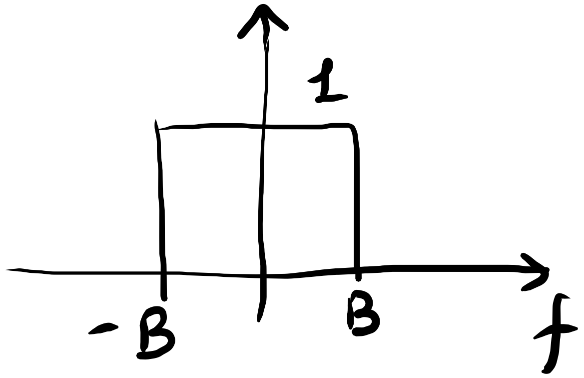
Lecture-12, DC

Suppose



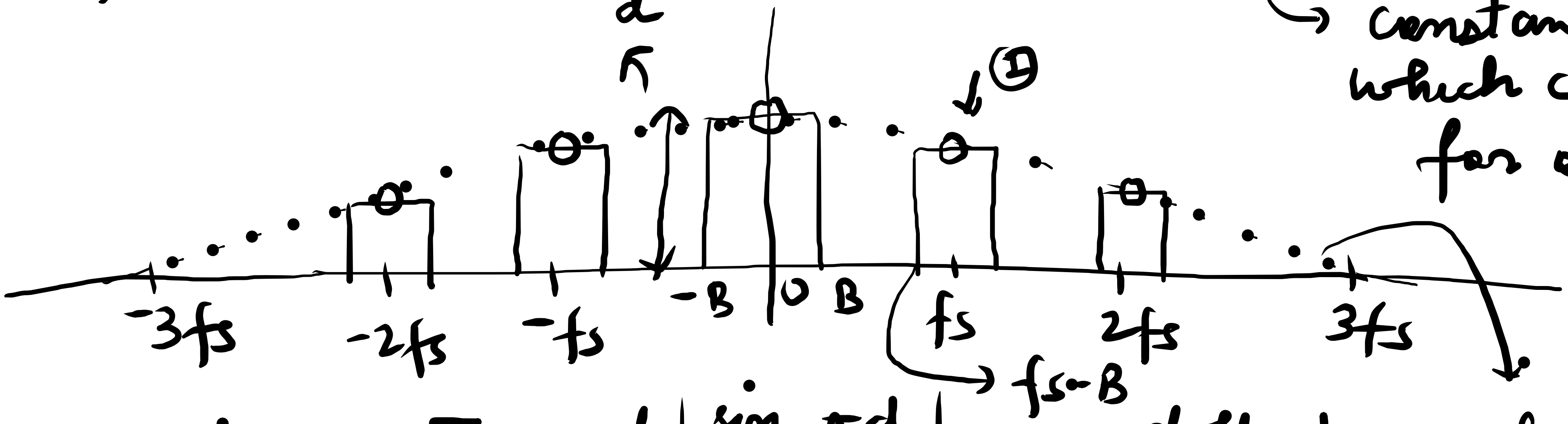
$|W(f)|$ mag. spectrum of LP analog WF

$$|W_s(f)| = \sum_{n=-\infty}^{\infty} \left\{ \underbrace{d\left|\frac{\sin n\pi d}{n\pi d}\right|}_{\text{constant value which changes for each } n} \cdot |W(f-nf_s)| \right\}$$

assuming $d = 1/3$

or $\frac{\tau}{T_s} = \frac{1}{3}$

or $T_s = 3\tau$



dotted envelop is of the sinc function

option 1: $2\sqrt{3}$

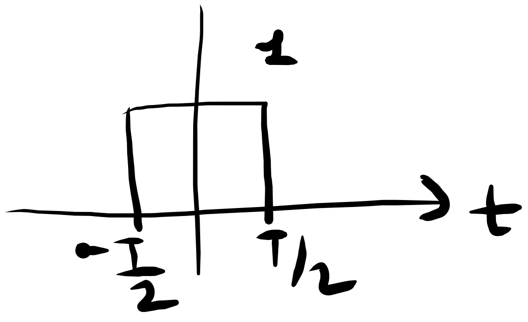
$$d\left|\frac{\sin \pi d}{\pi d}\right| = \frac{1}{3}\cdot\left|\frac{\sin \pi/3}{\pi/3}\right| = \frac{\sqrt{3}}{2\pi}$$

