PAGE NO.:
DATE: / /

Q3 $\cos(A-B) = \cos A\cos B + \sin A \sin B$

$$\sin\theta = \sin\left(\frac{\pi}{2}-\theta\right)$$

$$\sin(A+B) = \cos\left(\frac{\pi}{2}-(A+B)\right)$$

$$= \cos\left(\left(\frac{\pi}{2}-A\right)-B\right)$$

$$= \cos\left(\frac{\pi}{2}-A\right)\cos B + \sin\left(\frac{\pi}{2}-A\right)\sin B$$

$$= \boxed{\sin A\cos B + \cos A\sin B}$$

Q4 $\cos\left(x-\frac{\pi}{2}\right)$

$$\cos\left(-\left(\frac{\pi}{2}-x\right)\right)$$

$$= \cos\left(\frac{\pi}{2}-x\right) \quad \left(\because \cos \text{ is an even function}\right)$$

$= \boxed{\sin x}$ So here is the same old sine wave.

PAGE NO.:
DATE: / /

Q5 $\ln\left(\frac{t}{t-1}\right) = 2$

$$e^2 = \frac{t}{t-1}$$

$$e^2 t - e^2 = t$$

$$e^2 t - t - e^2 = 0$$

$$t(e^2-1) - e^2 = 0$$

$$\boxed{t = \frac{e^2}{e^2-1}}$$

Q6 $e^{2t} - 3e^t = 0$

$$e^t(e^t - 3) = 0$$

Then either $e^t = 0$ or $e^t - 3 = 0$

$e^t = 0$ ✗ | $e^t - 3 = 0$

Not possible | $e^t = 3$

PAGE NO. :
DATE : / /

$\Rightarrow$ ~~log~~ $\ln 3 = t$

$2.303 \log_{10} 3 = t$

→ To find log 3 to a good approx, we take the square root of 3, 13 times and then subtract 1 and multiply by 3558.

$2.303 \times 0.477188 = t$

$\boxed{1.098964 \approx t}$

A fair approximate of t

P(n):

Q8 $1^2 + 3^2 \ldots + (2n-1)^2 = \binom{2n+1}{3}$

$P(1) = (2-1)^2 = \underline{1}$ $\underline{LHS}$

$RHS = \binom{3}{3} = \underline{1}$

$\Rightarrow$ $\underline{LHS = RHS} \Rightarrow \underline{P(1) \text{ is true}}$

Assume P(K) to be true for some $K \in N$.

$\Rightarrow 1^2 + 3^2 \ldots + (2K-1)^2 = \binom{2K+1}{3}$

for $n = K+1$,

$\underline{LHS}$ $1^2 + 3^2 \ldots + (2K-1)^2 + (2K+1)^2$

$= \binom{2K+1}{3} + \binom{2K+1}{1}\binom{2K+1}{1}$

$(2K+1)^2$

$$= \frac{2k+1!}{3!(2k-2)!} + \frac{(2k+1)!}{1!\,2k!} \times \frac{(2k+1)!}{1!\,2k!}$$

$$= \frac{(2k+1)!}{(2k-2)!}\left[\frac{1}{3!} + \frac{1}{(2k)(2k-1)} \times \frac{1}{(2k)(2k-1)}\right]$$

$$(2k-1)(2k)(2k+1)\left[\frac{(2k)^2(2k-1)^2+3!}{(2k)^2(2k-1)^2\,3!}\right]$$

$$\cancel{(2k-1)}\cancel{(2k)}(2k+1)\left[\frac{(2k)^2(2k-1)^2+3!}{(2k)^2(2k-1)^2\,3!}\right]$$

$$(2k+1)\left[\frac{(2k)(2k-1)}{3!} + \frac{1}{(2k)(2k-1)}\right]$$

2k+2

2k+

2k+3

P(K)

$$Q = 1^2+3^2 \ldots + (2k-1)^2 = \binom{2k+1}{3}$$

P(1)

$$LHS = (2-1)^2 = \underline{1}$$

$$\underline{RHS} = \binom{3}{3} = \underline{1}$$

P(1) is true

Assu P(a) to be true for some $a \in N$.

$$① - 1^2+3^2 \ldots + (2a-1)^2 = \binom{2a+1}{3}$$

For P(a+1),

LHS

$$1^2+3^2 - \ldots + (2a-1)^2 + (2a+1)^2$$

$$= \binom{2a+1}{3} + \binom{2a+1}{1}\binom{2a+1}{1}$$

$$(2a+1)! \left[\frac{1}{3!(2a-2)!} + \frac{1}{(2a)!(2a)!} \right]$$

$$(2a+1)! \left[\frac{(2a)!(2a)! + 3!(2a-2)!}{\cancel{(2a)!}(2a)!\,3!\,(2a-2)!} \right]$$

$$(2a+1) \left[\frac{\cancel{(2a-2)!}\left[(2a-1)(2a)(2a)! + 3!\right]}{(2a)!\,3!\,\cancel{(2a-2)!}} \right]$$

$$(2a+1) \left[\frac{(2a-1)(2a)(2a)! + 3!}{2a!\,3!} \right]$$

$$\frac{(2a+1)(2a-1)(2a)}{3!} + \frac{(2a+1)}{2a!}$$

PAGE NO.:
DATE: / /

Q9 $\left(\frac{1+i}{1-i}\right)^2 + \left(\frac{1}{x+iy}\right) = 1+i$

$= \left(\frac{\cancel{2}i}{\cancel{2}}\right)^2 + \left(\frac{1}{x+iy}\right) = 1+i$

$= -x - iy + 1 = x + ix + iy - y$

$= (1-x) - iy = (x-y) + i(x+y)$

$(1-x) - iy = (x-y) - i(-x-y)$

Compare

$y = -x - y$

$\boxed{2y = -x}$ — 1

PAGE NO. :
DATE : / /

$$1 - x = x - y$$

$$1 + y = 2x$$

$$1 + y - 2x = 0 \qquad \boxed{\text{eq } 1}$$

$$1 + y + 4y = 0$$

$$\boxed{y = \frac{-1}{5}}$$

$$\boxed{x = -2y = \frac{2}{5}}$$

Q10 $x^4 + 4x^2 + 16 = 0$

Assume $t = x^2$ then

$$t^2 + 4t + 16 = 0$$

$$\cancel{t^2} +$$

$$D = 16 - 4 \times 16$$

$$= \underline{-48}$$

$$\cancel{t} = \frac{-4 \pm \sqrt{-48}}{2} = \frac{-4 \pm 4\sqrt{3}\, i}{2} = -2 \pm 2\sqrt{3}$$

$$-2 \pm 2\sqrt{3}i = t$$

$$x^2 = -2 + 2\sqrt{3}i, \quad x^2 = -2 - 2\sqrt{3}i$$

$$\cancel{x^4 = -4 \quad 12 - 2\sqrt{3}}$$

$$\boxed{x = \pm\sqrt{-2 + 2\sqrt{3}i}} \qquad \boxed{x = \pm\sqrt{-2 - 2\sqrt{3}i}}$$