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A.

1. $2A - B = 2\begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}-2\\-1\\0\end{bmatrix}$

2. $|A| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$
   The angle of A relative to the positive x axis is $\arccos\frac{1}{\sqrt{14}}$

3. The unit vector in the direction of A is $\hat{A} = \frac{1}{\sqrt{14}}\begin{bmatrix}1\\2\\3\end{bmatrix}$

4. The direction cosines of A are $(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}})$

5. $A\cdot B = B\cdot A = 1\cdot 4 + 2\cdot 5 + 3\cdot 6 = 32$

6. The angle between A and B is $\arccos\frac{A\cdot B}{|A|\cdot|B|} = \arccos\frac{32}{\sqrt{14\cdot 77}} = \frac{32}{7\sqrt{22}}$

7. $[x \ y \ z]\begin{bmatrix}1\\2\\3\end{bmatrix} = 0 \Rightarrow [x \ y \ z] = [1 \ 1 \ -1]$

8. $A \times B = \begin{bmatrix}-3\\6\\-3\end{bmatrix}$ $\quad B \times A = \begin{bmatrix}3\\-6\\3\end{bmatrix}$

9. A vector which is perpendicular to both A and B is
   $A \times B = \begin{bmatrix}-3\\6\\-3\end{bmatrix}$

10. $aA + bB + cC = 0 \Rightarrow 3A - B - C = 0$

11. $A^T B = [1 \ 2 \ 3]\cdot\begin{bmatrix}4\\5\\6\end{bmatrix} = 32$ $\quad AB^T = \begin{bmatrix}1\\2\\3\end{bmatrix}[4 \ 5 \ 6] = \begin{bmatrix}4 & 5 & 6\\8 & 10 & 12\\12 & 15 & 18\end{bmatrix}$

B.

1. $2A - B = 2\begin{bmatrix}1 & 2 & 3\\4 & -2 & 3\\0 & 5 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 1\\2 & 1 & -4\\3 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 5\\6 & -5 & 10\\-3 & 12 & -3\end{bmatrix}$

2. $A\cdot B = \begin{bmatrix}14 & -2 & -4\\9 & 0 & 15\\7 & 7 & -21\end{bmatrix}$ $\quad B\cdot A = \begin{bmatrix}9 & 3 & 8\\6 & -18 & 13\\-5 & 15 & 2\end{bmatrix}$

3. $(AB)^T = \begin{bmatrix}14 & 9 & 7\\-2 & 0 & 7\\-4 & 15 & -21\end{bmatrix}$ $\quad B^T A^T = (AB)^T$

4. $|A| = 55$ $\quad |C| = 0$

5. The matrix (A, B or C) in which the row vectors form an orthogonal set is the matrix B

6. $A^{-1} = \frac{1}{55}\begin{bmatrix}-13 & 17 & 12\\4 & -1 & 9\\20 & -5 & -10\end{bmatrix}$ $\quad B^{-1} = \begin{bmatrix}\frac{1}{6} & \frac{2}{21} & \frac{3}{14}\\ \frac{2}{6} & \frac{1}{21} & -\frac{2}{14}\\ \frac{1}{6} & -\frac{4}{21} & \frac{1}{14}\end{bmatrix}$

C.

1. $\lambda_1 = -1$ $\quad \lambda_2 = 4$ $\quad e_1 = \begin{bmatrix}-0.707\\0.707\end{bmatrix}$ $\quad e_2 = \begin{bmatrix}-0.555\\-0.832\end{bmatrix}$

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2. $V^{-1}AV = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$

3. $e_1 \cdot e_2 = -0.196$

4. $e_1 \cdot e_2 = 0$

5. Since B is symmetric real matrix, its eigenvectors are orthogonal

D.

1. $f'(x) = 2x \quad f''(x) = 2$

2. $\frac{\partial g}{\partial x} = 2x \quad \frac{\partial g}{\partial y} = 2y$

3. $\nabla g(x,y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

4. $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$