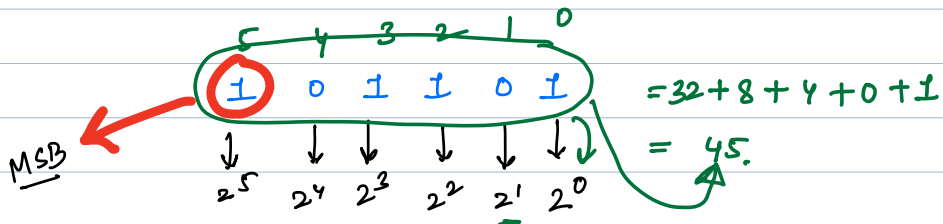


Binary number system [0, 1]



2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
	1	

$(45)_{10}$
 \downarrow
 $(101101)_2$

Quiz 1

$(1\ 1\ 0\ 1\ 0\ 1)_2 = (?)_{10}$
 $\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$
 $2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
 $\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$
 $32\ 16\ 0\ 4\ 0\ 1$
 $= 53$

Quiz 2

2	172	0
2	86	0
2	43	1
2	21	1
2	10	0
2	5	1
2	2	0
	1	

$(10101100)_2$
 \uparrow

Binary addition

$$\begin{array}{r} 453 \\ 969 \\ \hline 12210 \\ (2) \end{array}$$

$$\begin{array}{r} 111111 \\ 1011010 \\ 0110110 \\ \hline 100100010 \\ \uparrow \end{array}$$

$$\begin{array}{r} 01 \\ 11 \\ \hline 10 \\ 00 \\ 11 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 11 \\ 101001 \\ 010011 \\ \hline 111100 \\ \hline \end{array}$$

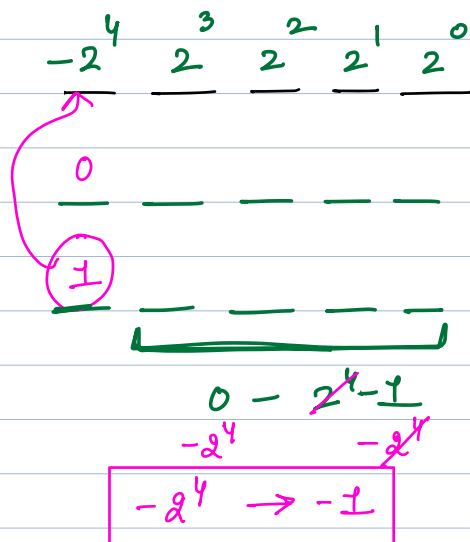
whole numbers

$$\begin{array}{c} 0/1 \quad 0/1 \quad 0/1 \quad 0/1 \quad 0/1 \\ \hline 2 \quad 2 \quad 2 \quad 2 \quad 2 \\ = 2^5 \end{array}$$

