Set 3: Modeling and Analysis of Logistic Equation with Harvesting and Agricultural Innovation Spread Dynamics

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CS302, Modeling and Simulation

In this report, we investigate the dynamics of two mathematical models. First, we modify the logistic equation to include a "harvesting" rate and examine its behavior through analytical and numerical solutions. We then study the spread of agricultural innovations among farmers through personal and impersonal communications, and recast the dynamic equation to understand the impact of social and environmental factors on the spread of innovation.

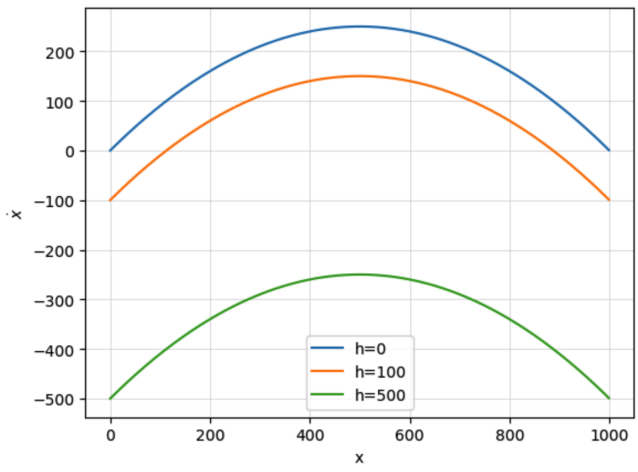
I. HARVESTING RATE MODIFICATION

$$x(t) = \frac{k}{1 + c^{-1}e^{(}- rt)} \tag{1}$$

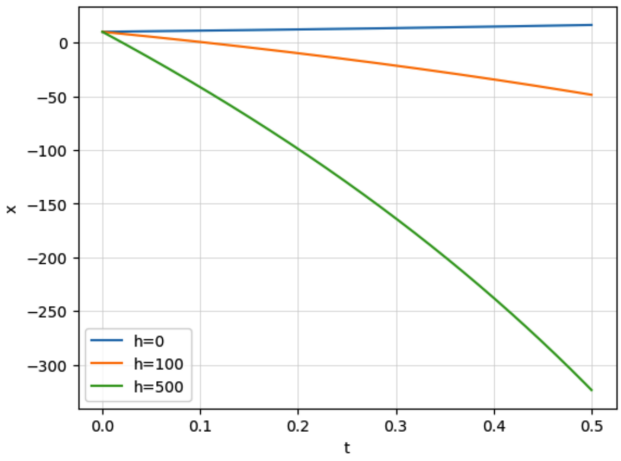
where,

$$c = \frac{x_0}{k - x_0} \tag{2}$$

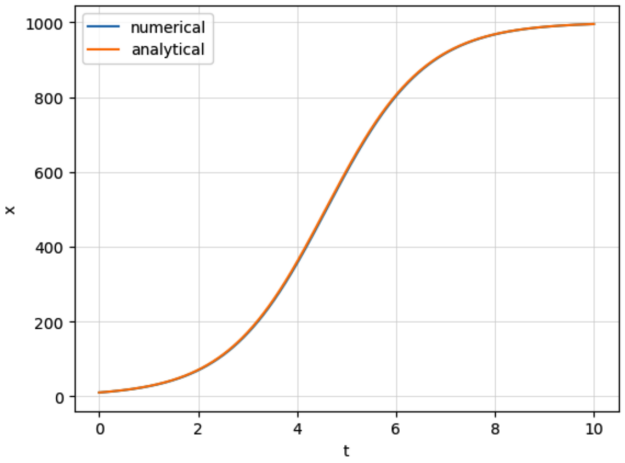
A. Graph of dx/dt versus x



B. Plot of Euler's Solution for x(t)



C. Comparison of Analytical and Numerical Solution



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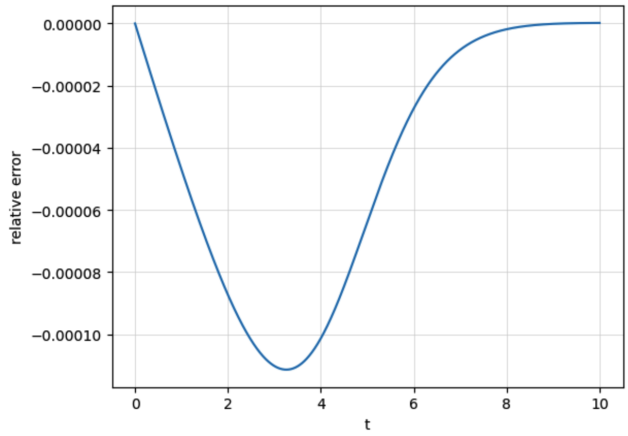
D. Plot of relative error between two solutions



