Group14_Lab4

September 29, 2022

[1]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn import datasets
from sklearn.datasets import fetch_openml
from sklearn.naive_bayes import GaussianNB
```

1 Question 1

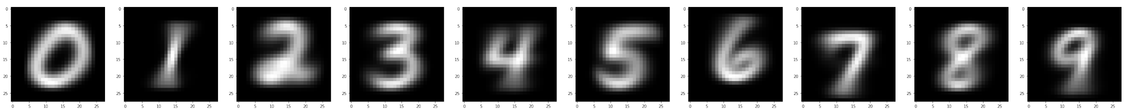
The MNIST database of handwritten digits, has a training set of 60,000 examples, and a test set of 10,000 examples. It is a subset of a larger set available from NIST. It is a good database for people who want to try learning techniques and pattern recognition methods on real-world data while spending minimal efforts on pre-processing and formatting.

1.1 Code

[2]:
```
mnist = fetch_openml('mnist_784')
```

[3]:
```
x = mnist.data
y = mnist.target
X_train , X_test, Y_train, Y_test = train_test_split(mnist.data,mnist.
 ↪target,test_size = 0.1)
X_train , X_test = (X_train.to_numpy()) , (X_test.to_numpy())
Y_train  = Y_train.cat.codes
Y_train = Y_train.to_numpy()
Y_test = Y_test.cat.codes
Y_test = Y_test.to_numpy()
meanImgArray = []

for i in range(10):
    tempArray = np.vstack(np.mean(X_train[np.where(Y_train==i)],axis=0))
    meanImgArray.append(tempArray)
    tempArray = []

fig,axes=plt.subplots(1,10,figsize=[50,5])
```

```
for i in range(10):
    meanImgArray[i] = meanImgArray[i].reshape(28,28)
    axes[i].imshow(meanImgArray[i],interpolation=None,cmap='gray')

plt.show()
```



1.2 Result

The images have been displayed above.

[ ]:
```
#Mean image for all classes
```

2 Question 2

Perform Linear Discriminant Analysis (LDA) on the MNIST dataset* for binary as well as for multiclass classification. Plot confusion matric and find out the combinations where the classifier is confused in predicting the right label.

2.1 Code

[4]:
```
import numpy as np
import pprint
from sklearn.model_selection import train_test_split
from sklearn.datasets import load_digits
import matplotlib.pyplot as plt
from sklearn.metrics import classification_report
import seaborn as sns
from sklearn.metrics import confusion_matrix
```

[5]:
```
def covv(x1,x2):
  m1 = np.sum(x1)/len(x1)
  m2 = np.sum(x2)/len(x2)
  temp = (x1-m1)*(x2-m2)
  return np.sum(temp)/len(temp)
# Check if matrix is singular or not
# If matrix is singular then add some noise to it
def sing(c):
  if(np.linalg.det(c) == 0):
    noise = np.random.normal(0,0.000000000001,len(c)**2)
    noise = noise.reshape(len(c),len(c))
```

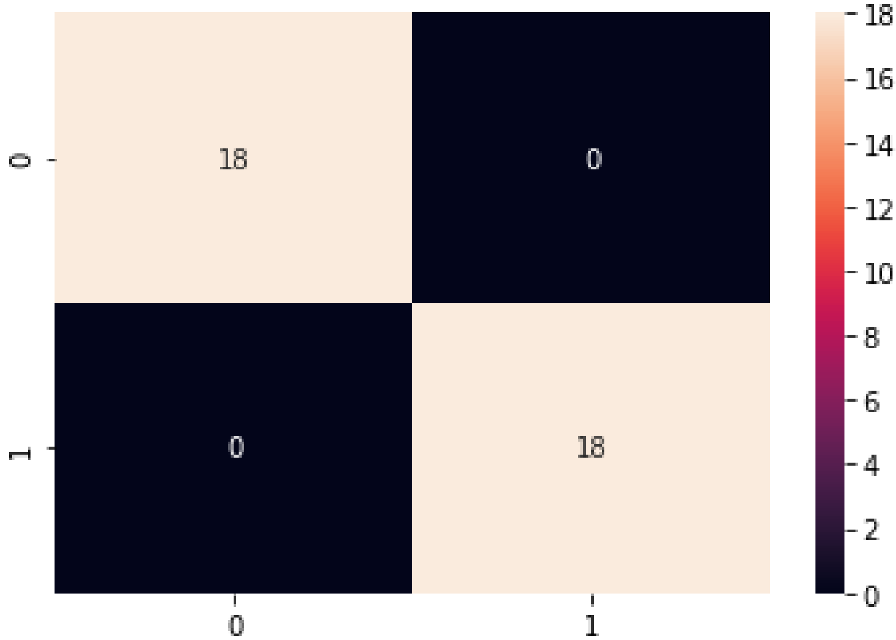
```
    c = c + noise
  return c
# Calculating delta for LDA
def LDA(x,m,c,p):
  t = x - m
  c = np.linalg.inv(c)
  return ( (-1/2)*(np.dot( np.dot(t.T,c) , t)) + np.log(p) )
# Calculating delta for QDA
def QDA(x,m,c,p):
  t = x - m
  d = np.log(abs(np.linalg.det(c)))
  c = np.linalg.inv(c)
  return (-1/2)*( d + (np.dot( np.dot(t.T,c) , t)) ) + np.log(p)
```

[11]:
```
digits = load_digits()
X = digits.data
y = digits.target
# Select images of 0 and 1 only
X1 = X[( y == 0)]
y1 = y[( y == 0)]
X2 = X[( y == 1)]
y2 = y[( y == 1)]

X1, X_test1, y1, y_test1 = train_test_split(X1, y1, test_size = 0.1,random_state␣
↪= 42)
X2, X_test2, y2, y_test2 = train_test_split(X2, y2, test_size = 0.1,random_state␣
↪= 42)
X1 = np.transpose(X1)
X2 = np.transpose(X2)

covM1 = []
for i in X1:
  t = []
  for j in X1:
    t.append(covv(i,j))
  covM1.append(t)

covM2 = []
for i in X2:
  t = []
  for j in X2:
    t.append(covv(i,j))
  covM2.append(t)



# print(covM1.shape)
```

```
# print(covM2.shape)

