PSet

Q1 $\frac{x-1}{x+1} = \frac{(x-1)^2}{x^2-1} = \frac{x^2-2x+1}{x^2-1}$

$$\left(\frac{x^2+1}{x^2-1}\right) + \left(\frac{-2x}{x^2-1}\right)$$

Analyzing both we see that,

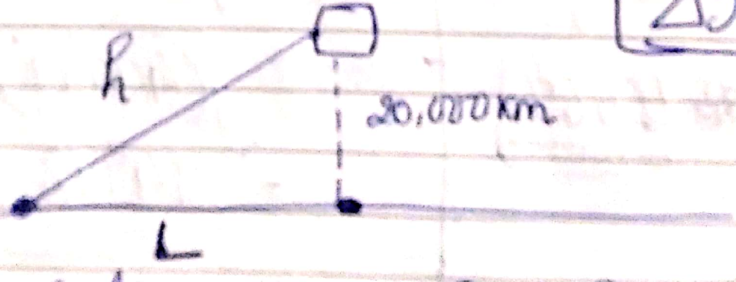
$\frac{x^2+1}{x^2-1}$ is even function since putting -ve values give the same answer.

$\frac{-2x}{x^2-1}$ is odd since putting -ve values of x gives -value of $f(x)$.

even + odd

Q2 $x = x_0 + \Delta x$

$\Delta f = f(x) - f(x_0)$ $\boxed{\frac{\Delta f}{\Delta x}}$

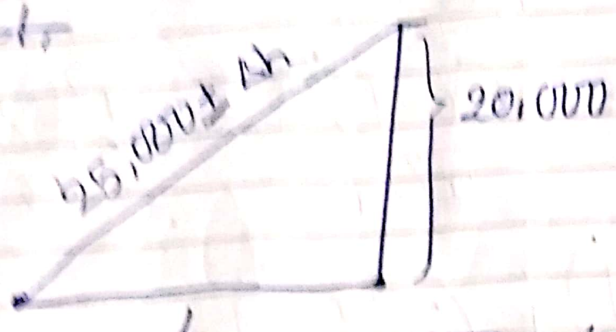


(a) calculate $\frac{\Delta L}{\Delta h}$ $h = h_0 \pm \Delta h$

$= 25,000 \pm \Delta h$

$h = h_0 \pm \Delta h = 25,000 \pm \Delta h$

~~$\Delta h = 1$~~



Case I we see that 20,000 is constant, so L depends only on h using pythagoras theory.

$$L = \sqrt{h^2 - (20{,}000)^2}$$

+1

$\Delta h = 1$ we know that earlier h was 25000 and changed to 25001

$h_0 = 25000 \quad L_0 = 15000$

$h = 25001 \quad L = \underline{15000.1666}$

$$\frac{L - L_0}{\Delta h} = \frac{1.6667}{1}$$

$$\boxed{1.6667}$$

−1

~~we know~~ $h = 24999$

$h_0 = 25000$

$$L_0 = \sqrt{(25000 + 20000)(25000 - 20000)}$$
$$= \sqrt{(45000)(5000)}$$
$$= \sqrt{225000000}$$
$$= \underline{15000}$$

$$L = \sqrt{(24999 + 20000)(24999 - 20000)}$$
$$= \sqrt{(44999)(4999)}$$
$$= 14998.333$$

$$\frac{\Delta L}{\Delta h} = \frac{L - L_0}{-1} = \frac{\cancel{-}1.6667}{\cancel{-}1}$$

$$= \boxed{1.6667}$$

Case II $\Delta h = \pm 0.1$

$h_0 = 25000 \quad L_0 = 15000$

+	−
$h = 25000.1$	$h = 24999.9$
$L = 15000.1666..$	$L = 14999.8333$
$\frac{\Delta L}{\Delta h} = \frac{15000.1666 - 15000}{0.1}$	$\frac{\Delta L}{\Delta h} = \frac{-0.16667}{-0.1}$
$= 1.6667$	$= 1.6667$

