Problem Set 0 : Prerequisites

18.01 Single Variable Calculus

19th September, 2021

1 Problems

1.1 Functions and their Graphs

1. For a curve to be *symmetric about the x-axis*, the point (x, y) must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the x-axis is not the graph of a function, unless the function is $y = 0$.

2. Graph this piecewise function:

$$F(x) = \begin{cases} 4 - x^{2x}, & x \le 1 \\ x^2 + 2x, & x > 1 \end{cases}$$

1.2 Trigonometry

3. Apply the formula for $\cos(A - B)$ to the identity $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ to obtain the addition formula for $\sin(A + B)$.

4. Graph the function $\cos\left(x - \frac{\pi}{2}\right)$ and find its period.

1.3 Transcendental Functions (e and $\ln x$)

Solve for t:

5. $\ln \left(\dfrac{t}{t-1} \right) = 2$

6. $e^{2t} - 3e^t = 0$

1.4 The Binomial Theorem

7. Give an algebraic and combinatorial proof of the identity[1]

$$\binom{n}{k}\binom{n-2}{k-2} = \binom{n}{k}\binom{k}{2}$$

8. For $n \geq 1$, verify that

$$1^2 + 3^2 + 5^2 \cdots + (2n+1)^2 = \binom{2n+1}{3}$$

1.5 Complex Numbers [Optional][2]

9. Solve for real numbers x and y:

$$\left(\dfrac{1+i}{1-i} \right)^2 + \dfrac{1}{x+iy} = 1+i$$

10. Find all solutions of the equation $x^4 + 4x^2 + 16 = 0$.

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[1]Here algebraic proof means just using the binomial theorem and *algebraically* proving the given identity. But the combinatorial proof is one where you prove the identity using combinatorial arguments, i.e. by coming up with several ways of counting.

[2]The following problems are not necessarily required for 18.01 and thus they're optional for those who are interested