Perceived Intensity Model of Dual-Frequency Superimposed Vibration: Pythagorean Sum

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***Abstract*—This paper presents a model for estimating the perceived intensity of a superimposed dual-frequency vibration from the perceived intensities of its two component vibrations. Based on the previous findings in the literature, we hypothesize that the three variables follow the Pythagorean relationship. Two psychophysical experiments were performed for verification with a wide range of single-frequency and superimposed vibrations applied to the fingertip. In Experiment I, we measured the perceived intensities of a large number of single-frequency vibrations and found a psychophysical magnitude function. Experiment II was designed based on the results of Experiment I in order to test the research hypothesis. For the 108 dual-frequency vibrations tested, the Pythagorean model showed 4.0% of average error in estimating the perceived intensity of a dual-frequency vibration from those of its two components. This model is robust and practical, and can be useful for any tactile interaction applications that make use of superimposed vibrations.**

***Index Terms*—Vibrotactile perception, superimposed vibration, perceived intensity, magnitude estimation.**

I. Introduction

In the past two decades, researchers have studied effective methods for providing diverse vibrotactile sensations to users for interaction purposes. Research efforts began with the various design parameters of single-frequency vibrations; see [1], [2] for reviews. Soon they have been extended to the superimposed vibrations that include several frequency components [3]–[7]. However, our current knowledge on the perceptual properties of superimposed vibrations is limited, and it hinders us from utilizing them in many applications.

In this paper, we address one of the basic properties of superimposed vibrations: *perceived intensity*. We propose a computational model that estimates the perceived intensity of a dual-frequency vibration from the perceived intensities of its two component vibrations, and then test the model's validity.

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A. Superimposed Vibrations

A superimposed signal $x_s(t)$ that consists of two sinusoidal signals, $x_1(t)$ and $x_2(t)$, can be written as

$$x_s(t) = a_1 \sin(2\pi f_1 t) + a_2 \sin(2\pi f_2 (t-\theta)), \tag{1}$$

where a_1 and a_2 are the amplitudes, f_1 and f_2 are the frequencies ($f_1 < f_2$), and θ is the phase offset. In this paper, any $x(t)$ represents the acceleration of the vibration signal stimulating the skin, unless specified otherwise. $x_s(t)$ can express diverse waveforms, as illustrated in Fig. 1.

Superimposed vibrations often exhibit easily perceptible beats (Fig. 1(a)). The frequency of beat is $|f_2 - f_1|$ for the superimposed vibration in (1). If the beat frequency is substantially lower than the two component frequencies, the beat is easily perceptible and very salient. This is an important feature distinguishing superimposed vibrations from single-frequency vibrations [3], [4], [6], [7].

The two component frequencies, f_1 and f_2, can also be represented by the base frequency $f_b = f_1$ and the frequency ratio $f_r = f_2/f_1 > 1$, which play key roles in the perception [6]. f_b has similar effects to the frequency of a single-frequency vibration: a low-frequency stimulus below 80–100 Hz feels as a rough, fluttering oscillation (mediated more by the fast adapting (FA)-1 channel), whereas a stimulus of a higher frequency is perceived as a smooth vibration (mostly mediated by the FA-2 (Pacinian) channel) [8]–[10]. This basic sensation determined by f_b is modulated by f_r. A low f_r close to 1 decreases the beat frequency, $f_2 - f_1$, and it generates the beat of a low-frequency fluctuating sensation. For a high f_r, the beat has a greater frequency, and the overall sensation becomes smoother. For example, both the (50 Hz + 60 Hz) superimposed vibration and (150 Hz + 180 Hz) vibration ($f_r = 1.2$) feel rough and intrusive, whereas the (70 Hz + 140 Hz) and (150 Hz + 300 Hz) vibrations ($f_r = 2.0$) feel smooth and vibrational; see Fig. 1.

The effects of a_1 and a_2 are relatively straightforward. If $a_1 \gg a_2$, the perceptual identity of $x_s(t)$ is determined by $x_1(t)$, and vice versa. If $a_1 \simeq a_2$, the frequency difference has a more dominant perceptual effect, as described in the preceding paragraph. The phase offset θ was shown to have little influence on the discriminability of superimposed vibrations [11].

Also see [7] for the analysis of perceptual relationships between superimposed vibrations made using perceptual spaces.

B. Vibrotactile Intensity Perception

The strength that humans feel from a single-frequency vibrotactile stimulus has been studied comprehensively. Vibrotactile

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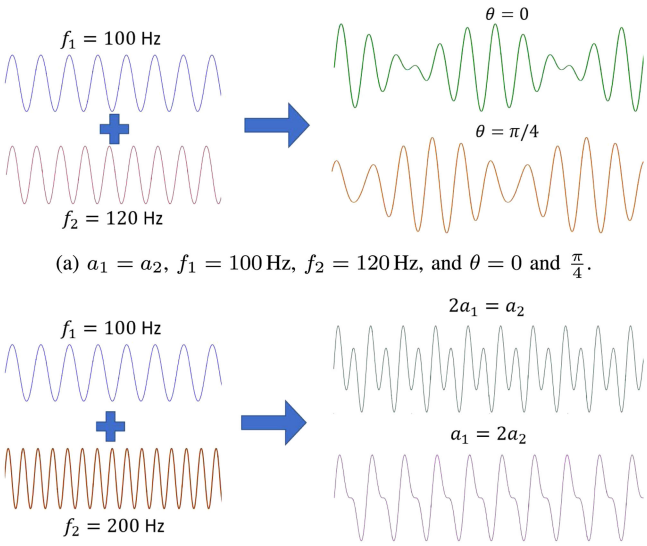

(a) $a_1 = a_2$, $f_1 = 100\,\text{Hz}$, $f_2 = 120\,\text{Hz}$, and $\theta = 0$ and $\frac{\pi}{4}$.

(b) $a_1 = 2a_2$ and $2a_1 = a_2$, $f_1 = 100\,\text{Hz}$, $f_2 = 200\,\text{Hz}$, and $\theta = 0$.

Fig. 1. Examples of superimposed vibrations (acceleration profiles).

intensity perception depends on many factors: vibration amplitude, frequency, duration, direction, body site, contact area, stimulator's weight, and age [12]–[15]. Among them, the frequency and amplitude are the most elementary variables. The perceived intensity of a single-frequency vibration increases with the increase in amplitude following Stevens' power law [16]. The exponent of the power function, which determines the rate of increase of perceived intensity, forms a U-shaped function of frequency [13]–[15].

A psychophysical magnitude function for single-frequency vibrations is a mapping from the frequency and amplitude of a vibration to its perceived intensity. A psychophysical magnitude function can be transformed to equal sensation contours, where each contour represents a set of the vibration frequencies and amplitudes that result in the same perceived intensity [17]. They are analogous to the equal loudness contours in sound perception [18]. A psychophysical magnitude function and associated equal sensation contours can serve as a reference for vibrotactile stimulus design in a variety of applications. Our group estimated several psychophysical magnitude functions of single-frequency vibrations transmitted through a mobile device to the hand [14], while taking into account the effects of vibration direction [15] and mechanical ground [19].

