Electrostatics

(Charges at rest)

Charge :- [Q]

⇒ Properties of charge :-

→ Types of Charges — Two types
- → Positive
- → Negative

→ Unit of Charge is Coulomb

→ Charge is quantised.
i.e., charge of a body is integral multiple of charge of electron.

$\boxed{Q = ne}$, where n = integer.

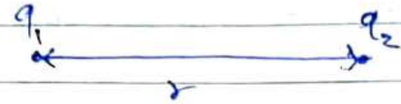
→ Total charge of an isolated system is always conserved.

→ Like charge — repulsion
Unlike charge — attraction

Methods of Charging :-

a) Charging by friction
b) Charging by conduction
c) Charging by Induction
d) Thermionic emission
e) Photoelectric effect
f) field emission

Coulomb's Law :- (only for point charges)



$F_{electrostatic} \propto q_1 q_2$

$F_{electrostatic} \propto \frac{1}{r^2}$

$\therefore \quad \boxed{F_{electrostatic\ (magnitude)} = \frac{K q_1 q_2}{r^2}}$ $\quad q_1, q_2$ → without sign

where, $\boxed{K = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9}$

ε_0 → Permittivity of free space / vacuum

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for other medium,

$K = \frac{1}{4\pi \varepsilon}$

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ → ε: Permittivity of medium

ε_r → Relative Permittivity of medium / dielectric constant of medium (ε_r)

$\therefore \quad \boxed{K = \frac{1}{4\pi \varepsilon_0 \varepsilon_r}}$

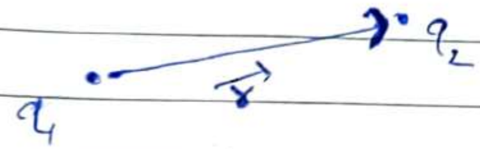
$\Longrightarrow$

$$\boxed{\vec{F} = \frac{K q_1 q_2}{r^3}\vec{r}}$$

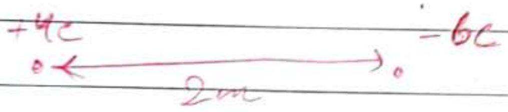
$\hookrightarrow$ on q_2 by q_1

$q_1 q_2$ — with sign

#



Question:-



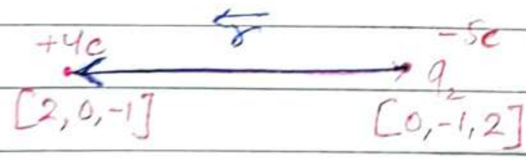
Find the magnitude of force exerted by one charge on another

Sol:-

$$F = \frac{(9\times10^9)\times 4\times 6}{4}$$

$$= 54\times10^9\ N$$

Question:-

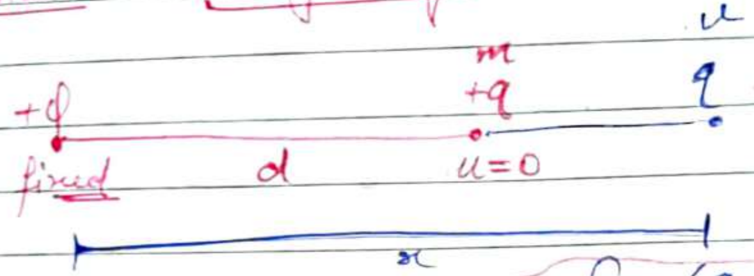


Find the force on q_1 by q_2 in vector form.

$\vec{r} = [+2,+1,-3]$

$$F_{\text{on } q_1 \text{ by } q_2} = \frac{(9\times10^9)(4)(-5)}{\left(\sqrt{4+1+9}\right)^3}\vec{r}\ N$$

Question:- $\boxed{\text{No gravity}}$



find the speed of q when separation becomes 3d.

$$\cancel{F = \frac{(9\times10^{9})\times(Q)\times(q)}{d^2}}$$

$$\cancel{\boxed{a = \frac{F}{m}}}$$

~~at q = x~~

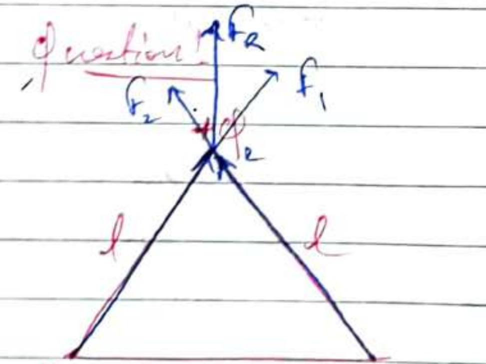
~~a =~~

At $q = x$

$$\therefore\ a = \frac{KQq}{mx^2} = \frac{v\,dv}{dx}$$

$$\Rightarrow \int_d^{3d} \frac{KQq}{mx^2}\,dx = \int_0^v v\,dv \Rightarrow \frac{v^2}{2} = \frac{-KQq}{3m}\left[\frac{1}{x^3}\right]_d^{3d}$$

Question:



find the resultant electrostatic force on one charge

Sol:

$$F_1 = \frac{KQ^2}{l^2}$$

$$F_2 = \frac{KQ^2}{l^2}$$

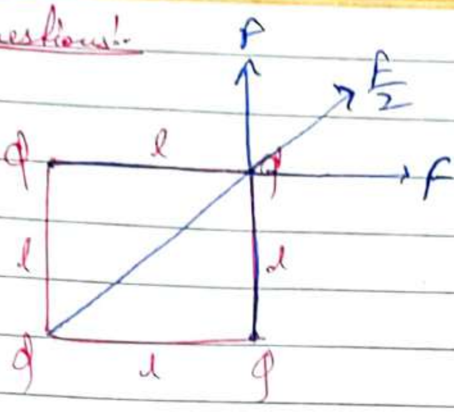
$\therefore$ Resultant force

$$= \sqrt{F^2 + F^2 + \frac{2F^2}{2}}$$

$$= \sqrt{3}\,F$$

$$\cancel{\therefore \text{ Total force on } Q_2 = \sqrt{F_1 + F_2}}$$

Question:-

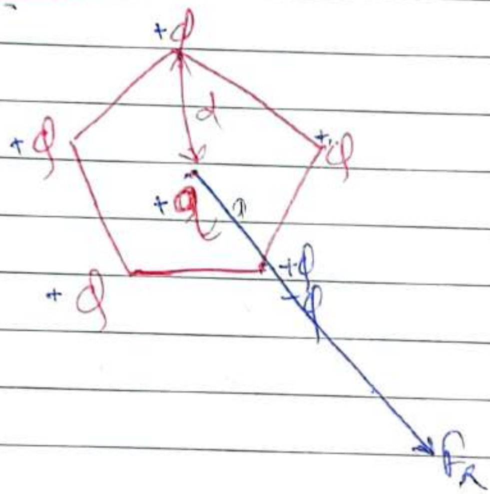


Find resultant force one one charge

$$F = \frac{Kq^2}{l^2}$$

$$\therefore F_R = \sqrt{2}F + \frac{F}{2}$$
$$= \frac{(2\sqrt{2}+1)F}{2}$$

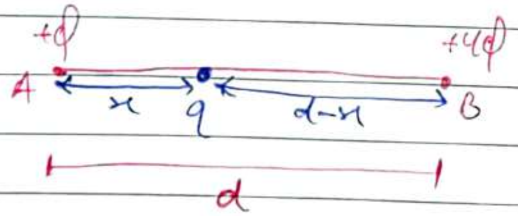
Question:-



Find the F_R on +q

$$F = \frac{Kqq}{d^2}$$

Question:-



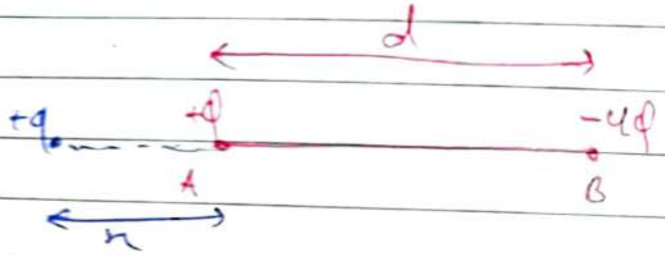
Find the Eq^m position of a third unknown of a charge on line AB.