$$\begin{array}{c} \text{5 bits} \\ \hline 32 \end{array}$$

$$[0-31] \Rightarrow [0-2^5-1]$$

$$\underline{n \text{ bits}} \rightarrow 2^n \Rightarrow [0-2^n-1]$$



$$2^5 \quad 0 - 2^5 - 1$$

$$0 - 2^4 - 1$$

$$0 - 31$$

$$\swarrow \quad \searrow$$

$$-16 \rightarrow -1 \quad 0 - 15$$

$$\underbrace{\hspace{10em}}$$

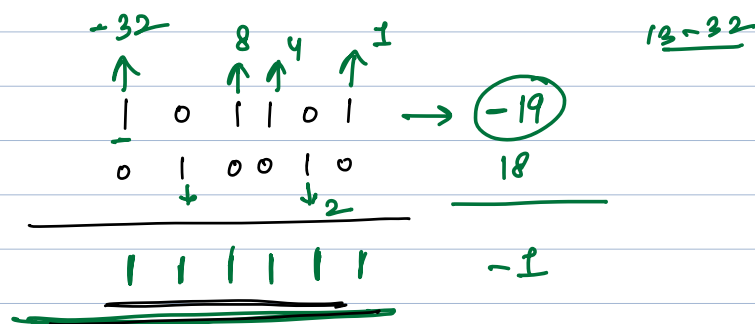
$$\downarrow$$

$-16 \rightarrow 15$

5 bits $-2^4 \rightarrow 2^4 - 1$

8 bits $-2^7 \rightarrow 2^7 - 1$

int \equiv 4 bytes \equiv 32 bits \Rightarrow $-2^{31} \rightarrow 2^{31} - 1$



$$19 - 32$$

$$\rightarrow -19$$

$$18$$

$$-1$$

2nd complement :- represent -ve numbers to Binary

-10

1st complement

0 0 0 0 1 0 1 0

↓ ↓

1 1 1 1 0 1 0 1 } +1

0 0 0 0 0 0 0 1

1 1 1 1 0 1 1 0

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

-128 64 32 16 0 4 2 0 = -10

-x

{ x $\xrightarrow{1^{st} \text{ comp}}$ \square +1 \longrightarrow -x }

-25

8 bits

0 0 0 1 1 0 0 1

↓

1 1 1 0 0 1 1 0

+1

1 1 1 0 0 1 1 1

= -25

1 0 0 1 0
1 0 1 0 0

0-31

overflow

18 + 20 = 38

1 0 0 1 1 0 \Rightarrow 6

Bitwise operators



$\&, |, ^, \sim, \ll, \gg$

a	b	$a \& b$	$a b$	$a \wedge b$	$\sim a$
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

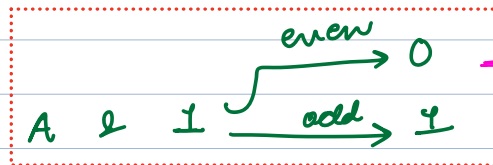
$a = 29, b = 18$

$$\begin{cases} a \& b \rightarrow 16 \\ a | b \rightarrow 31 \\ a \wedge b \rightarrow 15 \end{cases}$$

$$\begin{array}{r} 11101 \quad a \\ 10010 \quad b \\ \hline 10000 \\ \hline 11111 \quad a | b \\ \hline 01111 \quad a \wedge b \end{array}$$

Properties

•



→ This helps me to figure out if 0th bit is set/unset!

$$\begin{array}{r} 10110 \quad 0 \\ \times 00000 \quad 1 \\ \hline 00000 \quad 0 \end{array}$$

$$\begin{array}{r} 10110 \quad 1 \\ \times 00000 \quad 1 \\ \hline 00000 \quad 1 \end{array}$$

• $A \wedge A = 0$

• $A \wedge 0 = A$

0

$$\begin{array}{r} 101011 \\ 101011 \\ \hline 000000 \end{array}$$

$$\begin{array}{r} 101011 \\ \sim 000000 \\ \hline 101011 \end{array}$$

$$a \& b = b \& a$$

$$a | b = b | a$$

$$a \wedge b = b \wedge a$$

$$a \& (b \& c) = (a \& b) \& c$$

$$a | (b | c) = (a | b) | c$$

$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

left shift

$$a \ll 1$$

$$\begin{array}{cccccccc} 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{array} = 43 \quad \text{and} \quad \begin{array}{cccccccc} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{array} = 86 \quad \text{(*2)}$$

$$a \ll 1 = a \times 2$$

$$a \ll 2 = a \times 2^2$$

$$a \ll 3 = a \times 2^3$$

$$a \ll i = a \times 2^i$$

32 bits

$$1 \ll 34 = 2^{34}$$

$$a = 1$$

$$1 \ll i = 2^i$$

$$\begin{array}{cccccc} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \quad \begin{array}{r} 2^2 \\ \hline 4 \end{array}$$

Right shift

$$a \gg i = a / 2^i$$

$$\begin{array}{cccccccc} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{array}$$

keep on doing right shifts

$$00000000$$

x

x/2 → integer division

check if i^{th} bit is set or not?

