

Verifying Array Manipulating Program with Full-Program Induction

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Verify Properties of Programs with Arrays

- Arrays of parametric size N
- Compute values data dependent on values from previous iterations
- No trivial translation of loops to parallel assignments
- Quantified as well as quantifier-free properties, with possibly non-linear terms

Does $\{\varphi_N\} P_N \{\psi_N\}$ hold?

```
assume(true);  
  
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<N; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }  
  
assert( $\forall k \in [0, N), C[k] == k^3$ );
```

Challenges Faced by State-of-the-Art Tools & Techniques

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- Quantified invariants with non-linear terms difficult to synthesize
 - ▶ Loop invariants required by the respective loops in the program:
 - ▶ $\forall i \in [0 \dots x-1] (A[i] = 6i + 6)$
 - ▶ $\forall j \in [0 \dots y-1] (B[j] = 3j^2 + 3j + 1 \wedge A[j] = 6j + 6)$
 - ▶ $\forall k \in [0 \dots z-1] (C[k] = k^3 \wedge B[k] = 3k^2 + 3k + 1)$
 - ▶ FreqHorn[CAV'19], Tiler[SAS'17]

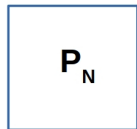
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 - ▶ FreqHorn[CAV'19], Tiler[SAS'17]
- Abstraction-based techniques are imprecise in presence of data dependence across loop iterations
 - ▶ VeriAbs[ASE'19], Vaphor[SAS'16]

Challenges Faced by State-of-the-Art Tools & Techniques

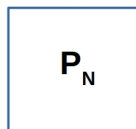
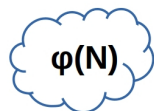
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 - ▶ FreqHorn[CAV'19], Tiler[SAS'17]
- Abstraction-based techniques are imprecise in presence of data dependence across loop iterations
 - ▶ VeriAbs[ASE'19], Vaphor[SAS'16]
- Difficult to solve (non-linear) recurrences when data flows across loops and loop iterations as well as difficult to find fix-points
 - ▶ VIAP[VSTTE'18], Booster[ATVA'14]

Full-Program Induction - Pictorial Overview

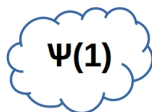
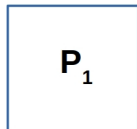
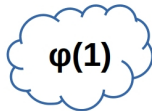


Holds?

Full-Program Induction - Pictorial Overview

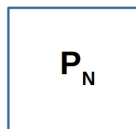
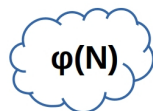


Holds?

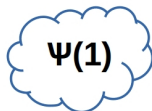
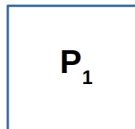
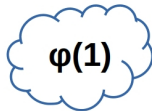


Base Case

Full-Program Induction - Pictorial Overview



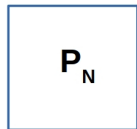
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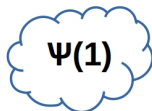
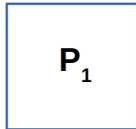
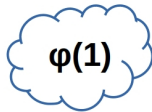
Base Case

X Property
Violation

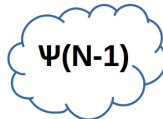
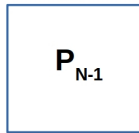
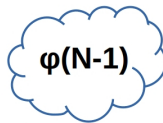
Full-Program Induction - Pictorial Overview



Holds?



