

1.1.1

A graph that is symmetric about the x-axis cannot be the graph of a function, because it cannot be solved for a unique y . A rule for relations in set theory is that a relation F from A to B cannot be called a function if it is not true that each element in the domain of F is paired with just one element in the range (see Ling 2006: 9)¹. If you pair the value of x from set X with the value of 0 from set Y , it does not violate this rule, as there can always only be one element 0 within a set.

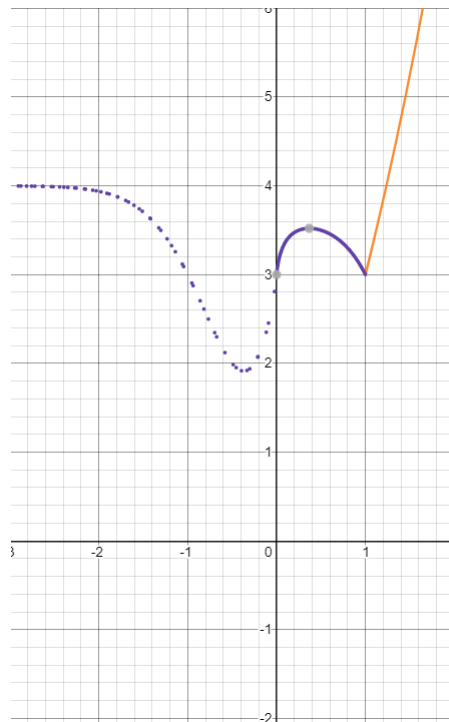
1.1.2

$$F(x) = \begin{cases} 4 - x^{2x}, & x \leq 1 \quad (W_0) \\ x^2 + 2x, & x > 1 \quad (W_1) \end{cases}$$

$x \ (W_0)$	$y \ (W_0)$
1	3
0	3
-1	3
-2	3,9375
-3	3,9986

4 is the asymptote for (A).

$x \ (W_1)$	$y \ (W_1)$
2	8
3	15
4	24
5	35
1000	1 002 000



Function graph of piecewise function $F(x)$ on $[-3, 2]$
<https://www.desmos.com/calculator/woigw9qykh>

1.2.3

$$\sin(\theta) = \cos(\pi/2 - \theta)$$

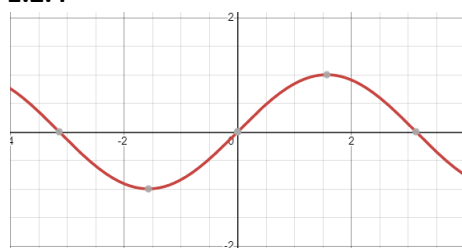
$$\sin(\theta) = \cos(\pi/2) \cdot \cos(\theta) + \sin(\pi/2) \cdot \sin(\theta)$$

$$\sin(\theta) = 0 \cdot \cos(\theta) + 1 \cdot \sin(\theta)$$

0 = 0 [I don't think this is how we were supposed to do it though.]

$$\begin{aligned} \cos(A - B) &= \cos(A) \cdot \cos(B) + \sin(A) \cdot \sin(B) \\ \sin(A + B) &= \sin(A) \cdot \cos(B) + \cos(A) \cdot \sin(B) \end{aligned}$$

1.2.4



$$y = a \cdot \cos(b \cdot x \pm c)$$

$$\text{in our case, } y = \cos(x - \pi/2)$$

natural period is 2π if coefficient of $b = 1$

if coefficient $\neq 1$, calculate $2\pi/b$ to get the period

Function graph of $y = \cos(x - \pi/2)$

<https://www.desmos.com/calculator>

¹ https://people.umass.edu/partee/NZ_2006/Set%20Theory%20Basics.pdf

1.3.5

$$\ln\left(\frac{t}{t-1}\right) = 2$$

$$\frac{t}{t-1} = e^2$$

$$t = e^2(t-1)$$

$$t = e^2t - e^2$$

$$t - e^2t = -e^2$$

$$t(1 - e^2) = -e^2$$

$$t = \frac{-e^2}{1 - e^2}$$

1.3.6

$$e^{2t} - 3e^t = 0$$

$$e^{2t} = 3e^t$$

$$2t \ln e = 3t \ln e$$

$$2t = 3t \quad \nrightarrow$$

no solution

1.4.7

$$\binom{n}{k} \binom{n-2}{k-2} = \binom{n}{k} \binom{k}{2}$$

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$$

I didn't really know what you wanted us to do here exactly. Should we just have used induction with $n+1$? Or should we have written about different ways of counting? I read some resources online² but I did not have enough time (and brainpower) left to wrap my head around what was expected of us.

1.4.8

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \binom{2n+1}{3} \text{ for } n \geq 1$$

$$[1] \quad (2n+1)! / ((2n+1)-k)! \cdot k! \rightarrow (3)! / (0! \cdot 3!) \rightarrow 6/1 \cdot 6 \rightarrow 1 = 1^2$$

$$[2] \quad (2n+1)! / ((2n+1)-k)! \cdot k! \rightarrow (5)! / (2! \cdot 3!) \rightarrow 120/2 \cdot 6 \rightarrow 10 = 1^2 + 3^2$$

$$[3] \quad (2n+1)! / ((2n+1)-k)! \cdot k! \rightarrow (7)! / (4! \cdot 3!) \rightarrow 5040/24 \cdot 6 \rightarrow 35 = 1^2 + 3^2 + 5^2$$

try to proof with $n+1$ (through induction)

$$[n+1] \quad (2(n+1)+1)! / ((2(n+1)+1)-k)! \cdot k! = (2n+1)! / ((2n+1)-k)! \cdot k! \\ (2n+3)! / ((2n+3)-k)! \cdot k! = (2n+1)! / ((2n+1)-k)! \cdot k!$$

² http://discrete.openmathbooks.org/dmoi2/sec_comb-proofs.html

$$\begin{aligned}
(2n+3)!/((2n+3)-3)! \cdot 3! &= (2n+1)!/((2n+1)-3)! \cdot 3! \\
(2n+3)!/((2n)! \cdot 3! &= (2n+1)!/((2n-2)! \cdot 3! \\
(2n+3)!/(2n)! &= (2n+1)!/(2n-2)! \\
(2n+3)!/(2n)! &= (2n+1)!/2(n-1)! \\
(2n+3)!/n! &= (2n+1)!/(n-1)! \\
(2n+3)! \cdot (n-1)!/n! &= (2n+1)! \\
(2n+3)! \cdot (n-1)! &= (2n+1)! \cdot n!
\end{aligned}$$

[I got stuck, must have made a mistake somewhere.]

Time ran out, so I didn't finish all of it, and some of it is wrong. So I have gotten rustier than I thought. Especially the combinatorics part was hard for me, as we never learned about induction in school, but only at university, and it was never my strong suit. The complex numbers part would have been doable I think, probably using polar coordinates to geometrically display complex roots. And I think the other one would probably have something to do with "complex composites" (if I recall the name correctly).

Would be happy if you did some kind of writeup on this. Or maybe upload the video. Thanks.