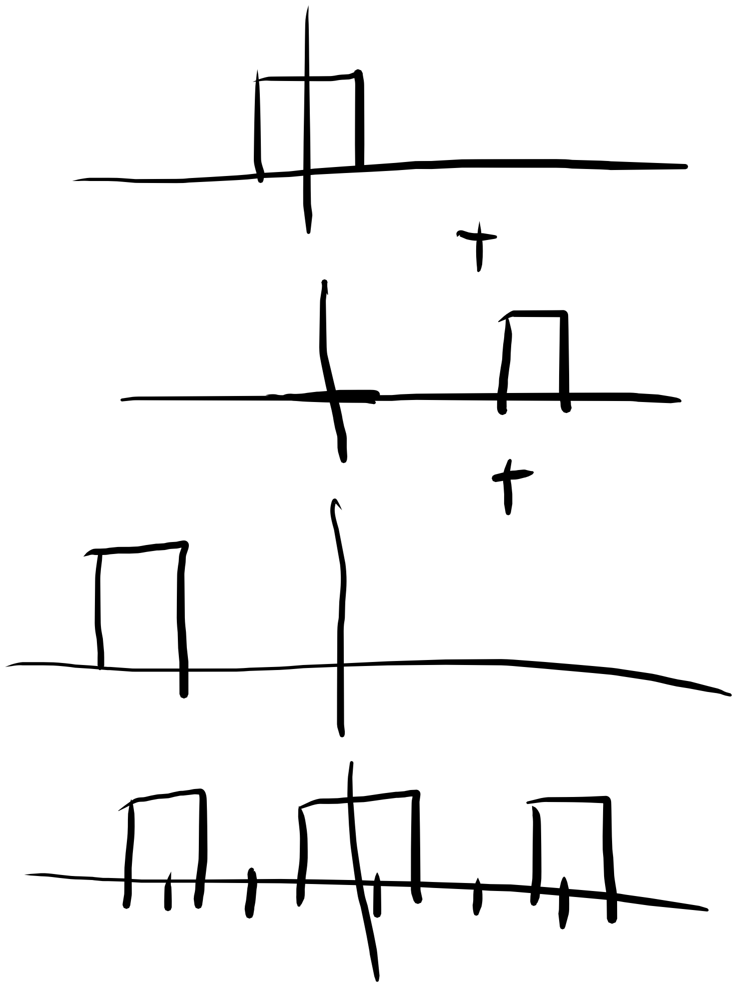
$$|W_s(f)| = \left| \frac{1}{T_s} H(f) \right| \sum_k |W(f-kf_s)|$$

$$W_s(f) = \sum_k T_k(f)$$

$$|W_s(f)| = \sum_k |T_k(f)|$$

$$z = \begin{matrix} a + ib \\ c + id \end{matrix} + = a + c + j(b+d)$$

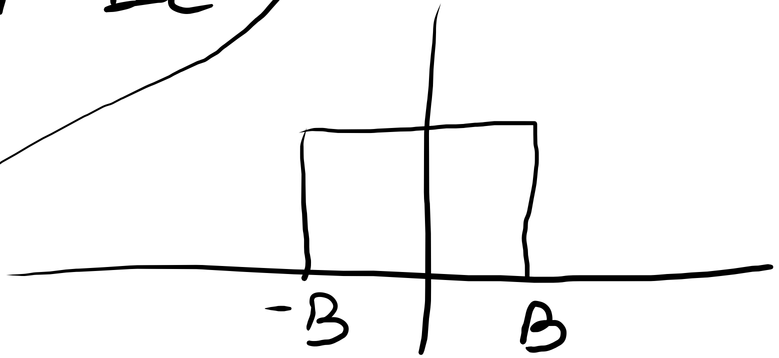
$$|z| = \sqrt{a^2+b^2} + \sqrt{c^2+d^2} \quad (?)$$