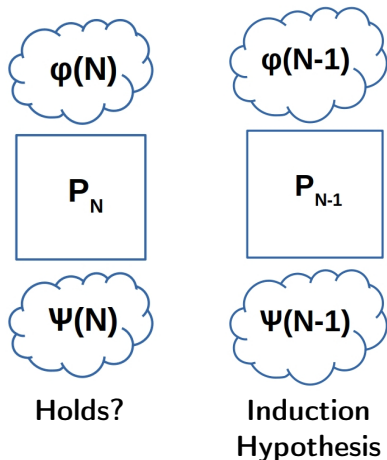
Base Case



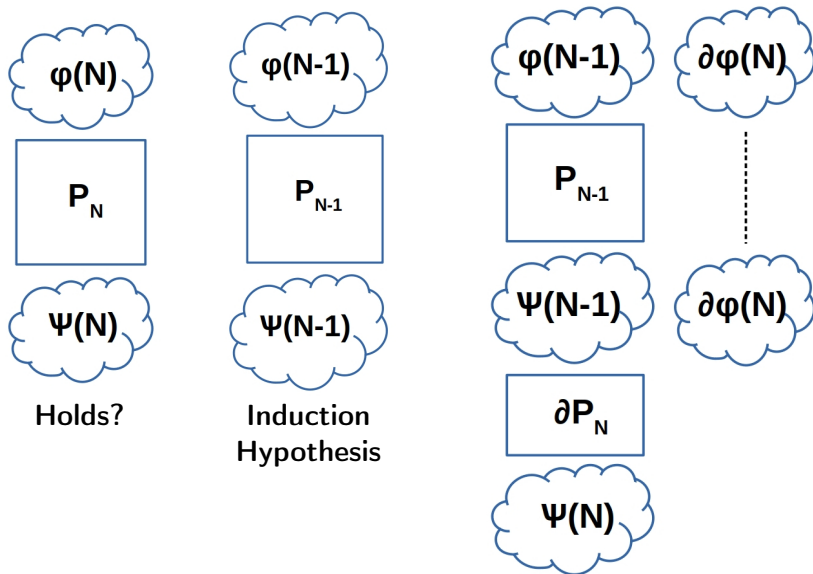
Induction
Hypothesis

X Property
Violation

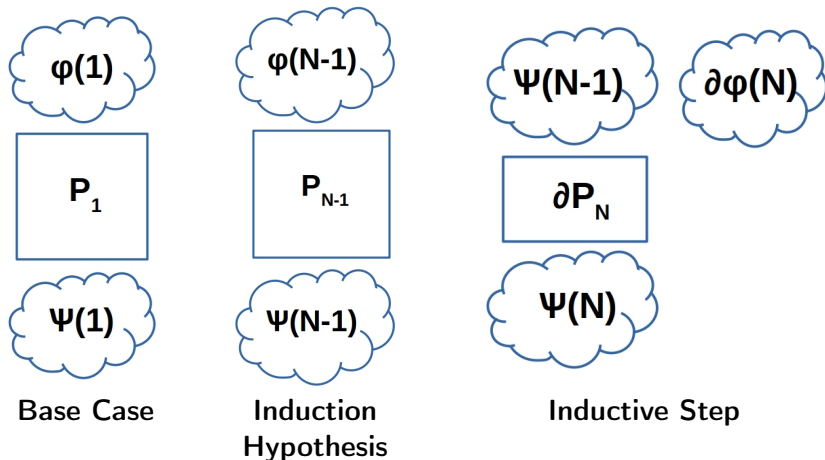
Full-Program Induction - Pictorial Overview



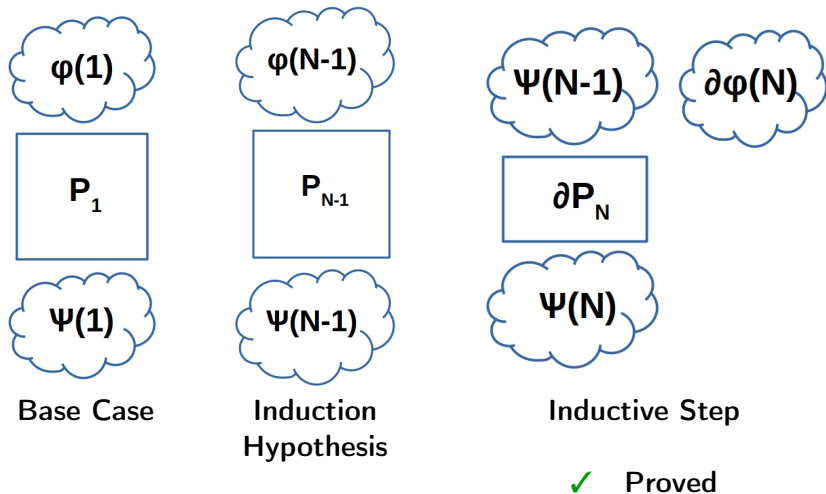
Full-Program Induction - Pictorial Overview



