3. Given $\omega_c = \omega_p = 2\times\pi\times 1000 = 2000\pi$ rad/sec

$\alpha_p = 3dB \qquad \alpha_s = 10dB$

$\omega_s = 2\times\pi\times 350 = 700\pi$ rad/sec

$T = \frac{1}{f} = \frac{1}{5000} = 2\times10^{-4}$ sec

$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi\times 2\times10^{-4}}{2}\right) = 10^4\tan(0.2\pi)$

$= 7265$ rad/sec

$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{700\pi\times2\times10^{-4}}{2}\right) = 2235$ rad/sec

The order of the filter

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log 3}{\log 3.25} = 0.932$$

$N = 1$

1st order Butterworth filter.

$\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

$\Omega_c = \Omega_p = 7265$ & $s = \frac{\Omega_c}{s} = \frac{7265}{s}$

$$H(s) = \frac{1}{s+1}\Big|_{s=\frac{7265}{s}} = \frac{s}{s+7265}$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{8}{s+7265}\Big|_{s=\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{0.5792-0.5792z^{-1}}{1-0.01584z^{-1}}$$