# To remove singularity
covM1 = sing(covM1)
covM2 = sing(covM2)

print(covM1.shape)
print(covM2.shape)
# Average
covM = (covM1 + covM2)/2

# Inverse of the covariance matrix
covInv = np.linalg.inv(covM)

# calculating mean vector
meanVector1T = []
meanVector2T = []

s = len(X1[0])
n = len(X1) #number of random variables 64
for i in range(n):
  meanVector1T.append(np.sum(X1[i])/s)
  s = len(X2[0])
  n = len(X2)

for i in range(n):
  meanVector2T.append(np.sum(X2[i])/s)

meanVector1 = np.transpose([meanVector1T])
meanVector2 = np.transpose([meanVector2T])
theta = np.dot(covInv ,(meanVector1-meanVector2))

# Prior probability of both will be 1/2 since there are equal number of samples
p0 = 1/2
apriory1 = 1/2
theta0 = np.log(apriory1/p0) - 1/2*( np.dot( np.dot(meanVector1T ,␣
 ↪covInv),meanVector1 )
- np.dot( np.dot(meanVector2T , covInv), meanVector2 ))


# Predict value using decision boundary
yPredicted = []
yActual = []

# predicting the value
for i in range(len(X_test1)):
  z1 = np.dot(X_test1[i],theta) + theta0
```

```
  if(z1 > 0):
    pred = 0
  else:
    pred = 1
  yPredicted.append(pred)
  yActual.append(0)
  z2 = np.dot(X_test2[i],theta) + theta0
  if(z2 > 0):
    pred = 0
  else:
    pred = 1
  yPredicted.append(pred)
  yActual.append(1)

# Confusion matrix
cm = confusion_matrix(yActual, yPredicted)
sns.heatmap(cm, annot = True)
print('----------Binary Classification Report -------------\n')
labels = ['Class 0', 'Class 1']
print(classification_report(yActual, yPredicted, target_names = labels))
```

```
(64, 64)
(64, 64)
----------Binary Classification Report -------------

              precision    recall  f1-score   support

     Class 0       1.00      1.00      1.00        18
     Class 1       1.00      1.00      1.00        18

    accuracy                           1.00        36
   macro avg       1.00      1.00      1.00        36
weighted avg       1.00      1.00      1.00        36
```

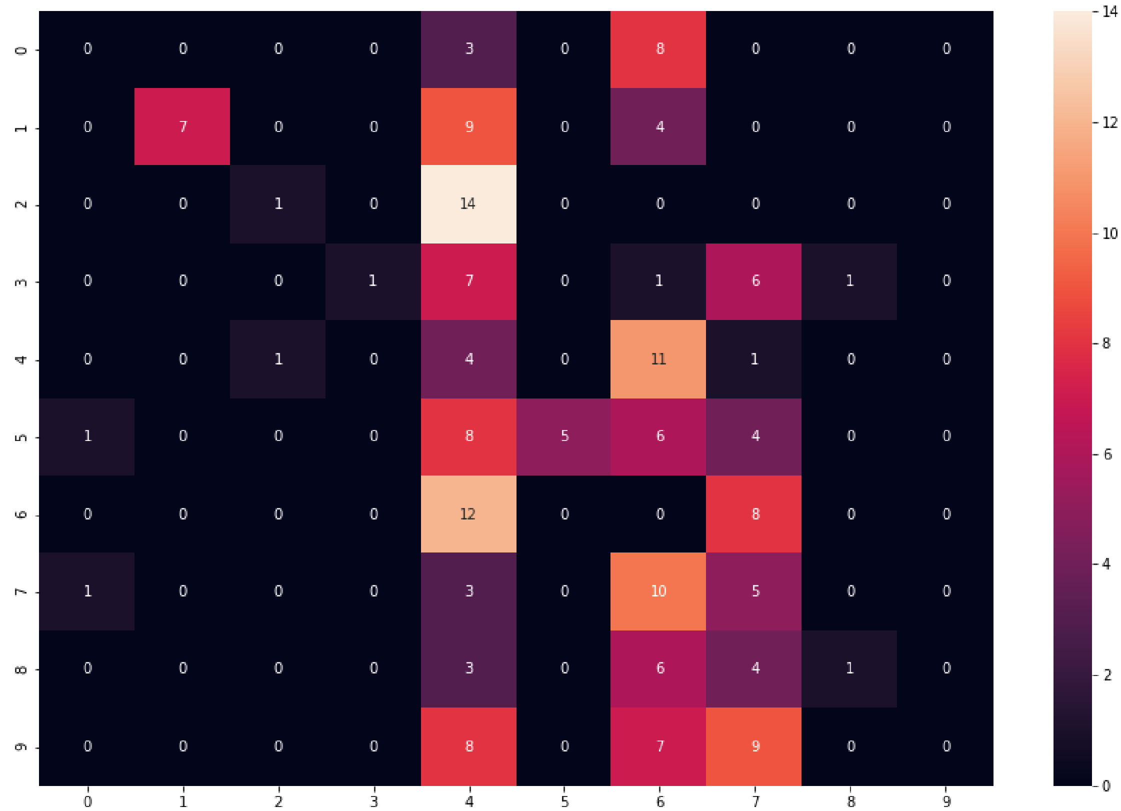


[12]:
```
digits = load_digits()
Xtemp = digits.data
ytemp = digits.target
# Number of classes
k = 10
# Splitting Train:test data into 9:1 ratio
Xtemp, X_test, ytemp, y_test = train_test_split(Xtemp, ytemp, test_size = 0.1)
# 3D arrays for storing data
X = []
total_samples = len(Xtemp)
for i in range(k):
  X.append( Xtemp[(ytemp == i)])

# Prior probability(1,10)
prior = []
for i in range(len(X)):
  prior.append(len(X[i])/total_samples)

meanVector = []
# mean vector: meanVector (10,64)
for i in range(len(X)):
  meanVectorT = []
  for j in range(len(X[0][0])):
    meanVectorT.append(np.sum(X[i][:,j])/len(X[0]))
  meanVector.append(meanVectorT)
```

```python
# covariance matrix (10,64,64)
covM = []
for i in range(len(X)):
  covM1 = []
  for j in range(len(X[0][0])):
    t = []
    for l in range(len(X[0][0])):
      t.append(covv(X[i][:,j] , X[i][:,l] ))
    covM1.append(t)
  covM1 = sing(covM1)
  covM.append(covM1)
n = len(X[0][0])

# taking average of all the covariance matrices
covariance = [[0 for i in range(n)] for j in range(n)]
for i in covM:
  covariance += i
covariance /= k