Lec-13, DC

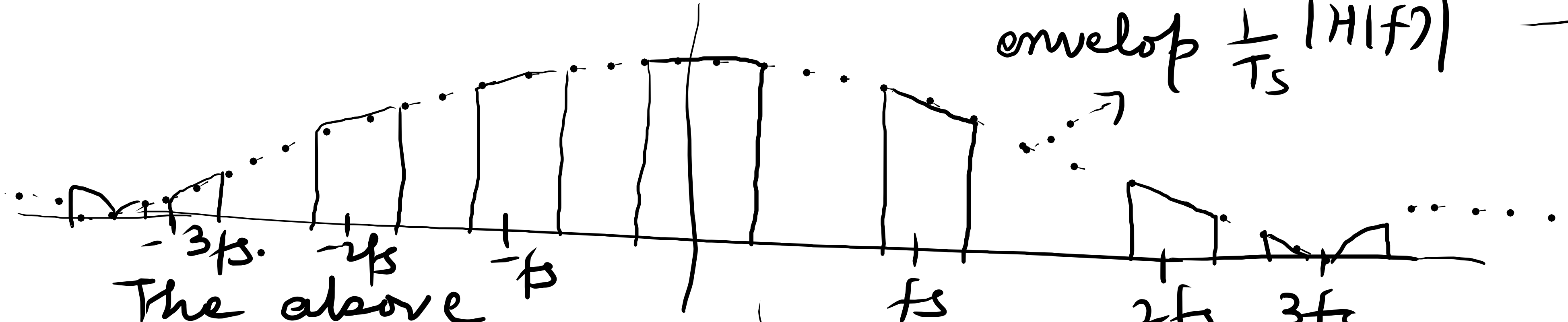
Magnitude Spectrum of input analog WF be

$$\frac{\tau}{T_s}\left|\frac{\sin \pi\tau f}{\pi\tau f}\right|$$



Instantaneous PAM / sampling

envelop $\frac{1}{T_s}|H(f)|$



$$\left|\frac{H(f)}{T}\right|$$

The above Case considers

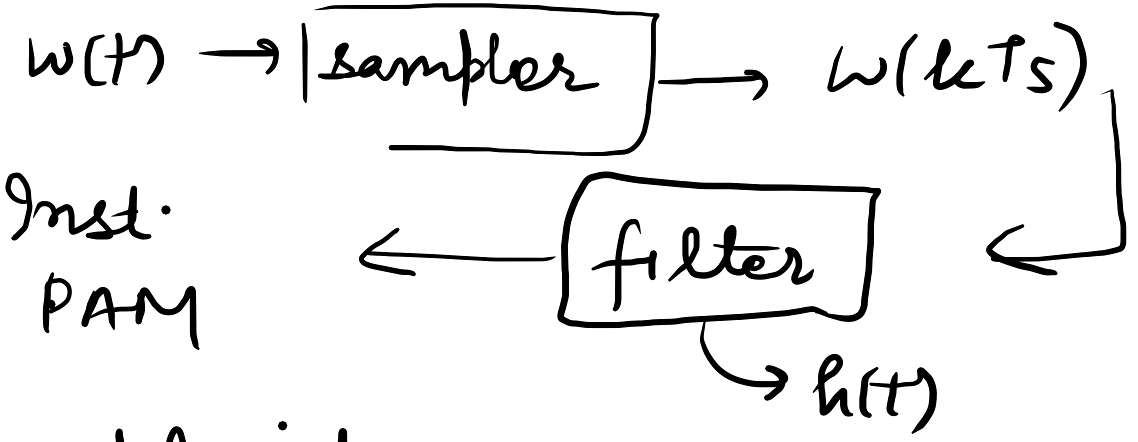
$\tau/T_s = 1/3$ & $f_s = 4B$

$$|W_s(f)| = \frac{1}{T_s}|H(f)| \sum_{n=-\infty}^{\infty} |W(f-nf_s)|$$

Class / home assignment :- Product detection of Inst. PAM from Pg 133, Haykin's TB

→ Inst. PAM is also called as "Sample & hold"

At some places, we have mentioned that Inst. PAM is sampled signal passed through a filter.

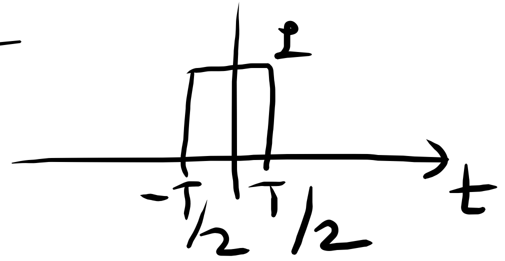


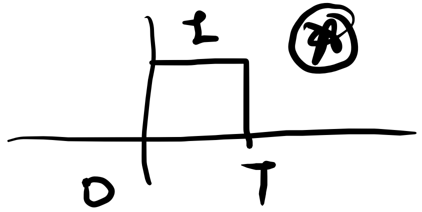
(flat-top Inst. PAM) PAM

Sim. Haykin's presentation of Inst. PAM takes ⊛ as the filter.

