Language Syntax, Semantics, Runtime Errors

CS 536: Science of Programming, Spring 2023

Due Thu Feb 9, 11:59 pm

Mon 2023-02-06 pp 1,2.

Problems [60 points]

Class 5: Language Syntax/Operational Semantics

1. [$12 = 2 * 6$ points] Translate the program below into our programming language.
   a. `m = x = 0; y = 1; while (m++ < n) { y = ++x; y *= x ; }; m = m*m`
   b. `m = n; p = y = 1; while (--m < n) { p = p *(y++); }`

For Problems 2 and 3, write out the operational semantics as a directed graph. With $\langle S, \sigma\rangle \rightarrow \langle S', \sigma\rangle$, the two configurations become nodes and the semantics $\rightarrow$ becomes a graph $\rightarrow$.. For these problems, it's okay to draw your answers on paper and scan it in to be part of your pdf. If the same configuration occurs more than once, don't write it out as two separate nodes; make it just one node.

2. [2023-02-06] [10 points] Let $S \equiv$ ***if*** $x > 0$ ***then*** $x := x * z$ ***fi***; $y := y * z$
   Evaluate $\langle S, \{x = -5, y = -2, z = -2\}\rangle$ to completion. [Hint: remember to expand the ***if-then***]

3. [8 points] Let $W \equiv$ ***while*** $m \neq n$ ***do*** S ***od*** where S is $m := m + 1;\ x := x + m * m$. Let $\sigma_0 = \{m = 0,$ $x = 1,\ n = 4$. Evaluate $\langle W, \sigma_0\rangle$ to completion. Show all configurations of the form $\langle W, state\rangle$ and the final $\langle E, state\rangle$. You can use $\rightarrow^n$ to skip other configurations if you like, or you can show them (your choice).

Class 6: Denotational Semantics, Runtime Errs

4. [$12 = 2 * 6$ points] As in Problem 2, let IF be ***if*** $x > 0$ ***then*** $x := x * z$ ***fi fi***; $y := y * z$. For both parts below, write your answer as a succession of $M(\ldots)$ steps. (See Example 1 in the Class 6 notes).
   a. What is $M(IF, \{x = 3, y = 5, z = 9\})$?
   b. What is $M(IF, \{x = 0, y = 2, z = 8\})$?

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5. [*14* points total] Let $W \equiv$ ***while*** $m \neq n$ ***do*** S ***od*** where S is $m := m + 1;\ x := x * x$.
   a. [*3* points] Let $\tau(m) = \alpha$ and $\tau(x) = \beta$. Calculate $M(S, \tau)$.
   b. [*3* points] Let $\sigma_0 = \{m = 0, x = 2, n = \delta\}$. What values of δ make $M(W, \sigma_0) = \perp_d$? Briefly explain.
   c. [*8* points] Consider the σ_0 where δ isn't one of the ones in part b. That is, $M(W, \sigma_0) = \tau$ for some $\tau \neq \perp$. Give a simple description of the $\tau(x)$ values — something at the level of "$\tau(x)$ is the sum of the even integers ≥ 0 and $< \delta$" (Except that that's a wrong answer.)

6. [*4* points] Let $S \equiv x := b[m+1] / sqrt(k)$ and let $\sigma = \{m = \alpha,\ k = \beta,\ b = \delta\}$. Let η be the length of b, so $\delta(0), \ldots, \delta(\eta - 1)$ are the values of $b[0], b[1]$, ... in σ. Describe the set of all σ that make $M(S, \sigma) = \{\perp_e\}$. [2023-02-06]

Solution: Language Syntax, Semantics, Runtime Errors

CS 536: Science of Programming, Spring 2023

Due Wed Sep 8, 11:59 pm

Class 5: Language Syntax/Operational Semantics

1. (Translate C code)
   a. $m := 0\,;\ x := 0\,;\ y := 1\,;$
   while $m < n$ ***do*** $m := m+1\,;\ x := x+1\,;\ y := y * x$ ***od;***
   $m := m+1\,;\ m := m * m$
   b. $m := n\,;\ y := 1\,;\ p := y\,;\ m := m - 1\,;$
   while $m < n$ ***do***
   $t := y\,;\ y := y+1\,;\ p := p * y\,;$ // also $p := p * y\,; y := y+1\,;$
   $m := m - 1$
   od

2. (Evaluate a conditional) Let $S \equiv$ ***if*** $x > 0$ ***then*** $x := x * z$ ***fi;*** $y := y * z$
   a. Let $\sigma = \{x = 3,\ y = 5,\ z = 8\}$. Then,
   $\langle S, \sigma \rangle$
   $\rightarrow \langle x := x * z\,;\ y := y * z, \sigma \rangle$ (Because $x > 0$ is true)
   $\rightarrow \langle y := y * z, \sigma[x \mapsto 24] \rangle$ (x is updated to $\sigma(x) * \sigma(z) = 3 * 8 = 24$)

   b. Let $\sigma = \{x = -5,\ y = -2,\ z = -2\}$. Then,
   $\langle S, \sigma \rangle$
   $\rightarrow \langle$ ***skip***; $y := y * z, \sigma \rangle$ ($x > 0$ is false and the false branch is ***skip***)
   $\rightarrow \langle y := y * z, \sigma \rangle$ (After executing ***skip***)
   $\rightarrow \langle E, \sigma[y \mapsto 4] \rangle$ (y is updated to $\sigma(y) * \sigma(z) = -2 * -2 = 4$)

