CHAPTER : 02
SCAN CONVERSION

1. ~~All rising lines have~~

DDA (Digital Differential Analyzer Algorithm)
Assume, we follow L → R convention

<u>Case I</u> : +ve slope

ⓐ $|m| < 1$

$m = \frac{\Delta y}{\Delta x}$

$\Delta x > \Delta y$

or $x_{k+1} = x_k + 1$

$\Delta x = 1$

$m = \frac{\Delta y}{\Delta x}$

$\therefore m = \Delta y$

$y_{k+1} = y_k + \Delta y$

$y_{k+1} = y_k + m$

$$\boxed{\begin{array}{l} y_{k+1} \longrightarrow y_k + m \\ x_{k+1} \longrightarrow x_{k+1} \end{array}}$$

ⓑ $|m| > 1$

$m = \frac{\Delta y}{\Delta x}$

$\Delta y > \Delta x$

or, $y_{k+1} = y_{k+1}$

$\therefore \Delta y = 1$

$m = \frac{\Delta y}{\Delta x}$

$\therefore \frac{1}{m} = \Delta x$

$x_{k+1} = x_k + \Delta x$

$x_{k+1} = x_k + 1/m$

$$\boxed{\begin{array}{l} y_{k+1} \rightarrow y_k + 1 \\ x_{k+1} \longrightarrow x_k + 1/m \end{array}}$$

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PAGE :
DATE : / /

Case II : -ve slope

(a) $|m| < 1$, $\quad +m = \frac{\Delta y}{\Delta x}$

$|\Delta x| > |\Delta y|$

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k + \Delta y$

$\Delta y = m \Delta x$

$\Delta y = m$

$y_{k+1} = y_k + m$

$$\boxed{\begin{array}{l} y_{k+1} \longrightarrow y_k + m \\ x_{k+1} \longrightarrow x_k + 1 \end{array}}$$

(b) $|m| > 1$

$|\Delta x| < |\Delta y|$

$y_{k+1} = y_k - 1$

$x_{k+1} = x_k - \frac{1}{m}$

$$\boxed{\begin{array}{l} y_{k+1} \longrightarrow y_k - 1 \\ x_{k+1} \longrightarrow x_k - 1/m \end{array}}$$

Digitalize the line at given points A(1,5) and B(6,12) by using DDA diagram

⇒ Here,

$$\text{Slope}(m) = \frac{12-5}{6-1} = 1.4$$

$\Delta x = x_2 - x_1 = 6 - 1 = 5$

$\Delta y = y_2 - y_1 = 12 - 5 = 7$

$\therefore$ Steps $|dy| = 7$

Calculated Points	K	(x_k, y_k)	$x_{k+1} = x_k + 1/m$	(x_{k+1}, y_{k+1})
(1, 5)	0			
(1, 5)	1	(1, 5)	1 + 0.714	

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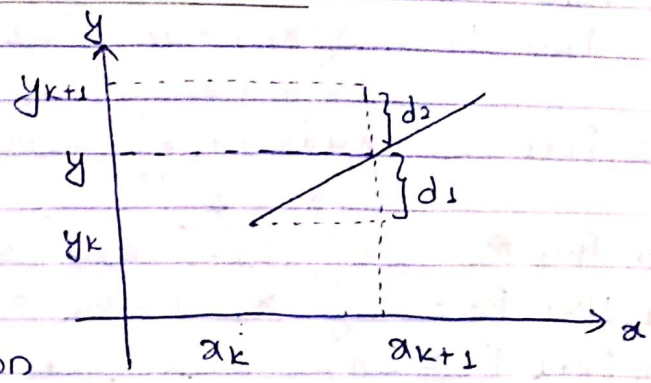
Bresenham's Line Drawing Algorithm (BSA)

For +ve slope $m < 1$ (L → R convention)



- Involves only integer calculation to determine the next pixel position to be plotted.
- The point is calculated by testing the sign of integer parameter called the decision parameter whose value is proportional to the diff betn the separation of two pixel position from the actual line path.
- Let (x_k, y_k) be the first pixel position, we sample at unit x interval

For next pixel position we have the value 'x' increased by 1 and corresponding value of y i.e,
next pixel position $(x_{k+1}, y_{k+1}) = (x_{k+1}, y_k \text{ or } y_{k+1})$

Choose the point (x_{k+1}, y) and set decision ~~pass~~ parameter at the point.

$$d_1 = y - y_k = m(x_k + 1) + c - y_k$$
$$d_2 = y_k + 1 - y = y_k + 1 - m(x_k + 1) - c$$

$$P_k = \Delta x (d_1 - d_2)$$
$$= \Delta x [m(x_k+1) + c - y_k - y_k - 1 + m(x_k+1) + c]$$
$$= \Delta x [2m(x_k+1) + 2c - (2y_k + 1)]$$
$$= \Delta x \left[2 \frac{\Delta y}{\Delta x}(x_k+1) + 2c - (2y_k+1)\right]$$
$$= 2\Delta y x_k + 2\Delta y + 2\Delta x c - 2y_k \Delta x - \Delta x \quad \text{——— (i)}$$

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Finding next successive decision parameter.