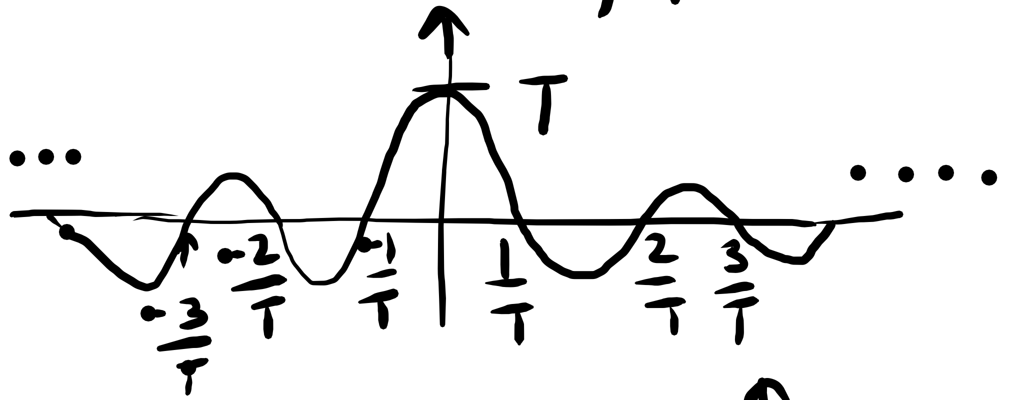
$$\frac{\sin \frac{n\pi}{3}}{n\pi/3}$$ at what values of n, will this be zero?

$n = 3k, \; k \in \mathbb{Z}, \; k \neq 0$

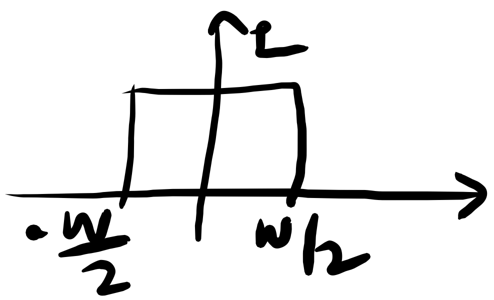
$$\Pi(t/T) \xrightarrow{FT} T\,\frac{\sin(\pi f T)}{\pi f T} \quad \text{or} \quad T\,\mathrm{sinc}(fT) \text{ if }$$

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

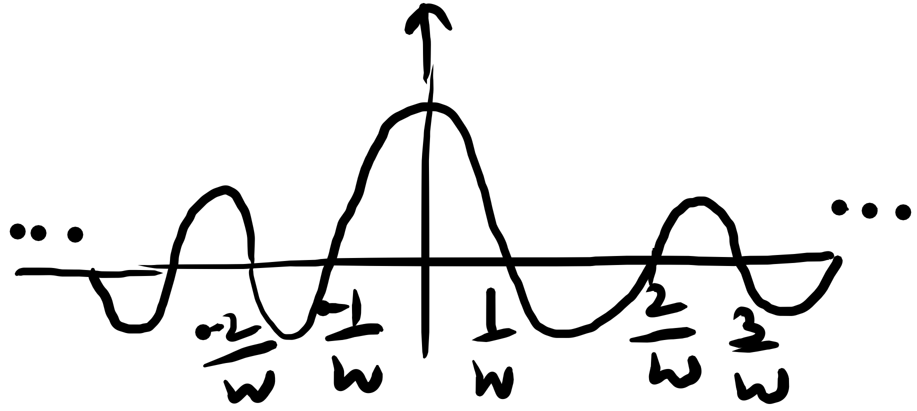




$\Pi(f/W)$



IFT



Recovery from a naturally sampled (PAM) $w(t)$

→ $w(t)$ can be recovered from $w_s(t)$ by passing the PAM signal through a LP filter where the cutoff freq. is $B < f_{cutoff} < f_s - B$