Sol:-

$$\frac{Kqq}{x^2} = \frac{4Kqq}{(d-x)^2}$$

$$\Rightarrow x = \frac{d}{3}$$

Question!



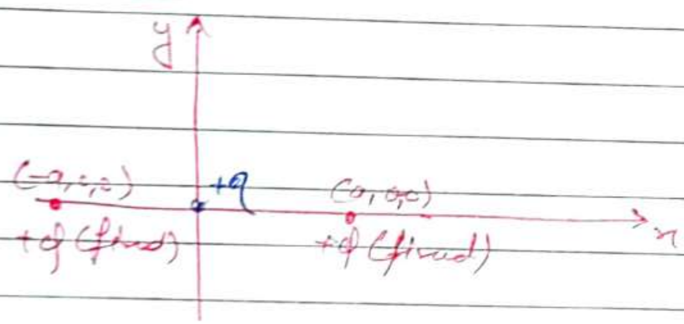
Find the Eq^n position of third charge on line AB.

Sol:

$$\frac{Kqq}{x^2} = \frac{K(4q)q}{(d+x)^2}$$

$$\boxed{x = d}$$

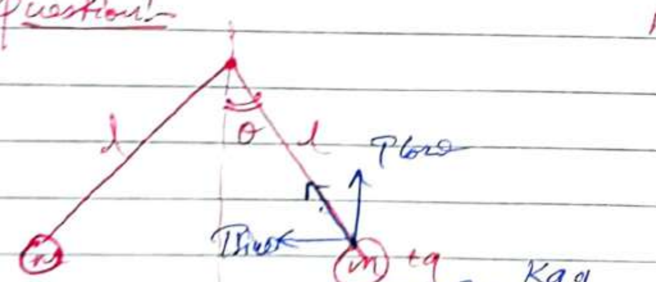
Question!



Find the nature of Eq^n of small q charge!
i) along x-axis
ii) along y-axis

Sol! i) stable Eq^n
ii) Unstable Eq^n

Question!



Find the value of θ in Eq^n position!

$$T\sin\theta = \frac{Kq_1q_2}{(2l\sin\theta)^2}$$

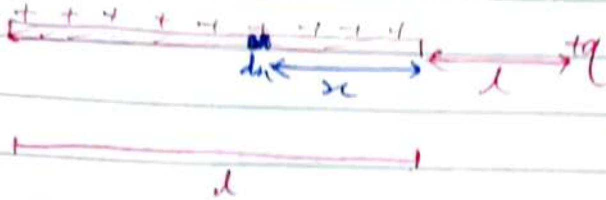
$$mg = T\cos\theta$$

$$\tan\theta = \frac{Kq_1q_2}{4l^2\sin^2\theta\, mg}$$

Question!-

Find force on 'q' due to rod

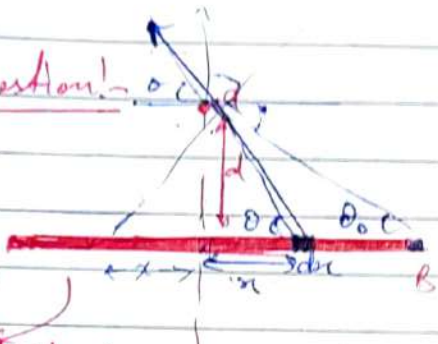
Uniform distribution



$$dF = \frac{Kq\left[\frac{Q}{l}\right]dx}{(x+l)^2}$$

$$F = \int_0^l Kq\left[\frac{Q}{l}\right]\frac{dx}{(x+l)^2}$$

Question!-



Find the resultant force on q due to rod

+ λ → Unit charge per unit length $\left[\frac{C}{m}\right]$

λ

$$F_{Res.} = \int 2dF\sin\theta$$

$$= \int_0^{\pi/2} 2\frac{Kq\lambda dx}{d^2+x^2}\sin\theta$$

$\tan\theta = \frac{d}{x}$

$x = d\cot\theta$

$dx = d(-\csc^2\theta)d\theta$

$$= \int_0^{\pi/2} -\frac{2Kq\lambda d\csc^2\theta\, d\theta\sin\theta}{d^2+d^2\cot^2\theta}$$

$$= \int_0^{\pi/2} \frac{-2Kq\lambda d\cancel{\csc^2\theta}\, d\theta\sin\theta}{d^2\cancel{\csc^2\theta}}$$

$\tan\theta_0 = \frac{2d}{l}$

$$= \int_{\pi/2}^{\theta_0} \frac{-2Kq\lambda}{d^2}\sin\theta\, d\theta$$

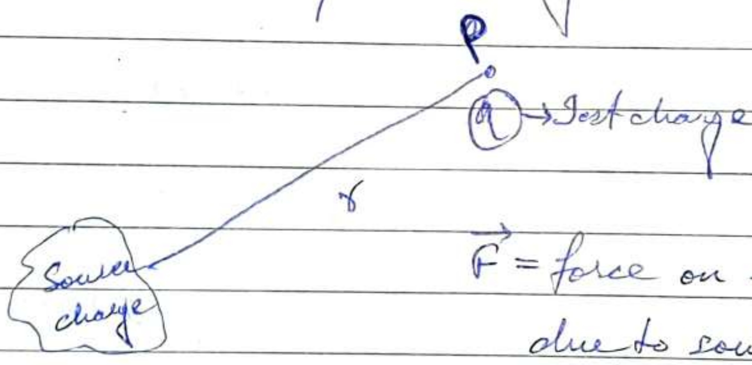
Electric field :-

~~Kisi source charge~~

A field around a source charge in which test charge experiences electrostatic force

Electric field Strength :- $[\vec{E}]$

Electrostatic force on a unit (+ve) charge is called electric field strength.



$\vec{F}$ = force on test charge 'q' due to source charge

$q \rightarrow \vec{F}$

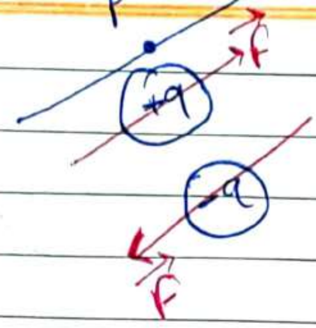
$1 \rightarrow \boxed{\dfrac{\vec{F}}{q} = \vec{E}_P}$ $\left[\dfrac{N}{C}\right]$ = unit

$\Rightarrow$ $\boxed{\vec{F} = q\,\vec{E}}$

Electric field Strength

force on 'q' due to electric field of source charge.

*



Source

Question:-

Source charges

$+q_1$ $-q_5$
$-q_2$
$+q_3$ $+q_4$

P

$q = 10C$

If Electric field due to Source charges at point P is $50\,N/C$, then

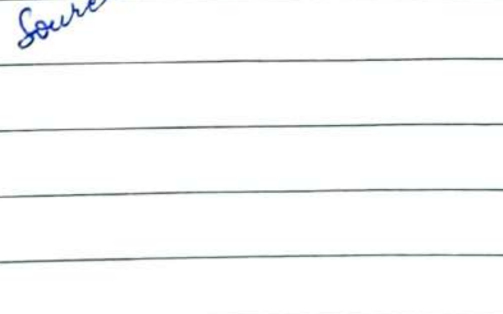
i) F_R (electrostatic) on q is 500 N

ii) F_R on q due to q only is 500 N.