a delay is imposed

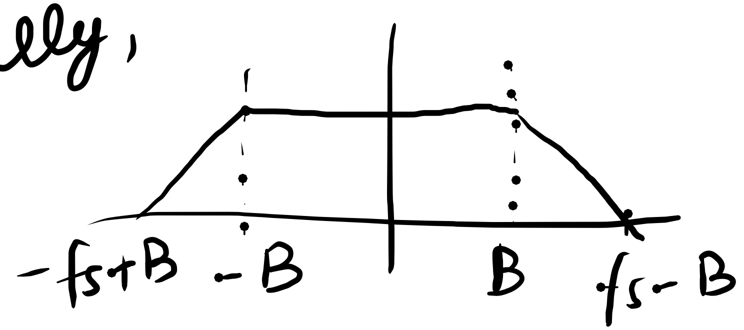
delay it by $T/2$





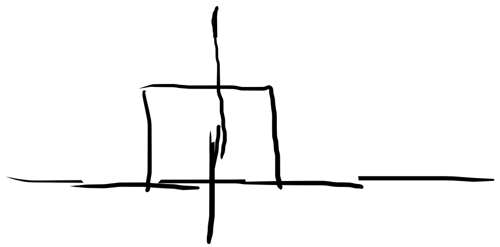
→ Natural sampling, a typical LP filter would look like

Ideally,

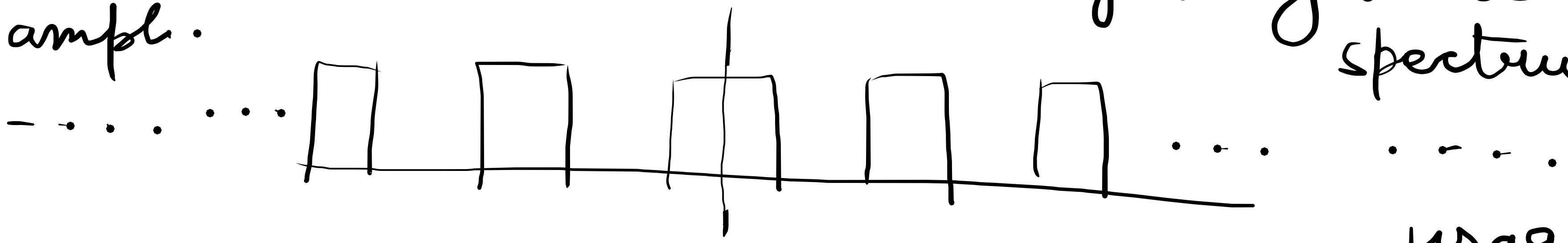


Ideal filter of BW $> B$ & $< f_s - B$ should work but since steep fall is diff. to realize, the used filter can die down slowly b/w B & $f_s - B$

One benefit of either form of PAM.



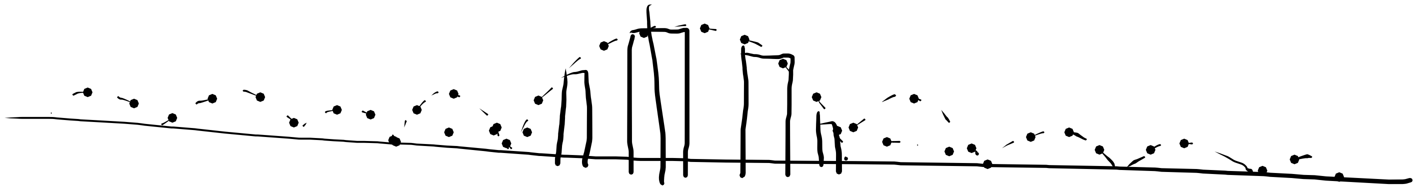
Ideal sampl. $\longrightarrow$



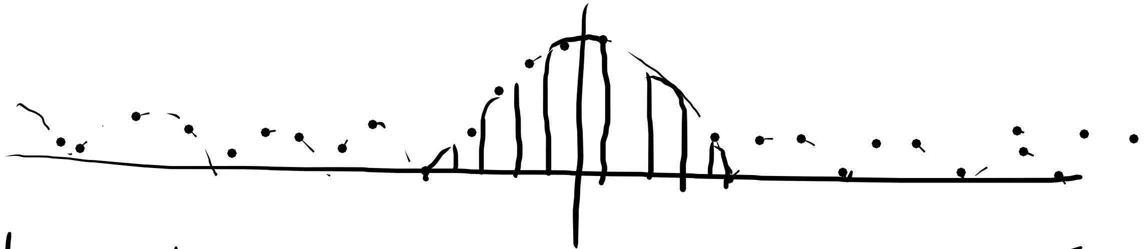
Very-very wide spectrum

usage.