############### Testing #######################
yPredicted = []
yActual = []
for i in range(len(y_test)):
  test = np.transpose([X_test[i]])
  delta = []
  for j in range(k):
    delta.append(LDA(test,np.transpose([meanVector[j]]),covM[j],prior[j]))
  yPredicted.append(delta.index(max(delta)))
  yActual.append(y_test[i])


# Confusion matrix
fig = plt.figure(figsize = (15,10))
cm = confusion_matrix(yActual, yPredicted)
sns.heatmap(cm, annot = True)
print('---------- Multiclass Classification Report -------------\n')
labels = ['Class 0', 'Class 1', 'Class 2', 'Class 3', 'Class 4', 'Class␣
 ↪5','Class 6','Class 7', 'Class 8', 'Class 9']
print(classification_report(yActual, yPredicted, target_names = labels))
```

```
---------- Multiclass Classification Report -------------


C:\Users\Aryan Shah\anaconda3\lib\site-
packages\sklearn\metrics\_classification.py:1245: UndefinedMetricWarning:
Precision and F-score are ill-defined and being set to 0.0 in labels with no
```

```
predicted samples. Use `zero_division` parameter to control this behavior.
  _warn_prf(average, modifier, msg_start, len(result))
C:\Users\Aryan Shah\anaconda3\lib\site-
packages\sklearn\metrics\_classification.py:1245: UndefinedMetricWarning:
Precision and F-score are ill-defined and being set to 0.0 in labels with no
predicted samples. Use `zero_division` parameter to control this behavior.
  _warn_prf(average, modifier, msg_start, len(result))
C:\Users\Aryan Shah\anaconda3\lib\site-
packages\sklearn\metrics\_classification.py:1245: UndefinedMetricWarning:
Precision and F-score are ill-defined and being set to 0.0 in labels with no
predicted samples. Use `zero_division` parameter to control this behavior.
  _warn_prf(average, modifier, msg_start, len(result))

              precision    recall  f1-score   support

     Class 0       0.00      0.00      0.00        11
     Class 1       1.00      0.35      0.52        20
     Class 2       0.50      0.07      0.12        15
     Class 3       1.00      0.06      0.12        16
     Class 4       0.06      0.24      0.09        17
     Class 5       1.00      0.21      0.34        24
     Class 6       0.00      0.00      0.00        20
     Class 7       0.14      0.26      0.18        19
     Class 8       0.50      0.07      0.12        14
     Class 9       0.00      0.00      0.00        24

    accuracy                           0.13       180
   macro avg       0.42      0.13      0.15       180
weighted avg       0.43      0.13      0.16       180
```



3 Question 3

Perform Quadratic Discriminant Analysis (QDA) on the MNIST dataset* for multiclass classification. Plot confusion matric and find out the combinations where the classifier is confused in predicting the right label.

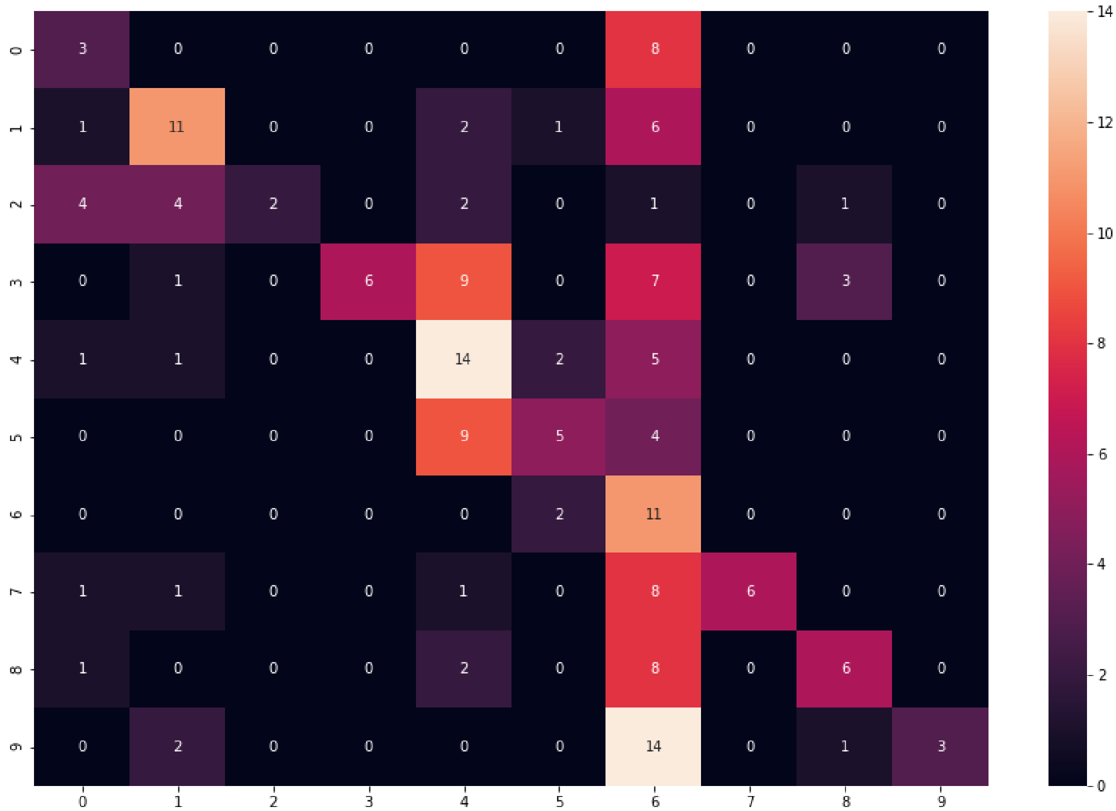
3.1 Code

[7]:
```
digits = load_digits()
Xtemp = digits.data
ytemp = digits.target
total_samples = len(Xtemp)
# total number of classes
k = 10

# splitting data into test and train
Xtemp, X_test, ytemp, y_test = train_test_split(Xtemp, ytemp, test_size = 0.1)
# 3D arrays for storing data
X = []
total_samples = len(Xtemp)
```

```
for i in range(k):
  X.append( Xtemp[( ytemp == i )])


# prior probabilitis
prior = []
for i in range(len(X)):
  prior.append(len(X[i])/total_samples)


meanVector = []
# mean vectors meanVector (10,64)
for i in range(len(X)):
  meanVectorT = []
  for j in range(len(X[0][0])):
    meanVectorT.append(np.sum(X[i][:,j])/len(X[0]))
  meanVector.append(meanVectorT)