iii) F_R on q due to q_4 only can be 500 N

iv) None of these

1) Electric field due to point charge :-

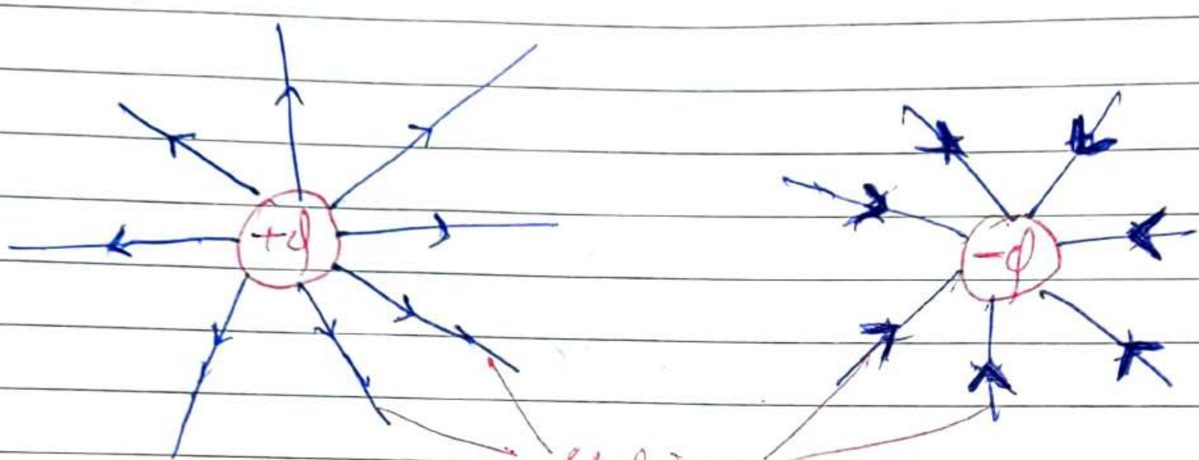


$q \rightarrow \frac{KQq}{r^2}$

$1 \rightarrow \boxed{\frac{KQ}{r^2} = E_P}$ (without sign)

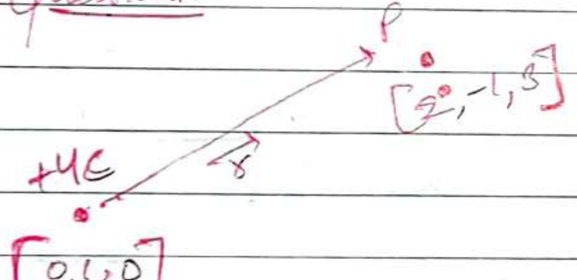
$\boxed{\vec{E}_P = \frac{KQ}{r^3}\vec{r}}$ (with sign)

$\vec{r}$ = Position vector of point 'P' wrt source charge.



Electric lines

Question:-



Find the electric field at point P due to source charge in vector form

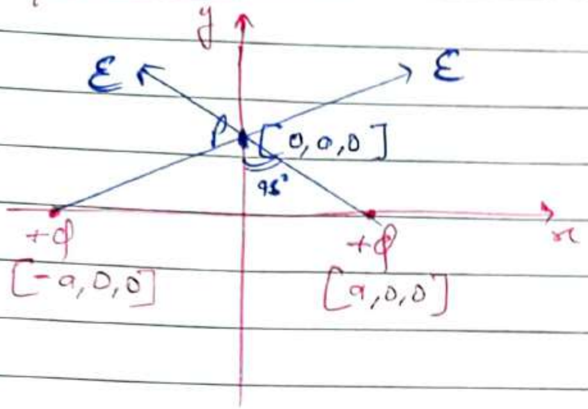
$$\boxed{\vec{E} = \frac{K(4C)}{r^3}\vec{r}} \qquad (1)$$

$$\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k})$$

$$r = \sqrt{4+4+9}$$
$$= \sqrt{17}$$

DPP - 17
Sec-1 I Sec A
II Sec A

Question -



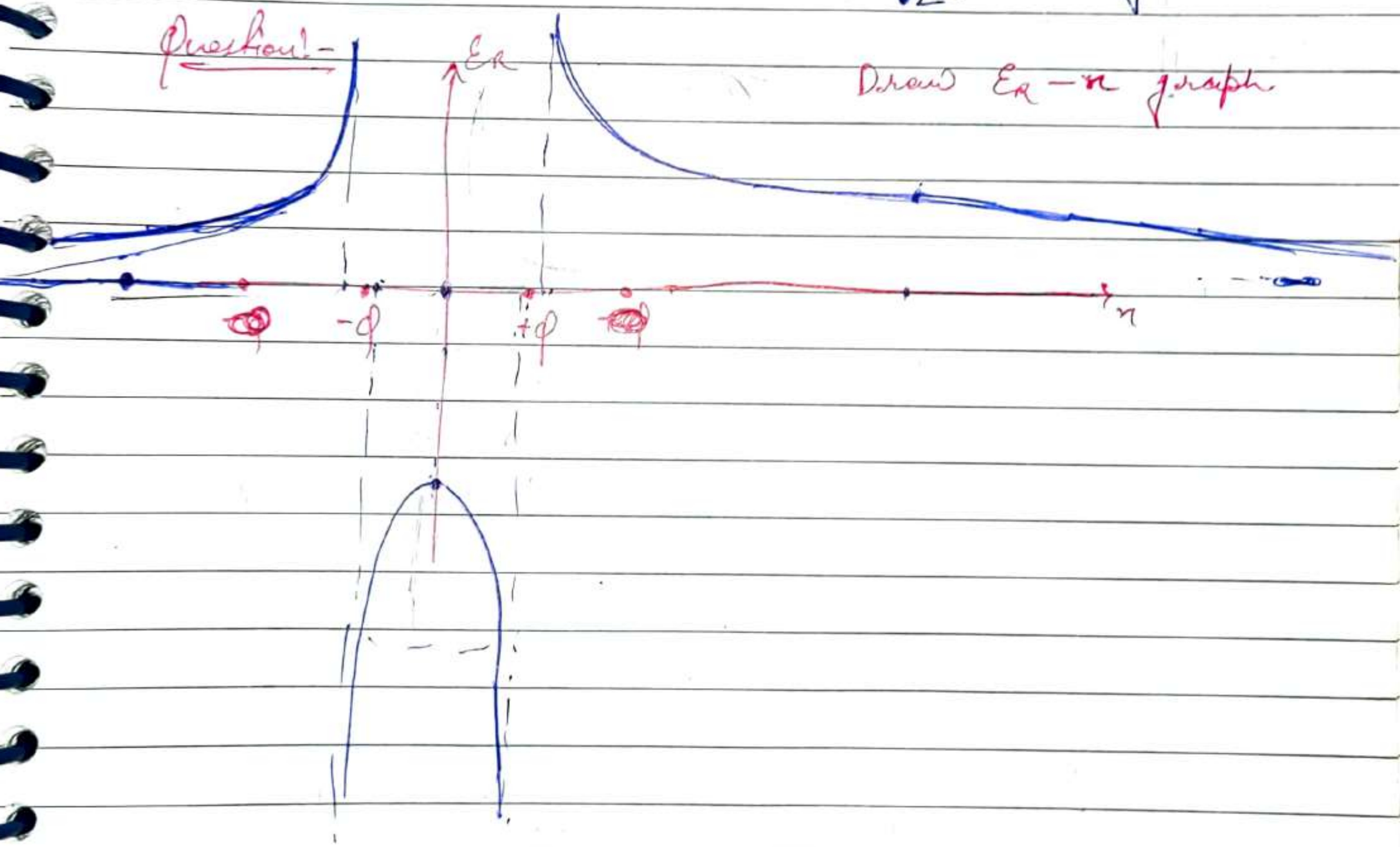
Find the Electric field at point [0, a, 0].

$$E_R = \frac{2Kq}{(a\sqrt{2})^2} \sin 45^\circ$$

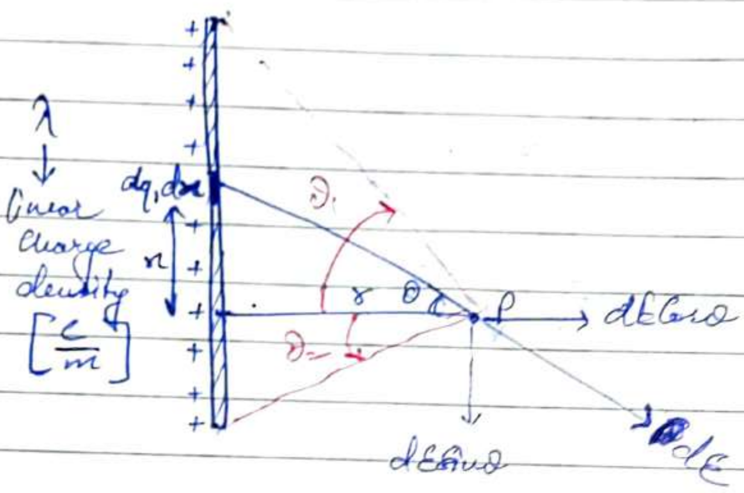
$$E_R = \frac{Kq}{\sqrt{2}a^2} \text{ in } y\text{-direction.}$$