Case III $\Delta h = \pm 0.001$

$h_0 = 25000, \quad L_0 = 15000$

+	−
$h = 25000.001$	$h = 24999.99$
$L = 15000.01666..$	$L = 14999.98333$
$\frac{\Delta L}{\Delta h} = \frac{0.016666}{0.01}$	$\Delta L = \frac{-0.016666}{-0.01}$
$= 1.667$	$\simeq 1.6667$

$$|L - L_0| = |\Delta L| \leq C |\Delta h|$$

$\cancel{L = 15000} =$

b) $20{,}001 \pm \Delta h$, $\Delta h = 1, 10^{-1}, 10^{-2}$

$$L(x) = \sqrt{h^2 - (20{,}000)^2}$$

$$\frac{dL}{dh} = \frac{1}{\cancel{2}}\left(h^2 - (20{,}000)^2\right)^{-1/2} \times (\cancel{2}h)$$

$$= \frac{h}{\left(h^2 - (20{,}000)^2\right)^{1/2}}$$

case I

$$\frac{\Delta L}{\Delta h} = \frac{20{,}0001}{(282.849)} = \frac{20{,}0001}{200.0024} = 100.003$$

$$L(20001) = \cancel{200.0024}\ 200.0024$$

$$L(20002) = 200.004999.$$

$$\cancel{\frac{\Delta L}{\Delta h} = \frac{200.004999 - 200.0024}{1}}$$

$$L(x) = \sqrt{h^2 - (20.000)^2}$$

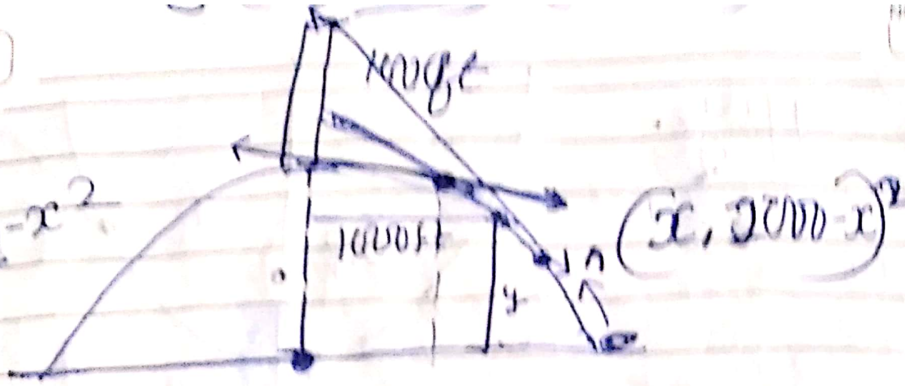
$$L(x) = \left(h^2 - 20{,}000^2\right)^{1/2}$$

$$\frac{dL}{dh} = \frac{1}{\cancel{2}}\left(h^2 - 20{,}000^2\right)^{-1/2} \times (\cancel{2}h)$$