# 3D covariance matrix
covM = []
# calculating covariance matrix for every class
for i in range(len(X)):
  covM1 = []
  for j in range(len(X[0][0])):
    t = []
    for l in range(len(X[0][0])):
      t.append(covv(X[i][:,j] , X[i][:,l] ))
    covM1.append(t)
  covM1 = sing(covM1)
  covM.append(covM1)

#number of random variables
n = len(X[0][0])

yPredicted = []
yActual = []
for i in range(len(y_test)):
  test = np.transpose([X_test[i]])
  delta = []
  for j in range(k):
    delta.append(QDA(test,np.transpose([meanVector[j]]),covM[j],prior[j]))
  yPredicted.append(delta.index(max(delta)))
  yActual.append(y_test[i])


# Confusion Matrix
fig = plt.figure(figsize = (15,10))
```

```python
cm = confusion_matrix(yActual, yPredicted)
sns.heatmap(cm, annot = True)
print('---------- Classification Report -------------\n')
labels = ['Class 0', 'Class 1', 'Class 2', 'Class 3', 'Class 4', 'Class 5',␣
 ↪'Class 6',
'Class 7', 'Class 8', 'Class 9']
print(classification_report(yActual, yPredicted, target_names = labels))
```

```
---------- Classification Report -------------

              precision    recall  f1-score   support

     Class 0       0.27      0.27      0.27        11
     Class 1       0.55      0.52      0.54        21
     Class 2       1.00      0.14      0.25        14
     Class 3       1.00      0.23      0.38        26
     Class 4       0.36      0.61      0.45        23
     Class 5       0.50      0.28      0.36        18
     Class 6       0.15      0.85      0.26        13
     Class 7       1.00      0.35      0.52        17
     Class 8       0.55      0.35      0.43        17
     Class 9       1.00      0.15      0.26        20

    accuracy                           0.37       180
   macro avg       0.64      0.38      0.37       180
weighted avg       0.67      0.37      0.38       180
```



3.2 Observation/Justification

The result is as shown in the graph/confusion matrix above. For classes 0,4,6 the classifier is not as precise in predicting the correct label/class as compared to other classes.

4 Question 4

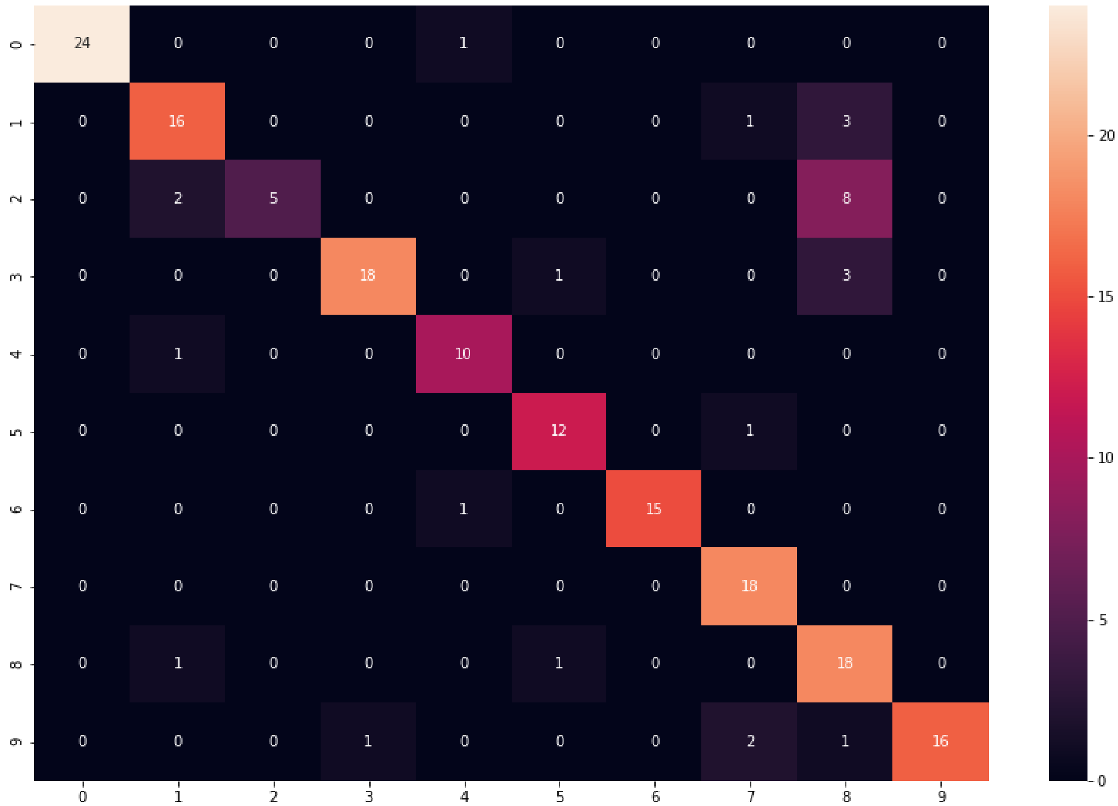
Perform Naïve-Bayes on the MNIST dataset* for multiclass classification. Plot confusion matric and find out the combinations where the classifier is confused in predicting the right label.

4.1 Code

[8]:
```
digits = load_digits()
Xtemp = digits.data
ytemp = digits.target
total_samples = len(Xtemp)

# total number of classes
k = 10

# splitting data into test and train
```

```python
Xtemp, X_test, ytemp, y_test = train_test_split(Xtemp, ytemp, test_size = 0.1)

# Using Naive-Bayes Classifier from sklearn
model = GaussianNB()
model.fit(Xtemp,ytemp)
yPredicted = model.predict(X_test)

# Confusion Matrix
fig = plt.figure(figsize = (15,10))
cm = confusion_matrix(y_test, yPredicted)
sns.heatmap(cm, annot = True)
print('---------- Classification Report -------------\n')
labels = ['Class 0', 'Class 1', 'Class 2', 'Class 3', 'Class 4', 'Class 5',␣
 ↪'Class 6',
'Class 7', 'Class 8', 'Class 9']
print(classification_report(y_test, yPredicted, target_names = labels))
```