$P_{k+1} = 2\Delta y x_{k+1} + 2\Delta y - 2\Delta x y_{k+1} + 2\Delta x c - \Delta x$

Now,

$P_{k+1} - P_k = [2\Delta y x_{k+1} + 2\Delta y - 2\Delta x y_{k+1} + 2\Delta x c - \Delta x] - [2\Delta y x_k + 2\Delta y - 2\Delta x y_k + 2\Delta x c - \Delta x]$

or, $P_{k+1} - P_k = 2\Delta y x_{k+1} + \cancel{2\Delta y} - 2\Delta x y_{k+1} + \cancel{2\Delta x c} - \cancel{\Delta x} - 2\Delta y x_k - \cancel{2\Delta y} + 2\Delta x y_k - \cancel{2\Delta x c} + \cancel{\Delta x}$

or, $P_{k+1} - P_k = 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k)$

or, $P_{k+1} - P_k = 2\Delta y (\cancel{x_k} + 1 - \cancel{x_k}) - 2\Delta x (y_{k+1} - y_k)$

or, $P_{k+1} - P_k = 2\Delta y - 2\Delta x (y_{k+1} - y_k)$

i.e, $\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)}$

If P_k is $-ve$,

$y_{k+1} = y_k$

$\therefore P_{k+1} = P_k + 2\Delta y$

If P_k is $+ve$,

$y_{k+1} = y_k + 1$

$\therefore P_{k+1} = P_k + 2\Delta y - 2\Delta x$

Initial Decision Parameter,

$P_0 = 2\Delta y x_0 + 2\Delta y - 2\Delta x y_0 + 2\Delta x c - \Delta x$

Put $y_0 = m x_0 + c$

$y_0 = \frac{\Delta y}{\Delta x} x_0 + c$

$\therefore P_0 = 2\Delta y x_0 + 2\Delta y - 2\Delta x \left[\frac{\Delta y}{\Delta x} x_0 + c\right] + 2\Delta x c - \Delta x$

or, $P_0 = \cancel{2\Delta y x_0} + 2\Delta y - \cancel{2\Delta y x_0} - \cancel{2\Delta x c} + \cancel{2\Delta x c} - \Delta x$

$\boxed{\text{i.e, } P_0 = 2\Delta y - \Delta x}$

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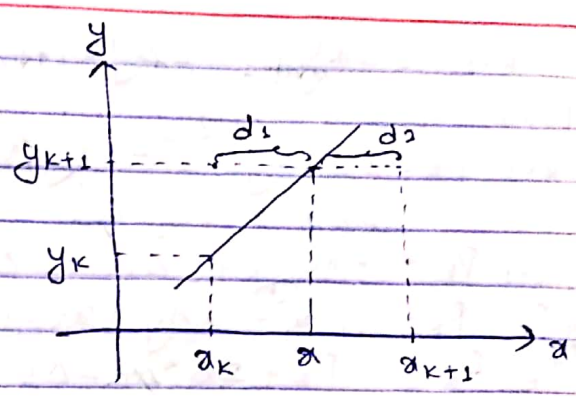
For +ve slope $m > 1$,



Let us assume that pixel (x_k, y_k) is already plotted assuming that the sampling direction is along 'y' direction.
Now for next pixel the value of y is increased by 1 and corresponding value of x i.e, next pixel position $(x_{k+1}, y_{k+1}) = (x_k \text{ or } x_k + 1, y_k + 1)$.
We have, eqn of line as:

$$y = mx + c \quad \text{—— (i)}$$

At point y_{k+1}, i.e, $y_{k+1} = y_k + 1$

$$y_{k+1} = mx + c$$

$$\text{or, } x = \frac{y_k + 1 - c}{m} \quad \text{—— (ii)}$$

Now,

$$d_1 = x - x_k = \frac{y_k + 1 - c}{m} - x_k$$

$$d_2 = x_{k+1} - x = x_k + 1 - \left(\frac{y_k + 1 - c}{m}\right)$$

Then,

$$P_k = \Delta y (d_1 - d_2)$$

$$= \Delta y \left[\frac{y_k + 1 - c}{m} - x_k - x_k - 1 + \frac{y_k + 1 - c}{m}\right]$$

$$= \Delta y \left[\left(y_k + 1 - c\right) \frac{\Delta x \times 2}{\Delta y} - 2x_k - 1 + \right]$$

$$= 2\Delta x y_k + 2\Delta x - 2\Delta x c - 2x_k \Delta y - \Delta y$$

i.e, $$\boxed{P_k = 2\Delta x y_k - 2\Delta y x_k + 2\Delta x (c - 1) - \Delta y} \quad \text{—— (iii)}$$

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$$P_{k+1} = 2\Delta x\, y_{k+1} - 2\Delta y\, x_{k+1} - 2\Delta x(c-1) - \Delta y$$

Now,

$$P_{k+1} - P_k = 2\Delta x(y_{k+1}) - 2\Delta y\, x_{k+1} - 2\Delta x(c-1) - \Delta y - 2\Delta x y_k + 2\Delta y\, x_k + 2\Delta x(c-1) + \Delta y$$

$$\text{or, } P_{k+1} - P_k = 2\Delta x y_k + 2\Delta x - 2\Delta y\, x_{k+1} - 2\Delta x y_k + 2\Delta y\, x_k$$

$$\text{or, } P_{k+1} - P_k = 2\Delta x - 2\Delta y(x_{k+1} - x_k)$$

$$\boxed{\text{i.e. } P_{k+1} = P_k + 2\Delta x - 2\Delta y(x_{k+1} - x_k)}$$

When P_k is -ve,

$$x_{k+1} = x_k$$

$$P_{k+1} = P_k + 2\Delta x$$

When P_k is +ve,

$$x_{k+1} = x_k + 1$$

$$P_{k+1} = P_k + 2\Delta x - 2\Delta y$$

Initial Decision Parameter

$$P_0 = 2\Delta x\, y_0 - 2\Delta y\, x_0 - 2\Delta x(c-1) - \Delta y$$

Put, $y_0 = m x_0 + c$

$$x_0 = \frac{y_0 - c}{\Delta x} \times \Delta y$$

$$P_0 = 2\Delta x\, y_0 - 2\Delta y\left(y_0 - c \times \frac{\Delta x}{\Delta y}\right) - 2\Delta x(c-1) - \Delta y$$