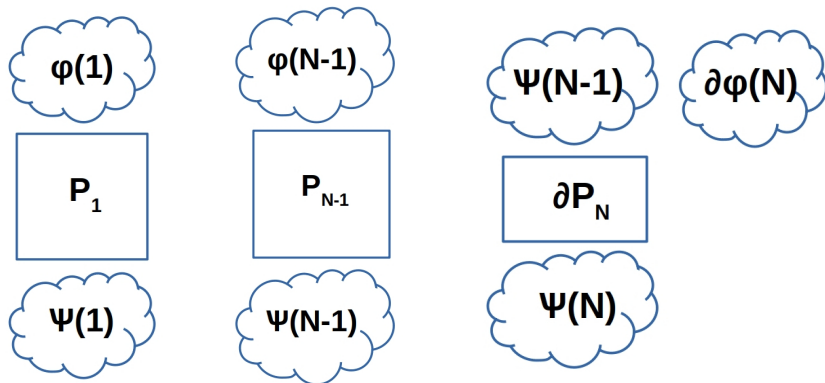
Full-Program Induction - Pictorial Overview



Full-Program Induction - Pictorial Overview



Full-Program Induction - Pictorial Overview



Base Case

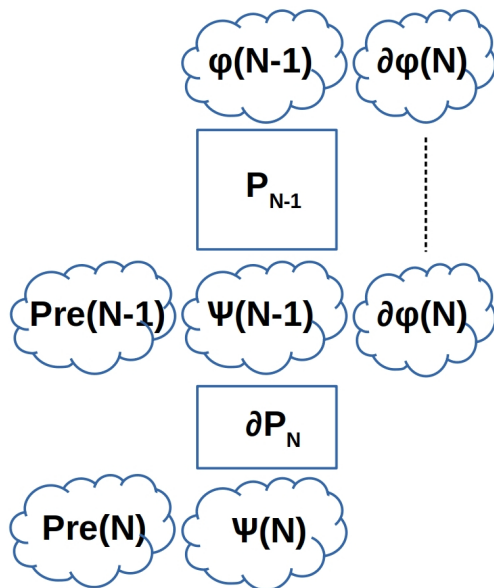
Induction
Hypothesis

Inductive Step

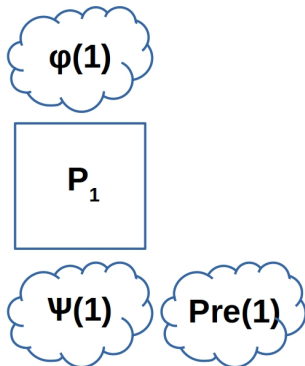
✓ Proved

✗ Infer New Sub-goals

Full-Program Induction - Pictorial Overview

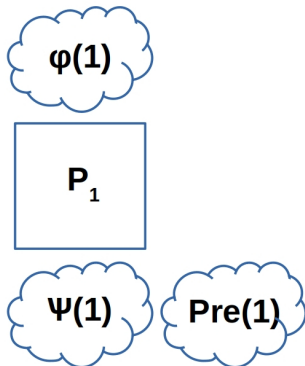


Full-Program Induction - Pictorial Overview



Base Case

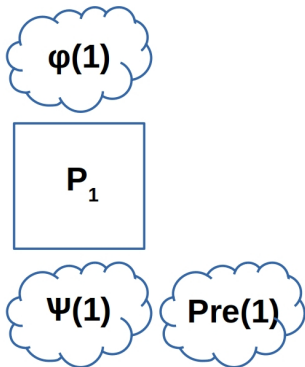
Full-Program Induction - Pictorial Overview