NS $\longrightarrow$



flat top Inst Sam. $\longrightarrow$



Recovery of PAM (Inst.) :- Low pass filter the signal (sampled) - but there is high freq. loss in the recovered analog WF due to the filtering effect $H(f)$. How, this loss can be reduced by $\downarrow \tau$ (How?) or using an equalizer with TF of $1/H(f)$

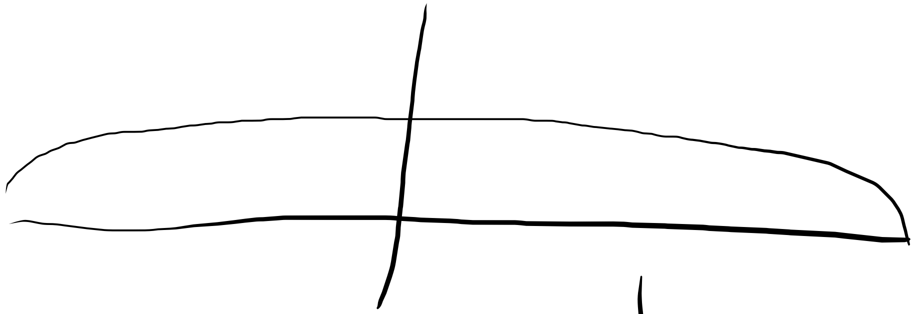
Sampled (Inst. PAM) $\rightarrow$ $\boxed{\text{LPF}}$ $\rightarrow$ $\boxed{1/H(f)}$ $\rightarrow$

subject to this is finite / exists

Pulse width τ is called aperture
since τ/T_s determines the gain
of the recovered analog signal

→ limitations of PAM:- (both NS & IS)

↳ If we Tx PAM signal over a wire / any channel
requires a very wide freq. response
because of narrow pulse width (how?)



→ BW reqd. is much larger than that
of the original analog signal

→ Noise perf. is not better than
direct analog sig. Tx.

PAM is used as the first building block of the PCM system. Also, PAM helps in efficient multiplexing of diff. signals.

Quantization process:— PAM gives us signal at discrete time but the amplitudes do not belong to a finite set

4 bit length can rep. $2^4 = 16$

→ The existence of a finite no. of discrete amplitude levels is a basic condition of PCM → pulse-code-modul^n

