Solution to Homework 7

1. $p_2 \equiv sp(p_1, x := x*2) \equiv (p \wedge k < n)[x_0/x] \wedge x = x_0 * 2$
   $\equiv ((x = 2 \,\hat{}\, k \wedge k \le n) \wedge k < n)[x_0/x] \wedge x = x_0 * 2$
   $\equiv (x_0 = 2 \,\hat{}\, k \wedge k \le n \wedge k < n \wedge x = x_0 * 2)$

   $p_3 \equiv sp(p_2, k := k+1) \equiv p_2[k_0/k] \wedge k = k_0 + 1$
   $\equiv (x_0 = 2 \,\hat{}\, k \wedge k \le n \wedge k < n \wedge x = x_0 * 2)[k_0/k] \wedge k = k_0 + 1$
   $\equiv (x_0 = 2 \,\hat{}\, k_0 \wedge k_0 \le n \wedge k_0 < n \wedge x = x_0 * 2 \wedge k = k_0 + 1)$

   $R_1 \equiv$ postcondition weakening 3, 4

   $R_2 \equiv$ while loop, 5

Quick sanity check: $p_3 \rightarrow p$?

$p_3 \equiv (x_0 = 2 \,\hat{}\, k_0 \wedge k_0 \le n \wedge k_0 < n \wedge x = x_0 * 2 \wedge k = k_0 + 1)$
$\Rightarrow x_0 * 2 = 2 \,\hat{}\, (k_0+1) \wedge k_0 + 1 < n+1 \wedge (x = x_0 * 2 \wedge k = k_0 + 1)$
$\Rightarrow x = 2 \,\hat{}\, k \wedge k < n+1$
$\Rightarrow x = 2 \,\hat{}\, k \wedge k \le n$
$\equiv p$

2. (Proof of if-else statement)

   $q_1 \equiv q \wedge odd(x)$
   $\equiv r = X*Y - x*y \wedge odd(x)$

   $R_1 \equiv$ assignment (forward)

   $q_2 \equiv (q \wedge odd(x))[r_0/r] \wedge r = (r + y)[r_0/r]$
   $\equiv r_0 = X*Y - x*y \wedge odd(x) \wedge r = r_0 + y$

   $R_2 \equiv$ postcondition weakening 2, 3

   $q_3 \equiv q_2[x_0/x] \wedge x = (x-1)[x_0/x]$
   $\equiv r_0 = X*Y - x_0*y \wedge odd(x_0) \wedge r = r_0 + y \wedge x = x_0 - 1$

   $R_3 \equiv$ assignment (backward)

   $q_4 \equiv q[(x/2)/x]$
   $\equiv r = X*Y - (x/2)*y$

   $R_4 \equiv$ assignment (backward)

   $q_5 \equiv q_4[2*y/y]$
   $\equiv r = X*Y - (x/2)*(2*y)$

   $R_5 \equiv$ precondition strengthening 8, 7

   $q_6 \equiv q \wedge even(x)$
   $\equiv r = X*Y - x*y \wedge even(x)$

   $R_6 \equiv$ if 10, *5*