```
---------- Classification Report -------------

              precision    recall  f1-score   support

     Class 0       1.00      0.96      0.98        25
     Class 1       0.80      0.80      0.80        20
     Class 2       1.00      0.33      0.50        15
     Class 3       0.95      0.82      0.88        22
     Class 4       0.83      0.91      0.87        11
     Class 5       0.86      0.92      0.89        13
     Class 6       1.00      0.94      0.97        16
     Class 7       0.82      1.00      0.90        18
     Class 8       0.55      0.90      0.68        20
     Class 9       1.00      0.80      0.89        20

    accuracy                           0.84       180
   macro avg       0.88      0.84      0.84       180
weighted avg       0.88      0.84      0.84       180
```



4.2 Observation/Justification

The result is as shown in the graph/confusion matrix above. For class 8 the classifier is not as precise in predicting the correct label/class as compared to other classes.

5 Question 5

Mean and variance of two classes are, Class_1 : $\mu = 8$, $\sigma^2 = 20$ Class_2 : $\mu = 16$, $\sigma^2 = 25$

a. Draw 50 random samples from N[5,20]
b. Draw 50 random samples from N[11,10]
c. Draw 50 random samples from N[20,8]

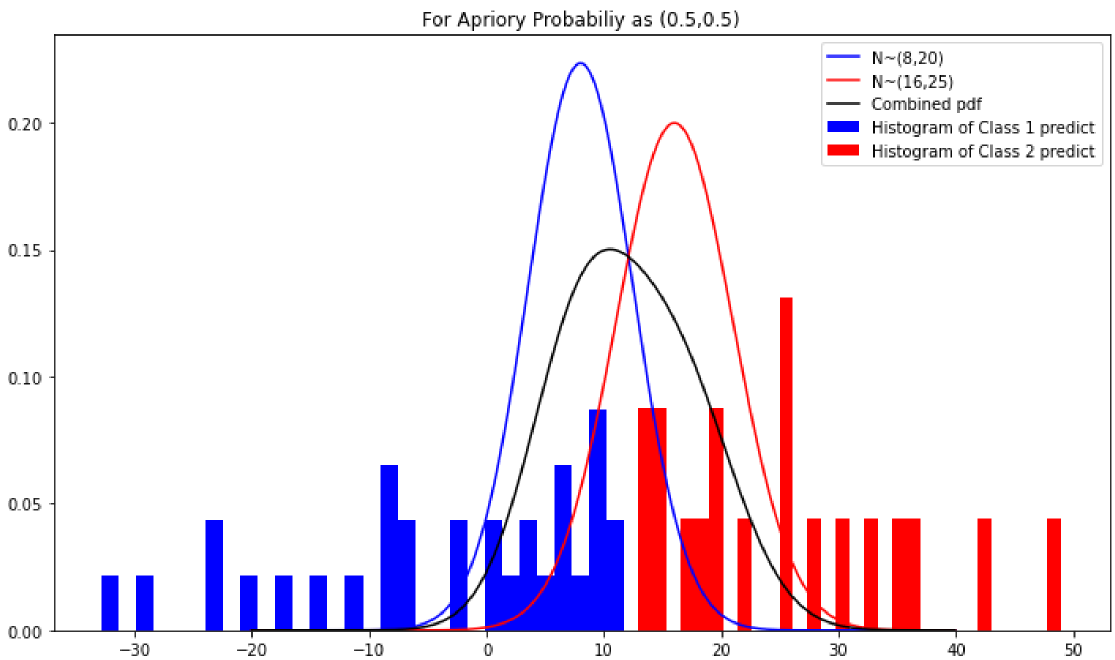
and classify using Naïve-Bayes classifier having apriory probabilities as (0.5,0.5), (0.3,0.7) and (0.7,0.3) and visualize data and class by plotting histogram.

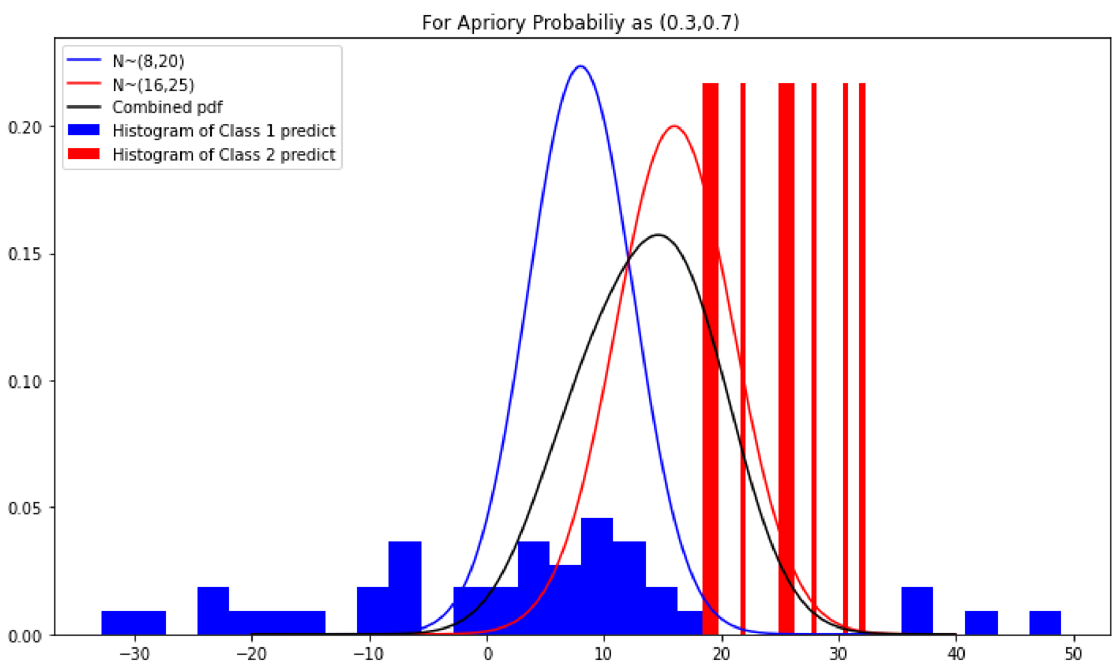
5.1 Code

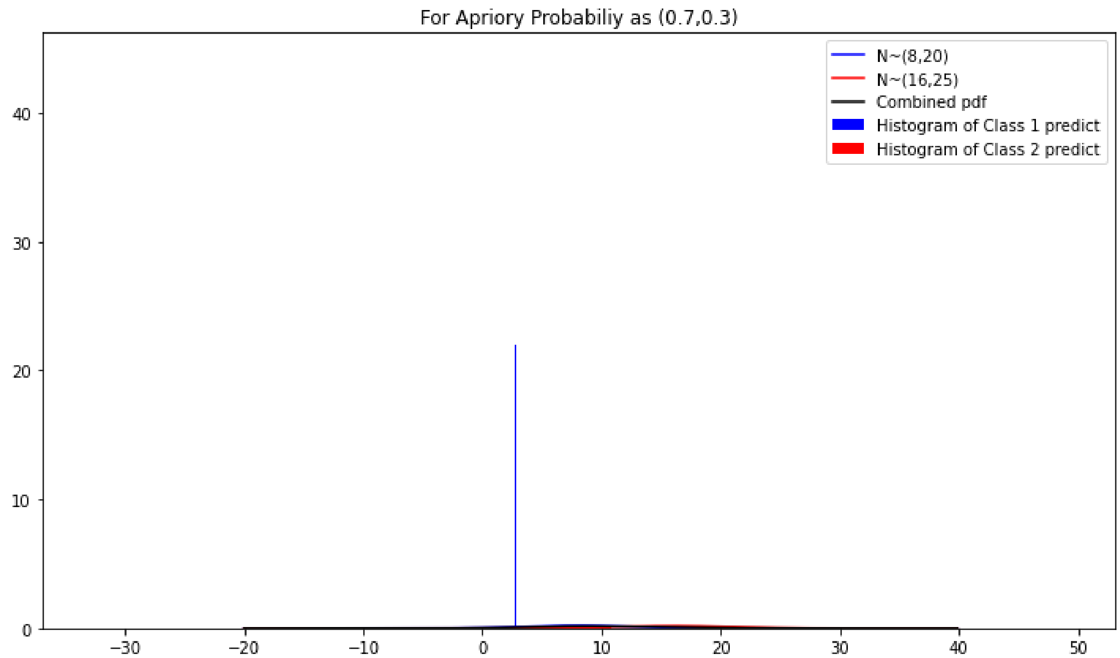
[9]:
```
import math
def plotHistograms(X, Y, apriory1, apriory2):
  x = np.arange(-20, 40, 0.1)
  pdf0 = ((1/math.sqrt(20))*np.exp((-1/2)*((x - 8)/math.sqrt(20))**2))
```

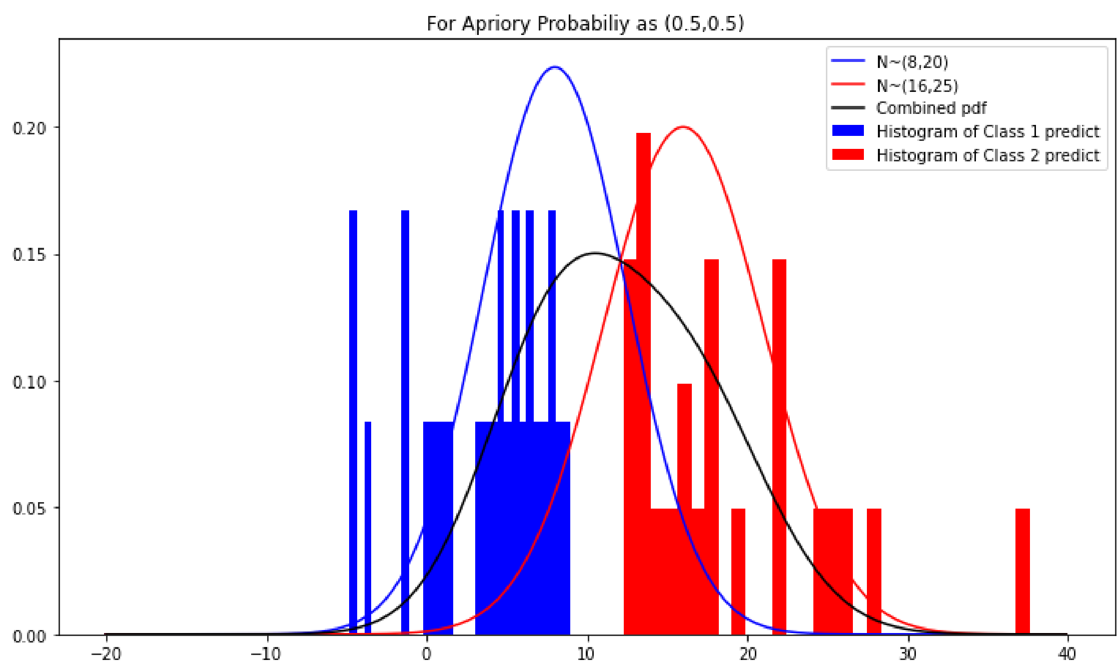
```
  pdf1 = ((1/math.sqrt(25))*np.exp((-1/2)*((x - 16)/math.sqrt(25))**2))
  plt.figure(figsize = (10, 6))