$$\text{or, } P_0 = 2\Delta x y_0 - 2\Delta x y_0 + 2c\Delta x - 2\Delta x c + 2\Delta x - \Delta y$$

$$\boxed{\text{i.e. } P_0 = 2\Delta x - \Delta y}$$

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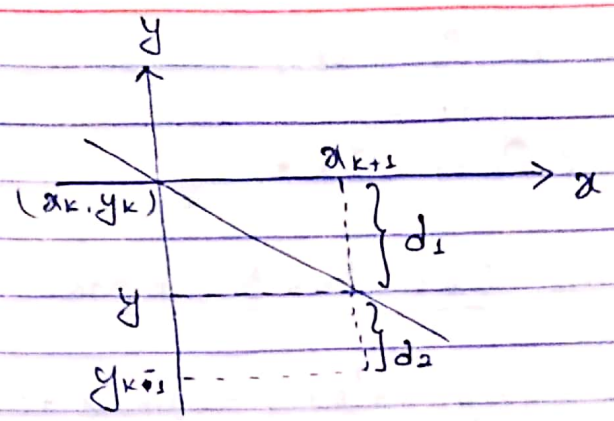
For -ve slope, $|m| < 1$



Here,

$x_{k+1} = x_k + 1$

$y = m(x_{k+1}) + c$

$d_1 = -y + y_k = -m(x_{k+1}) - c - y_k$

$d_2 = -(y_k - 1) + y = -(y_k - 1) + m(x_k + 1) + c$

Now,

$d_1 - d_2 = -m(x_k + 1) - c + y_k + y_k - 1 - m(x_k + 1) - c$

$= -2m(x_k + 1) + 2y_k - 2c - 1$

$= -2\frac{\Delta y}{\Delta x}(x_k + 1) + 2y_k - 2c - 1$

Here, decision parameter is given by,

$P_k = \Delta x(d_1 - d_2)$

$= \Delta x\left[-2\frac{\Delta y}{\Delta x}(x_k + 1) + 2y_k - 2c - 1\right]$

$\boxed{\text{or, } P_k = -2\Delta y\, x_k - 2\Delta y + 2y_k\Delta x - 2c\Delta x - \Delta x}$

Now,

$P_{k+1} = -2\Delta y\, x_{k+1} - 2\Delta y + 2y_{k+1}\Delta x - 2c\Delta x - \Delta x$

$= -2\Delta y(x_k + 1) - 2\Delta y + 2\Delta x\, y_{k+1} - 2\Delta x c - \Delta x$

Then,

$P_{k+1} - P_k = \cancel{-2\Delta y(x_k + 1)} - 2\Delta y + 2\Delta x y_{k+1} - \cancel{2\Delta x c} - \cancel{\Delta x} + \cancel{2\Delta y x_k}$

$+ \cancel{2\Delta y} - 2y_k\Delta x + \cancel{2c\Delta x} + \cancel{\Delta x}$

or, $P_{k+1} - P_k = -2\Delta y + 2\Delta x y_{k+1} - 2y_k\Delta x$

or, $P_{k+1} - P_k = -2\Delta y + 2\Delta x(y_{k+1} - y_k)$

$\boxed{\text{i.e. } P_{k+1} = P_k - 2\Delta y + 2\Delta x(y_{k+1} - y_k)}$

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When P_k is -ve,

$$y_{k+1} = y_k$$

$$\therefore P_{k+1} = P_k \mp 2\Delta y$$

When P_k is +ve,

$$y_{k+1} = y_k - 1$$

$$\therefore P_k = P_k \mp 2\Delta y \mp 2\Delta x$$

Initial Decision Parameter

$$P_0 = 2\Delta y x_0 + 2\Delta y - 2y_0\Delta x + 2c\Delta x + \Delta x$$

$$y_0 = m x_0 + c$$

$$y_0 = \frac{\Delta y}{\Delta x} x_0 + c$$

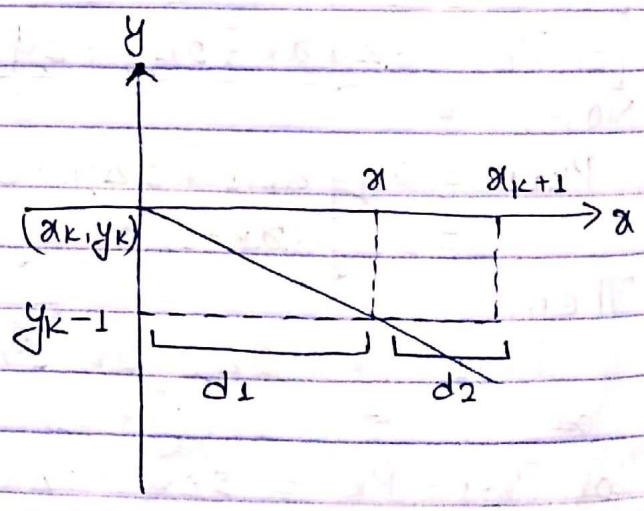
Now,

$$P_0 = -2\Delta y x_0 \mp 2\Delta y + 2y\left(\frac{\Delta y}{\cancel{\Delta x}} x_0 + c\right)\cancel{\Delta x} \mp 2c\Delta x \mp \Delta x$$

