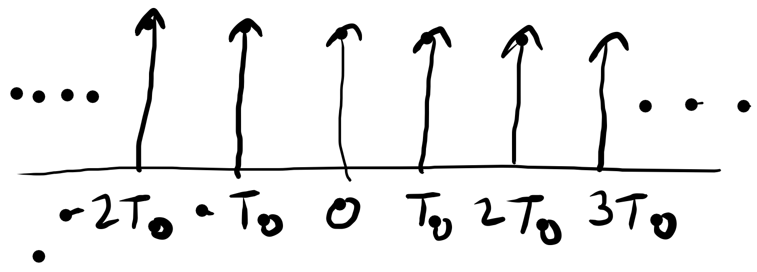
Lecture 5 - DC

$\sum_{i=-\infty}^{\infty} \delta(t - iT_0) \overset{?}{\neq}$ periodic signal



using its FS coeff. we can obtain its FT. # oppenh.. S&S TB

FS coeff are $1/T_0$

$a_k = 1/T_0 \;\forall k$

$$FT\left(\sum_{i=-\infty}^{\infty} \delta(t - iT_0)\right) = \frac{1}{T_0}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_0}\right)$$

From last lecture

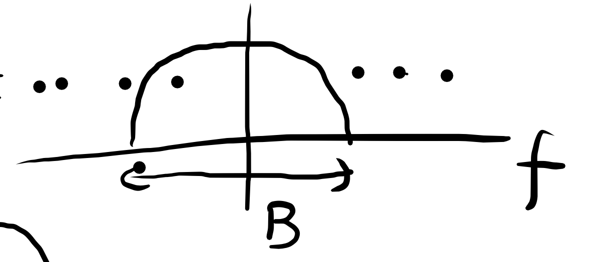
$$FT(g_S(t)) = G(f) * \frac{1}{T_s}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_s}\right) = \frac{1}{T_S}\sum_{m=-\infty}^{\infty} G\left(f - \frac{m}{T_S}\right)$$

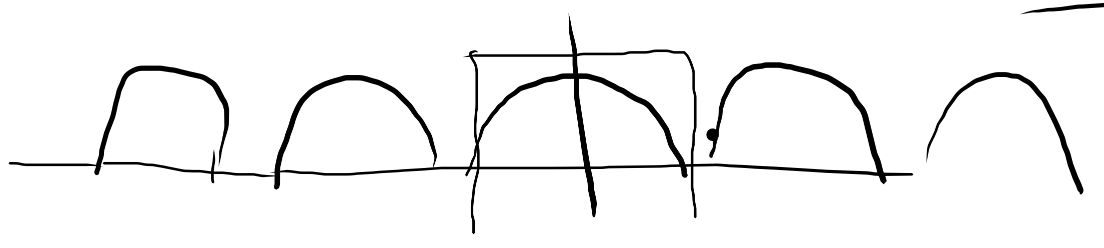
$G_S(f) \rightarrow$

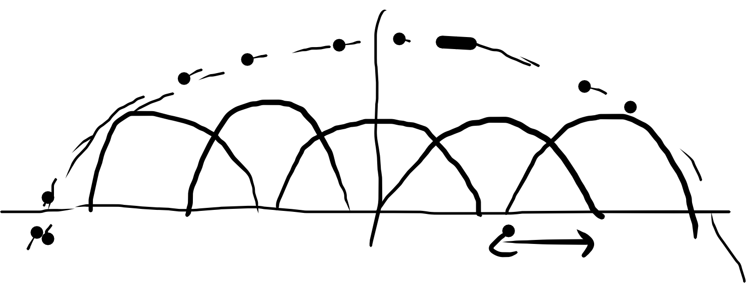
$\frac{1}{T_s} = f_s$;

$$f_s \sum_{m=-\infty}^{\infty} G(f - m f_s)$$

$G(f)$:







$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s) \; ; \quad \underbrace{FT(g_\delta(t))}_{} = FT \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\; FT\left[\delta(t-nT_s)\right]$$

exp1 :- $G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n f T_s}$ #

- This is DTFT of $\{g(nT_s)\}$
- It may be viewed as a complex FS represen^n of the periodic freq. func^n $G_\delta(f)$ with seq. of samples $\{g(nT_s)\}$ def. the coeff. of expansion.