Question -

Draw $E_R - x$ graph



Electric field due to line charge :-



$$\varepsilon_{\perp} = \int dE \cos\theta$$

$$= \int \frac{K\lambda\, dx}{x^2+r^2} \cos\theta \qquad \tan\theta = \frac{x}{r}$$

$$x = r\tan\theta$$

$$= \int \frac{K\lambda\, r\, \cancel{\sec^2\theta}\, d\theta}{r^2 \cancel{\sec^2\theta}} \cos\theta \qquad dx = r\sec^2\theta\, d\theta$$

$$= \int_{-\theta_2}^{\theta_1} \frac{K\lambda}{r} \cos\theta\, d\theta$$

$$\therefore \boxed{\varepsilon_{\perp} = \frac{K\lambda}{r}\left[\sin\theta_1 + \sin\theta_2\right]}$$

$$\boxed{\varepsilon_{\parallel} = \left|\frac{K\lambda}{r}\left[\cos\theta_2 - \cos\theta_1\right]\right|}$$

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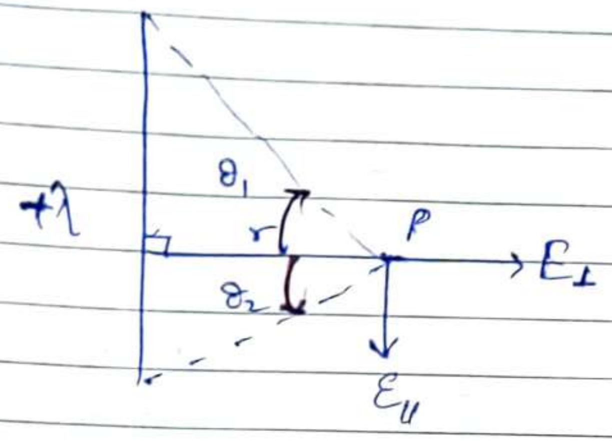
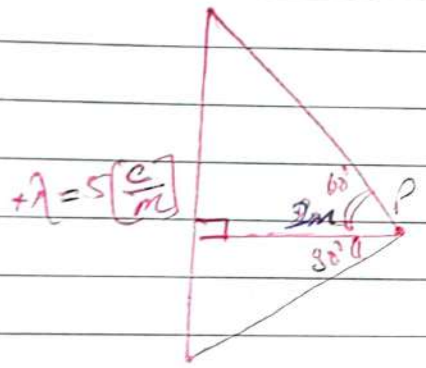


Question 1.



Find the Resultant Electric field at point P.

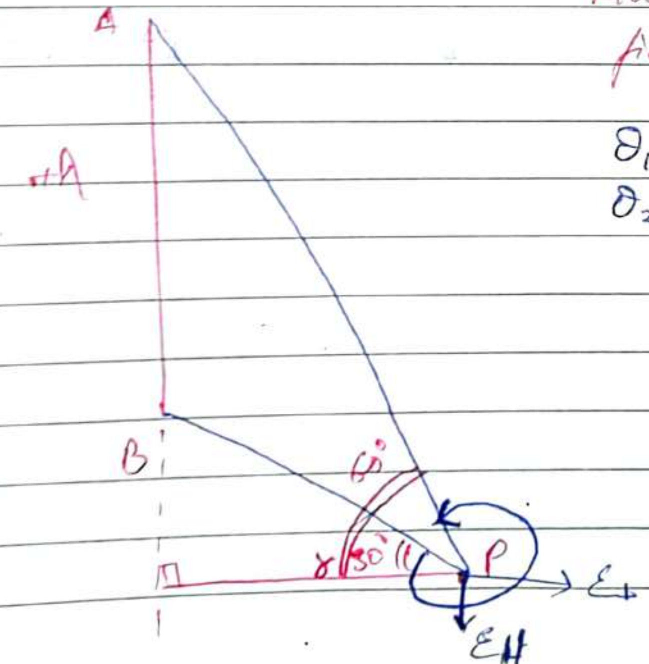
Sol!

$$E_\perp = \frac{5K}{2}\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right]$$

$$E_\parallel = \frac{5K}{2}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$\therefore E_R = \sqrt{E_\perp^2 + E_\parallel^2}$$

Question



Find the resultant Electric field.

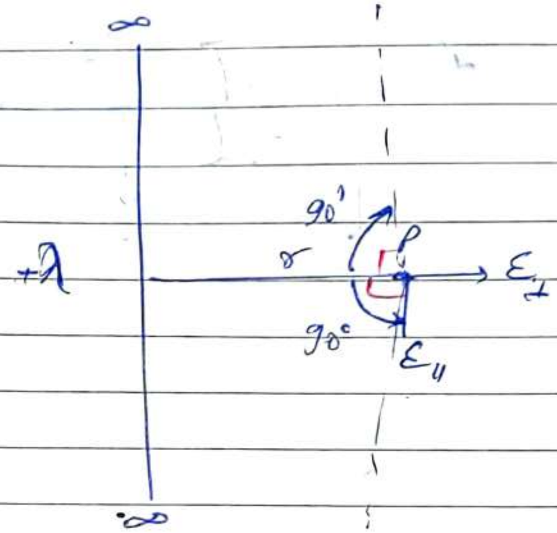
$\theta_1 = 60°$

$\theta_2 = -30°$ or $330°$

Special Cases:-

Case I :- ∞ wire

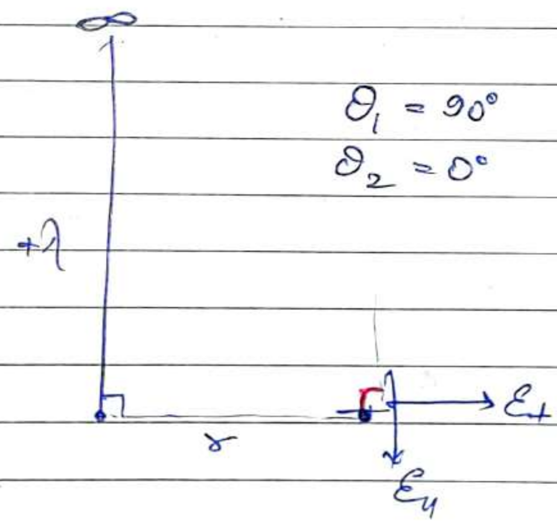
$$\mathcal{E}_\perp = \frac{2K\lambda}{r}, \quad \mathcal{E}_{\parallel} = 0$$

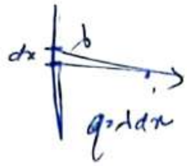


Case II :- Semi-∞ wire

$$\mathcal{E}_{\parallel} = \frac{K\lambda}{r}, \quad \mathcal{E}_\perp = \frac{K\lambda}{r}$$

$$\therefore \boxed{\mathcal{E}_R = \sqrt{2}\,\frac{K\lambda}{r}}$$





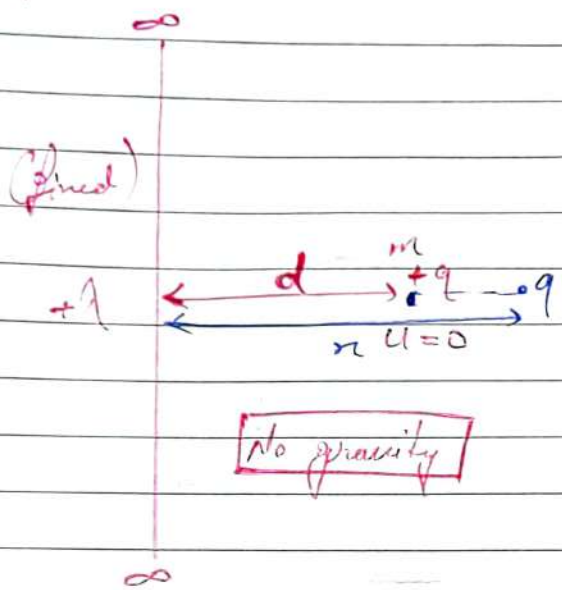
DPP-18

Ex-2 Q 1,2,4,5,

Q 1-6 (4,6)

Q

Question 1:



Find the speed of point charge when separation becomes 3d.