Lec-14, DC

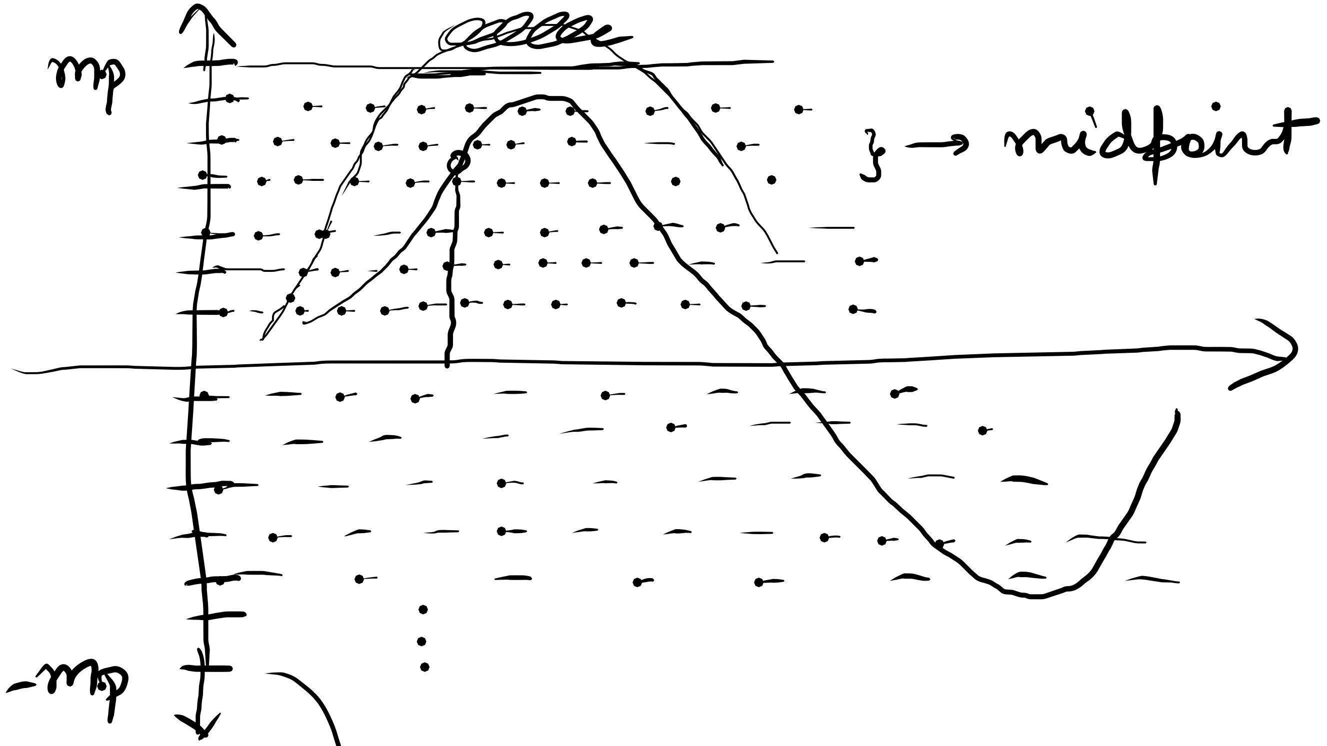
Amplitude quanti:- process of transforming the sample amplitude $m(nT_s)$ of a msg. sig. $m(t)$ at $t=nT_s$ into a discrete amplitude $V(nT_s)$ taken from a finite set of possible amplitudes.

→ process is assumed to be "memoryless & instantaneous"

Called as scalar quantizn

* Vector quantizn is also a well-studied concept

↳ for sample at $t=nT_s$, quantizn not affected by earlier or later samples



Allowed quantization levels be L ← $2^{\text{no. of bits}}$

gap b/w levels :- $\dfrac{2m_p}{L}$

(If uniformly spaced)

→ they can be equally spaced / uniformly spaced

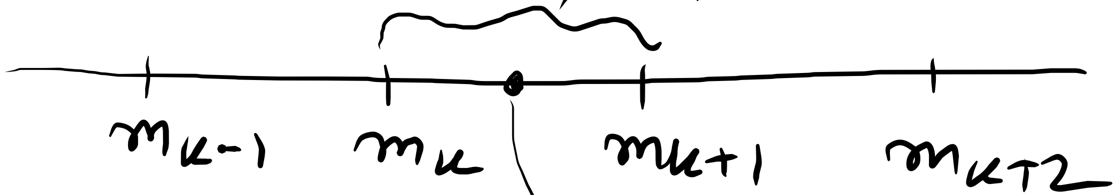
or non-uniformly spaced

→ anything above mp or below -mp is chopped off.

continuous sample m → $\boxed{\text{quantizer } g(\cdot)}$ → Discrete sample v

Few terms:-

m_ks :- decision levels or thresholds.