Several attempts were made to associate vibrotactile perceived intensity with physical measures. According to Bensmaïa and Hollins [20], the Pacinian-weighted power P_p can be computed as follows: for a sinusoidal vibration $y(t) = b\sin(2\pi ft)$, where b is the amplitude in a length unit (e.g., μm), not acceleration, the power mediated by the Pacinian system is the spectral power of the vibration weighted by the detection threshold $T(f)$, such that

$$P_p = \left(\frac{b}{T(f)}\right)^2. \tag{2}$$

This model is useful for predicting the perceived intensity. However, it remains unclear whether this metric is applicable to low-frequency vibrations processed by the FA-I system. How to generalize the Pacinian-weighted power model to superimposed vibrations is also an open problem.

In [15], our group noted the possibility that the vibrotactile stimulus energy absorbed to the hand is a good estimator for perceived intensity. Let the hand's mechanical impedance be $Z(f)$, such that

$$Z(f) = \frac{G(f)}{V(f)}, \tag{3}$$

where $G(f)$ and $V(f)$ are the Fourier transforms of the applied force and the skin movement velocity, respectively. The real part $Re[Z]$ is the damping term related to the vibration energy absorbed to the hand [21]. The imaginary part $Im[Z]$ contains the spring and mass terms associated with the energy stored in the hand-arm system and the energy transmitted to the movement of the hand-arm system, respectively [21]. In the frequency range of our interest (a few tens to 500 Hz), the imaginary part has greatly lower magnitudes than the real part.

The stimulus absorbed to the hand has the power as

$$P(f) = Re[G(f)\,V(f)] = Re[Z(f)]\,|V(f)|^2. \tag{4}$$

Thus, for a vibration with the acceleration profile $z(t) = a_z \sin(2\pi f_z t)$, the skin-absorbed power integrated over all frequencies is [15]

$$P_A = Re[Z(f_z)]\left|\frac{a_z}{2\pi f_z}\right|^2. \tag{5}$$

The perceived intensity of $z(t)$ could be expressed by a monotonic (quadratic) function of P_A [15]. It suggests that the skin-absorbed stimulus power is an adequate indicator of vibrotactile perceived intensity. This metric uses only physical variables and values as input and can be applied to vibrotactile stimuli of all frequencies.

The summation of stimulus power has also been considered in intensity perception of whole-body vibrations; see [22] for a review. For example, this issue was investigated in the temporal domain [23] and the frequency domain [24] using a chair generating whole-body vibrations. In both studies, stimulus energy and perceived intensity were proportional to each other, except for short pulses ($<$ 300 ms).

C. Perception of Complex Vibration

Complex sound waves consists of many spectral components, and its perception is generally modeled by a bank of auditory filters based on the notion of frequency channel. Each filter has a critical band, which is the smallest band of frequencies that stimulate the same area of the cochlea in the ear [25]. The loudness resulted from the stimulus energy within each channel (critical band) grows nonlinearly with increase in intensity, and the total loudness is explained by summating the neural activity across the frequency channels [26]–[28]. In particular, Howes demonstrated that the auditory loudness of multiple-tone noise can be estimated from the sum of the powers of the individual channels [28].

Researchers applied similar ideas to studying perception of vibrotactile stimuli. Makous *et al.* demonstrated that low- and high-frequency bands behave like critical bands [8]. Participants robustly discriminated low- (25 Hz) and high-frequency (200 Hz) vibrations, without mutual interference in perception. Bensmaïa *et al.* proposed the Pacinian minichannel theory to explain the perception of multiple-frequency superimposed vibrations [3]. This theory states that the Pacinian channel consists of many mini-channels, and each of them is responsible for perception in a narrow frequency range. For verification, they made intensive and spectral models of perception and quantified the contributions of the two cues to discriminating superimposed vibrations. The intensive model summed up the perceived intensities (represented by Stevens' power law) of individual frequency components, assuming that the components contribute independently. The spectral model assumed that the Weber fraction of frequency is constant and that the general perceived intensity of complex vibration follows Stevens' power law. Mini-channels in the spectral model were modeled as Gaussian filters that have log-equidistant central frequencies. Then, the intensity of each mini-channel was modeled as a power function of the Pacinian-weighted power in (2). Lastly, the model adds the intensities of all mini-channels to obtain the overall intensity. This model could explain 77%–79% of variance in the distribution of sensitivity index d', while the intensive model could explain 52%–60% of variance.

D. Research Hypothesis

The literature review suggests the following two important facts. First, the perceived intensity of a sinusoidal vibration is proportional to the square of skin-absorbed stimulus power [15]. Second, the spectral energies of different frequency components are independent and additive in vibrotactile intensity perception [3], [8]. Based on these, we hypothesize that *the perceived intensity of a superimposed vibration is represented by the Pythagorean equation of the perceived intensities of its component vibrations*.

For clarity, we denote

$$x_1(t) = a_1 \sin(2\pi f_1 t), \tag{6}$$

$$x_2(t) = a_2 \sin(2\pi f_2 t), \tag{7}$$

$$x_s(t) = x_1(t) + x_2(t). \tag{8}$$

A phase offset between $x_1(t)$ and $x_2(t)$ is not considered due to its negligible effect on the discrimination of superimposed vibrations [11]. We also define notations for perceived intensity as follows:

- $\psi(f, a)$: Perceived intensity of a single-frequency vibration (x_1 or x_2) with frequency f and amplitude a.
- $\psi(f_1, a_1, f_2, a_2)$: Perceived intensity of a dual-frequency superimposed vibration x_s.

Then, our research hypothesis can be expressed by

$$\psi(f_1, a_1, f_2, a_2)^2 = \psi(f_1, a_1)^2 + \psi(f_2, a_2)^2. \tag{9}$$

This equation means that the perceived intensities between two component vibrations and their superimposed vibration follow the Pythagorean equation, indicating a sort of perceptual independence. If the hypothesis is true, we can easily predict the perceived intensity of a dual-frequency vibration from the perceived intensities of its single-frequency component vibrations. This relationship can be a useful tool that facilitates scientific research and application development involving tactile superimposed vibrations.

E. Research Overview

To evaluate the validity of the research hypothesis in (9), we conducted two perceptual experiments in which participants rated the perceived intensities of vibrations in the absolute magnitude estimation paradigm. Exp. I was designed to collect the perceived intensities of single-frequency vibrations presented to the fingertip. These data were used as the baseline for designing the next experiment. Exp. II compared the measured and predicted perceived intensities of many dual-frequency superimposed vibrations, where the latter were computed using our research hypothesis of the Pythagorean relationship.

In Section II, we describe the general methods used for both experiments. The next two section III and IV report the designs and results of Exp. I and II, respectively. We conclude this paper in Section V.

II. General Methods

This section describes the experimental methods used for both Exp. I and II.[1]

A. Apparatus

Fig. 2 shows the device setup used in both experiments. We used a mini-shaker (Brüel-Kjær; model 4809; frequency bandwidth from DC to 18 kHz) and an amplifier (Brüel-Kjær, model 2719) to generate vibrotactile stimuli. A metal plate (16 cm × 16 cm × 8 mm; Teflon-coated) was attached to the mini-shaker using a 15 cm-long bar-shaped bracket. The metal plate was a mockup of current commercial tablets[2]. The total mass of the metal structures was approximately 550 g. This large moving mass enabled the system to be robust against the loading effect of one lightly-placed finger. The frequency response of the shaker system is shown in Fig. 3, where no structural resonances are observed.