or, $P_0 = \cancel{-2\Delta y x_0} \mp 2\Delta y + \cancel{2x_0\Delta y} + \cancel{2\Delta x c} \mp \cancel{2\Delta x c} \mp \Delta x$

$$\boxed{\text{i.e, } P_0 = -2\Delta y \mp \Delta x}$$

For -ve slope $|m| > 1$,



Here,

$$y_{k+1} = y_k - 1$$

$$y_{k+1} = mx + c$$

or, $x = \dfrac{y_{k+1} - c}{m}$

i.e, $x = \dfrac{y_k - 1 - c}{m}$

Now,

$$d_1 = x - x_k = \frac{y_k - 1 - c}{m} - x_k$$

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$$d_2 = x_{k+1} - x = x_{k+1} - \left(\frac{y_k - 1 - c}{m}\right)$$

Then,

$$d_1 - d_2 = \frac{y_k - 1 - c}{m} - x_k - x_{k-1} + \left(\frac{y_k - 1 - c}{m}\right)$$

or, $$d_1 - d_2 = 2\left(\frac{y_k - 1 - c}{m}\right) - 2x_k - 1$$

Decision parameter,

$$P_k = -\Delta y\,(d_1 - d_2)$$

or, $$P_k = -\Delta y\left[2\left(y_k - 1 - c\right) \times \frac{\Delta x}{\Delta y} - 2x_k - 1\right]$$

or, $$P_k = -2\Delta x y_k + 2\Delta x + 2\Delta x c + 2x_k \Delta y + \Delta y$$

$$\boxed{\text{or, } P_k = -2\Delta x y_k + 2\Delta x + 2\Delta x c + 2\Delta y x_k + \Delta y}$$

Now,

$$P_{k+1} = 2\Delta x y_{k+1} - 2\Delta x - 2\Delta x c - 2\Delta y x_{k+1} - \Delta y$$
$$= -2\Delta x\left(y_k - 1\right) + 2\Delta x + 2\Delta x c + 2\Delta y x_{k+1} + \Delta y$$

$$P_{k+1} - P_k = -2\cancel{\Delta x y_k} + 2\cancel{\Delta x} + 2\Delta x + 2\cancel{\Delta x c} + 2\Delta y x_{k+1} + \cancel{\Delta y} + 2\cancel{\Delta x y_k} - 2\cancel{\Delta x}$$
$$- 2\cancel{\Delta x c} - 2\Delta y x_k - \cancel{\Delta y}$$

or, $$P_{k+1} - P_k = +2\Delta x + 2\Delta y\left(x_{k+1} - x_k\right)$$

$$\boxed{\text{or, } P_{k+1} = P_k + 2\Delta x + 2\Delta y\left(x_{k+1} - x_k\right)}$$

When P_k is -ve,

$\cancel{y_{k+1} = y}$

$x_{k+1} = x_k$

$P_{k+1} = P_k + 2\Delta x$

When P_k is +ve,

$x_{k+1} = x_k + 1$

$P_{k+1} = P_k + 2\Delta x + 2\Delta y$

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Initial Decision Parameter

$$P_0 = -2\Delta x y_0 + 2\Delta x + 2\Delta x c + 2\Delta y x_0 + \Delta y$$

$$y_0 = m x_0 + c$$

$$x_0 = \frac{y_0 - c}{m} = (y_0 - c) \times \frac{\Delta x}{\Delta y}$$

$$P_0 = -2\Delta x y_0 + 2\Delta x + 2\Delta x c + 2\cancel{\Delta y}\left(y_0 - c \times \frac{\Delta x}{\cancel{\Delta y}}\right) + \Delta y$$

$$\text{or, } P_0 = \cancel{-2\Delta x y_0} + 2\Delta x + \cancel{2\Delta x c} + \cancel{2\Delta x y_0} \cancel{+ 2\Delta x c} + \Delta y$$

$$\text{or, } P_0 = +2\Delta x + \Delta y$$

$$\boxed{\text{i.e, } P_0 = +2\Delta x + \Delta y}$$

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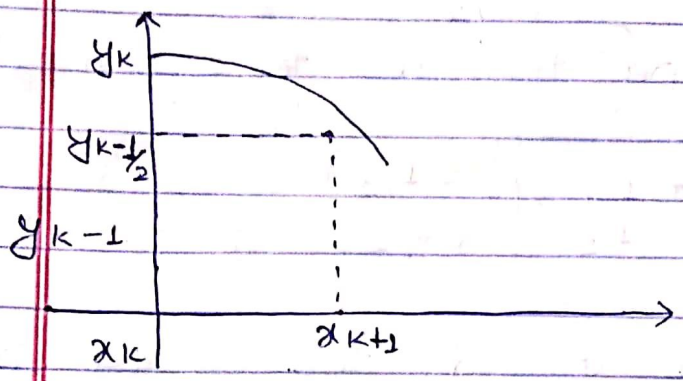
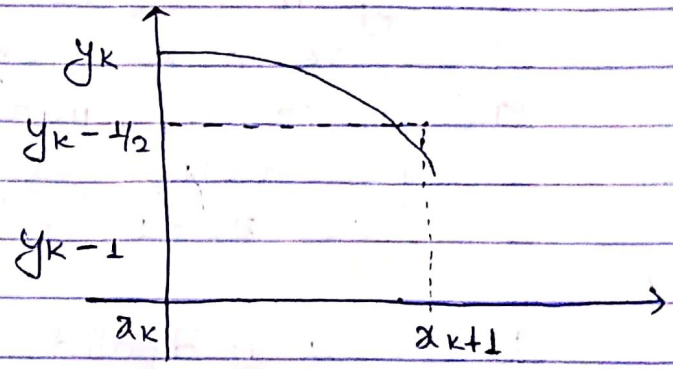
PAGE :
DATE : / /

Mid point Circle ~~Theor~~ Algorithm

8 point Symmetry



Inside ($P_k < 0$)



Outside ($P_k > 0$)

Using concept of BSA and 8 point symmetry, we calculate points that ~~w~~ use to be drawn in circle.

