SYIT APPLIED MATHEMATICS 2023-24 QUESTION BANK

UNIT 1

1) Find the inverse of the given matrix using adjoint method: $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

2) Verify Cayley Hamilton theorem and find A^T, where $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

3) Find the characteristic values (eigenvalues) of the following: $B = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

4) Examine for non-trivial solution and solve them:

   $5x + 2y - 3z = 0$; $3x + y + z = 0$; $2x + y + 6z = 0$

5) Reduce $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 0 \end{bmatrix}$ row echelon form. Hence find the rank of A.

6) Express the given complex number $z = \sqrt{3} + i$ in polar and exponential form.

7) Find modulus and amplitude for $\sqrt{-5 + 12i}$

8) Show that $A = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ is an orthogonal matrix.

9) Test the consistency of the equations and solve if consistent:

   $2x - y + z = 8$; $3x - y + z = 6$; $4x - y + 2z = 7$: $-x + y - z = 4$

10) If $2\, cos\theta = x + \frac{1}{x}$ $prove\ that 2 \cos r\theta = x^r + \frac{1}{x^r}$

11) Using Euler's formula prove that $\sin^2\theta + \cos^2\theta = 1$

12) Simplify using De Moivre's Theorem:

$$\frac{(cos\,3\theta - i\,sin\,3\theta)^2 (cos\,2\theta + i\,sin\,2\theta)^{\frac{3}{2}}}{(cos\,5\theta + i\,sin\,5\theta)\left(cos\,\frac{5\theta}{2} - i\,sin\,\frac{5\theta}{2}\right)^4}$$

13) Express in Polar from $(-1 - \sqrt{3}\, i)$

14) Evaluate $(1 + i)^{100} + (1 - i)^{100}$

15) Evaluate $\frac{(\cos 2\theta + i \sin 2\theta)^3 (\cos 3\theta - i \sin 3\theta)^2}{(\cos 4\theta + i \sin 4\theta)^5 (\cos 5\theta - i \sin 5\theta)^5}$

16) Find the inverse of the matrix by adjoint method $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$

17) Express the following matrix as a sum of symmetric and skew-symmetric matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

18) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 5 & 7 & 8 \\ 5 & 6 & 6 & 4 \end{bmatrix}$

UNIT 2

1) Solve $\frac{dy}{dx} = \tan\left(\frac{y}{x}\right) + \frac{y}{x}$
2) Solve $(x^4 + y^4)dx - xy^3\, dy = 0$
3) Solve: $\sin(px - y) = p$
4) Solve: $(D^3 - 2D^2 - 5D + 6)\, y = \sin(x)$, where $D = \frac{d}{dx}$
5) Solve : $(x - 4xy - 2y^2)\, dx + (y^2 - 4xy - 2x^2)\, dy = 0$
6) Solve: $2e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$
7) Solve: $xp^2 - 2yp + ax = 0$
8) Solve : $\frac{d^4y}{dx^4} - \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0$
9) Solve $\frac{dy}{dx} = x\sqrt{25 - x^2}$
10) Solve $(e^{2x} + \sin x \cos y)dx + (\tan y + \cos x \sin y)dy = 0$
11) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$
12) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 0$
13) Solve : $x\frac{dy}{dx} + y = x^3y^6$
14) Solve : $\frac{dy}{dx} = \frac{x^3 + y^3}{x\, y^2}$
15) Solve : $\sec^2 x \,.\tan y\, dx + \sec^2 y \,.\tan x\, dy = 0$
16) Solve : $\frac{dy}{dx} = \frac{x + y + 1}{x + y - 1}$
17) Solve : $\frac{dy}{dx} - \frac{2y}{(x + 1)} = (x + 1)^3$
18) Solve : $(D^3 + 2D^2 + D)\, y = e^{-x} + \sin 2x$

UNIT 3

1) Find Laplace transform of $f(t) = \sin at$

2) Find inverse Laplace transform of $\tan^{-1}\left(\frac{1}{s}\right)$

3) Find inverse Laplace transform of $\frac{1}{s^3(s^2+1)}$

4) Find inverse Laplace transform by using convolution theorem $\frac{1}{(s+1)(s^2+1)}$

5) Find Laplace transform of $\frac{1-\cos t}{t}$

6) Find inverse Laplace transform of $\frac{s}{(s-2)^4}$

7) Find the Laplace transform of : $f(t) = [cos(3t)]^2$

8) Find the Laplace transform of :

$$f(t) = \int_0^t \frac{sin\,x}{x} dx$$

9) Find the Laplace transform of :$f(t) = 5t^2 + 4e^{-t}$

10) Find the Laplace transform of :$f(t) = t^3 + 5t - 2$

11) Find the inverse Laplace transform of : $F(s) = \frac{1}{s-2} - \frac{3s}{s^2+16} + \frac{5}{s^2+9}$

12) Find the inverse Laplace transform of : $F(s) = \frac{1}{s^2-8s+25}$

13) Find the Inverse Laplace transform of $\left(\frac{1}{s^2-4s-5}\right)$

14) Find the Laplace transformation of the function $f(t) = te^{-2t}\cos 3t$

15) Find the inverse Laplace of $\varphi\,(\mathrm{s}) = \frac{\mathrm{s}}{(\mathrm{s}^2+4)(\mathrm{s}^2+9)}$