The mini-shaker was placed inside a custom-made table. The metal plate protruded from the table and was covered with a (blue) plastic hand rest providing participants with stable contact in a comfortable posture. Participants sat in front of the mini-shaker table and placed their left index finger on the metal plate through a hole in the hand rest. A computer controlled the shaker using a data acquisition card (National Instruments; USB-6251) at a sampling rate of 10 kHz. A 50-ms Hanning window was applied to the start and stop intervals of vibration signals to obtain smooth and continuous output.

[1] All the experimental procedures were approved by the Institutional Review Board at POSTECH (PIRL-2016-E042).

[2] The use of the tablet mockup was due to the applied nature of this research supported by Samsung Electronics.

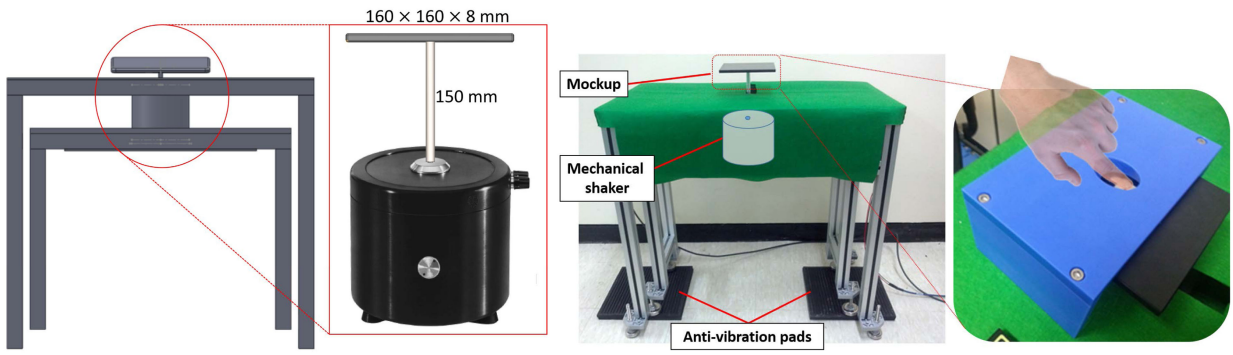

Fig. 2. Experimental setup.

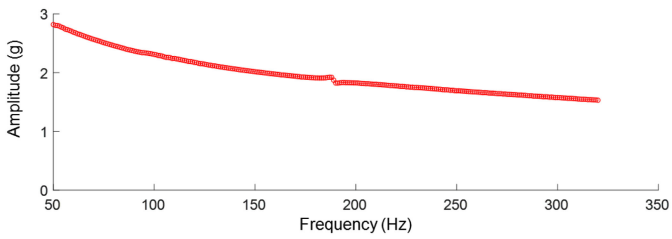

Fig. 3. Frequency response at the shaker system measured at the center of the metal plate (input amplitude 1 V).

TABLE I
FREQUENCIES AND AMPLITUDES OF VIBRATIONS USED IN EXP. I

Frequency (Hz)	Amplitude (g)				
50	0.300	0.575	0.850	1.125	1.400
80	0.321	0.612	0.903	1.195	1.486
120	0.343	0.652	0.960	1.268	1.577
160	0.368	0.694	1.020	1.347	1.673
200	0.393	0.739	1.085	1.430	1.776
250	0.421	0.787	1.153	1.519	1.885
300	0.450	0.838	1.225	1.613	2.000

The shaker was calibrated by the standard procedure described in [29]. For each vibration frequency used, we measured the shaker's vibration amplitudes while varying the input voltage amplitude in a small step size (0.01 V). The vibration acceleration was measured using a triaxial accelerometer (Kistler; model 8765 A) at the center of the metal plate, where the participants' fingertips were to contact. Then, the input-output (I/O) relationship was estimated by fitting a linear function between the input voltage amplitude and the output vibration amplitude. The inverse of the I/O function was used to calculate the input voltage level for the the desired vibration amplitude.

B. Procedure

We used the psychophysical procedure of absolute magnitude estimation [30], [31] as a rating method of perceived intensity. Prior to the experiments, participants were given the standard instructions of absolute magnitude estimation taken from [17, p. 254]. In each trial, participants perceived a vibration, represented its perceived intensity by a positive number in a free scale without a standard stimulus, and entered the number using a keypad with their right hand. Participants wore headphones to block the noise produced by the mini-shaker.

Both experiments consisted of four sessions. Each session included all the vibrations in the stimulus set; so each stimulus was evaluated four times. The presentation order of the vibrations was randomized per session and participant. Participants were given a 2-min rest between sessions to prevent tactile adaptation and fatigue. They could also take a rest whenever they wanted.

C. Data Analysis

In each experiment, the first session was considered as training, and its data were not used for data analysis. Participants' responses collected in the next three sessions were standardized using the procedure in [31]. We computed the geometric mean M_i over all the stimuli calculated for each participant i and the grand geometric mean M_g across all the stimuli and participants. Then, a scaling constant for each participant, $M_i = M_g/M_i$, was multiplied to the participant's scores. These standardized perceived intensities were used for all further data analyses.

To find psychophysical magnitude functions from the collected data, we used the nonlinear function fitting tool with the Newton method in SAS 9.3 for both experiments. We used R 4.0 for other statistical tests reported in Exp. II.

III. EXP. I: SINGLE-FREQUENCY VIBRATIONS

In this experiment, we gathered the perceived intensities of single-frequency sinusoidal vibrations transmitted to the index fingertip through a flat tablet-like surface (Fig. 2). From the collected data, we derived a psychophysical magnitude function that maps the vibration frequency and amplitude to the perceived intensity. These results were prerequisite for Exp. II that evaluated the validity of our Pythagorean hypothesis on the perceived intensity of superimposed vibration.

A. Participants

Twelve volunteers (6 females and 6 males; $M = 21.8$ years with $SD = 3.04$ years) participated in this experiment. They reported no known sensorimotor abnormalities. They signed a consent form after listening to explanations about the experiment's goal and procedure before the experiment. Each participant was paid 10,000 KRW ($\simeq$9 USD) after the experiment.

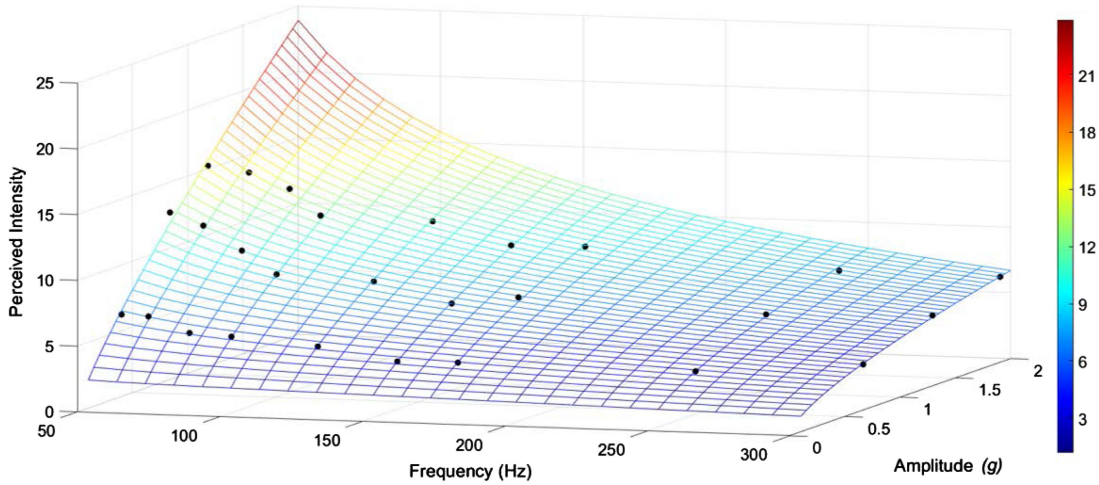

Fig. 4. Perceived intensities of single-frequency vibrations obtained in Exp. I. The black dots indicate the measured points. The colored mesh represents the psychophysical magnitude function fit to the points.