Eqn of circle,

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

For centre at origin,

$$x^2 + y^2 = r^2 \quad \text{—— (i)}$$

If eqn (i) = 0, on the boundary of circle
If eqn (i) > 0, outside of circle
If eqn (i) < 0, inside of circle)

By eight point symmetry we know that we only have to calculate first octant of the circle i.e (0, r) upto $x < y$ (45°)

For first octant $|m| < 1$

So, $|\Delta x| > |\Delta y|$ i.e, sampling occurs at x-axis also +ve sampling.

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i.e. $x_{k+1} = x_k + 1$
& $y_{k+1} = y_k$ or $y_k - 1$

Taking mid point i.e $y_k - 1/2$
To determine next position we take mid point as decision parameter.

$P_k = f_{circle}(x_k + 1, y_k - 1/2)$
$P_k = (x_k + 1)^2 + (y_k - 1/2)^2 - r^2$ ——— (ii)

If $P_k > 0$ midpoint lies outside the circle and next point to be plotted will be $(x_k + 1, y_k - 1)$

If $P_k < 0$ midpoint lies ~~outside~~ inside the circle and next point to be plotted will be $(x_k + 1, y_k)$

$P_{k+1} = f_{circle}(x_{k+1} + 1, y_{k+1} - 1/2)$
$= [(x_{k+1})^2 + 1]^2 + (y_{k+1} - 1/2)^2 - r^2$
$P_{k+1} - P_k = \{[(x_{k+1})^2 + 1]^2 + (y_{k+1} - 1/2)^2 - r^2\} - [(x_k + 1)^2 + (y_k - 1/2)^2 - r^2]$

or, $P_{k+1} - P_k = \cancel{(x_{k+1})^2} + 2(x_k + 1) + 1 + (y_{k+1} - 1/2)^2 \cancel{- r^2} - \cancel{(x_{k+1})^2} - (y_k - 1/2)^2 \cancel{+ r^2}$

or, $P_{k+1} - P_k = 2x_k + 2 + 1 + (y_{k+1})^2 - y_{k+1} + \cancel{1/4} - y_k^2 + y_k \cancel{- 1/4}$

$\therefore P_{k+1} - P_k = 2x_k + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 3$ ——— (iii)

$\because$ For $P_k < 0$; $y_{k+1} = y_k$

$P_{k+1} - P_k = 2x_k + (y_k^2 - y_k^2) - (y_k - y_k) + 3$
$\therefore P_{k+1} - P_k = 2x_k + 3$
$\therefore P_{k+1} = P_k + 2x_k + 3$

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Case II: For $P_k > 0$; $y_{k+1} = y_k - 1$

$P_{k+1} - P_k = 2x_k + \{(y_k - 1)^2 - (y_k)^2\} - \{y_k - 1 - y_k\} + 1$

or, $P_{k+1} - P_k = 2x_k + \cancel{y_k^2} - 2y_k + 1 - \cancel{y_k^2} - \cancel{y_k} + 1 + \cancel{y_k} + 3$

$\therefore P_{k+1} = P_k + 2x_k - 2y_k + 5$

For initial decision parameter, we calculate circle function at $(x_0, y_0) = (0, r)$

$P_0 = f_{circle}(0+1, r - 1/2)$

$= f_{circle}(1, r - 1/2)$

$= 1^2 + (r - 1/2)^2 - r^2$

$= 1 + \cancel{r^2} - 2 \cdot r \cdot 1/2 + 1/4 - \cancel{r^2}$

$= 1 - r + 1/4$

$= 5/4 - r \approx \boxed{1 - r}$

Q. Digitize first octant of a circle with $r = 10$ and center at origin.

=) Soln:

Initial points $(x_0, y_0) = (0, r) = (0, 10)$

$P_0 = 1 - r = 1 - 10 = -9$

$x_{k+1} = x_k + 1$

$P_k \geq 0$; $y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2x_k - 2y_k + 5$

For $P_k < 0$, $y_{k+1} = y_k$

$P_{k+1} = P_k + 2x_k + 3$

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PAGE :
DATE : / /

K	(x_k, y_k)	P_k	(x_{k+1}, y_{k+1})	P_{k+1}
0	(0,10)	-9	(1,10)	-6
1	(1,10)	-6	(2,10)	-1
2	(2,10)	-1	(3,10)	6
3	(3,10)	6	(4,9)	-3
4	(4,9)	-3	(5,9)	8
5	(5,9)	8	(6,8)	5
6	(6,8)	5	(7,7)	