Soln:

$$F = \frac{2Kq\lambda}{x}$$

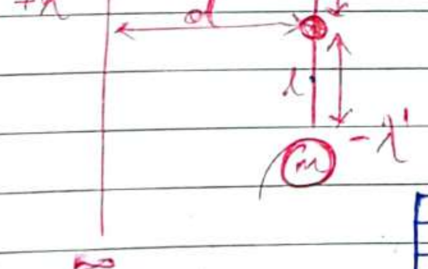
$$a = \frac{2Kq\lambda}{mx} = \frac{v\,dv}{dx}$$

$$\int_d^{3d} \frac{2Kq\lambda}{mx}\,dx = \int_0^v v\,dv$$

$$\frac{v^2}{2} = \frac{2Kq\lambda}{m}\ln 3$$

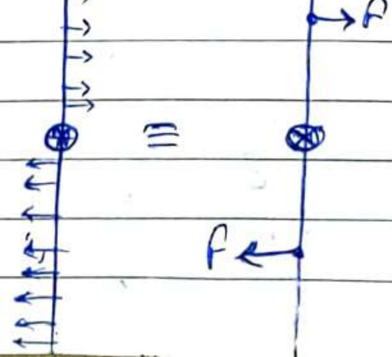
$$\Rightarrow v = \sqrt{\frac{4Kq\lambda}{m}\ln 3}$$

Question:



Find the α of wire just after release

Soln:

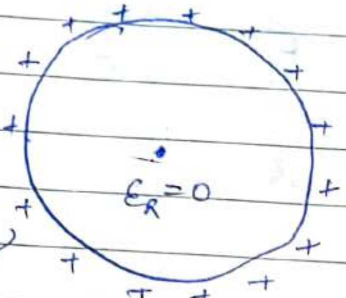


$$2Fl = \frac{4ml^2}{12}\alpha$$

$$2\left(\frac{2K\lambda}{d}\right)l = \frac{4ml^2}{12}\alpha$$

Electric field due to Circular Ring :-

a) Electric field at the Centre :



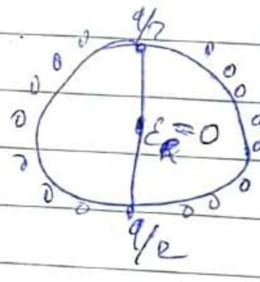
Uniform charge distribution

Q. T/F

Electric field at the centre of a non uniformly charge ring is non-zero.

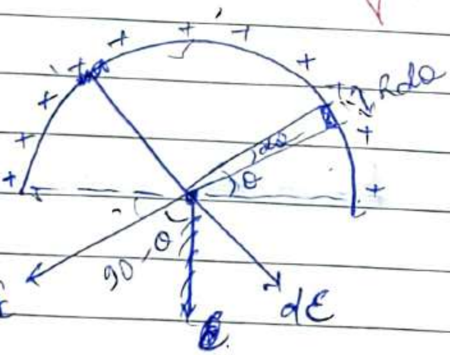
$\rightarrow$ False

Eg:



$\Rightarrow$ Semi-Circular ring :-

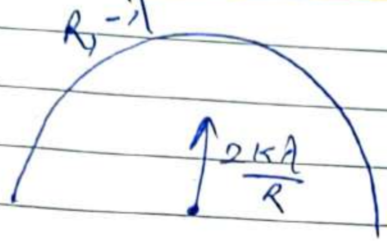
Semi-Circular ring 'R', λ



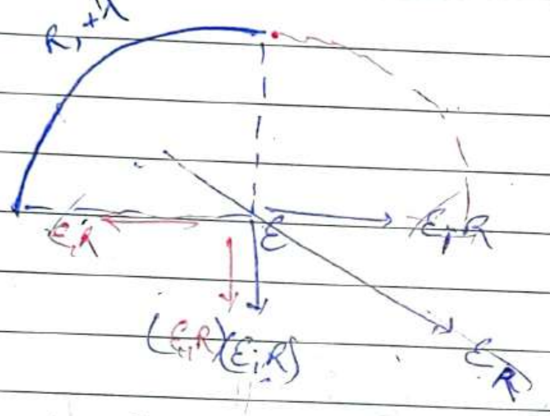
$$\therefore E = \int 2\,dE \sin\theta = \int_0^{\pi/2} 2\,\frac{K(\lambda R\,d\theta)}{R^2}\sin\theta$$

$$\therefore \boxed{E = \frac{2K\lambda}{R}}$$

#



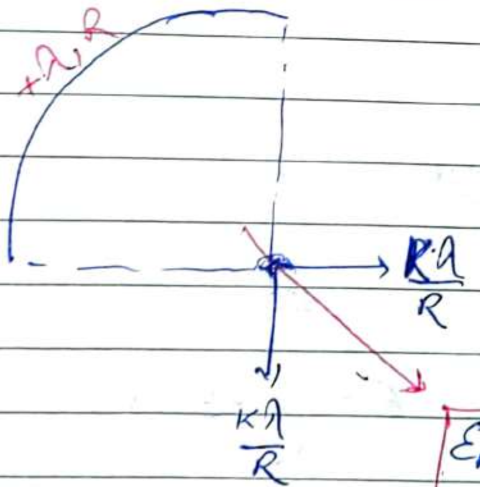
⇒ Quarter ring :-



$$\therefore \sqrt{2}\, E, R = \frac{2K\lambda}{R}$$

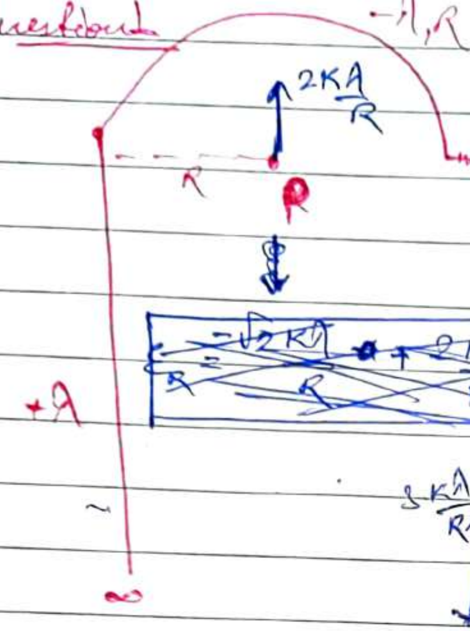
$$\Rightarrow E, R = \frac{\sqrt{2}\, K\lambda}{R}$$

⇒



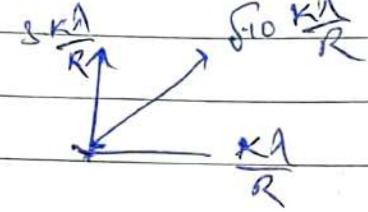
$$\boxed{E_R = \sqrt{2}\,\frac{K\lambda}{R}}$$

Question:

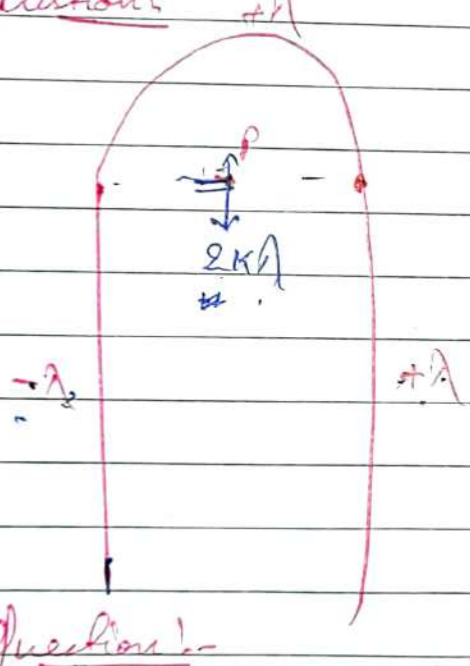


Find the Electric field at point P.