TABLE II
COEFFICIENTS OF THE MAGNITUDE FUNCTION IN (10) AND (11)

i	0	1	2	3
α_i	225.2	-276.7	126.2	-20.37
β_i	3.718	2.311	-3.801	0.9818

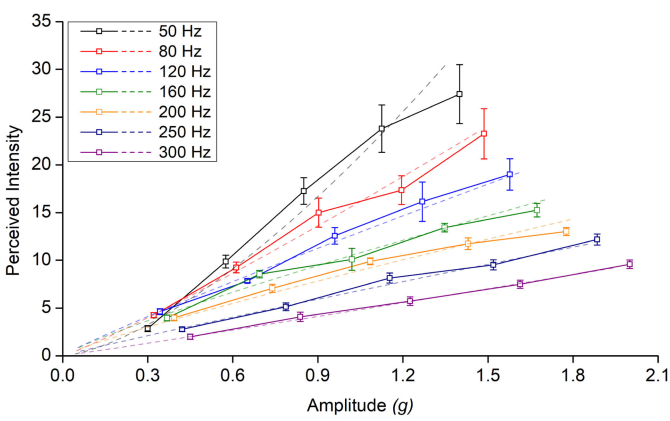

Fig. 5. Perceived intensity comparison between measured values (solid lines with square markers) and model-estimated values (dashed lines). Error bars indicate standard errors.

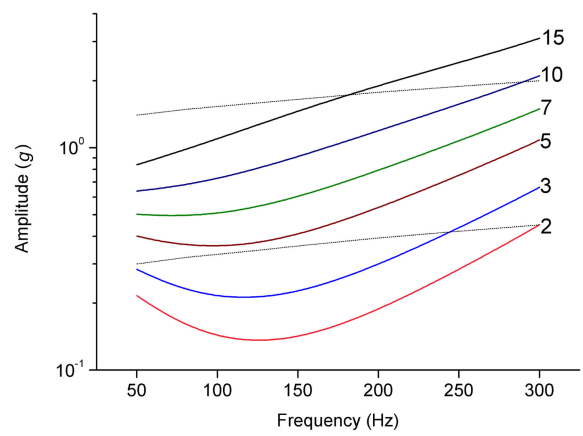

Fig. 6. Equal sensation contours for fingertip vibrations (Exp. I). Note that the logarithmic scale in the ordinate. The dotted lines represent the lower and upper bounds of the amplitudes used in the experiment. The contours outside those bounds should be considered as extrapolated values.

B. Stimuli

We designed 35 single-frequency vibrations, as specified in Table I, to cover a wide range of frequencies and amplitudes. We first selected seven frequencies: 50, 80, 120, 160, 200, 250, and 300 Hz. For each frequency, we chose five amplitudes as follows: (1) We determined five equidistant amplitudes from 0.3 g[3] to 1.4 g for the lowest frequency 50 Hz and another five equidistant amplitudes from 0.45 g to 2.0 g for the highest frequency 300 Hz; (2) The lowest amplitudes for the other five frequencies from 80 Hz to 250 Hz were computed using a geometric sequence with a common ratio of 1.07; (3) The same was applied to the highest amplitudes with a common ratio of 1.0613; and (4) For each frequency, we computed three other equidistant amplitudes between the lowest and highest amplitudes. The overall frequency and amplitude range was determine to encompass the vibration feedback range provided by commercial smartphones and tablets.

We increased the vibration amplitude with frequency to compensate for the fact that a vibration of a higher frequency is perceived to be weaker than one of a lower frequency for the same physical amplitude [14], [15], as listed in Table I. The duration was 1 s for all vibrations. This value is sufficiently long to prevent the perceived intensity underestimation caused for short vibrotactile stimuli by the temporal summation of the Pacinian channel [12].

[3] In this paper, vibration amplitudes are represented in an acceleration unit g (gravitational acceleration; $\simeq 9.8\,\mathrm{m/s^2}$) unless specified otherwise. This notation follows the IEEE's convention and is different from nonitalic g (gram).

C. Results

Fig. 4 shows the perceived intensities of the 35 single-frequency vibrations and a psychophysical magnitude function

TABLE III
SINGLE-FREQUENCY VIBRATIONS USED IN EXP. II

Frequency (Hz)		50	60	75	90	120	150	170	250	300
Weak	Perceived Intensity	5	5	5	5	5	4	4	3	3
	Amplitude (g)	0.401	0.386	0.371	0.363	0.371	0.317	0.349	0.437	0.664
Medium	Perceived Intensity	15	15	13	11	11	9	9	7	6
	Amplitude (g)	0.839	0.883	0.845	0.772	0.878	0.809	0.897	1.075	1.291
Strong	Perceived Intensity	25	25	21	17	17	14	14	11	9
	Amplitude (g)	1.182	1.296	1.277	1.17	1.411	1.347	1.501	1.737	1.904

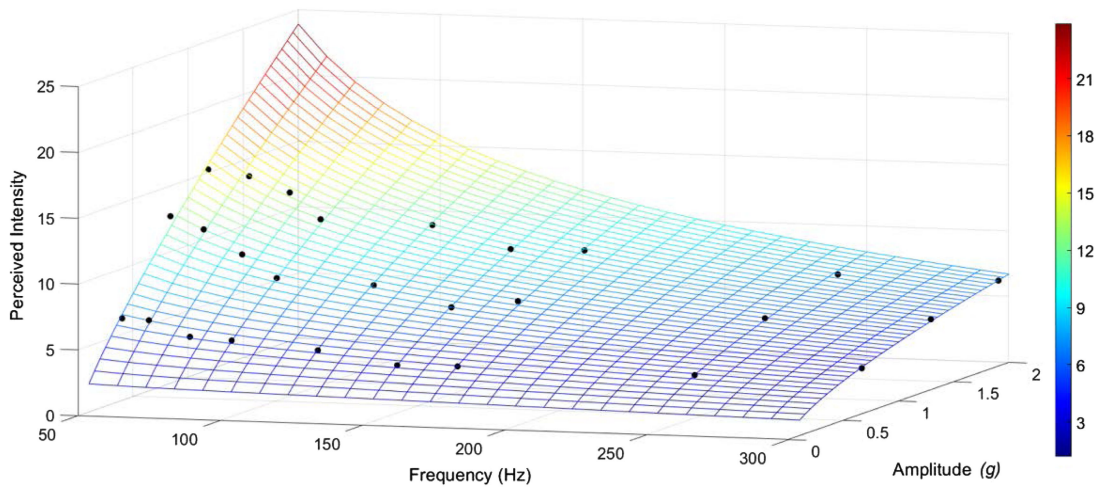

Fig. 7. Perceived intensities of single-frequency vibrations obtained in Exp. II. The black dots indicate the measured points. The colored mesh represents the psychophysical magnitude function fit to the points in (10) and (11).

derived from the experimental data. Following Stevens' power law, the magnitude function of a sinusoidal vibration with amplitude a and frequency f has the following form:

$$\psi(f, a) = k(f)a^{e(f)}. \tag{10}$$

Both the slope k and exponent e depend on f, such that

$$k(f) = \sum_{i=0}^{3} \alpha_i (\log_{10} f)^i, \; e(f) = \sum_{i=0}^{3} \beta_i (\log_{10} f)^i. \tag{11}$$

The two functions in (11) express the coefficient k and the exponent e by third-order polynomials of f. They were found to be adequate in a previous study [15]. This psychophysical magnitude function fitted the collected data very well ($R^2 = 0.999$) with the coefficients shown in Table II. The measured data and model-estimated values are also shown as 2D plots in Fig. 5 for easier comparison.