exp2 :- $G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f-mf_s)$

process of uniform sampling a CT signal of finite energy results in a periodic spectrum with period = sampling rate

For periodic signal $x(t)$ with period T,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\frac{2\pi}{T}t} \quad \text{or} \quad e^{jk\omega_0 t} \rightarrow \frac{2\pi}{T}$$

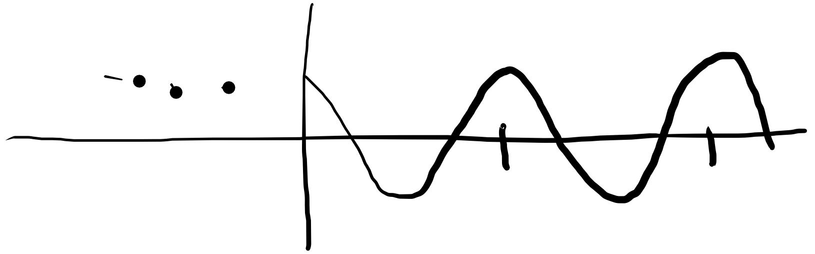
verify.

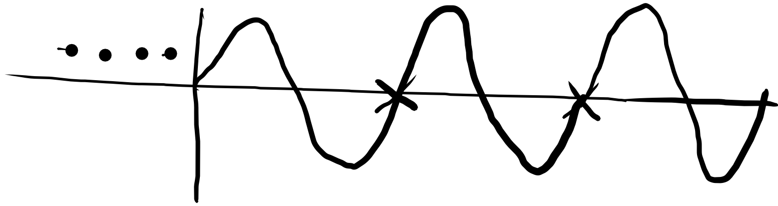
$$G_\delta(f+f_s) = G_\delta(f)$$

$f_s \rightarrow 1/T_s$

$$e^{-j2\pi n(f+f_s)T_s} = e^{-j2\pi n f T_s}\, \underbrace{e^{-j2\pi n}}_{1}$$

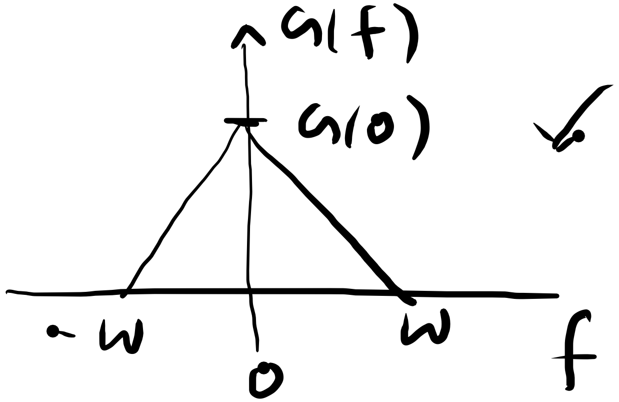
$$\underbrace{\cos(2\pi n)}_{1} - j\, \underbrace{\sin 2\pi n}_{0} \qquad n \in \mathbb{Z}$$



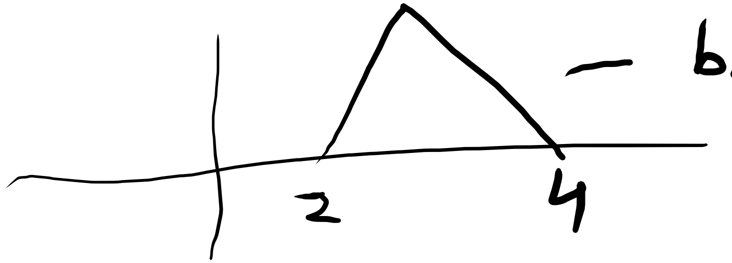


$$G_\delta(f+f_s) = G_\delta(f) \quad \forall f$$

→ we assumed that $g(t)$ is of finite energy & infinite duration. Now, suppose it is also band-limited (strictly) to W Hz



✓ a sample



— bandpass

for such a signal, choose the sampling rate as 2W samples/sec or $T_s = 1/2W$, then

① $$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\frac{\pi n f}{W}} \quad \text{— (5a)}$$

also, ② $$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f-mf_s) \quad \text{— (5)}$$