$$\cancel{\boxed{\frac{-\sqrt{2}K\lambda}{R} + \frac{2K\lambda}{R} = \frac{\sqrt{2}K\lambda(-1+\sqrt{2})}{R} \text{ along +ve y-axis}}}$$

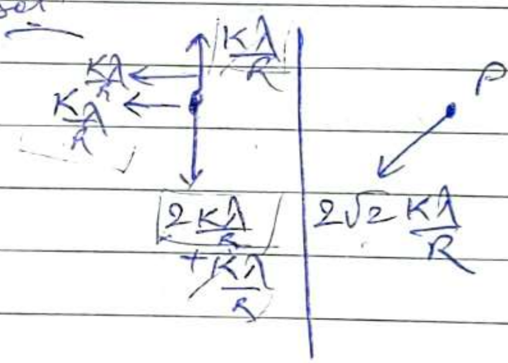


Question: $+\lambda$

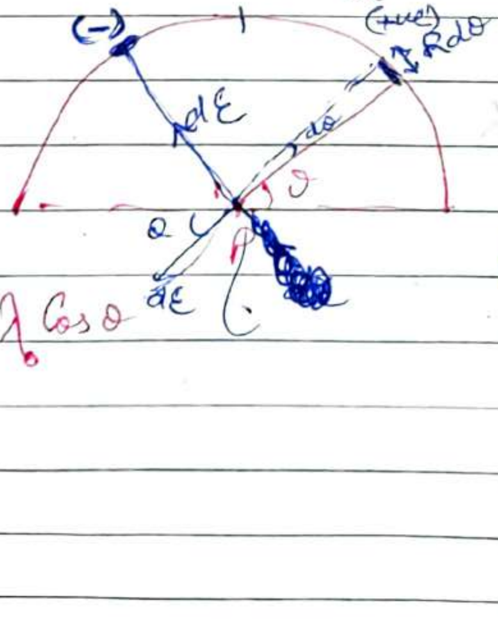


Find the electric field at P.

Sol:



Question:-



$\lambda = \lambda_0 \cos\theta$

Find the Electric field at point P

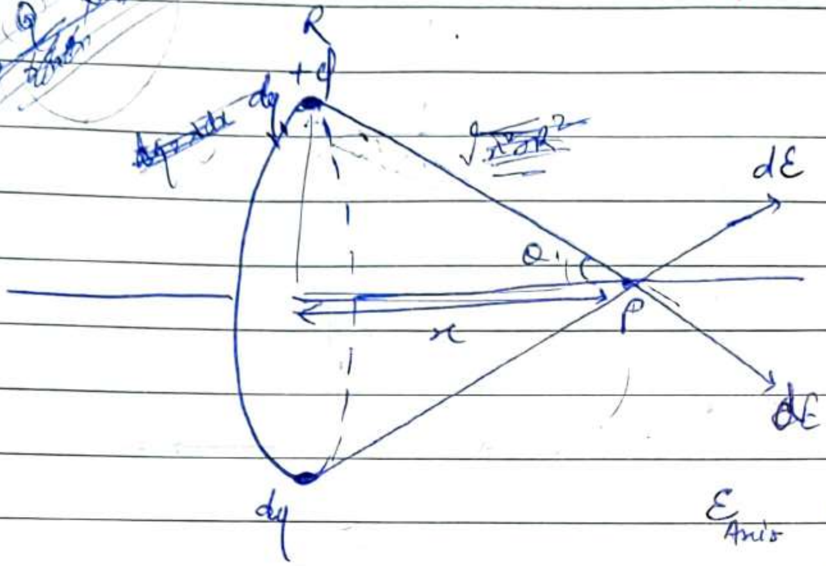
Sol:

$$\mathcal{E} = 2\, dE \cos\theta$$

$$= \int_0^{\pi/2} 2\, \frac{K\lambda_0 \cos\theta\, R\, d\theta}{R^2} \cos\theta$$

$$= \frac{2K\lambda_0}{R} \int_0^{\pi/2} \cos^2\theta\, d\theta$$

b) Electric field on the axis of a uniformly charged ring:-



$$E_{Axis} = \int_0^{\pi} 2\, dE \cos\theta$$

$$= 2 \frac{K\, dq}{} \frac{x}{(R^2+x^2)^{3/2}}$$

$$\int dq = \frac{Q}{2}$$

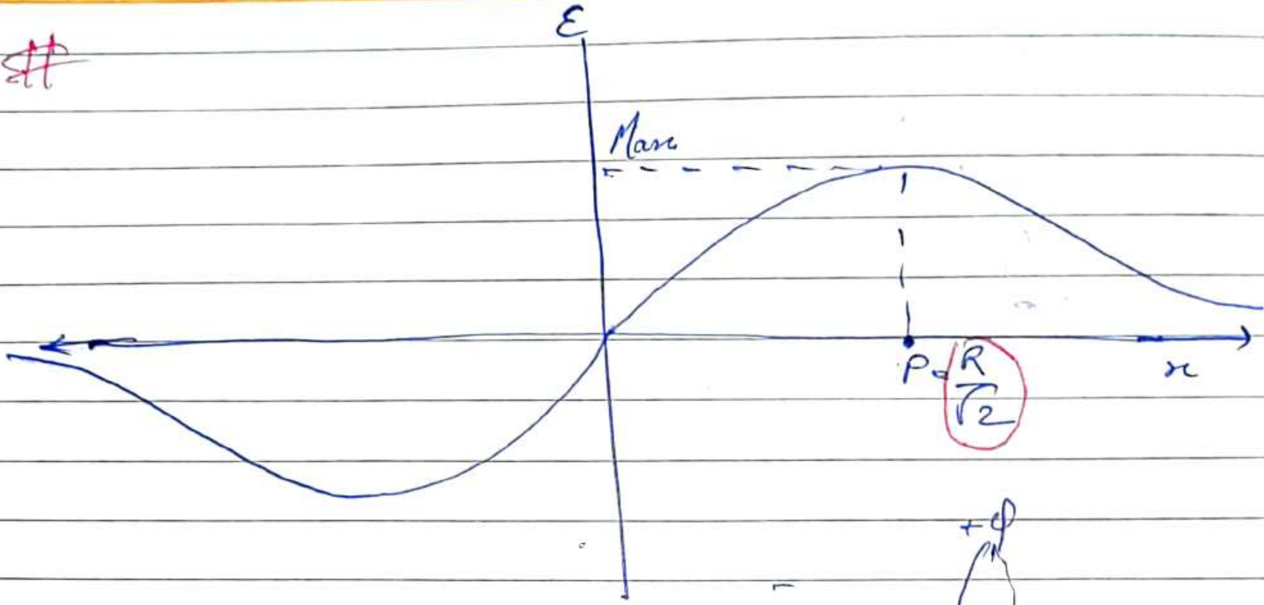
$$\boxed{E_{axis} = \frac{Kqx}{(R^2+x^2)^{3/2}}}$$

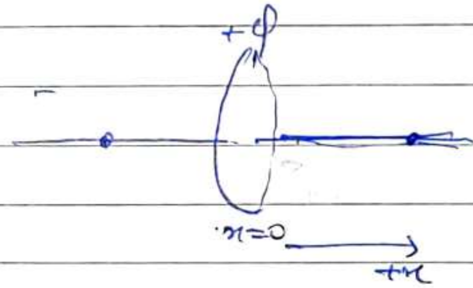
Special Case:-

If $x <<< R$

$$\boxed{E_{axis} = \frac{Kq}{R^3} x}$$

#





For $\mathcal{E}_{max}$,

$$\frac{d\mathcal{E}}{dx} = 0$$

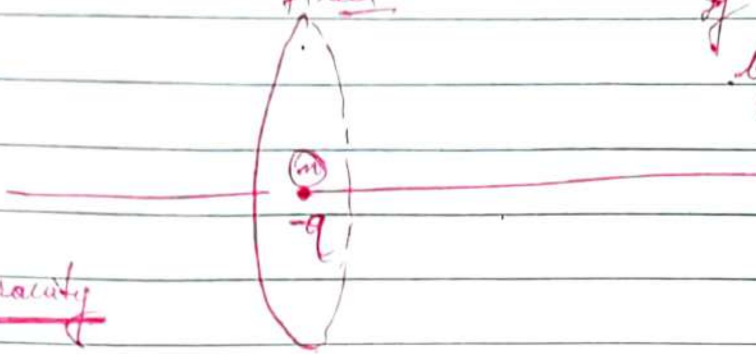
$$\Rightarrow \frac{d}{dx} \frac{K\phi x}{(R^2+x^2)^{3/2}} = 0$$

$$\boxed{x = \frac{R}{\sqrt{2}}}$$

$$\Rightarrow \cancel{\frac{d}{dx} \frac{K\phi}{R^2+x^2} \frac{x}{\sqrt{R^2+x^2}}} = 0$$