$P_1 = P_0 + 2x_0 + 3$
$= -9 + 0 + 3 = -6$

$P_2 = P_1 + 2x_1 + 3$
$= -6 + 2 + 3 = -1$

$P_3 = P_2 + 2x_2 + 3$
$= -1 + 2\times 2 + 3 = 6$

$P_4 = P_3 + 2x_3 + 5 - 2y_3$
$= 6 + 6 + 5 - 20$
$= -3$

$P_5 = P_4 + 2x_4 + 3$
$= -3 + 8 + 3$
$= 8$

$P_6 = P_5 + 2x_5 + 5 - 2y_5$
$= 8 + 10 + 5 - 18$
$= 5$

Q. Digitize complete circle with eq^n $(x-2)^2 + (y-3)^2 = 25$

⇒ Soln:

Initial points $(x_0, y_0) = (0, r) = (0, 5)$

$P_0 = 1 - r = 1 - 5 = -4$

$x_{k+1} = x_k + 1$

$P \geq 0$; $y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2x_k + 5 - 2y_k$

$P < 0$; $y_{k+1} = y_k$

$P_{k+1} = P_k + 2x_k + 3$

$P_1 = P_0 + 2x_0 + 3$
$= -4 + 2\times 0 + 3$
$= -1$

$P_2 = P_1 + 2x_1 + 3$
$= -1 + 2\times 1 + 3$
$= -1 + 2 + 3$
$= 4$

$P_3 = P_2 + 2x_2 + 5 - 2y_2$
$= 4 + 2\times 2 + 5 - 2\times 5$
$= 4 + 4 + 5 - 10$
$= 3$

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PAGE :
DATE : / /

K	(x_k, y_k)	P_K	(x_{k+1}, y_{k+1})	P_{K+1}
0	(0, 5)	-4	(1, 5)	-1
1	(1, 5)	-1	(2, 5)	4
2	(2, 5)	4	(3, 4)	3
3	(3, 4)	3	(4, 3)	

Points are (1, 5) (2, 5) (3, 4) (4, 3)

Applying 8-point symmetry in (1, 5)

$(x, y) = (1, 5)$
$(x, -y) = (1, -5)$
$(-x, y) = (-1, 5)$
$(-x, -y) = (-1, -5)$
$(y, x) = (5, 1)$
$(-y, x) = (-5, 1)$
$(-y, -x) = (-5, -1)$
$(y, -x) = (5, -1)$

Now,

Translate with centre (2, 3)

$(x, y) = (3, 8)$
$(x, -y) = (3, -2)$
$(-x, y) = (1, 8)$
$(-x, -y) = (1, -2)$
$(y, x) = (7, 4)$
$(-y, x) = (-3, 4)$
$(-y, -x) = (-3, 2)$
$(y, -x) = (7, 2)$

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Midpoint Ellipse Algorithm

Eqn of ellipse

$$\left(\frac{x-x_c}{r_x}\right)^2 + \left(\frac{y-y_c}{r_y}\right)^2 = 1$$

For center at origin,

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

or, $x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0$ ——— (i)

$f(\text{ellipse}) = x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0$ ——— (ii)

For region-1, slope $|m| < 1$

So, sampling is done in x-axis i.e. $x_{k+1} = x_k + 1$ and based on decision parameter next y increment is y_k or $y_k - 1$.

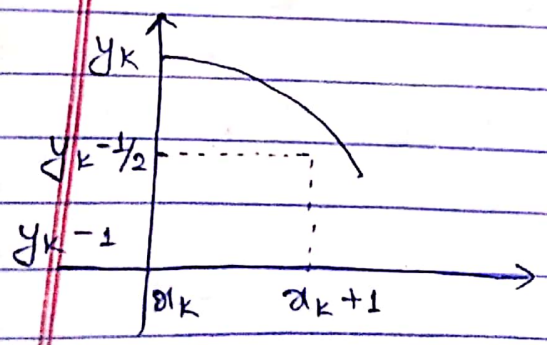
For region-2, Slope $|m| > 1$

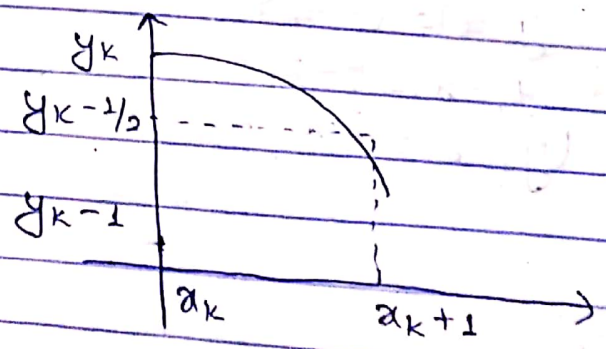
So, sampling is done in y-axis i.e, $y_{k+1} = y_k - 1$ and $x_{k+1} = x_k$ or x_{k+1} based on decision parameter

For region R_1

Using midpoint $y_k - 1/2$

$P_{1k} = f_{ellipse}(x_{k+1}, y_k - 1/2)$





$P_{1k} < 0$ (point lies inside)

$y_{k+1} = y_k$

$P_{1k} > 0$ (outside)

$y_{k+1} = y_k - 1$

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PAGE :
DATE : / /

$$P_{1K} = (x_K + 1)^2 r_y^2 + (y_K - 1/2)^2 r_x^2 - \cancel{r_y^2}\, r_y^2 r_x^2 \quad \text{—— (iii)}$$

For incremental recursive decision parameter.