16) Find the Laplace transformation of the function $\mathrm{f(t)} = \frac{\cos 3\mathrm{t}}{\mathrm{t}}$

17) Find the Inverse Laplace transform of the function $\frac{\mathrm{s}}{(\mathrm{s}-2)^4}$

18) Find the Laplace Transform of the function $\mathrm{f(t)} = \mathrm{t}^3\,\mathrm{e}^{-2\mathrm{t}}$

UNIT 4

1) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y\, dy\, dx$

2) Evaluate $\int_0^1 \int_0^y xy\, e^{-x^2}\, dx\, dy$

3) Evaluate $\iiint (x+y+z)\, dx\, dy\, dz$ over the tetrahedron bounded by the planes x=0, y=0, z=0 and x + y + z = 1.

4) Evaluate $\iint y\ dx\, dy$ over the area bounded by $x = 0, y = x^2\ and\ x + y = 2$ in the first quadrant

5) Evaluate $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\ dx\, dy\, dz$

6) Find $\int_0^3 \int_0^2 (1+(x-1)^2 y + 4y^2)\, dy\, dx$

7) Find $\iint_R 1\, dA$where the region R is bounded by the curves $y^2 = 2x$, $y = x$

8) Show that $\int_0^{\frac{\pi}{2}} \int_0^1 y \sin x\, dy\, dx = \frac{1}{2}$

9) Evaluate $\int_0^1 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$

10) Find the area of the region bounded by the curves $x = y^2 - 1$ and $y = 1 - x$ over the constant function $f(x, y) = 1$

11) Find the volume of the solid bounded by:

$0 \leq x \leq 1, 0 \leq y \leq x$ and $x + y \leq z \leq e^{x+y}$.

12) Find the volume of the solid bounded by the four planes:

$$x = 0, y = 0, z = x + y, z = 1 - x - y$$

13) Evaluate $\int_0^1 \int_0^2 e^{x+y}\ dx\, dy$

14) Evaluate $\int_0^1 \int_0^1 \frac{dx\ dy}{\sqrt{(1-x^2)(1-y^2)}}$

15) Evaluate $\iiint \frac{dx\ dy\ dz}{\sqrt{1-x^2-y^2-z^2}}$ taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$in the positive quadrant.

16) Evaluate $\iint y\,dx\,dy$ over the area bounded by $y = x^2$ and x + y = 2.

17) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z\ dx\ dy\ dz$

18) Evaluate $\iiint \frac{dx\ dy\ dz}{(1+x+y+z)^3}$over the volume of the tetrahedron x = 0, y = 0, z = 0 and x + y + z = 1.

UNIT 5

1) Evaluate : $\int_0^\infty e^{-x^3}\, dx$

2) Evaluate: $\int_0^\infty \sqrt{x} \cdot e^{-\sqrt[5]{x}}\, dx$

3) Prove that: $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \pi\sqrt{2}$

4) Evaluate: $\int_0^1 \left(\frac{1-x^\alpha}{\log x}\right) dx\,, \alpha \geq 0$

5) Using $\int_0^\infty \quad e^{-xy} = \frac{1}{y}, y \geq 0$, deduce that $\int_0^\infty \frac{e^{-\alpha x} - e^{-\beta x}}{x} dx = log\left(\frac{b}{a}\right); a > 0; b > 0$

6) Prove that $\int_0^t erf(ax)\, dx + \int_0^t erf_c(ax)\, dx = t$

7) Define error function and prove that $erf(0) = 0$

8) Prove that: $\beta(m, n) = \beta(n, m)$.

9) Evaluate $\int_0^\pi x \sin^6 x \;\, dx$

10) Show that: $\int_0^\infty \frac{1-e^{-ax}}{x}.\; e^{-x}\,.\; dx = \log{(a + 1)}$

11) Show that: $\mathrm{erf}(x) + erf_c(x) = 1$

12) Define gamma function and show that $\Gamma_n = \int_0^1 (\log\frac{1}{y})^{n-1} \; dy$

13) Evaluate $\int_0^1 \frac{\mathrm{x}^7}{(1-\mathrm{x}^4)^{1/2}} \; \mathrm{dx}$

14) Evaluate: $\mathrm{erf}\;(\mathrm{x}) \;+\; \mathrm{erf_c}\;(\mathrm{x})$

15) Evaluate $\int_0^1 \frac{\mathrm{x}^\propto \;-\; 1}{\log \mathrm{x}} \;\; \mathrm{dx}$

16) Evaluate $\int_0^1 \frac{\mathrm{x}^\mathrm{a} \;-\; \mathrm{x}^\mathrm{b}}{\log \mathrm{x}} \;\; \mathrm{dx}$

17) Evaluate $\int_0^{\pi/2} sin^5\, 2x \;\;\, dx$

18) Find : $\frac{d}{dx}[erf\;(x) \;+\; erf_c\;(ax)]$