Additionally, equal sensation contours reconstructed from the psychophysical magnitude function are shown in Fig. 6. The contour shape is consistent with that reported previously [15].

IV. EXP. II: SUPERIMPOSED VIBRATIONS

In Exp. II, we examined whether our research hypothesis expressed by (9) is indeed valid. We measured the perceived intensities of both single- and dual-frequency vibrations in a comprehensive parameter range. The collected data were compared to each other in the form of the Pythagorean equation.

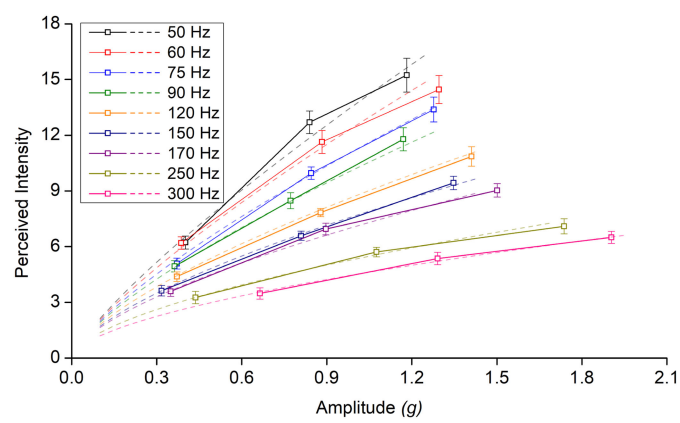

Fig. 8. Perceived intensity comparison between measured values (solid lines with square markers) and model-estimated values (dashed lines). Error bars indicate standard errors.

A. Participants

We recruited 20 adults (9 females and 11 males; age $M = 23.7$ years with $SD = 4.36$ years) as participants. None of them reported known sensory disorders. Before the experiment, they signed a consent form after listening to explanations about the experiment's goal and procedure. Each participant was paid 15,000 KRW ($\simeq$14 USD) after the experiment.

B. Stimuli

Both single- and dual-frequency sinusoidal vibrations were included in the stimuli. For single-frequency stimuli, we chose

TABLE IV
DUAL-FREQUENCY SUPERIMPOSED VIBRATIONS USED IN EXP II

No.	Frequency f_1 Hz + f_2 Hz	Each cell shows the individual perceived intensity ψ and the amplitude a (g) as ψ (a).						
		Weak				Strong		
		$\psi(f_1, a_1, f_2, a_2)$	$\psi(f_1, a_1) : \psi(f_2, a_2)$			$\psi(f_1, a_1, f_2, a_2)$	$\psi(f_1, a_1) : \psi(f_2, a_2)$		
			1:3	1:1	3:1		1:3	1:1	3:1
1	50 + 60	10	3.16 (0.295) 9.48 (0.625)	7.07 (0.506) 7.07 (0.501)	9.48 (0.616) 3.16 (0.274)	20	6.32 (0.469) 18.97 (1.053)	14.16 (0.806) 14.16 (0.845)	18.97 (0.982) 6.32 (0.461)
2	50 + 75	9.5	3.00 (0.285) 9.01 (0.616)	6.72 (0.489) 6.72 (0.478)	9.01 (0.595) 3.00 (0.239)	18.5	5.85 (0.445) 17.55 (1.094)	13.08 (0.765) 13.08 (0.849)	17.55 (0.932) 5.85 (0.425)
3	50 + 90	9	2.85 (0.274) 8.54 (0.606)	6.36 (0.471) 6.36 (0.457)	8.54 (0.574) 2.85 (0.212)	17	5.38 (0.421) 16.13 (1.112)	12.03 (0.723) 12.03 (0.840)	16.13 (0.880) 5.38 (0.389)
4	60 + 75	9.5	3.00 (0.263) 9.01 (0.616)	6.72 (0.482) 6.72 (0.478)	9.01 (0.602) 3.00 (0.239)	18.5	5.85 (0.435) 17.55 (1.094)	13.08 (0.796) 13.08 (0.849)	17.55 (0.993) 5.85 (0.425)
5	60 + 90	9	2.85 (0.253) 8.54 (0.606)	6.36 (0.463) 6.36 (0.457)	8.54 (0.578) 2.85 (0.212)	17	5.38 (0.408) 16.13 (1.112)	12.03 (0.747) 12.03 (0.840)	16.13 (0.932) 5.38 (0.389)
6	75 + 90	8.5	2.69 (0.217) 8.06 (0.574)	6.01 (0.435) 6.01 (0.433)	8.06 (0.560) 2.69 (0.201)	15.5	4.90 (0.365) 14.70 (1.018)	10.96 (0.729) 10.96 (0.769)	4.90 (0.940) 14.70 (0.356)
7	50 + 120	9	2.85 (0.274) 8.54 (0.666)	6.36 (0.471) 6.36 (0.483)	8.54 (0.574) 2.85 (0.201)	17	5.38 (0.421) 16.13 (1.332)	12.03 (0.723) 12.03 (0.967)	16.13 (0.880) 5.38 (0.402)
8	50 + 170	8.25	2.61 (0.259) 7.83 (0.762)	5.83 (0.445) 5.83 (0.541)	7.83 (0.542) 2.61 (0.212)	15.75	4.98 (0.400) 14.94 (1.619)	11.14 (0.686) 11.14 (1.150)	14.94 (0.836) 4.98 (0.450)
9	50 + 250	7.5	2.37 (0.243) 7.12 (1.094)	5.30 (0.417) 5.30 (0.801)	7.12 (0.508) 2.37 (0.341)	14.5	4.59 (0.378) 13.76 (2.202)	10.25 (0.649) 10.25 (1.612)	13.76 (0.791) 4.59 (0.686)
10	90 + 120	8	2.53 (0.189) 7.59 (0.586)	5.66 (0.409) 5.66 (0.425)	7.59 (0.541) 2.53 (0.177)	14	4.43 (0.323) 13.28 (1.078)	9.90 (0.698) 9.90 (0.782)	13.28 (0.924) 4.43 (0.325)
11	90 + 170	7.25	2.29 (0.172) 6.88 (0.656)	5.13 (0.372) 5.13 (0.466)	6.88 (0.493) 2.29 (0.182)	12.75	4.03 (0.296) 12.10 (1.266)	9.02 (0.638) 9.02 (0.899)	12.10 (0.845) 4.03 (0.352)
12	90 + 250	6.5	2.06 (0.155) 6.17 (0.940)	4.60 (0.335) 4.60 (0.688)	6.17 (0.444) 2.06 (0.293)	11.5	3.64 (0.268) 10.91 (1.722)	8.13 (0.578) 8.13 (1.261)	10.91 (0.766) 3.64 (0.537)
13	150 + 170	6.5	2.06 (0.144) 6.17 (0.577)	4.60 (0.372) 4.60 (0.410)	6.17 (0.523) 2.06 (0.161)	11.5	3.64 (0.284) 10.91 (1.122)	8.13 (0.719) 8.13 (0.797)	10.91 (1.010) 3.64 (0.312)
14	150 + 250	5.75	1.82 (0.128) 5.45 (0.825)	4.07 (0.323) 4.07 (0.604)	5.45 (0.454) 1.82 (0.257)	10.25	3.24 (0.249) 9.72 (1.524)	7.25 (0.630) 7.25 (1.116)	9.72 (0.884) 3.24 (0.475)
15	150 + 300	5.5	1.74 (0.121) 5.22 (1.129)	3.89 (0.307) 3.89 (0.852)	5.22 (0.431) 1.74 (0.394)	9.5	3.00 (0.228) 9.01 (1.906)	6.72 (0.577) 6.72 (1.438)	9.01 (0.810) 3.00 (0.665)
16	170 + 250	5.75	1.82 (0.139) 5.45 (0.825)	4.07 (0.355) 4.07 (0.604)	5.45 (0.501) 1.82 (0.257)	10.25	3.24 (0.273) 9.72 (1.524)	7.25 (0.697) 7.25 (1.116)	9.72 (0.982) 3.24 (0.475)
17	170 + 300	5.5	1.74 (0.132) 5.22 (1.129)	3.89 (0.337) 3.89 (0.852)	5.22(0.475) 1.74 (0.394)	9.5	3.00 (0.250) 9.01 (1.906)	6.72 (0.638) 6.72 (1.438)	9.01 (0.898) 3.00 (0.665)
18	250 + 300	4.75	1.50 (0.210) 4.51 (0.981)	3.36 (0.493) 3.36 (0.740)	4.51 (0.674) 1.50 (0.342)	8.25	2.61 (0.377) 7.83 (1.665)	5.83 (0.886) 5.83 (1.256)	7.83 (1.210) 2.61 (0.581)