3. We're given $W \equiv$ ***while*** $m \neq n$ ***do*** S ***od***, $S \equiv m := m+1\,;\ x := x + m * m$, $\sigma_0 = \{m = 0,\ x = 1,\ n = 4\}$
   $\langle W, \sigma_0 \rangle$
   $\rightarrow \langle S\,; W, \sigma_0 \rangle$ (Here's some detail for the first iteration)
   $= \langle m := m+1\,;\ x := x + m * m\,;\ W, \sigma_0 \rangle$
   $\rightarrow \langle x := x + m * m\,;\ W, \sigma_0[m \mapsto \sigma_0(m) + 1] \rangle$
   $\rightarrow \langle x := x + m * m\,;\ W, \sigma_0[m \mapsto 1] \rangle$
   $\rightarrow \langle W, \sigma_1 \rangle$ where $\sigma_1 = \sigma_0[m \mapsto 1][x \mapsto 2] = \{m = 1,\ x = 2,\ n = 4\}$
   $\rightarrow^3 \langle W, \sigma_2 \rangle$ where $\sigma_2 = \sigma_1[m \mapsto 2][x \mapsto 6] = \{m = 2,\ x = 6,\ n = 4\}$

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$\rightarrow^3 \langle W, \sigma_3 \rangle$ where $\sigma_3 = \sigma_2[m \mapsto 4][x \mapsto 31] = \{m = 4, x = 31, n = 4\}$
$\rightarrow \langle E, \sigma_3 \rangle$

Class 6: Denotational Semantics, Runtime Errs

4. (Denotational semantics of sequence with ***if-then***) Let IF be ***if*** $x > 0$ ***then*** $x := x * z$ ***fi***; $y := y * z$.
   a. $M(IF, \{x = 3, y = 5, z = 9\})$
   $= M(x := x * z; y := y * z, \{x = 3, y = 5, z = 9\})$ (Since $x > 0$)
   $= M(y := y * z, \{x = 27, y = 5, z = 9\})$ (After $x := x * z$)
   $= M(E, \{x = 27, y = 45, z = 9\})$ (After $y := y * z$)
   $= \{x = 27, y = 45, z = 9\}$ (Defn M)
   b. $M(IF, \{x = 0, y = 2, z = 8\})$
   $= M($***skip***$; y := y * z, \{x = 0, y = 2, z = 8\})$ ($x \le 0$ and the false branch is ***skip***)
   $= M(y := y * z, \{x = 0, y = 2, z = 8\})$ (After ***skip***)
   $= M(E, \{x = 0, y = 16, z = 8\})$ (After $y := y * z$)
   $= \{x = 0, y = 16, z = 8\}$ (Defn M)

5. (Denotational semantics of loop) Let $W \equiv$ ***while*** $m \ne n$ ***do*** S ***od*** where S is $m := m + 1;\ x := x * x$.
   a. $M(S, \tau)$ (where $\tau(m) = \alpha$ and $\tau(x) = \beta$)
   $= M(m := m + 1;\ x := x * x, \tau)$ (defn S)
   $= M(x := x * x, \tau[m \mapsto \alpha + 1])$ (Since $\tau(m) = \alpha$)
   $= M(E, \tau[m \mapsto \alpha + 1][x \mapsto \beta^2])$ (Since $\tau(x) = \beta$)
   b. We have $\sigma_0 = \{m = 0, x = 2, n = \delta\}$ and want to know what δ cause divergence of W.
   Since each iteration increases m by 1, we'll reach $m = n$ iff originally $m \le n$, so $\delta < 0$ causes nontermination.
   c. To figure out what $M(W, \sigma_0)$ looks like, let's start by figuring out the sequence of states at the loop tests: σ_0, $\sigma_1 = M(S, \sigma_0)$, $\sigma_2 = M(S, \sigma_1)$, …. Applying part (a) above, we get
   $\sigma_1 = \sigma_0[m \mapsto \sigma_0(m) + 1][x \mapsto \sigma_0(x)^2]$ (From part a)
   $= \sigma_0[m \mapsto 1][x \mapsto 4]$ (Since $\sigma_0(m) = 0$ and $\sigma_0(x) = 2$)
   $\sigma_2 = \sigma_1[m \mapsto 2][x \mapsto 8]$ (Increment m, multiply x by itself)
   $\sigma_3 = \sigma_2[m \mapsto 3][x \mapsto 16]$ …
   The sequence of values $2, 4, 8, 16, 32, \ldots$ can be described as $2 \wedge 1, 2 \wedge 2, 2 \wedge 4, 2 \wedge 8, \ldots,$ which is $2 \wedge (2 \wedge 0), 2 \wedge (2 \wedge 1), 2 \wedge (2 \wedge 2), 2 \wedge (2 \wedge 3), \ldots, 2 \wedge (2 \wedge k), \ldots$ So in general, $\sigma_k = \sigma_0[m \mapsto k][x \mapsto 2 \wedge (2 \wedge k)]$.
   So if $\delta \ge 0$, then we halt with at iteration δ, with state σ_δ, with m having value δ and x having value $2 \wedge (2 \wedge \delta)$.

6. The expression $b[m + 1] / sqrt(k)$ evaluates to $\perp_e$ when $m + 1$ is out of range (< 0 or $\ge \eta$) or k is negative. So with $\sigma = \{m = \alpha, k = \beta, b = \delta\}$, the expression fails when $\alpha < -1$, $\alpha \ge \eta$, or $\beta < 0$.

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