Set 2 - Compartment Modeling of Linear Systems

Divya Patel [202001420][*] and Aryan Shah [202001430][†]
Dhirubhai Ambani Institute of Information & Communication Technology,
Gandhinagar, Gujarat 382007, India
CS302, Modeling and Simulation

Abstract In this lab we apply compartment model to different types of linear model such as concentration of pollutants in lake, single dose of medicine and a course of medicine.

I. LAKE POLLUTION

Q1. The concentration $C(t)$ of pollutants in a lake follows the equation $\dot{C} = a - bC$, in which $a = FC_{in}/V$ and $b = F/V$. Here C_{in} in the constant pollutant concentration of inflow into the lake, F is the fixed volumetric flow rate and V is the fixed volume of the lake (as the lake also drains out). Take $F = 5 \times 10^8 m^3/day$, $V = 10^{12} m^3$ and $C_{in} = 3$ unit and $C(0) = C_0 = 10$ unit.
After solving $\dot{C} = a - bC$,

$$C(t) = C_{in} - (C_{in} - C_0)e^{-\frac{F}{V}t} \tag{1}$$

A. Plot C(t) versus t

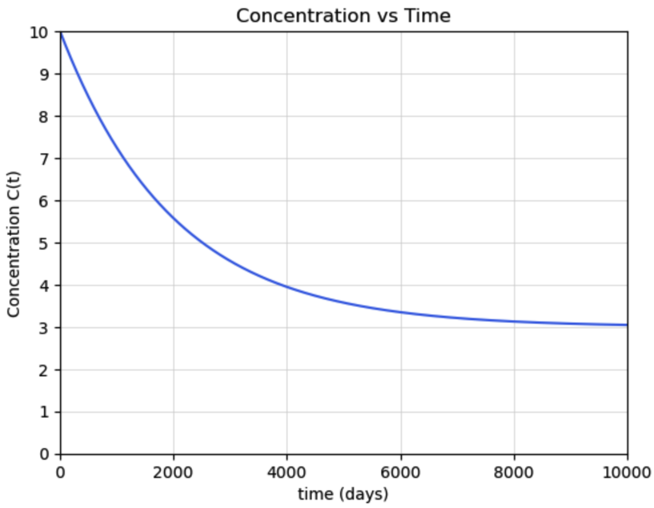

Figure 1: The plot of concentration $C(t)$ vs time (in days).

B. Estimating the time take for $C = 0.5C_0$

Putting $C(t) = 0.5C_0$, $C_{in} = 3$ unit, $F = 5 \times 10^8 m^3/day$ and $V = 10^{12} m^3$ in Eq. (1),

$$5 = 3 - (3 - 10)e^{-\frac{5\times10^8}{10^{12}}t}$$
$$\therefore t \approx 2506 \; days$$

Hence, time taken for the concentration of the pollutant in the lake to be $0.5C_0$ is **2506 days**.

[*]Electronic address: 202001420@daiict.ac.in
[†]Electronic address: 202001430@daiict.ac.in

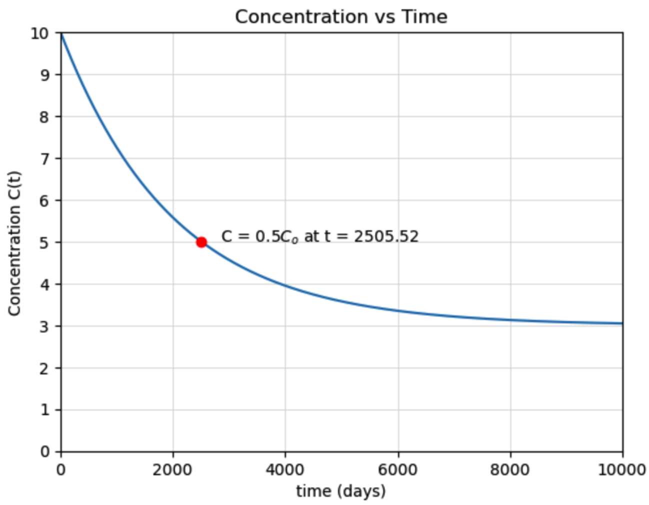

Figure 2: $C(t)$ vs time (in days). The point corresponding to $C = 0.5C_0$ has been marked.