Question! $R, +Q$

fixed



No gravity

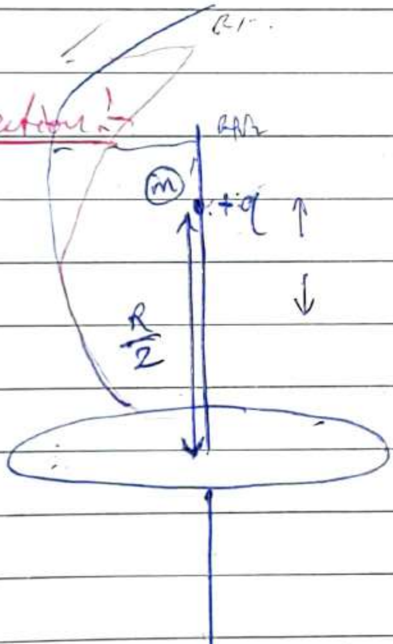
If -q charge is displaced by very small amount along the axis and then released then find time period of oscillation.

$$\underset{max}{F} = q\frac{KQ\cancel{A}}{R^3} = m\omega^2\cancel{A}$$

$$\omega = \sqrt{\frac{qKQ}{mR^3}}$$

$$\therefore T = \frac{2\pi}{\omega}$$

Question! →



fixed $+Q, R$

If small q charge is in eqm then find the nature of eqm along the axis.

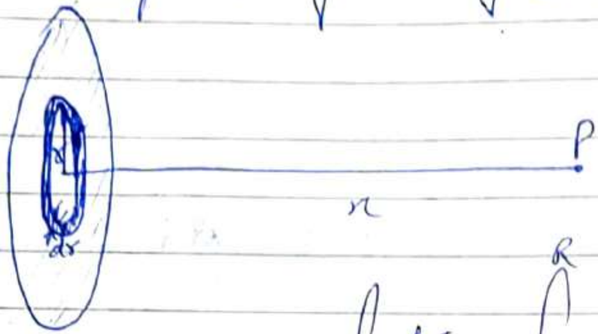
$$\boxed{\cancel{F = q\frac{KQR}{2\left(R^2+\frac{R^2}{4}\right)^{3/2}} = \frac{KqQR}{2}\left(\frac{4}{5R^2}\right)^{3/2}}}$$

→ Unstable Eqm

Only stable at $R/\sqrt{2}$.

Electric field due to uniformly charged disc :-

σ – Surface charge density $\left[\frac{C}{m^2}\right]$



$$\int dE = \int_0^R \frac{K[\sigma 2\pi r\,dr]\,x}{(x^2+r^2)^{3/2}}$$

$$E = K\sigma 2\pi x \int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

$x^2 + r^2 = t^2$

$\cancel{2}r\,dr = \cancel{2}t\,dt$

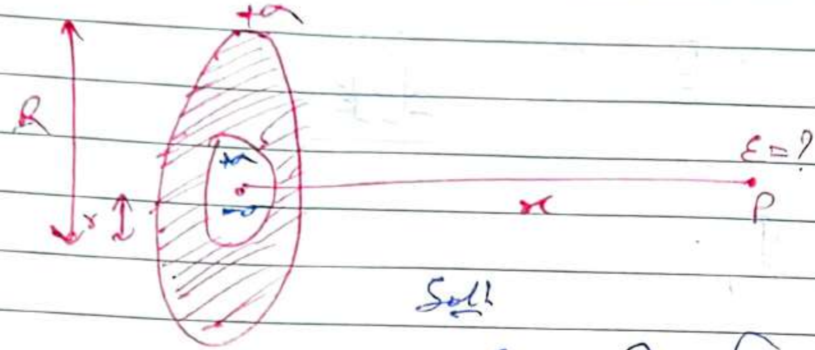
$$= K\sigma 2\pi x \int \frac{t\,dt}{t^{3/2}}$$

$$= K\sigma 2\pi x \int \frac{-1}{t}$$

$$= K\sigma 2\pi x \left[\frac{-1}{\sqrt{r^2+x^2}}\right]_0^R$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]}$$

Question:-



Find the Electric field at point P.

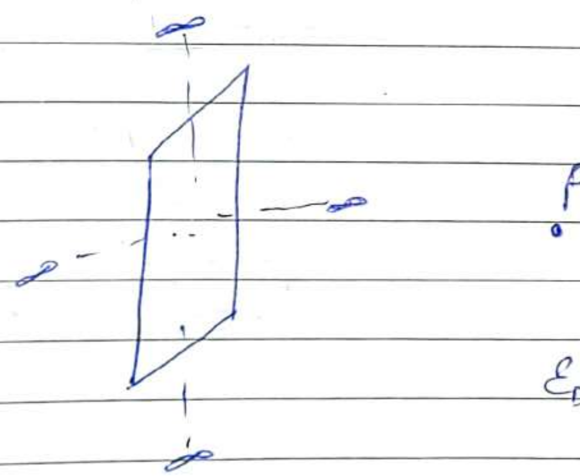
Sol:

$$\mathcal{E} = \underbrace{\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]}_{\text{full disc } (+\sigma)} - \underbrace{\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{r^2+x^2}}\right]}_{\text{Small part } (-\sigma)}$$

$$\Rightarrow \mathcal{E} = \frac{\sigma}{2\varepsilon_0}\left[\cancel{1} - \frac{x}{\sqrt{R^2+x^2}} \cancel{-1} + \frac{x}{\sqrt{r^2+x^2}}\right]$$

$$\Rightarrow \mathcal{E} = \frac{\sigma}{2\varepsilon_0}\left[\frac{x}{\sqrt{r^2+x^2}} - \frac{x}{\sqrt{R^2+x^2}}\right]$$

Electric field due to uniformly charged infinite plate:-

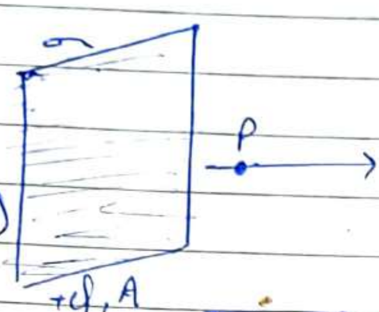


$$\boxed{\mathcal{E} = \frac{\sigma}{2\varepsilon_0}}$$

$$\mathcal{E}_{Disc} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

↳ if $R \to \infty$

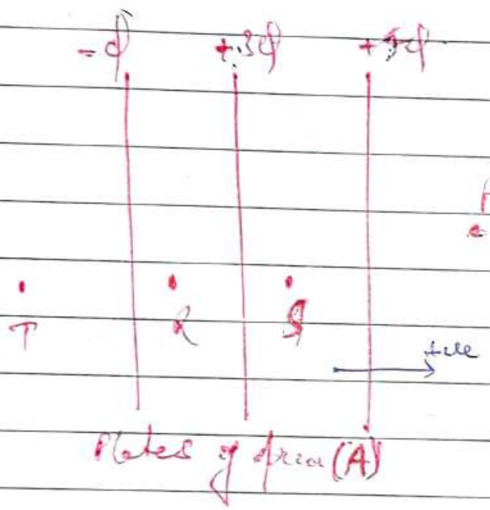
#



$$\boxed{E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0}}$$

$$\because \boxed{\sigma = \frac{q}{A}}$$

Question:-



Find the Electric field at Each given points.

$$E_P = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0}$$

$$= \frac{[4+3-1]q}{2A\varepsilon_0} = \frac{3q}{A\varepsilon_0}$$

$$E_S = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0}$$

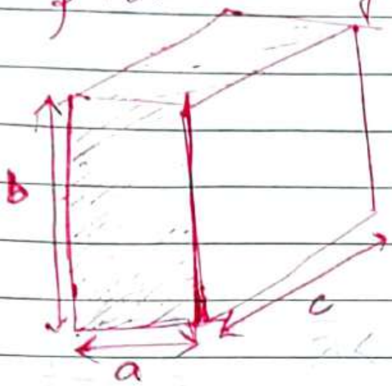
$$= \frac{[-4+3-1]q}{2A\varepsilon_0} = \frac{-q}{A\varepsilon_0}$$

$$E_R = \frac{[-4-3-1]}{2A\varepsilon_0} = \frac{-4q}{A\varepsilon_0}$$

$$E_T = \frac{[-4-3+1]}{2A\varepsilon_0} = \frac{-3q}{A\varepsilon_0}$$

Question:-

ρ = Volume charge density $\left[\frac{C}{m^3}\right]$



find the Electric field at point P.