$$P_{1K+1} = \text{fellipse}(x_{K+1} + 1, y_{K+1} - 1/2)$$
$$= \text{fellipse}(x_K + 1 + 1, y_{K+1} - 1/2)$$

$$P_{1K+1} = (x_K + 1 + 1)^2 r_y^2 + (y_{K+1} - 1/2) r_x^2 - r_y^2 r_x^2 \quad \text{—— (iv)}$$

$$P_{1K+1} - P_{1K} = r_y^2 (x_K + 1 + 1)^2 + (y_{K+1} - 1/2)^2 r_x^2 - \cancel{r_x^2 r_y^2} - [r_y^2 (x_K + 1)^2 + (y_K - 1/2) r_x^2 - \cancel{r_x^2 r_y^2}]$$

$$= r_y^2 [\cancel{(x_K+1)^2} + 2(x_K+1) + 1 - \cancel{(x_K+1)^2}] + r_x^2 [(y_{K+1} - 1/2)^2 - (y_K - 1/2)^2] \quad \text{—— (v)}$$

For $P_{1K} < 0$; $y_{K+1} = y_K$

$$P_{1K+1} - P_{1K} = 2 r_y^2 (x_K + 1) + r_y^2$$
$$= 2 r_y^2 x_{K+1} + r_y^2$$

For $P_{1K} > 0$; $y_{K+1} = y_K - 1$

$$P_{1K+1} - P_{1K} = 2 r_y^2 (x_K + 1) + r_y^2 + r_x^2 [(y_K - 1 - 1/2)^2 - (y_K - 1/2)^2]$$

$$= 2 r_y^2 (x_K + 1) + r_y^2 + r_x^2 [\cancel{(y_K - 1/2)^2} - 2(y_K - 1/2) + 1 - \cancel{(y_K - 1/2)^2}]$$
$$= 2 r_y^2 (x_K + 1) + r_y^2 + r_x^2 (-2y_K + 2)$$
$$= 2 r_y^2 (x_K + 1) + r_y^2 - 2 r_x^2 (y_K - 1)$$

$$P_{1K+1} = P_{1K} + 2 r_y^2 x_{K+1} + r_y^2 - 2 r_x^2 y_{K+1} \quad \text{—— (vi)}$$

Scanned with CamScanner

PAGE
DATE / /

For initial decision parameter,
P_{10} ; initial points are $(0, r_y)$
$P_{10}(1, r_y - 1/2)$

$$P_{10} = r_y^2 + r_x^2 (r_y - 1/2)^2 - r_y^2 r_x^2$$
$$= r_y^2 + r_x^2 [r_y^2 - \cancel{2} r_y \cdot 1/\cancel{2} + 1/4] - r_y^2 r_x^2$$
$$= r_y^2 + \cancel{r_x^2 r_y^2} - r_x^2 r_y + \frac{1}{4} r_x^2 - \cancel{r_x^2 r_y^2}$$

$$= r_y^2 - r_y r_x^2 + \frac{1}{4} r_x^2 \quad \text{——— (vii)}$$

<u>Ending condition for region R_1.</u>

R_1 lasts till $m = -1$
i.e. $\frac{dy}{dx} = -1$

Diff eqn (i) w.r.t x
$$2x r_y^2 + r_x^2 \, 2y \frac{dy}{dx} = 0$$

or, $\frac{dy}{dx} = -\frac{2x r_y^2}{2y r_x^2}$

or, $\frac{dy}{dx} = -1 = -\frac{2x r_y^2}{2y r_x^2}$

or, $2r_y^2 x = 2r_x^2 y$

i.e., Region lasts till
$$2r_x^2 y \geq 2r_y^2 x$$

Scanned with CamScanner

PAGE :
DATE : / /

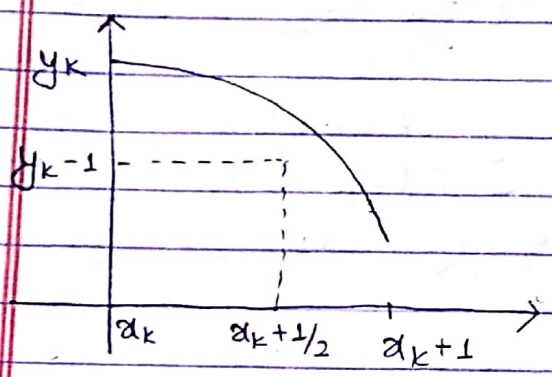
Consider region R_1 ends at (x_0, y_0)

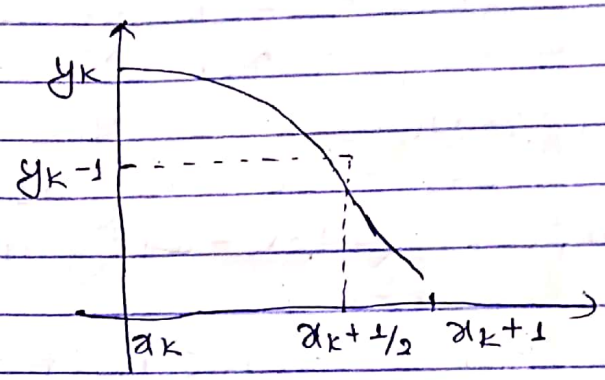
$|m| > 1$ so $|\Delta x| < |\Delta y|$

$\therefore y_{k+1} = y_k - 1$ i.e. sampling occurs in y-axis.