C. Estimating the time take for $C = 0.5C_0$ for clean fresh water flow ($C_{in} = 0$)

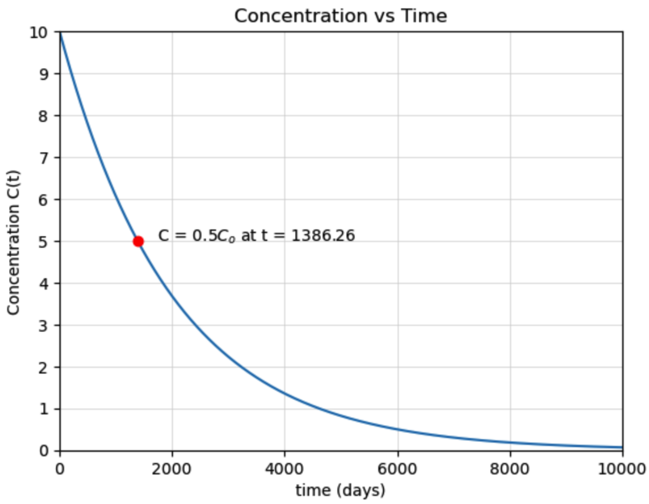

Figure 3: Concentration $C(t)$ vs time (in days) when there's a fresh water inflow ($C_{in} = 0$)

Changing $C_{in} = 0$ unit and keeping the rest similar as in part B., we get in Eq. (1),

$$5 = -(-10)e^{-\frac{5\times10^8}{10^{12}}t}$$
$$\therefore t \approx 1386 \; days$$

Hence, time taken for the concentration of the pollutant in the lake to be $0.5C_0$ is **1386 days**.

II. SINGLE DOSE OF MEDICINE

Q2. A single dose of a medicine is administered to a patient. The dynamics of the medicine follows the equation $\dot{x} = -k_1 x$, $x(0) = x_0$ in the GI tract, and $\dot{y} = k_1 x - k_2 y$, $y(0) = 0$ in the blood stream. Take $k_1 = 0.6931\ hr^{-1}$, $k_2 = 0.0231\ hr^{-1}$ and $x_0 = 1$ unit. After solving $\dot{x} = -k_1 x$ and $\dot{y} = k_1 x - k_2 y$,

$$C(t) = C_{in} - (C_{in} - C_0)e^{-\frac{F}{V}t} \tag{2}$$

$$C(t) = C_{in} - (C_{in} - C_0)e^{-\frac{F}{V}t} \tag{3}$$

A. Plot of x(t) and y(t)

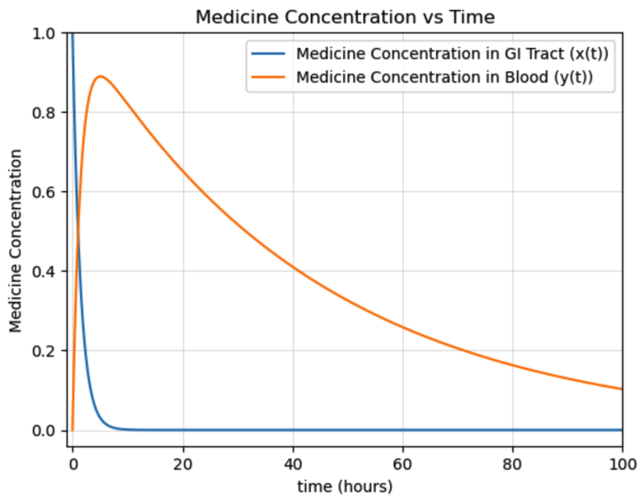

Figure 4: Plot of concentration of medicine in GI tract $x(t)$ and in Blood $y(t)$

B. Plot of x(t) and y(t) with $k_1 = k_2 = 0.6931$

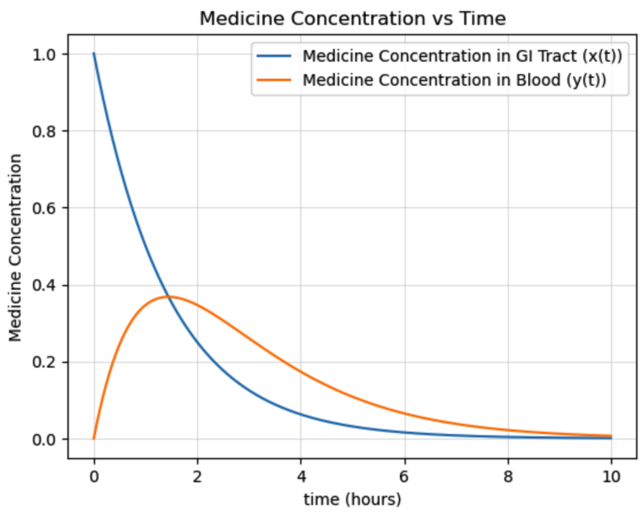

Figure 5: Plot of concentration of medicine in GI tract $x(t)$ and in Blood $y(t)$