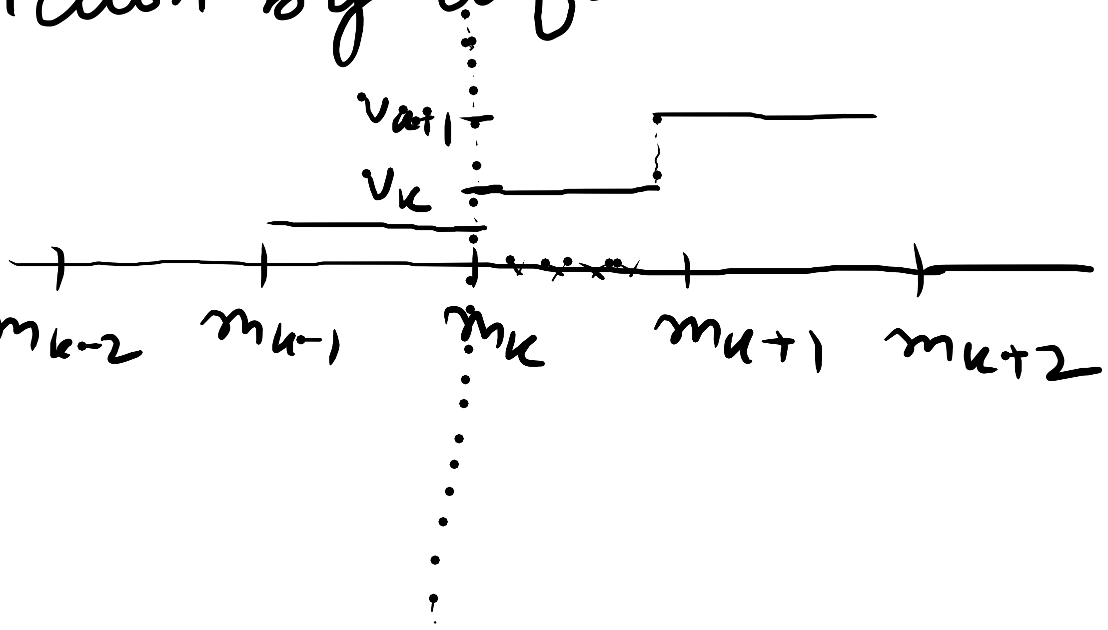
Partition cell, denoted as I_k, $k = 1, 2, 3 \ldots L$

L → total no. of amplitude levels used in quantizer

all amplitudes in $[m_k, m_{k+1})$ are represented by v_k

v_k

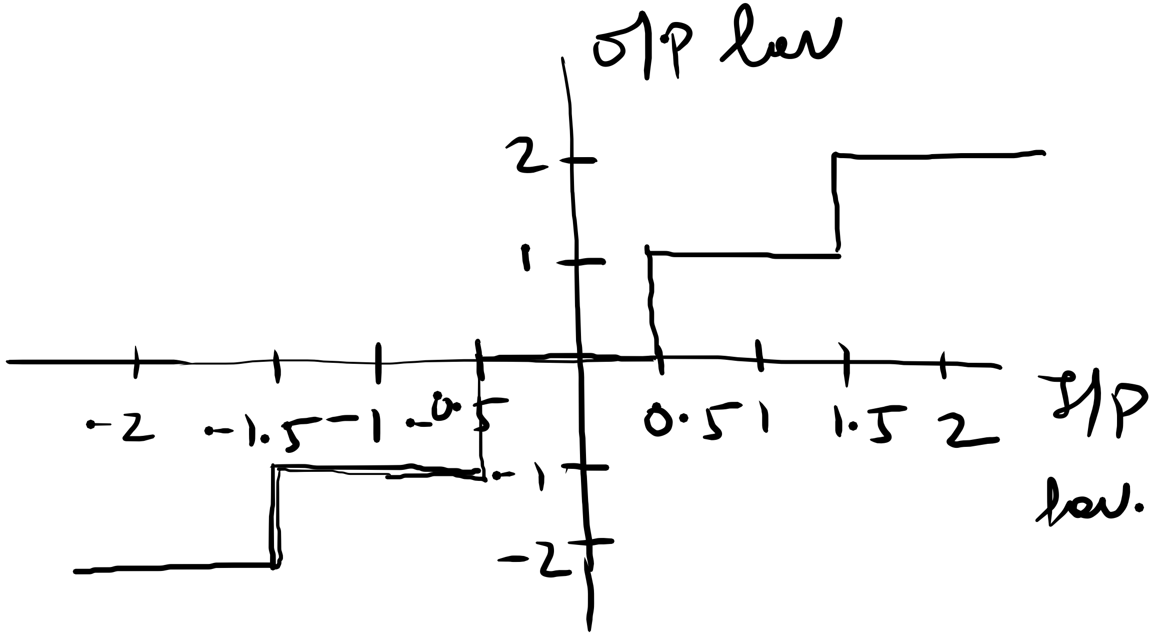
mapping $V = g(m)$ is the quantizer characteristic, which is a staircase function by definition