Base Case

X Infer New $\text{Pre}'(N)$
or

Full-Program Induction - Pictorial Overview



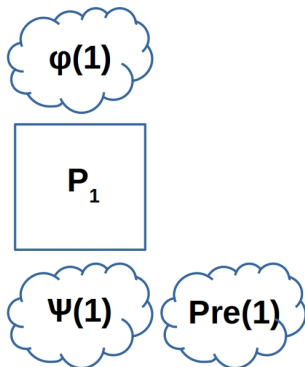
Base Case

X Infer New $\text{Pre}'(N)$

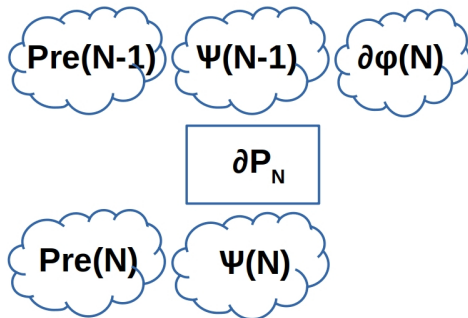
or

Split $\text{Pre}(N)$ and/or $\partial\phi(N)$

Full-Program Induction - Pictorial Overview



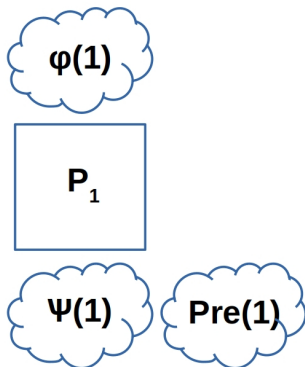
Base Case



Inductive Step

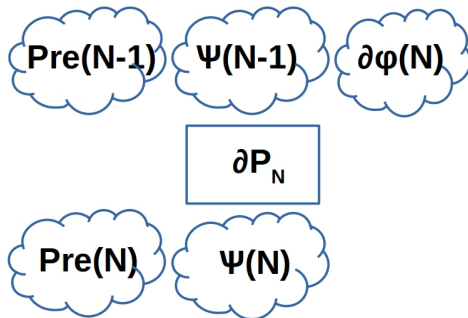
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or
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Full-Program Induction - Pictorial Overview



Base Case

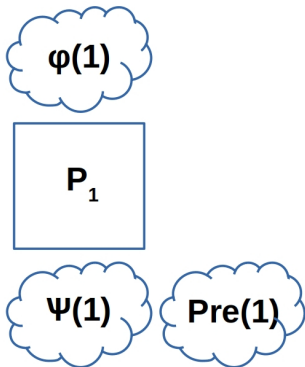
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Inductive Step

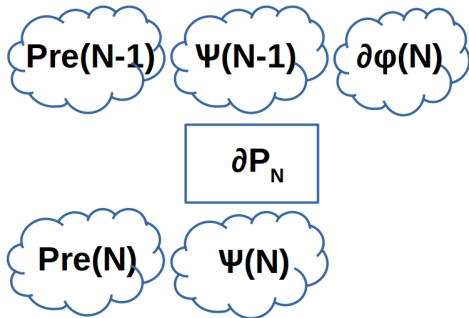
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Full-Program Induction - Pictorial Overview



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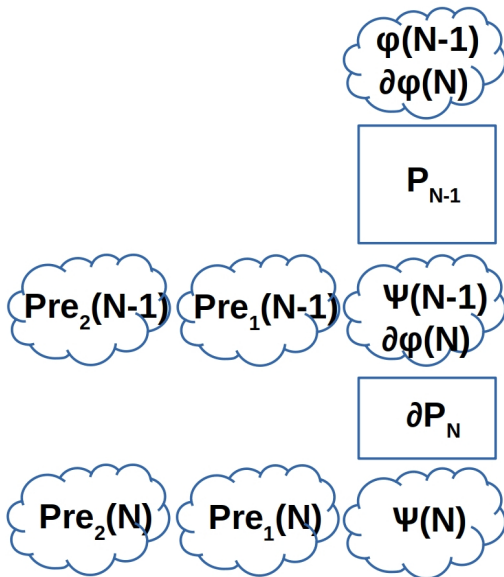
✗ Infer New $\text{Pre}'(N)$
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Split $\text{Pre}(N)$ and/or $\partial\varphi(N)$



Inductive Step

✓ Proved
✗ Infer New Sub-goals

Full-Program Induction - Pictorial Overview



Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
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assert($\forall k \in [0, N), C[k] == k^3$);

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assert( $\forall k \in [0, N), C[k] == k^3$ );
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Base Case: Substitute $N=1$

```
assume(true);
```

```
1. void PolyCompute(int N) {  
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4.   for (int x=1; x<1; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<1; y++)  
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8.   for (int z=1; z<1; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