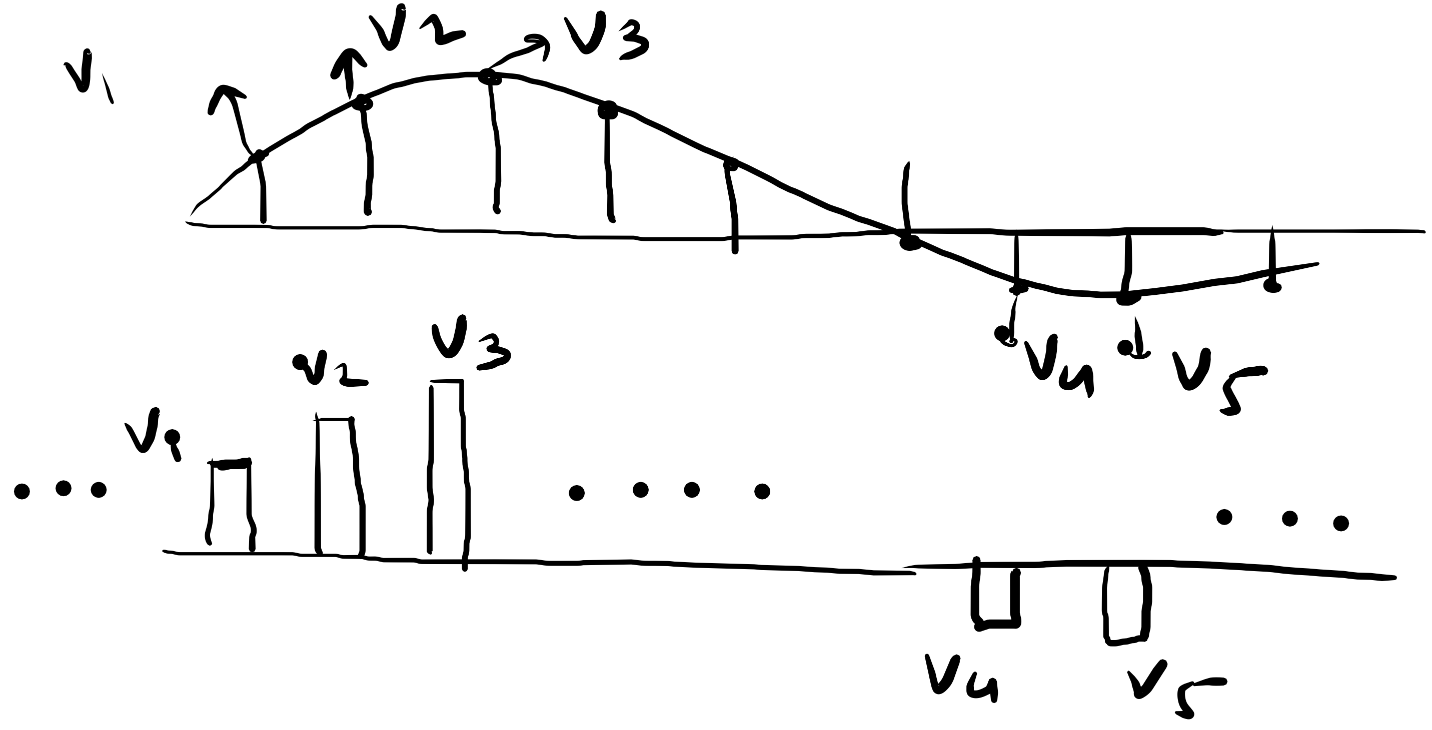
Also, $f_s \geq 2B$ (Nyquist rate) is reqd. to avoid aliasing

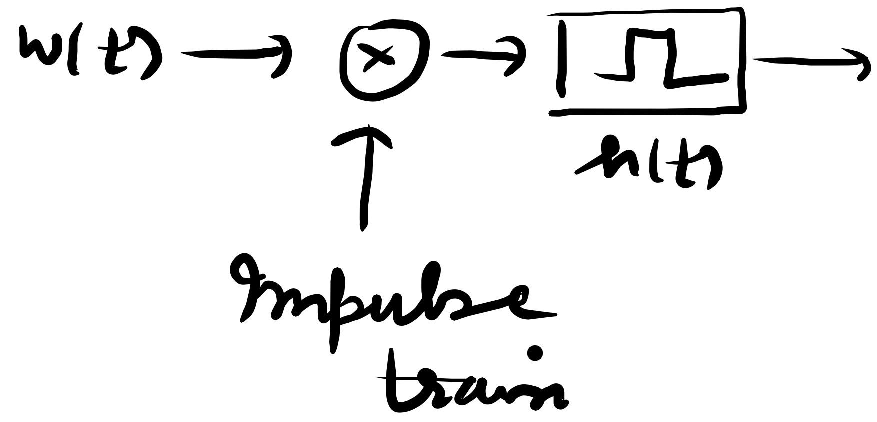
→ Prefiltering $w(t)$ before is reqd. because of TL- BL paradox.