II. AGRICULTURAL INNOVATION

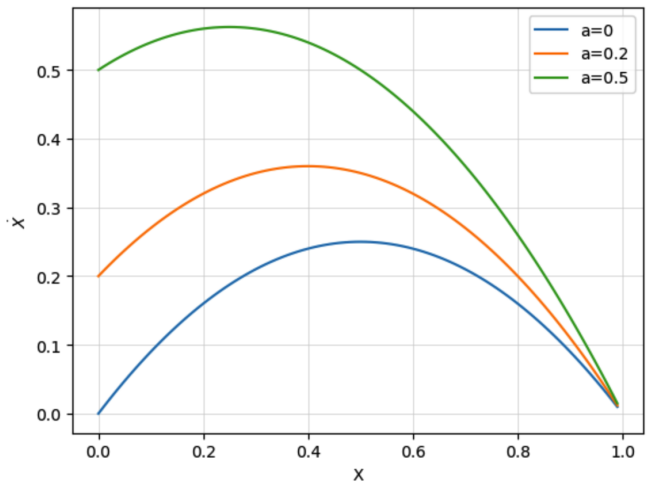
$$X(T) = \frac{1}{2}((1-a) + (1+a)(\frac{e^{\alpha}-1}{e^{\alpha}+1})) \tag{3}$$

where,

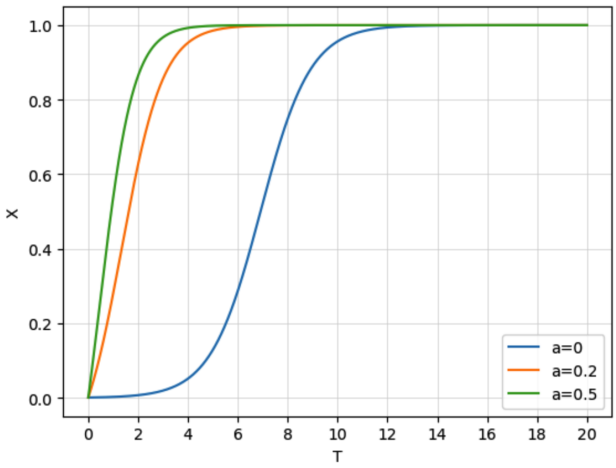
$$\alpha = (1+a)T + c \tag{4}$$

$$c = ln(\frac{a+X_0}{1-X_0}) \tag{5}$$

A. Plot of dX/dT versus X



B. Plot of X(T) versus T



III. CONCLUSIONS

- Rescaling the integral solution can simplify the analysis of dynamical systems. By rescaling the solution in terms of X and T, we were able to obtain a simpler form of the dynamical equation that only depends on the parameter a.

- The results showed that harvesting rate significantly impacts the population dynamics, and as h increases, the population size decreases faster.

- Through experimentation with different values of 'a', it has been observed that the rate of innovation diffusion in agriculture increases as 'a' increases. Additionally, it has been noted that agricultural innovation 'X' will eventually reach saturation, as evidenced by a decreasing rate of change over time, as represented by the derivative dX/dT.

IV. REFERENCES

[1] Arnab K. Ray *Lectures notes on Agricultural Innovation*