```
assert( $\forall k \in [0, 1), C[k] == k^3$ );
```

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
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8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Inductive Step

assume($\forall k \in [0, N-1), C[k] == k^3$);

1. $A[N-1] = A[N-2] + 6$;
2. $B[N-1] = B[N-2] + A[N-2]$;
3. $C[N-1] = C[N-2] + B[N-2]$;

assert($\forall k \in [0, N), C[k] == k^3$);

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

```
assume(true);
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7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

```
assert( $\forall k \in [0, N), C[k] == k^3$ );
```

Inductive Step

```
assume( $\forall k \in [0, N-1), C[k] == k^3$ );
```

1. $A[N-1] = A[N-2] + 6$;
2. $B[N-1] = B[N-2] + A[N-2]$;
3. $C[N-1] = C[N-2] + B[N-2]$;

```
assert( $C[N-1] == (N-1)^3$ );
```

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
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8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Inferred Pre_1

$\text{assume}(B[N-2] == (N-1)^3 - (N-2)^3);$
 $\text{assume}(\forall k \in [0, N-1), C[k] == k^3);$

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

$\text{assert}(C[N-1] == (N-1)^3);$

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

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8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Quantify Inferred Pre_1

assume($\forall j \in [0, N-1), B[j] == (j+1)^3 - j^3$);
assume($\forall k \in [0, N-1), C[k] == k^3$);

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

assert($C[N-1] == (N-1)^3$);

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

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Base Case: Substitute $N=1$

```
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5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<1; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<1; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

```
assert( $\forall k \in [0, 1), C[k] == k^3$ );  
assert( $\forall j \in [0, 1), B[j] == (j+1)^3 - j^3$ );
```

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<N; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Inductive Step

assume($\forall j \in [0, N-1), B[j] == (j+1)^3 - j^3$);
assume($\forall k \in [0, N-1), C[k] == k^3$);

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

assert($C[N-1] == (N-1)^3$);
assert($B[N-1] == N^3 - (N-1)^3$);

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
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8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Inferred Pre_2

$\text{assume}(A[N-2] == N^3 - 2 * (N-1)^3 + (N-2)^3);$
 $\text{assume}(\forall j \in [0, N-1), B[j] == (j+1)^3 - j^3);$
 $\text{assume}(\forall k \in [0, N-1), C[k] == k^3);$

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

$\text{assert}(C[N-1] == (N-1)^3);$
 $\text{assert}(B[N-1] == N^3 - (N-1)^3);$

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<N; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Quantify Inferred Pre_2

assume($\forall i \in [0, N-1), A[i] == (i+2)^3 - 2*(i+1)^3 + i^3$);
assume($\forall j \in [0, N-1), B[j] == (j+1)^3 - j^3$);
assume($\forall k \in [0, N-1), C[k] == k^3$);

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

assert($C[N-1] == (N-1)^3$);
assert($B[N-1] == N^3 - (N-1)^3$);

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

Base case: Substitute $N=1$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<N; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[1], B[1], C[1];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<1; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<1; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<1; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }  
  
assert( $\forall k \in [0, 1), C[k] == k^3$ );  
assert( $\forall j \in [0, 1), B[j] == (j+1)^3 - j^3$ );  
assert( $\forall i \in [0, 1), A[i] == (i+2)^3 - 2*(i+1)^3 + i^3$ );
```

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
3.   A[0]=6; B[0]=1; C[0]=0;  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
6.   for (int y=1; y<N; y++)  
7.     B[y] = B[y-1] + A[y-1];  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Inductive Step

assume($\forall i \in [0, N-1), A[i] == (i+2)^3 - 2*(i+1)^3 + i^3$);
assume($\forall j \in [0, N-1), B[j] == (j+1)^3 - j^3$);
assume($\forall k \in [0, N-1), C[k] == k^3$);

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

assert($C[N-1] == (N-1)^3$);
assert($B[N-1] == N^3 - (N-1)^3$);
assert($A[N-1] == (N+1)^3 - 2*N^3 + (N-1)^3$);