Now,

x_{k+1} is either x_k or $x_k + 1$





For $P_{2k} < 0$ (inside)

$x_{k+1} = x_k + 1$

For $P_{2k} \geq 0$ (outside)

$x_{k+1} = x_k$

$P_{2k} = f_{ellipse}(x_k + 1/2, y_k - 1)$

$P_{2k} = r_y^2 (x_k + 1/2)^2 + r_x^2 (y_k - 1)^2 - r_y^2 r_x^2$ ——— (viii)

So,

$P_{2k+1} = r_y^2 (x_{k+1} + 1/2)^2 + r_x^2 (y_{k+1} - 1)^2 - r_y^2 r_x^2$

$= r_y^2 (x_{k+1} + 1/2)^2 + r_x^2 (y_k - 1 - 1)^2 - r_y^2 r_x^2$

$P_{2k+1} - P_{2k} = r_y^2 [(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2] + r_x^2 [(y_k - 1 - 1)^2 - (y_k - 1)^2] - \cancel{r_x^2 r_y^2} + \cancel{r_x^2 r_y^2}$

or, $P_{2k+1} - P_{2k} = r_y^2 [(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2] + r_x^2 [\cancel{(y_k - 1)^2} - 2(y_k - 1) + 1 - \cancel{(y_k - 1)^2}]$

, $P_{2k+1} - P_{2k} = r_y^2 [(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2] + r_x^2 [-2(y_k - 1) + 1]$

, $P_{2k+1} = P_{2k} + r_y^2 [(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2] - 2r_x^2 (y_k - 1) + r_x^2$

e, $P_{2k+1} = P_{2k} + r_y^2 [(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2] - 2r_x^2 y_{k+1} + r_x^2$ ——— (ix)

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For $P_{2K} > 0$ $\quad x_{K+1} = x_K$

$\therefore P_{2K+1} = P_{2K} + r_x^2 - 2r_x^2 y_{K+1}$ —— (x)

For $P_{2K} < 0$ $\quad x_{K+1} = x_K + 1$

$P_{2K+1} = P_{2K} - 2r_x^2 y_{K+1} + r_x^2 + r_y^2 [(x_K + 1/2 + 1)^2 - (x_K + 1/2)^2]$

or, $P_{2K+1} = P_{2K} - 2r_x^2 y_{K+1} + r_x^2 + r_y^2 [\cancel{(x_K + 1/2)^2} + 2(x_K + 1/2).1 + 1$
$- \cancel{(x_K + 1/2)^2}$

or, $P_{2K+1} = P_{2K} - 2r_x^2 y_{K+1} + r_x^2 + r_y^2 (2x_K + 2)$

or, $P_{2K+1} = P_{2K} - 2r_x^2 y_{K+1} + r_x^2 + 2r_y^2 (x_K + 1)$

or, $P_{2K+1} = P_{2K} - 2r_x^2 y_{K+1} + r_x^2 + 2r_y^2 x_{K+1}$ —— (xi)

For initial condition,

Let us assume region-2 starts at (x_0, y_0)

$\therefore P_{20} = f_{ellipse}(x_0 + 1/2, y_0 - 1)$

i.e, $P_{20} = r_y^2 (x_0 + 1/2)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$ —— (xii)

Q. Digitize an ellipse with eqn

$$\left(\frac{x+3}{10}\right)^2 + \left(\frac{y+4}{8}\right)^2 = 0$$

Comparing with standard eqn

$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 0$$

$\Rightarrow$ Soln:

$r_x = 10$, $r_y = 8$

$x_c = -3$ $\quad y_c = -4$

For region-1,

Initial points $(x_0, y_0) = (0, r_y) = (0, 8)$

$P_0 = r_y^2 - r_y^2 r_x^2 + \frac{1}{4} r_x^2$

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PAGE :
DATE : / /

$P_0 = (8)^2 - (8)^2 \times (10)^2 + \frac{1}{4} \times 10^2$

$= 64 - 800 + 25$

$= -711$

K	P_{1K}	x_{K+1}, y_{K+1}	$2r_y^2 x_{K+1}$	$2r_x^2 y_{K+1}$
0	-711	(1,8)	128	1600
1	-519	(2,8)	256	1600
2	-199	(3,8)	384	1600
3	~~-~~ 249	(4,7)	-512	1400
4	-575	(5,7)	640	1400
5	129	(6,6)	768	1200
6	-239	(7,6)	896	1200
7	721	(8,5)	1024	1000

For region-2,

starting points are $(8,5) = (x_0, y_0)$

$y_{K+1} = y_K - 1$

$P_{20} = r_y^2 (x_0 + 1/2)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$

$= (8)^2 \times (8 + 1/2)^2 + (10)^2 \times (5-1)^2 - (10)^2 \times (8)^2$

$= -176$

K	P_K	x_{K+1}, y_{K+1}	$2r_y^2 x_{K+1}$	$2r_x^2 y_{K+1}$
0	-176	(9,4)	1152	800
1	276	(9,3)	1152	600
2	~~328~~ -224	(10,2)	1280	400
3	7~~8~~56	(10,1)	1280	200
4	~~1936~~ 656	(10,0)	1280	0

Scanned with CamScanner

PAGE :
DATE : / /

Now, using four point symmetry we draw ellipse at center ~~argin~~ origin.

$(-3, -4) = (x_c, y_c)$

Choosing any point $(1, 8)$

$(x, y) = (1, 8)$

$(x, -y) = (1, -8)$

$(-x, -y) = (-1, -8)$

$(-x, y) = (-1, 8)$

Now shifting as per center $(-3, -4)$

$(-3+1, 8-4) = (-2, 4)$

$(-3+1, -4-8) = (-2, -12)$

$(-1-3, -8-4) = (-4, -12)$

$(-1-3, 8-4) = (-4, 4)$

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