  plt.title(f"For Apriory Probabiliy as ({apriory1},{apriory2})")

  plt.hist(X[Y==0], bins=30, density=True, color='blue', label='Histogram of Class 1 predict')
  plt.plot(x, pdf0, 'b', label='N~(8,20)')


  plt.hist(X[Y==1], bins=30, density=True, color='red', label='Histogram of Class 2 predict')
  plt.plot(x, pdf1, 'r', label='N~(16,25)')


  pdf2 = apriory1*pdf0 + apriory2*pdf1
  plt.plot(x, pdf2, 'k', label='Combined pdf')
  plt.tight_layout()
  plt.legend()
  plt.show()


def predict(X_i, apriory1, apriory2):
  value1 = ((1/math.sqrt(20))*np.exp((-1/2)*((X_i - 8)/math.sqrt(20))**2))**apriory1
  value2 = ((1/math.sqrt(25))*np.exp((-1/2)*((X_i - 16)/math.sqrt(25))**2))**apriory2
  if value1 > value2:
    return 0
  else:
    return 1


def naive_bayes(X, apriory1, apriory2):
  Y = []
  for i in range(50):
    Y.append(predict(X[i], apriory1, apriory2))
  Y = np.array(Y)
  #Calling plot histogram function
  plotHistograms(X, Y, apriory1, apriory2)