Full-Program Induction - Concrete Example

Verify $\{\varphi(N)\} P_N \{\psi(N)\}$

assume(true);

```
1. void PolyCompute(int N) {  
2.   int A[N], B[N], C[N];  
  
3.   A[0]=6; B[0]=1; C[0]=0;  
  
4.   for (int x=1; x<N; x++)  
5.     A[x] = A[x-1] + 6;  
  
6.   for (int y=1; y<N; y++)  
7.     B[y] = B[y-1] + A[y-1];  
  
8.   for (int z=1; z<N; z++)  
9.     C[z] = C[z-1] + B[z-1];  
  
10. }
```

assert($\forall k \in [0, N), C[k] == k^3$);

Eliminate Quantifiers in Pre

```
assume(A[N-2]==N3-2*(N-1)3+(N-2)3);  
assume(B[N-2]=(N-1)3-(N-2)3);  
assume(C[N-2]=(N-2)3);
```

```
1. A[N-1] = A[N-2] + 6;  
2. B[N-1] = B[N-2] + A[N-2];  
3. C[N-1] = C[N-2] + B[N-2];
```

```
assert(C[N-1]==(N-1)3);  
assert(B[N-1]==N3-(N-1)3);  
assert(A[N-1]==(N+1)3-2*N3+(N-1)3);
```

Validity proved by Z3

Computing the “Difference” Pre-Condition - $\partial\varphi(N)$

- Need to compute $\partial\varphi(N)$ such that
 - (a) $\varphi(N) \rightarrow \varphi(N-1) \wedge \partial\varphi(N)$ holds
 - (b) $\partial\varphi(N)$ does not refer to scalars and array elements modified in P_{N-1}

Computing the “Difference” Pre-Condition - $\partial\varphi(N)$

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 - (a) $\varphi(N) \rightarrow \varphi(N-1) \wedge \partial\varphi(N)$ holds
 - (b) $\partial\varphi(N)$ does not refer to scalars and array elements modified in P_{N-1}
- Test for existence of $\partial\varphi(N)$
 - ▶ Validity of $\varphi(N) \rightarrow \varphi(N-1)$
 - ▶ Difference cannot be computed if above formula is invalid!

Computing the “Difference” Pre-Condition - $\partial\varphi(N)$

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 - (b) $\partial\varphi(N)$ does not refer to scalars and array elements modified in P_{N-1}
- Test for existence of $\partial\varphi(N)$
 - ▶ Validity of $\varphi(N) \rightarrow \varphi(N-1)$
 - ▶ Difference cannot be computed if above formula is invalid!
- Computed based on the pattern of $\varphi(N)$
 - ▶ **If** $\varphi(N) := \forall i (0 \leq i \leq N) \rightarrow \widehat{\varphi(i)}$ **then** $\partial\varphi(N) := \widehat{\varphi(N)}$
 - ★ $\varphi(N) := \forall i (0 \leq i \leq N) \rightarrow A[i] > 0$ $\partial\varphi(N) := A[N] > 0$
 - ▶ **If** $\varphi(N) := \varphi^1(N) \wedge \dots \wedge \varphi^k(N)$ **then**
 $\partial\varphi(N) := \partial\varphi^1(N) \wedge \dots \wedge \partial\varphi^k(N)$
 - ▶ Otherwise $\partial\varphi(N) := \text{True}$

Computing the “Difference” Program - ∂P_N

- $\partial P_N := \text{PeelLoops}(P_N)$;
- Replace assignments in the peeled loops with “difference” statements
 - ▶ $A[i] = C$;
is replaced with
 $A[i] = A_{Nm1}[i] + (C - C) ;$
 - ▶ $A[i] = B[i] + v$;
is replaced with
 $A[i] = A_{Nm1}[i] + (B[i] - B_{Nm1}[i]) + (v - v_{Nm1}) ;$
- “Simplify” generated difference terms, “Accelerate” loops
- Remove loops that simply copy values from $N-1^{th}$ to N^{th} version

Soundness Guarantee

Theorem

Soundness Guarantee

Theorem

Suppose

$$1) \{ \varphi(N) \} P_N \{ \psi(N) \} \iff \{ \varphi(N) \} P_{N-1}; \partial P_N \{ \psi(N) \}$$

