1.1 Find the points of discontinuity of the following functions:
A point of discontinuity occurs when a number *a* is both zero of the numerator and denominator.

A)

$$\boldsymbol{\frac{x}{x^2+1}}$$

[Multiply with conjugate]

$$\boldsymbol{\frac{x}{x^2+1} \cdot \frac{x^2+1}{x^2+1} = \frac{x(x^2+1)}{(x^2+1)(x^2+1)}}$$

Either x = 0 or $x^2 + 1 = 0$

[Use quadratic formula]

$$ax^2 + bx + c = 0$$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ https://en.wikipedia.org/wiki/Quadratic_formula

$b^2 - 4ac > 0$ two real solutions
$b^2 - 4ac < 0$ complex solution
$b^2 - 4ac = 0$ one real solution

For $x^2 + 1 = 0$ this is $0 - 4 \cdot 1 \cdot 1 < 0$ therefore there is no real solution, since x = $\sqrt{-1} = \pm i$
Also the nominator and denominator cannot be 0 at the same time, so there is no discontinuity.

B)

$$\boldsymbol{\frac{1}{\sqrt{x}}}$$

[Multiply with conjugate]

$$\boldsymbol{\frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}}$$

If x = 0, both nominator and denominator are 0, which is undefined (nonremovable discontinuity).

1.2 Evaluate the following limits:
Quotient Law of Limits (Rule of l'Hospital)
Regel von l'Hospital

$$\lim_{x \to x_0} \frac{g(x)}{h(x)} = \lim_{x \to x_0} \frac{g'(x)}{h'(x)}$$

C)

$$\lim_{x \to 0} \frac{\sin x}{3\sqrt{x}}$$

Problem: Since denominator would go to 0, this would be undefined.
Solution: Use Rule of l'Hospital.

$$= \lim_{x\to 0}\left(\left(\frac{\frac{\cos(x)}{3}}{2\sqrt{x}}\right)\right) = \frac{1}{3}\cdot\lim_{x\to 0}\left(\left(\frac{\frac{\cos(x)}{1}}{2\sqrt{x}}\right)\right) = \frac{1}{3}\cdot\lim_{x\to 0}\left(2\sqrt{x}\cdot cos(x)\right)$$

for x = 0 this leads to $\frac{1}{3}\cdot\left(2\sqrt{0}\cdot cos(0)\right) = \frac{1}{3}\cdot(0\cdot 1) = 0$

D)

$$\lim_{x\to 0}\frac{3x^2+\sin x}{x}$$

Use Rule of l'Hospital.

$$\lim_{x\to 0}\frac{6x+\cos(x)}{1}$$

for x = 0 this leads to $\left(6\cdot 0 + cos(0)\right) = (6\cdot 0+1) = 1$

2.3 Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

[Differentiate with respect to x]

$$y' = 18x^2 + 5$$

[Equate y' to 4]

$$4 = 18x^2 + 5$$

$$18x^2 + 1 = 0$$

[Use quadratic formula]

$$0 - 4\cdot 18\cdot 1 < 0$$

Therefore no real solution exists, so there is no tangent line with slope 4.

2.4 Differentiate the following functions:

E)

$$f(t) = \sqrt{t} - 1/\sqrt{t}$$

[Apply sum rule]

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$F'(t) = (\sqrt{t})' - (1/\sqrt{t})'$$

$$F'(t) = \frac{1}{2t^{1/2}} - \left(-\frac{1}{2t^{3/2}}\right)$$

$F'(t) = \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}^3}$

F)

$y = \frac{x^2 - 2\sqrt{x}}{x}$

[Apply quotient rule]

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)] \cdot g(x) - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$y' = \frac{(x^2 - 2\sqrt{x})' \cdot x - (x^2 - 2\sqrt{x}) \cdot (\boldsymbol{x})'}{x^2}$

$y' = \frac{(2x - 1/\sqrt{x}) \cdot x - (x^2 - 2\sqrt{x}) \cdot (\mathbf{1})}{x^2}$

$y' = \frac{(2x^2 - x/\sqrt{x}) - (x^2 - 2\sqrt{x})}{x^2}$

$y' = \frac{x^2 - x/\sqrt{x} + 2\sqrt{x}}{x^2}$

$y' = \frac{x^2 + \sqrt{x}}{x^2}$

G)

$y = \sqrt{x}(x - 1)$

[Use product rule]

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$$

$y' = (\sqrt{x})' \cdot (x - 1) + (\sqrt{x}) \cdot (x - 1)'$

$y' = (1/2\sqrt{x}) \cdot (x - 1) + (\sqrt{x}) \cdot (1)$

$y' = \left(\frac{x-1}{2}\sqrt{x}\right) + \sqrt{x}$

$y' = \sqrt{x}\left(\frac{x-1}{2}\right) + \sqrt{x}$

$y' = \sqrt{x}(\frac{x-1}{2} + 1)$

$y' = \sqrt{x}(\frac{x-1}{2} + \frac{2}{2})$

$y' = \sqrt{x}(\frac{x+1}{2})$