C. Plot of x(t) and y(t) with $k_1 = k_2 = 0.0231$

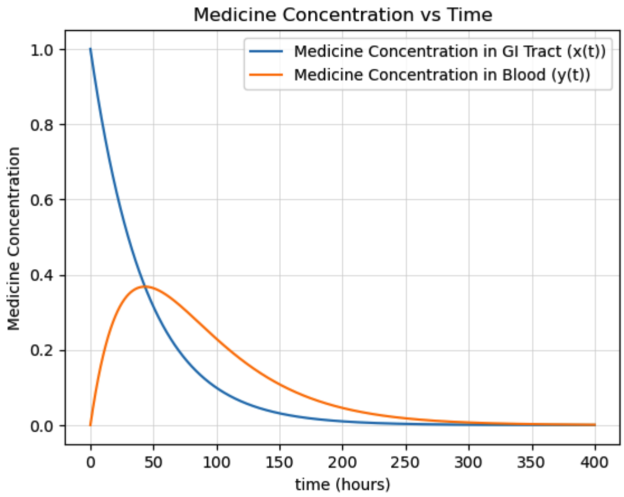

Figure 6: Plot of concentration of medicine in GI tract $x(t)$ and in Blood $y(t)$

III. COURSE OF MEDICINE

Q3. A course of a medicine is administered to a patient. The dynamics of the medicine follows the equation $\dot{x} = -k_1 x$, $x(0) = x_0$ in the GI tract, and $\dot{y} = k_1 x - k_2 y$, $y(0) = 0$ in the blood stream. Take $k_1 = 0.6931\ hr^{-1}$, $k_2 = 0.0231\ hr^{-1}$ and $x_0 = 1$ unit.