nine frequencies from 50 Hz to 300 Hz. For each frequency, we selected three levels of perceived intensity representing weak, medium, and strong, respectively, by referring to the results of Exp. I. For each pair of vibration frequency and perceived intensity, the amplitude was determined from the results of Exp. I using (11) and Table II. This combination resulted in 27 single-frequency vibrations, as detailed in Table III.

For dual-frequency superimposed vibrations, we began with selecting 18 pairs of two frequencies f_1 and f_2. As shown in Table IV, six pairs (no. 1–6) combined two low-frequency vibrations between 50 Hz and 90 Hz, other six (no. 13–18) two high-frequency vibrations between 150 Hz and 300 Hz, and the rest six (no. 7–12) one low-frequency and one middle- or high- frequency vibration. This was to distinguish the presumable effects of the two neural mechanoreceptive channels (FA-I and PC) involved in vibrotactile perception.

For each frequency pair, we chose two target perceived intensities of $\psi(f_1, a_1, f_2, a_2)$ representing weak and strong ones, respectively. Each combination of a frequency pair and a target perceived intensity was also paired with a ratio between the perceived intensities of the two component vibrations, i.e., $\psi(f_1, a_1) : \psi(f_2, a_2)$. Three ratios, 1:3, 1:1 (2:2), and 3:1, were used for that. For example, a superimposed vibration with the perceived intensity ratio of 1:3 satisfies $\psi(f_1, a_1) : \psi(f_2, a_2) = 1 : 3$, so $\psi(f_2, a_2) = 3\,\psi(f_1, a_1)$. Then, we determined the values of $\psi(f_1, a_1)$ and $\psi(f_2, a_2)$ from $\psi(f_1, a_1, f_2, a_2)$ using our Pythgorean hypothesis in (9). Finally, the vibration amplitudes expected to give $\psi(f_1, a_1)$ and $\psi(f_2, a_2)$ were computed using the psychophysical magnitude function obtained in Exp. I: (11) and Table II. Table IV shows all the vibration parameters determined as explained above. We carefully selected the values of $\psi(f_1, a_1, f_2, a_2)$, $\psi(f_1, a_1)$, and $\psi(f_2, a_2)$ so as not to extrapolate the results of Exp. I while covering wide parameter ranges. This procedure (18 frequency conditions, 2 perceived intensity levels, and 3 superposition ratios) resulted in 108 superimposed vibrations.

TABLE V
COEFFICIENTS OF THE MAGNITUDE FUNCTION IN (10) AND (11) FOR SINGLE-FREQUENCY VIBRATIONS IN EXP. II

i	0	1	2	3
α_i	78.985	-72.4829	26.3773	-3.7367
β_i	-0.5272	2.5681	-1.4698	0.2475

As a result, the stimulus set of Exp. II had a total of 135 vibrations (27 single-frequency and 108 dual-frequency). Inclusion of the single-frequency vibrations in this