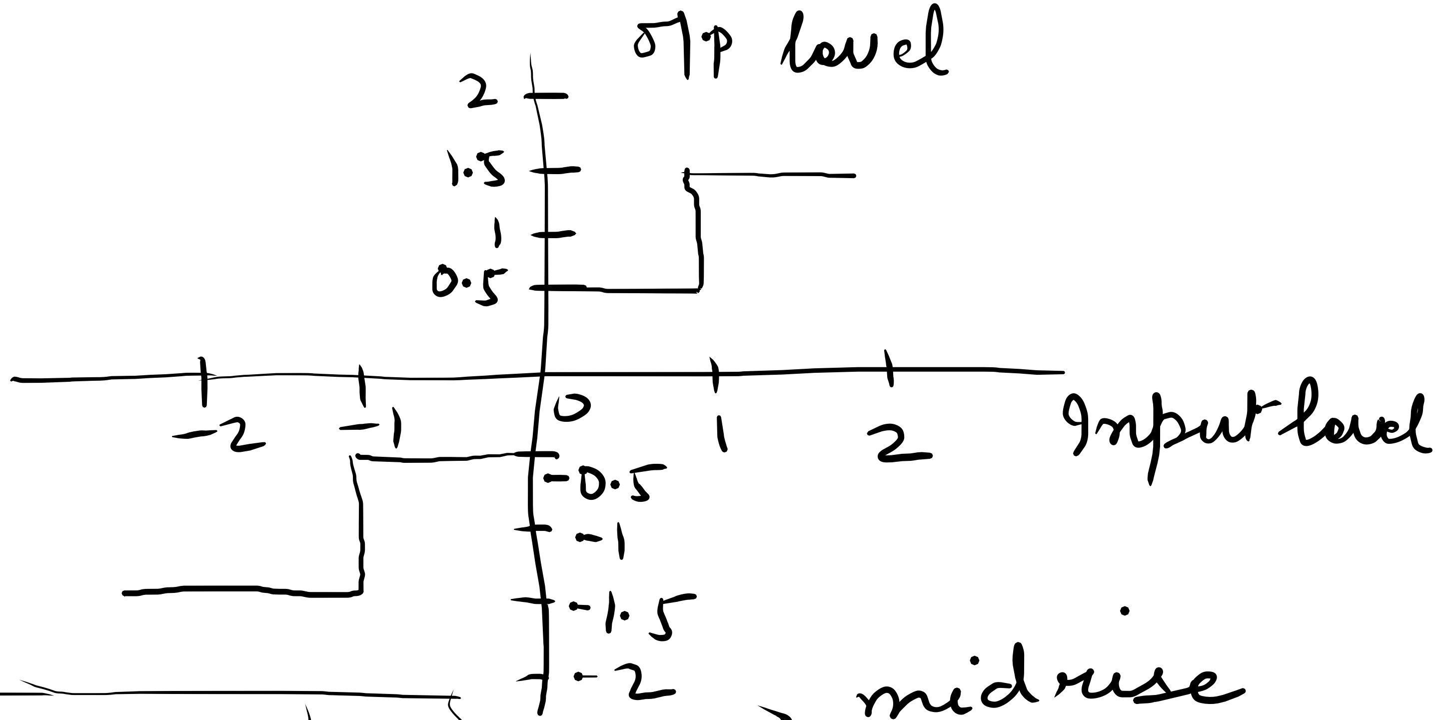


V_ks :- representation levels or reconstruction levels.

midtread
midrise.

} quantizer charac. can be of two types.





midtread

↳ origin lies in the middle of a tread of the staircase like graph

midrise

↳ origin lies in the middle of the rising part.

both are symmetric about the origin

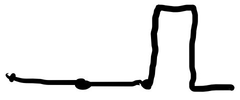
Quantizers
- uniform
- non-uniform.

→ The quantized samples are coded and Txd as binary pulses.

At the rx, some pulse maybe detected incorrectly.