Soundness Guarantee

Theorem

Suppose

- 1) $\{\varphi(N)\} P_N \{\psi(N)\} \iff \{\varphi(N)\} P_{N-1}; \partial P_N \{\psi(N)\}$
- 2) *Formula $\partial\varphi(N)$ exists such that*
 - (a) $\varphi(N) \rightarrow \varphi(N-1) \wedge \partial\varphi(N)$
 - (b) $\{\partial\varphi(N)\} P_{N-1} \{\partial\varphi(N)\}$

Soundness Guarantee

Theorem

Suppose

- 1) $\{\varphi(N)\} P_N \{\psi(N)\} \iff \{\varphi(N)\} P_{N-1}; \partial P_N \{\psi(N)\}$
- 2) *Formula $\partial\varphi(N)$ exists such that*
 - (a) $\varphi(N) \rightarrow \varphi(N-1) \wedge \partial\varphi(N)$
 - (b) $\{\partial\varphi(N)\} P_{N-1} \{\partial\varphi(N)\}$
- 3) *Formula $\text{Pre}(M)$ exists such that for $M \geq 1$*
 - (a) $\{\varphi(N)\} P_N \{\psi(N)\}$ for $0 < N \leq M$
 - (b) $\{\varphi(M)\} P_M \{\psi(M) \wedge \text{Pre}(M)\}$
 - (c) $\{\partial\varphi(N) \wedge \psi(N-1) \wedge \text{Pre}(N-1)\} \partial P_N \{\psi(N) \wedge \text{Pre}(N)\}$ for $N > M$

Soundness Guarantee

Theorem

Suppose

- 1) $\{\varphi(N)\} P_N \{\psi(N)\} \iff \{\varphi(N)\} P_{N-1}; \partial P_N \{\psi(N)\}$
- 2) *Formula $\partial\varphi(N)$ exists such that*
 - (a) $\varphi(N) \rightarrow \varphi(N-1) \wedge \partial\varphi(N)$
 - (b) $\{\partial\varphi(N)\} P_{N-1} \{\partial\varphi(N)\}$
- 3) *Formula $\text{Pre}(M)$ exists such that for $M \geq 1$*
 - (a) $\{\varphi(N)\} P_N \{\psi(N)\}$ for $0 < N \leq M$
 - (b) $\{\varphi(M)\} P_M \{\psi(M) \wedge \text{Pre}(M)\}$
 - (c) $\{\partial\varphi(N) \wedge \psi(N-1) \wedge \text{Pre}(N-1)\} \partial P_N \{\psi(N) \wedge \text{Pre}(N)\}$ for $N > M$

Then $\{\varphi(N)\} P_N \{\psi(N)\}$ holds for all $N \geq 1$.

Implemented in a prototype tool - **Vajra**



Permanent Archive



[https://doi.org/10.6084/
m9.figshare.11875428.v1](https://doi.org/10.6084/m9.figshare.11875428.v1)

- Evaluated on 231 challenging array benchmarks
- Proved 110/121 safe, 108/110 unsafe and inconclusive on 13 programs

Vajra Participated in SV-COMP 2020

- **Vajra** integrated into TCS verification tool VeriAbs
 - ▶ Bundled with VeriAbs **v1.4** as a part of its SV-COMP 2020 archive

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- “**Gold** ● **Medal**” in the Reach-safety category
 - ▶ Vajra improved the score in *Arrays sub-category*
 - ▶ **1st** place in 2020, with 694/759 points, solved 410/436 programs
 - Map2Check - **2nd** place in 2020, with 379/759 points
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- *Publication* in SV-COMP/TACAS 2020 - “VeriAbs: Verification by Abstraction and Test Generation (Competition Contribution)”
 - ▶ *Main novelty*: Full-Program Induction using Vajra

Conclusion

- Presented the novel *Full-Program Induction* technique that
 - ▶ proves quantified as well as quantifier-free assertions of programs
 - ▶ computes the “difference” of program and property in the inductive step
 - ▶ uses weakest-pre computation to infer new facts that aid induction
 - ▶ is property driven and efficient
- Vajra verifies a large class of challenging array benchmarks

Thank You