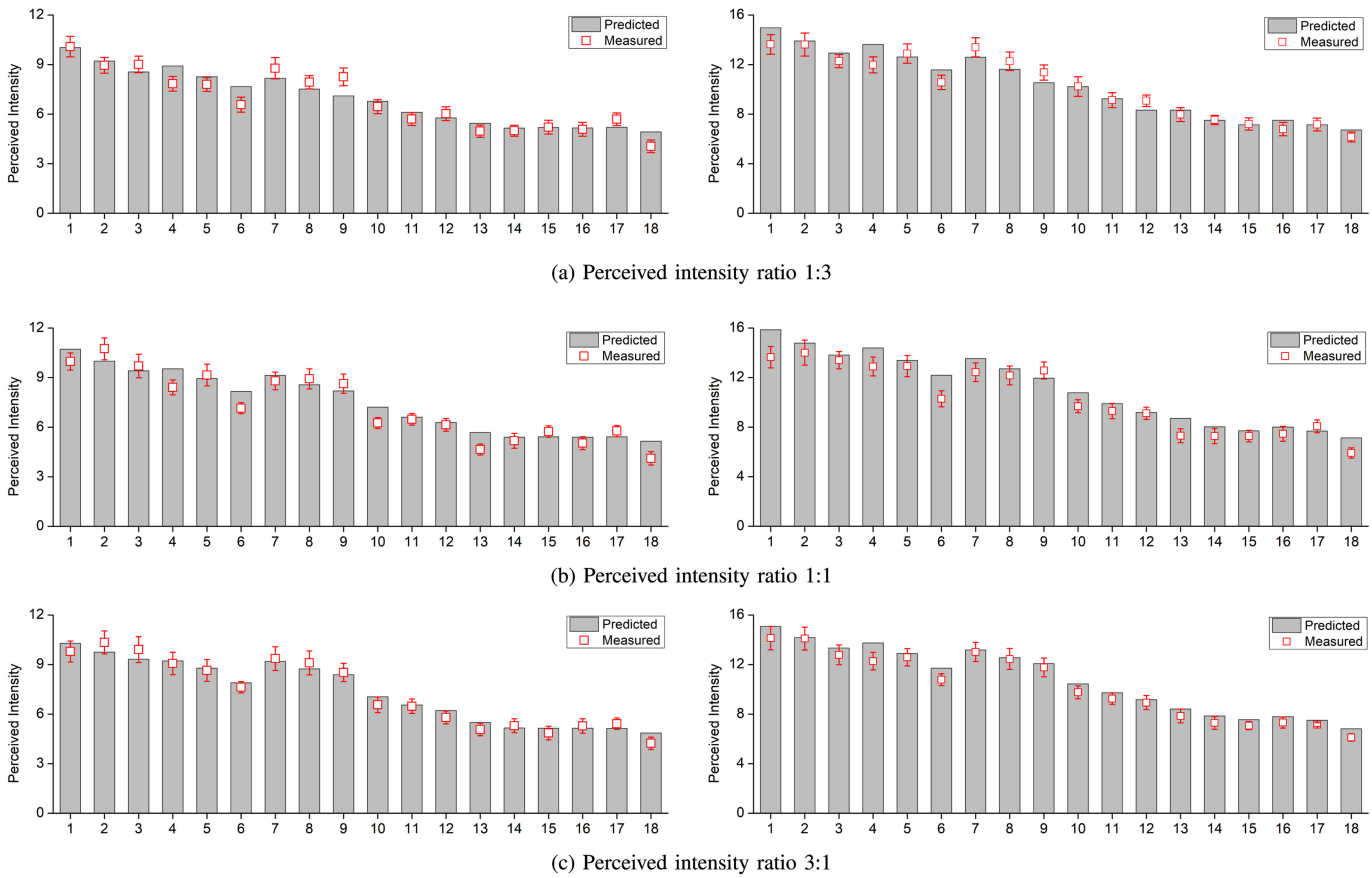

Fig. 9. Perceived intensities of superimposed vibrations measured in Exp. II. Left: weak vibrations and right: strong vibrations. The abscissa shows the superimposed vibration numbers in Table IV. The averaged measured perceived intensities ψ are represented by red squares, and its standard errors are by error bars. The predicted perceived intensities $\widetilde{\psi}$ are shown by gray bar graphs.

experiment, instead of just using the results of Exp. I, was to prevent the stimulus context effect[4], which is generally present with magnitude estimation procedures, from the analysis of results.

C. Results

The perceived intensities of the single-frequency vibrations collected in Exp. II are shown in Fig. 7. When compared to Exp. I (Fig. 4), the general trends are very similar, but the range is smaller presumably because of the narrower parameter ranges.

We fit another psychophysical magnitude function using the same form in (10) and (11), and obtained a very good result ($R^2 > .999$) with the coefficients α_k and β_k in Table V. Fig. 8 shows 2D comparisons between the measured data and the model-fit values. We used this data and magnitude function for the component perceived intensities $\psi(f_1, a_1)$ and $\psi(f_2, a_2)$ in all following analyses.

Let $\widetilde{\psi}(f_1, a_1, f_2, a_2)$ be the perceived intensity of the dual-frequency vibration predicted by the Pythagorean equation (our hypothesis), that is,

$$\widetilde{\psi}(f_1, a_1, f_2, a_2) = \left\{\psi(f_1, a_1)^2 + \psi(f_2, a_2)^2\right\}^{\frac{1}{2}} \tag{12}$$

We also define the percentage error E of prediction as

$$E = \frac{\widetilde{\psi}(f_1, a_1, f_2, a_2) - \psi(f_1, a_1, f_2, a_2)}{\psi(f_1, a_1, f_2, a_2)} \times 100, \tag{13}$$

where $\psi(f_1, a_1, f_2, a_2)$ is a measured value in Exp. II.

Fig. 9 shows the measured and predicted perceived intensities of all the 108 dual-frequency superimposed vibrations. The plots demonstrate good agreement between the measured and predicted values. The prediction error E had $M = 4.0\%$ and $SD = 7.55\%$ with the median (Mdn) of 3.69% and the interquartile range of $[-1.62\%, 8.34\%]$. For the absolute prediction error $|E|$, $M = 6.7\%$, $SD = 5.25\%$, and $Mdn = 5.56\%$, with the interquartile range of $[2.73\%, 8.71\%]$. This can be well acceptable considering that the Weber fraction of vibration discrimination based on amplitude differences is around 20% [2].[5]

[4] The numerical scaling result of a stimulus also depends on the other stimuli included in the experiment, i.e., the stimulus context [32].

[5] This speculation is based on the fact that the definition of E in (13) has the same form as the Weber fraction. It is noted that E uses the perceived intensity whereas the Weber fraction for vibrotactile perception uses the physical amplitude.

TABLE VI
LOGIT MODEL ANALYSIS RESULTS FOR E AND $|E|$

	Coefficient					
	E			$\|E\|$		
	Est.±StdErr	t	p	Est.±StdErr	t	p
(Intercept)	0.03±0.66	0.05	0.960	-1.27±0.68	-1.87	0.065
Intensity: W (Weak) compared to Strong						
W	-0.12±0.93	-0.13	0.900	1.21±0.96	1.26	0.213
Ratio: compared to (1:3)						
(1:1)	1.21±0.93	1.30	0.196	2.62±0.96	2.72	0.008*
(3:1)	0.82±0.93	0.88	0.380	1.88±0.96	1.95	0.055
Frequency Group: compared to H (high-frequency, no. 13–18)						
L(1–6)	0.56±0.93	0.60	0.547	1.49±0.96	1.55	0.125
M(7–12)	-2.19±0.93	-2.35	0.021*	0.42±0.96	0.44	0.662
Interactions						
W:(1:1)	-0.38±1.32	-0.29	0.772	-1.11±1.36	-0.81	0.419
W:(3:1)	-0.85±1.32	-0.65	0.528	-1.65±1.36	-1.21	0.228
W:L	-0.09±1.32	-0.07	0.945	-1.37±1.36	-1.00	0.319
W:M	0.62±1.32	0.47	0.641	-0.00±1.36	-0.01	0.998
(1:1):L	-0.58±1.32	-0.44	0.660	-1.97±1.36	-1.44	0.153
(3:1):L	-0.96±1.32	-0.73	0.466	-2.35±1.36	-1.72	0.088
(1:1):M	1.12±1.32	0.85	0.397	-1.88±1.36	-1.38	0.172
(3:1):M	1.22±1.32	0.93	0.356	-2.13±1.36	-1.56	0.122
W:(1:1):L	-0.72±1.86	-0.39	0.698	0.89±1.93	0.46	0.644
W:(3:1):L	-0.29±1.86	-0.16	0.875	1.29±1.93	0.67	0.506
W:(1:1):M	-0.55±1.86	-0.30	0.768	-0.40±1.93	-0.21	0.836
W:(3:1):M	0.05±1.86	0.03	0.980	0.69±1.93	0.36	0.721

This GLM analysis compares the data of the first level of each factor (strong intensity, ratio of 1:3, and high frequency) with the data of the given level to find the corresponding coefficients of the generalized linear model. Significant cases are marked by *.

We further analyzed whether the frequency group (no. 1–6, 7–12, or 13–18), perceived intensity level (strong or weak), and perceived intensity ratio (1:3, 1:1, or 3:1) affected the prediction errors E and $|E|$. Since the data resulted from ratios, we used a logit model [33]. To this end, we converted E (or $|E|$) to its percentile q and computed its its logit, $\ln\frac{q}{1-q}$. Then, we calculated a three-way generalized linear model (GLM) on the logit values. Table VI summarize the results. For E, the only significant difference was observed between the high- and mid-frequency groups. For $|E|$, one only case was significant between ratios of 1:1 and 1:3. No interaction terms were significant for either E or $|E|$. The adjusted R^2 values of the GLMs were low, 0.118 and 0.052, for E and $|E|$. These results of logit model analysis indicate that the parameter levels did not have considerable effects on the estimation errors and our estimation model was robust in a wide range of parameters.