def dataGen(mu, sigma):
  #Generating data for given mu and sigma
  X = np.random.normal(mu, sigma, 50)
  naive_bayes(X, 0.5, 0.5)
  naive_bayes(X, 0.3, 0.7)
```

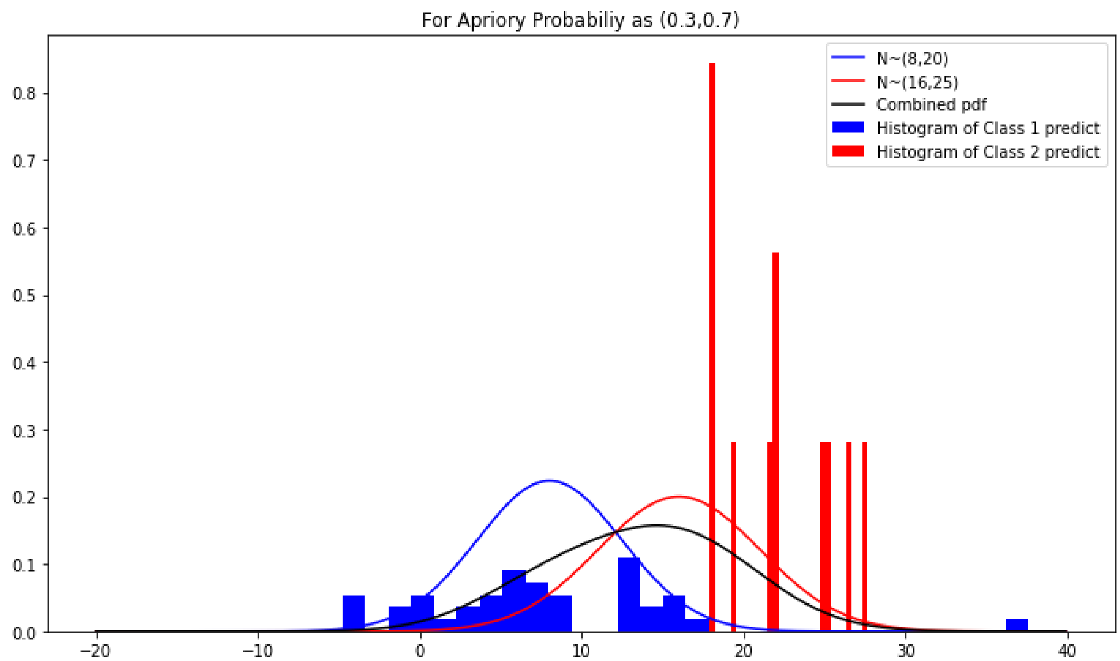
```
  naive_bayes(X, 0.7, 0.3)