$$= \frac{h}{\sqrt{h^2 - (20{,}000)^2}}$$

Q3

curve = $1000 - x^2$



$\frac{dy}{dx} = -2x$

$(0, 0)$

$$y = 1000 - x^2$$

~~$\frac{dy}{dx} = 2x$~~ B ~~$1100 = 1000 - x^2$~~

A

$$(x, 1000-x^2) \quad (0, 1100)$$

$\frac{dy}{dx} = -2x$

$$\frac{1100 - (1000 - x^2)}{-x} = -2x$$

$$1100 - 1000 + x^2 = 2x^2$$

$$100 = x^2$$

$$\Rightarrow \boxed{x = \pm 10}$$

On both sides, we can climb.
the respective points are

$$(10, 900) \quad (-10, 900)$$

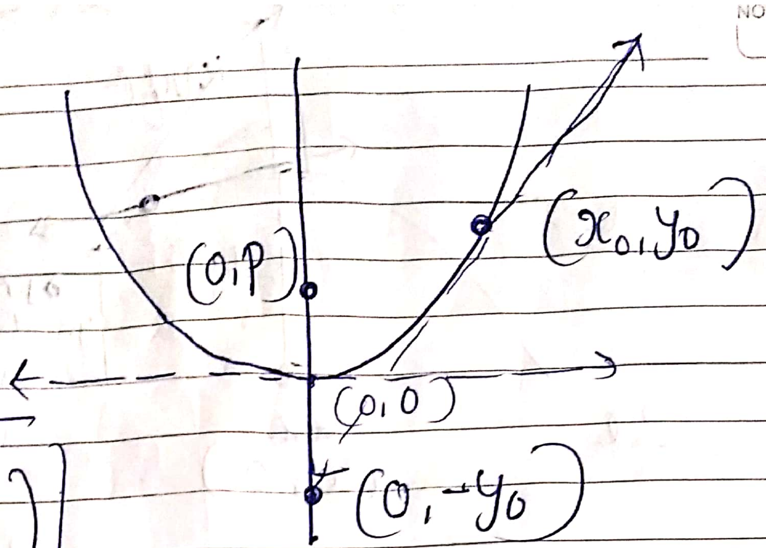
Q4

$$x^2 = 4py$$

$$x^2 = 4py$$

$$\frac{x^2}{4p} = y$$

$$\boxed{\frac{dy}{dx} = \frac{x}{2p}}$$



$$\boxed{\left(x_0, \frac{x_0^2}{4p}\right)}$$

$$(0, y)$$

$$\frac{dy}{dx} = \cancel{\frac{x_0}{2p} = \frac{x_0^2}{4p} - y = \frac{x_0^2}{4p} = -x_0}$$

$$y \cancel{=} - y_1 = m(x - x_1)$$

7

Q5 $V = \frac{(10-t)^2}{5}$

Average rate $= \dfrac{\frac{(10-5)^2}{5} - \frac{(10-0)^2}{5}}{5}$

$= \frac{25-100}{5\times5} = \frac{-75}{25} = -3 \text{ litre/min}$

(b) V

(b) $\cancel{\frac{dV}{dt}} = \cancel{2(10-t)}\ \frac{100+t^2-20t}{5}$

$V = \frac{t^2}{5} - 4t + 20$

$\frac{dV}{dt} = \frac{2t}{5} - 4$

Put $t=5$, $\frac{2(\cancel{5})}{\cancel{5}} - 4 = \boxed{-2 \text{ lit/min}}$

Q6 (19(a)), (f) & (g)

20 (c), (g)

22 (a)

22 (b)

19 $\lim_{x\to\infty} x \sin\frac{1}{x}$ $\left(\because \text{since as } x\to\infty,\ \frac{1}{x}\to 0\right)$

$\lim_{x\to\infty} \frac{\sin 1/x}{1/x} = \lim_{1/x\to 0} \frac{\sin 1/x}{1/x}$

$= 1$

DAY MONTH YEAR NOTE /

(f) $\lim_{x\to 0} \frac{\sin^2 x}{3x^2}$

$\lim_{x\to 0} \frac{\sin x}{x} \times \frac{\sin x}{x} \times \frac{1}{3}$

$\lim_{x\to 0} = 1 \times 1 \times \frac{1}{3} = \frac{1}{3}$

(g) $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \frac{2\sin x\cos x}{\sin(2x+x)}$

$= \cancel{\frac{2\sin x\cos x}{2\sin x\cos^2 x + }} = \frac{2\sin x\cos x}{3\sin x - 4\sin^3 x}$

$= \frac{2\cos x}{3-4\sin^2 x} = \frac{2\cos x}{3-4(1-\cos^2 x)}$

$= \frac{2\cos x}{3-4+4\cos^2 x} = \boxed{\frac{2\cos x}{4\cos^2 x - 1}}$

$\lim_{x\to 0} \frac{2\cos x}{4\cos^2 x - 1} = \frac{2\cos(0)}{4\cos^2(0)-1} = \frac{2}{4-1} = \frac{2}{3}$