D. Discussion

The low prediction errors observed in Exp. II validate our research hypothesis that the perceived intensities of component and superimposed vibrations follow the Pythagorean relationship. This finding can be useful for designing superimposed vibrations for a variety of interactive applications that benefit from rich and diverse vibrotactile stimuli [5], [6], [34]–[37].

To obtain more insights, we further examined the prediction errors of the superimposed vibrations using Fig. 10. Large absolute prediction errors over 5% were observed with superimposed vibration 1 (50 Hz + 60 Hz), 4 (60 Hz + 75 Hz), 6 (75 Hz + 90 Hz), 10 (90 Hz + 120 Hz), 13 (150 Hz + 170 Hz), and 18 (250 Hz + 300 Hz). Each of these vibrations had a low

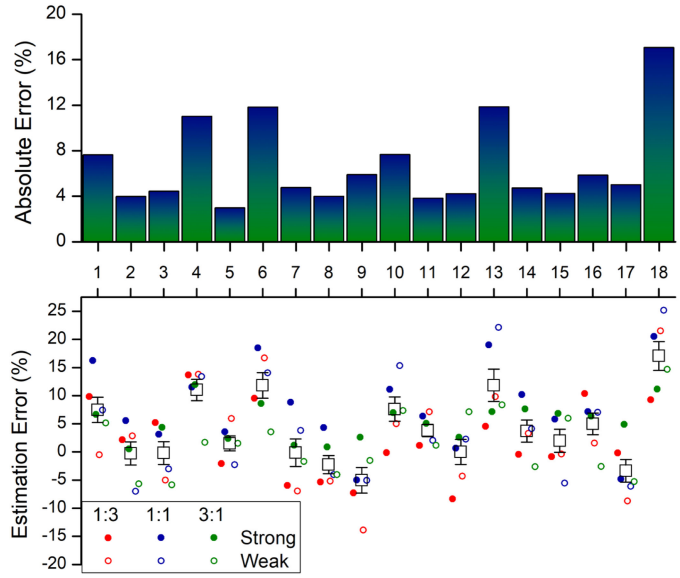

Fig. 10. Predictions errors of individual vibrations. In both plots, the abscissa represents the vibration number in Table IV. The top plot shows the means of the absolute errors E. The bottom plot shows all values of the absolute prediction error $|E|$. Squares and error bars represent means and standard errors for each vibration.

beat frequency compared to the lower frequency of the pair; $f_2 - f_1$ (Hz) / f_1 (Hz) = 10 / 50, 15 / 60, 15 / 75, 30 / 90, 20 / 150, and 50 / 250, respectively. A beat frequency less than around 30% of the lower frequency caused a large prediction error.

Similar observations are made from Fig. 11, which shows two scatter plots for the prediction error E and the absolute error $|E|$ against the frequency difference (beat frequency) $f_2 - f_1$. The results of linear regression, shown in the upper-right corners, demonstrate that both E and $|E|$ generally decrease as the frequency difference increases. This pattern can explain the significantly lower prediction errors of the mixed-frequency group than the low- and the high-frequency group. No superimposed vibrations with very low beat frequencies belonged to the mixed-frequency group; see no. 7–12 in Table IV.

Our results showed that a small difference between the two frequencies of a superimposed vibration causes a large positive prediction error deviating from our Pythagorean hypothesis. It means that the predicted perceived intensity is greater than the measured (true) perceived intensity. It appears that two close spectral components have an effect similar to masking, which makes the individual components in magnitude perception less independent. This result is consistent with the general masking effects based on the critical band theory in both sound and tactile perception [8], [20].

Finally, we made an additional analysis by fitting the following p-norm-based relationship to the experimental data:

$$\psi(f_1, a_1, f_2, a_2)^p = \psi(f_1, a_1)^p + \psi(f_2, a_2)^p. \tag{14}$$

When p was varied from 1.0 to 3.0 in a step size of 0.1, the numerical fitting errors were the smallest at $p = 2.3$ with $E = 0.4\%$ and $p = 2.4$ with $|E| = 5.6\%$. Therefore, our Pythagorean hypothesis corresponding to the integer $p = 2$ is the closest form

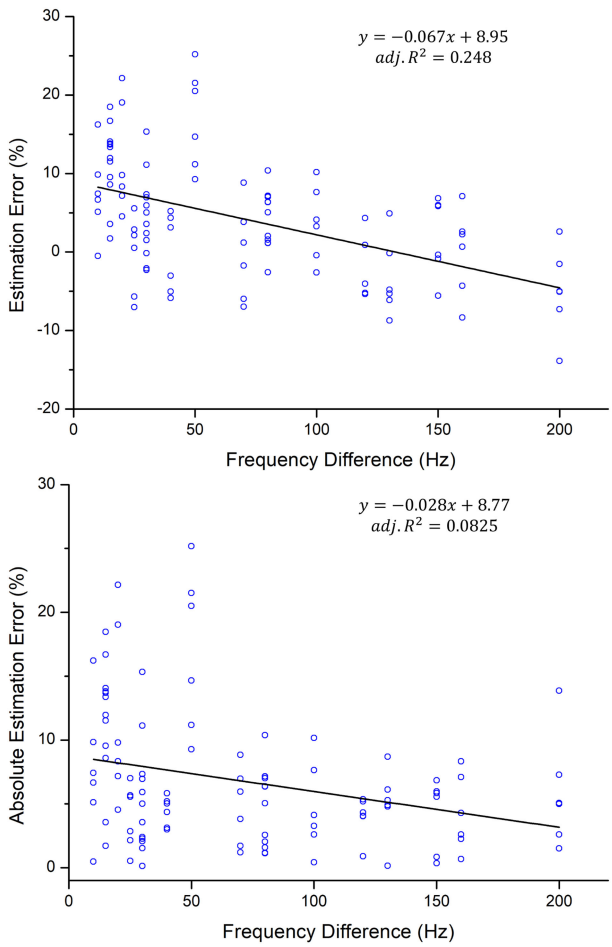

Fig. 11. Prediction error E vs. frequency difference (top). Absolute prediction error $|E|$ vs. frequency difference (bottom).

to the behavior of the experimental data, providing an intuitive power-based interpretation.

V. Conclusion

Superimposed vibrations offer unique tactile sensations that cannot be elicited by single-frequency vibrations. This advantage is attractive, but it requires an increased number of parameters that need to be turned for desired tactile impressions. In this present study, we attempted to find a convenient method that predicts the perceived intensity of a superimposed vibration from the perceived intensities of its component vibrations. Based on the previous literature, we derived a hypothesis that the perceived intensity of a superimposed vibration can be represented by those of its component vibrations by the Pythagorean equation. The hypothesis' validity was assessed by two magnitude estimation experiments with a large number of single- and dual-frequency vibrations. The experimental results support that the hypothesis provides a robust and convenient way for the perceived intensity estimation of superimposed vibrations.

The hypothesis was rather encountered in our course of research on utilizing superimposed tactile vibrations. At the moment, we are unaware of solid perceptual or neurophysiological theories to explain the perceptual phenomenon. We hope that this paper could stimulate further research interests on this intriguing topic from the haptics community.

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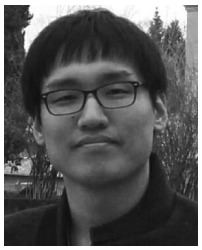

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