dataGen(5, 20)
dataGen(11, 10)
dataGen(20, 8)
```

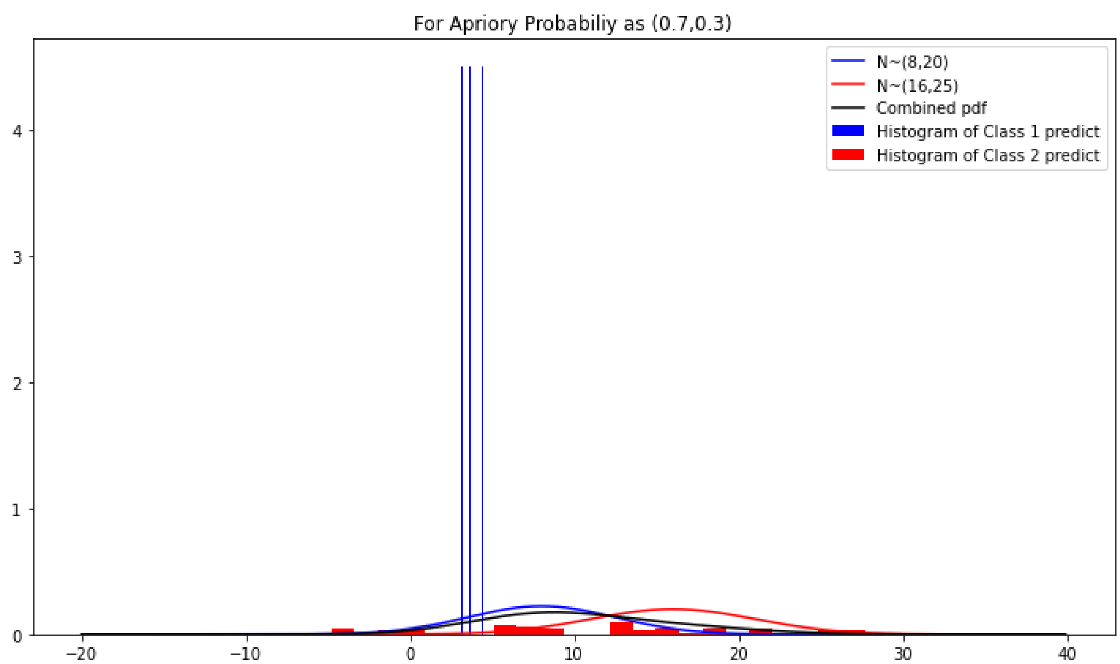


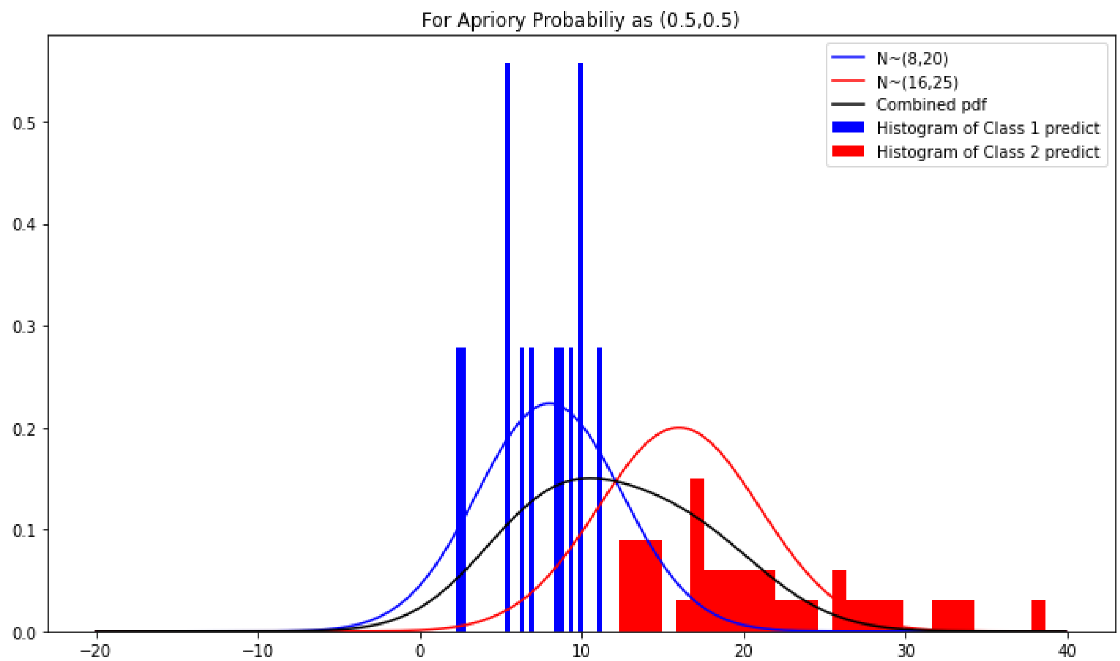


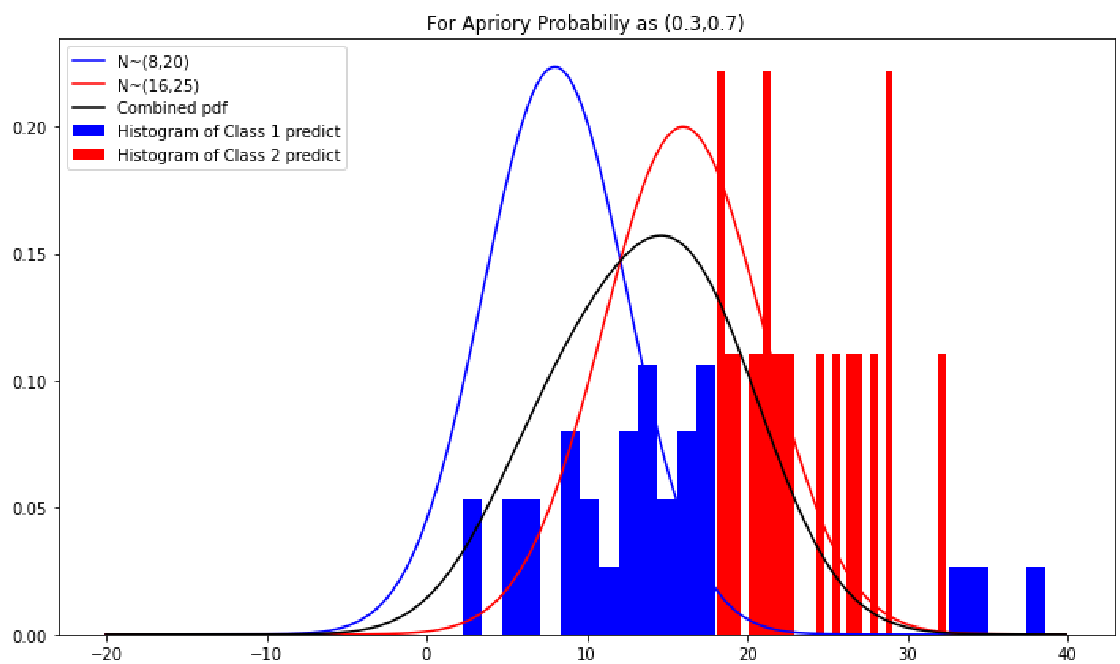

For Apriory Probabiliy as (0.7,0.3)
N~(8,20)
N~(16,25)
Combined pdf
Histogram of Class 1 predict
Histogram of Class 2 predict

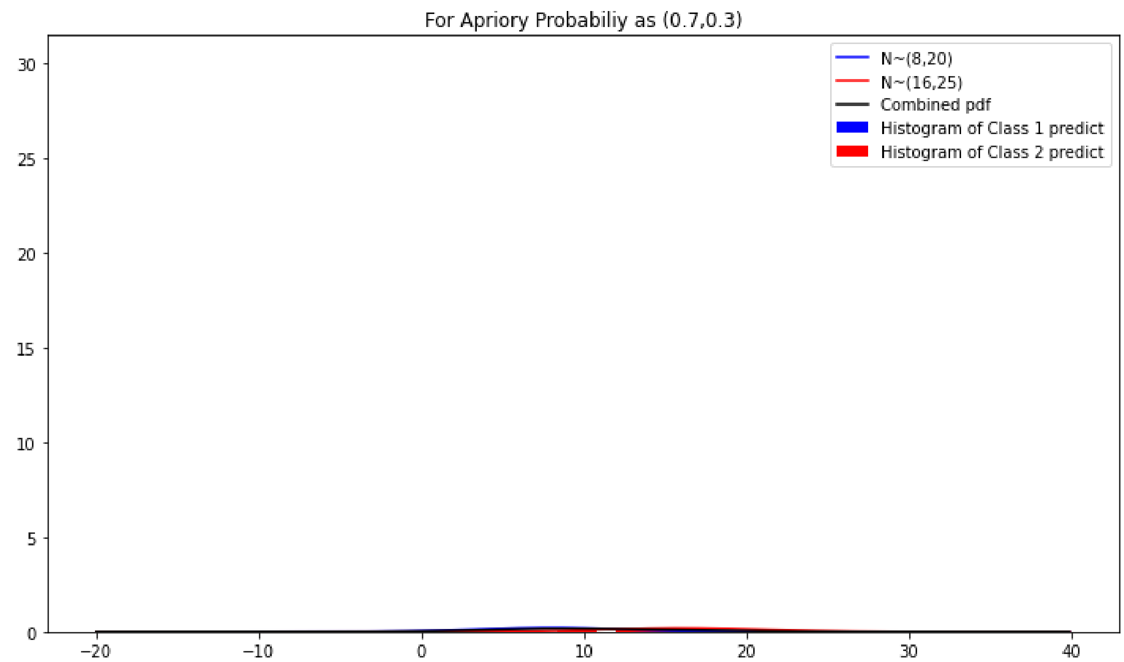

For Apriory Probabiliy as (0.5,0.5)
N~(8,20)
N~(16,25)
Combined pdf
Histogram of Class 1 predict
Histogram of Class 2 predict


For Apriory Probabiliy as (0.3,0.7)
N~(8,20)
N~(16,25)
Combined pdf
Histogram of Class 1 predict
Histogram of Class 2 predict


For Apriory Probabiliy as (0.7,0.3)
N~(8,20)
N~(16,25)
Combined pdf
Histogram of Class 1 predict
Histogram of Class 2 predict


For Apriory Probabiliy as (0.5,0.5)
N~(8,20)
N~(16,25)
Combined pdf
Histogram of Class 1 predict
Histogram of Class 2 predict


For Apriory Probabiliy as (0.3,0.7)
N~(8,20)
N~(16,25)
Combined pdf
Histogram of Class 1 predict
Histogram of Class 2 predict



5.2 Result

The plotted graphs have been shown above.