A. Plot of x(t) and y(t)

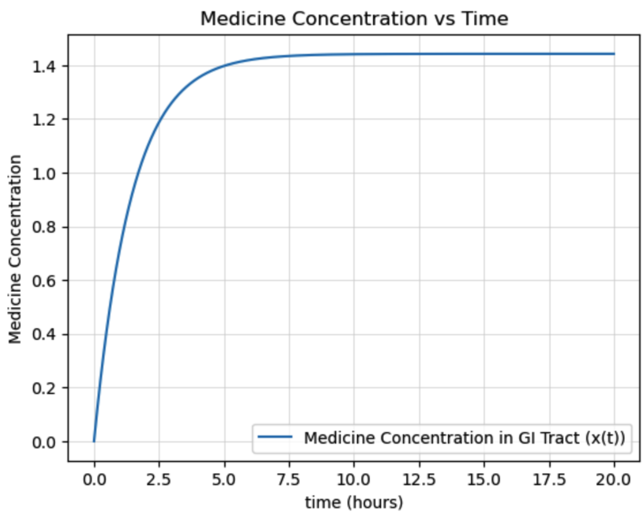

Figure 7: Plot of concentration of medicine in GI tract $x(t)$ vs time

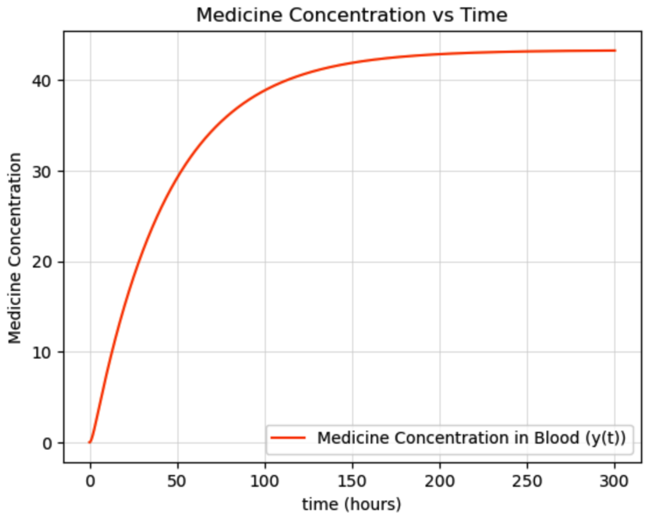

Figure 8: Plot of concentration of medicine in Blood $y(t)$ vs time

B. Plot of x(t) and y(t) with $k_1 = k_2$

Plots for $k_1 = k_2$ = 0.6931

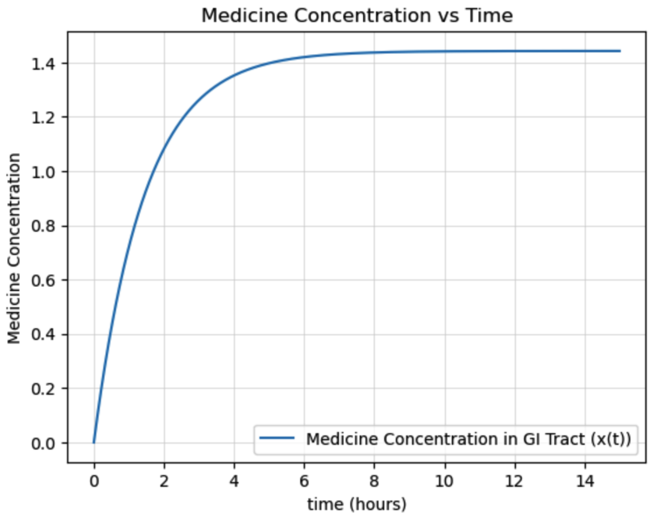

Figure 9: Plot of concentration of medicine in GI tract$x(t)$ vs time

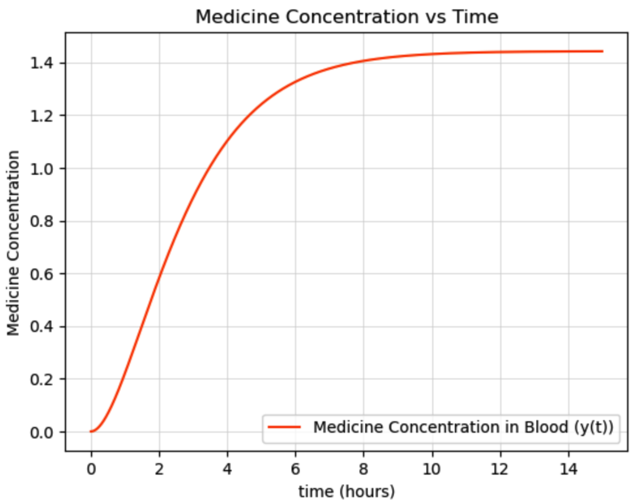

Figure 10: Plot of concentration of medicine in Blood $y(t)$ vs time

C. Plot of x(t) and y(t) with $k_1 = k_2$

Plots for $k_1 = k_2$ = 0.0231

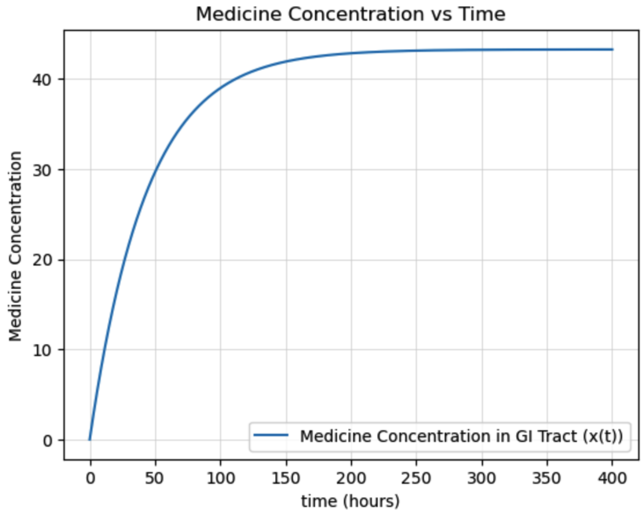

Figure 11: Plot of concentration of medicine in GI tract$x(t)$ vs time

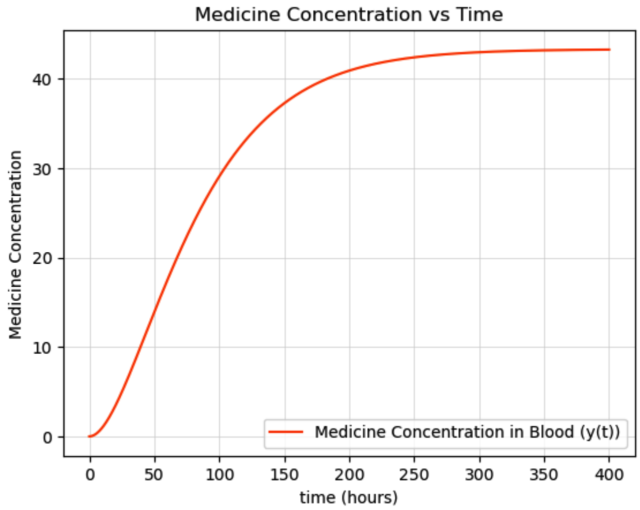

Figure 12: Plot of concentration of medicine in Blood $y(t)$ vs time

IV. CONCLUSION

- The concentration of pollutants in the lake decreases exponentially over time, and the rate of decrease depends on the values of a and b. If the inflow concentration remains constant, then the concentration of pollutants in the lake will approach a steady-state value. .

- In the case of a single dose, the concentration of medicine in the GI tract decreases exponentially over time, while the concentration of medicine in the bloodstream increases until it reaches a maximum and then decreases exponentially as the medicine is eliminated from the body.

- In the case of a course of medicine, the concentration of medicine in the bloodstream increases as each dose is administered, but eventually approaches a steady-state value.

V. REFERENCES

[1] Arnab K. Ray *Lectures notes on Lake Pollution*

[2] Arnab K. Ray *Lectures notes on drug dosage*