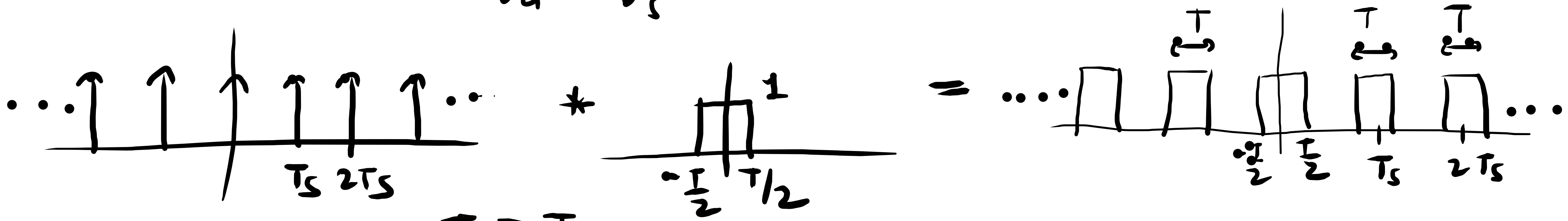
You need to compensate the gain factor of d by using an amplifier.

Instantaneous sampling (flat-top PAM)

→ generalization of the impulse train sampling technique.





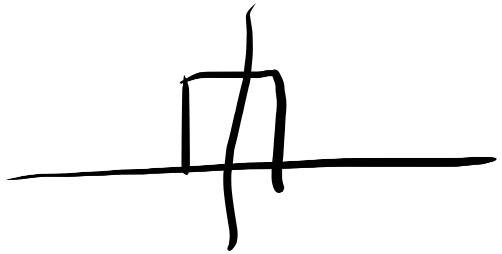


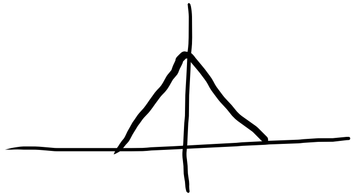
$T_s > T$

For BL WF $w(t)$, (to B Hz)

$$w_s(t) = \sum_{k=-\infty}^{\infty} w(kT_s)\, h(t-kT_s) \quad -①$$

$h(t)$ = sampling pulse shape





for "flat-top" sampling, it is

$$h(t) = \Pi(t/\tau) = \begin{cases} 1, & |t| < \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

where $\tau \le T_s = 1/f_s$ & $f_s \geq 2B$

Spectrum of $w_s(t)$ is

$$W_s(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} W(f-kf_s) \; ; \quad H(f) = FT[h(t)] = \frac{\tau \sin(\pi\tau f)}{\pi\tau f}$$

<u>Derivation</u>:- from ①

$$w_s(t) = \sum_k w(kT_s)\left[h(t) * \delta(t-kT_s)\right]$$

$$= h(t) * \left[\sum_k w(kT_s)\,\delta(t-kT_s)\right]$$

Hence, $w_s(t) = h(t) * \left[w(t) \times \sum_k \delta(t - kT_s) \right]$

Spectrum

$$W_s(f) = H(f) \left[W(f) * \left(\underbrace{\sum_k e^{-j2\pi f k T_s}} \right) \right]$$

But the sum of the exponential func^ns is equivalent to a FS expansion (in freq. domain) where the periodic func^n is an impulse train

(*) That is, $\frac{1}{T_s} \sum_k \delta(f - kf_s) = \frac{1}{T_s} \sum_n c_n e^{j2\pi n T_s f}$

$\frac{c_n}{T_s} = 1$

(*) $c_n = 1/f_s$

$$W_s(f) = H(f) \left[w(t) * \frac{1}{T_s} \sum_k \delta(f - kf_s) \right]$$

$$= \frac{H(f)}{T_s} \sum_k \left[W(f) * \delta(f - kf_s) \right]$$

$$= \frac{H(f)}{T_s} \sum_k W(f - kf_s) \quad (\#)$$

$$|W_s(f)| = \left| \frac{1}{T_s} H(f) \right| \sum_k |W(f - kf_s)|$$

$$w_s(t) = \sum_k T_k(t)$$

$$|w_s(t)| = \sum_k |T_k(t)|$$

$$z = \begin{matrix} a+ib \\ c+id \end{matrix} \; + \; = a+c+j(b+d)$$

$$|z| = \sqrt{a^2+b^2} + \sqrt{c^2+d^2} \; (?)$$