7 6 5 4 3 2 1 0
 0 1 0 1 1 0 1 1
 & 0 0 0 0 0 0 0 1

$1 \ll 3$

& 0 0 0 0 1 0 0 0

0 0 0 0 0 0 0 0

0^{th} bit
 $a \& 1$
 0 → unset
 1 → set

① $a \& (1 \ll i)$
 0 → unset
 > 0 → set

② $a \gg i \& 1$
 0 → unset
 1 → set

i^{th} bit has
 become 0^{th}
 bit

Q count total set bits in a number No ^{int}

for (int $i=0$; $i<32$; $i++$)

if ($a \& (1 \ll i) \neq 0$)
 cnt++;

0 1 1 0 1 0 1 → $a \gg 1$

0110101
 0011010
 0001101
 0000110
 0000011
 0000001
 0000000 }
 ←

cnt = 1
 a >> 1
 cnt = 2
 cnt = 3
 cnt = 4

② while (a > 0)
 {
 if (a & 1 == 1)
 cnt++;
 a = a >> 1; // a = a / 2;
 }

$O(\text{bits})$
 \downarrow
 $O(\log_2 N)$

Q Array of size N. Every number occurs twice/even
 except for 1 number.
 Find this unique number.

~~3~~ ~~1~~ ~~3~~ ~~4~~ ~~4~~ 5 ~~1~~ ~~6~~ ~~7~~ ~~6~~ ~~7~~

$a \oplus a = 0$

Take xor of complete array

T.C: $O(N)$, S.C: $O(1)$

for HN $\left\{ \begin{array}{l} O(n) - T.C \\ O(n) SC \end{array} \right.$

n^2
 \downarrow


```
    ans = 0;  
    for (i = 0; i < n; i++)  
        ans ^= arr[i];  
    return ans;
```

Q

Array of size N . Every element occurs twice except for one number. Find this unique number.

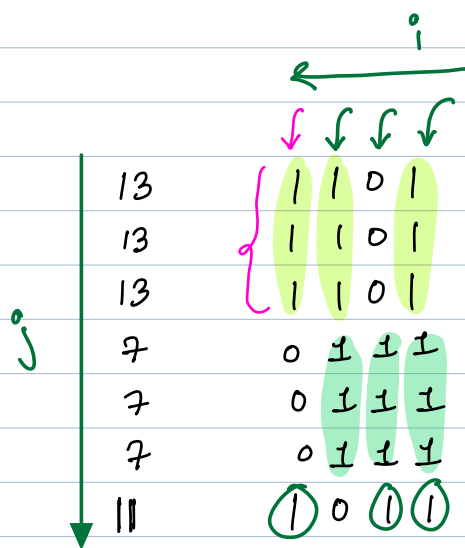
no. of bits
odd

1 2 1 3 2 1 3 2 5 3

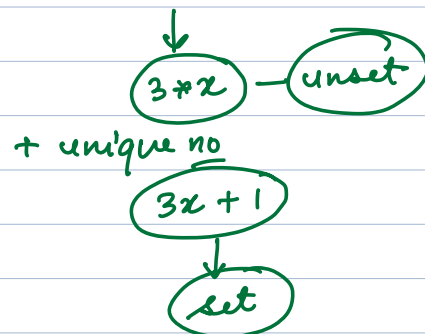
↑

freq Hashmap

T.C: $O(N)$
S.C: $O(N)$



In how many
no i^{th} bit is set



3 * 2 + 1

$\xrightarrow{\text{location}}$
 $\text{for}(i=0; i<32; i++)$
 $\{$

$\text{ans} = 0;$

$\text{int cnt} = 0;$

$\text{for}(j=0; j<n; j++)$
 $\{$

$\text{if}(\text{arr}[j] \& (1 \ll i) \neq 0)$
 $\text{cnt}++;$

$\}$

$\text{if}(\text{cnt} \% 3 \neq 0)$
 $\text{ans} = \text{ans} + (1 \ll i);$

$\}$

T.C: $O(\underline{n \times 32})$
 S.C: $O(1)$