The Principle of Competitive Exclusion in Population Biology

> In nature, the struggle for existence between two similar species, competing for the same limited food supply and living space, nearly always ends in the complete extinction of one species.
>
> — The Principle of Competitive Exclusion
>
> Charles Darwin (1859)

The Logistic Equation: $\boxed{\frac{dx}{dt} = ax - bx^2}$

$$\Rightarrow \boxed{\frac{dx}{dt} = a\left(1 - \frac{x}{a/b}\right)x = a\left(1 - \frac{x}{K}\right)x} \quad a,b>0$$

We write $\boxed{\frac{dx}{dt} = rx}$ where $\boxed{r = a\left(1 - \frac{x}{K}\right)}$

$K \to$ Carrying capacity. $\boxed{\text{When } x \to K,\ r \to 0}$.

$\boxed{\text{When } x \ll K,\ r \simeq a} \Rightarrow \boxed{\frac{dx}{dt} \simeq ax}$ $\boxed{ax \to \text{Biotic potential.}}$

$\boxed{1 - \frac{x}{K} = \frac{K-x}{K}} \to$ Fraction of the environment that can still be accessed.

Represents competition WITHIN the species.

Now consider two similar species with population size x and y. They intrude

(P.T.O.)

on each other's space. As a result competition arises between x and y.

We write $\boxed{\dfrac{dx}{dt} = a_x\left(\dfrac{K_x - x - \alpha y}{K_x}\right)x}$ and

$\boxed{\dfrac{dy}{dt} = a_y\left(\dfrac{K_y - y - \beta x}{K_y}\right)y}$ $\alpha, \beta \rightarrow$ Parameters of competition. $(\alpha, \beta > 0)$

When $\boxed{\alpha = \beta = 0} \Rightarrow$ No competition $(a_x, a_y > 0)$

$K_x \rightarrow$ Carrying capacity of species x.
$K_y \rightarrow$ Carrying capacity of species y.

If $\boxed{\alpha = \beta} \Rightarrow$ Intense competition (both are equally strong)

Expanding the $\boxed{\dfrac{dx}{dt}}$ and $\boxed{\dfrac{dy}{dt}}$ equations,

$$\boxed{\frac{dx}{dt} = a_x x - \frac{a_x}{K_x}x^2 - \alpha\frac{a_x}{K_x}xy}$$ and,

$$\boxed{\frac{dy}{dt} = a_y y - \frac{a_y}{K_y}y^2 - \frac{\beta a_y xy}{K_y}}$$. When $\boxed{\alpha = \beta = 0}$,

we get uncoupled logistic equations for x and y.

The competitive interaction is only active through the coupling of x and y in the cross terms $\boxed{\alpha \frac{a_x}{K_x} xy}$ and $\boxed{\beta \frac{a_y}{K_y} xy}$.

i.) With more species, more such equations can be set down (n-order system).
ii.) The effect of seasons and natural calamities has been ignored.