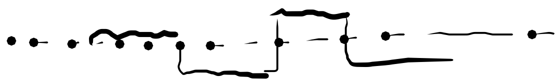
$L = 3$

$v_k \to 001$



Two sources of error
- → Quantiz^n process
- → Pulse detec^n

⊕ noise



1 0 1

$m \to q(m)$ $\quad |m - q(m)|$ — irreducible

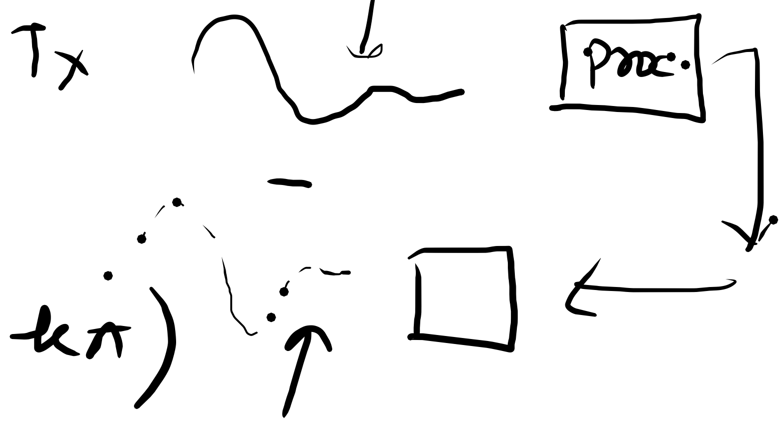
In most cases, pulse detec^n error is quite small compared to quantiz^n error. Basically, for the deriv^n ahead, we assume that the error in the rxd sig. is caused exclusively by quantiz^n.

$$m(t) = \sum_k m(kT_s)\, \mathrm{sinc}(2\pi B t - k\pi)$$ → standard interpolation formula.

$m(kT_s)$ ↳ sample values of $m(t)$

$$\hat{m}(t) = \sum_k \hat{m}(kT_s)\, \mathrm{sinc}(2\pi B t - k\pi)$$

$\hat{m}(kT_s)$ ↳ quantized sample.



distortion component

$$q(t) = \sum_k \left[\hat{m}(kT_s) - m(kT_s)\right] \mathrm{sinc}(2\pi B t - k\pi)$$

$$= \sum_k q(kT_s)\, \mathrm{sinc}(2\pi B t - k\pi)$$

$q(t) = \hat{m}(t) - m(t)$

$q(t)$:- undesired signal called as quantization noise

↳ let us obtain its power or the mean square value.

$\hat{m}(t) = m(t) + q(t)$ (?)

$$\widetilde{q^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} q^2(t)\,dt \longrightarrow \text{mod 1 of S\&S}$$

$\downarrow$

mean-square value of $q(t)$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left[\sum_k q(kT_s)\,\text{sinc}(2\pi Bt - k\pi)\right]^2 dt \quad -①$$

using, $$\int_{-\infty}^{\infty} \text{sinc}(2\pi Bt - m\pi)\,\text{sinc}(2\pi Bt - n\pi)\,dt = \begin{cases} 0, & m \neq n \\ 1/2B, & m = n \end{cases}$$

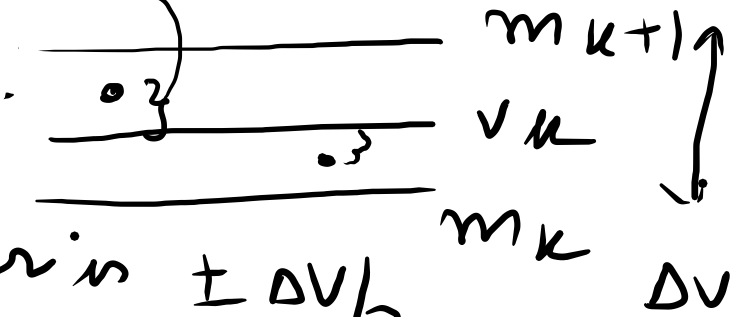
$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \sum_k q^2(kT_s)\,\text{sinc}^2(2\pi Bt - k\pi)\,dt \quad -②$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_k q^2(kT_s) \int_{-T/2}^{T/2} \text{sinc}^2(2\pi Bt - k\pi)\,dt$$

$$\widetilde{q^2(t)} = \lim_{T\to\infty} \frac{1}{2BT} \sum_k q^2(kT_s)$$

→ quantizn levels are separated by $\Delta V = \frac{2m_p}{L}$. Since a sample value is approx. by the midpoint of the subin-terval (of height ΔV) in which the sample falls, the max. quantizn error is $\pm \Delta V/2$

Thus, the quantizn error lies in the range $\left[-\frac{\Delta V}{2}, \frac{\Delta V}{2}\right]$ where max poss. error is $\pm \Delta V/2$



$$\Delta V = \frac{2m_p}{L}$$