20: (c) $\lim_{x\to 0} \frac{x^2}{1-\cos^2 x} = \frac{x^2}{\sin^2 x}$

$\lim_{x\to 0} \frac{1}{\frac{\sin^2 x}{x^2}} = \frac{1}{\frac{\sin x}{x} \times \frac{\sin x}{x}}$

$\lim_{x\to 0} = 1$

DAY MONTH YEAR NOTE /

(g) $\lim_{x \to 0} \frac{3x^2+4x}{\sin 2x}$

$= \frac{3x^2+4x}{2\sin x \cos x}$

$\frac{3x^2}{2\sin x \cos x} + \frac{\cancel{4}^{2}x}{\cancel{2}\sin x \cos x}$

$\lim_{x \to 0} : \frac{3x}{2 \times \frac{\sin x}{x} \times \cos x} + \frac{2 \times 0}{\frac{\sin x}{x} \times \cos x}$

$= \frac{3(0)}{2 \times 1 \times 1} + \frac{2}{1 \times 1}$

$= 0 + 2 = 2$

22a

$\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$

Q7 u, v, w

(a) ~~$u.v.w$~~ $D((uv) \times w)$

$= (uv)'w + (uv)w'$

$= (u'v + v'u)w + (uv)w'$

$= \underline{u'vw + v'uw + w'vu}$

Q (b) $f_1, f_2, \ldots f_n$ function

$D((f_1 f_2 \ldots f_{n-1}) f_n)$

$= (f_1 f_2 \ldots f_{n-1})' f_n + f_n' (f_1 f_2 \ldots f_{n-1})$

$\cancel{f_1 (f_1 f_2 \ldots f_{n-2})' f_{n-1} + f_n' (f_1 f_2 \ldots f_{n-1})}$

$+ \cancel{(f_1 f_2 \ldots f_{n-2}) f_{n-1}' f_n}$

$= \left[(f_1 f_2 \ldots f_{n-2})' f_{n-1} + f_{n-1}' (f_1 f_2 \ldots f_{n-2}) \right] f_n$

$+ f_n' (f_1 \ldots f_{n-1})$

n-1 element

where $\boxed{1 \le i \le n}$

$$\sum_{i=1,\ cyc}^{n} f_i' \overbrace{(f_1 f_2 f_3 \ldots f_n)}^{n-1 \text{ element}} = (f_1 f_2 \ldots f_n)'$$

(b) $P(n) = \sum_{i=1}^{n} f_i'(\overbrace{f_1 f_2 \cdot \cancel{f_i} \cdots f_n}^{\text{except } i})$ where $1 \le i \le n$

$= D(f_1 f_2 \cdot f_i \cdots f_n)$

For $n=1$

$RHS = f'$

$LHS = \sum_{i=1}^{1} f'(= \cancel{f}'$

$LHS = RHS \Rightarrow P(1)$ is true

Assume $P(K)$ is true for some $K \in N$.

$P(K) = \sum_{i=1}^{K} f_i'(f_1 f_2 \cdots f_n)$

$= (f_1 f_2 \cdot f_i \cdots f_n)'$

where $1 \le i \le K$

For $K+1$

$\sum^{K+1}$

DAY MONTH YEAR NOTE

$$RHS = (f_1 f_2 f_3 \cdots f_k f_{k+1})'$$

$$= (f_1 f_2 f_3 \cdots f_k)' f_{k+1} + (f_1 f_2 f_3 \cdots f_k) f_{k+1}'$$

$$\left[\sum_{i=1}^{k} (f_i' (f_1 \cdots f_k))\right] f_{k+1} + (f_1 f_2 \cdots f_{k+1}')$$

$$= \sum_{i=1}^{k+1} f_i' (f_1 \cdots f_{k+1})$$

$\therefore$ Proved