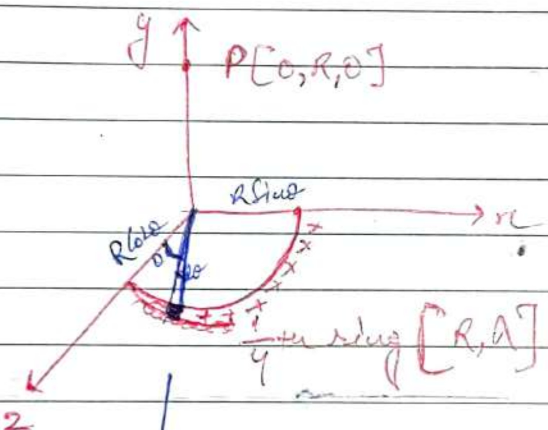
$$\mathcal{E}_P = \frac{[illegible]}{\varepsilon_0} = \frac{\sigma}{2A\varepsilon_0}$$

$$= \frac{\rho abc}{2bc\varepsilon_0}$$

$$= \frac{\rho a}{2\varepsilon_0}$$

Revision

Question!



find the Electric field at point P.

Sol:-

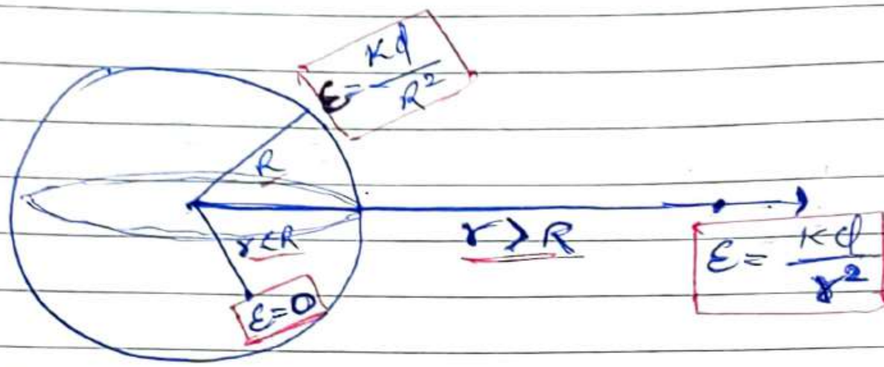
$[-R\sin\theta, 0, R\cos\theta]$

$$\int d\vec{E} = \int \frac{K(\lambda R d\theta)}{r^3}\vec{r}$$

$\therefore \vec{r} = -R\sin\theta\,\hat{i} + R\hat{j} - R\cos\theta\,\hat{k}$

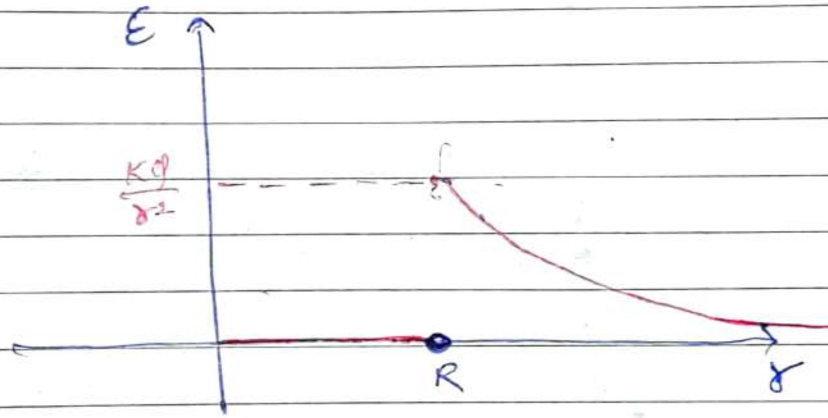
$$\boxed{\frac{2K\lambda}{r}\sin\theta/2}$$

Electric field due to uniformly charged hollow sphere :-



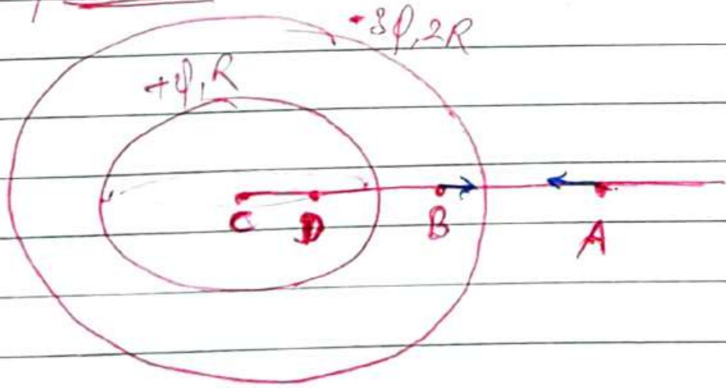
hollow sphere
[+q, R]

Graph



Discontinuous at $\boxed{r=R}$

Question:-



find the Electric field at given points

$E_C = 0$

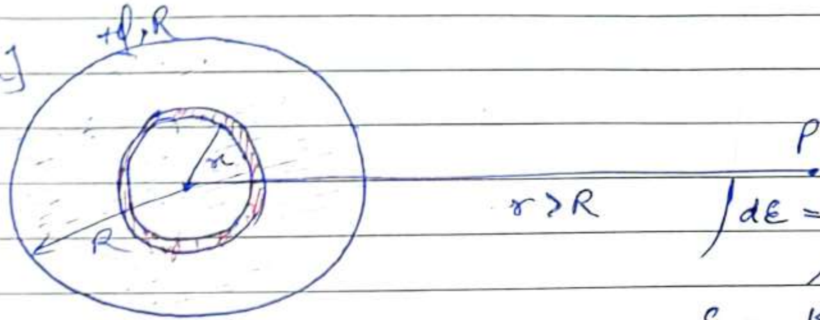
$E_A = \frac{-K(3q)}{(CA)^2} + \frac{Kq}{(CA)^2}$

$= \frac{-K(2q)}{(CA)^2}$

$\mathcal{E}_B = \frac{Kq}{(CB)^2}$, $\mathcal{E}_D = 0$

Electric field due to uniformly charged Solid sphere :-

[uniform]



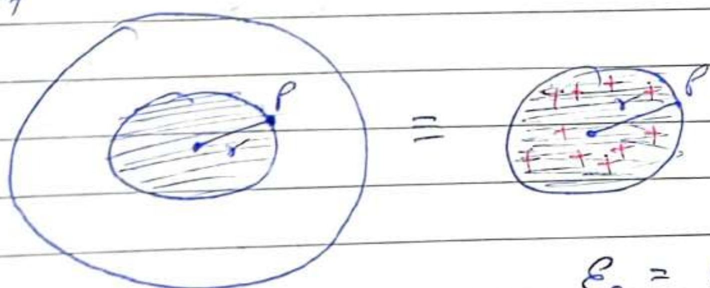
$\int d\mathcal{E} = \int \frac{K\,dq}{r^2}$

$\mathcal{E} = \frac{K}{r^2} \int dq$

$\boxed{\mathcal{E} = \frac{Kq}{r^2}}$

→ Electric field at surface, $\boxed{\mathcal{E} = \frac{Kq}{R^2}}$

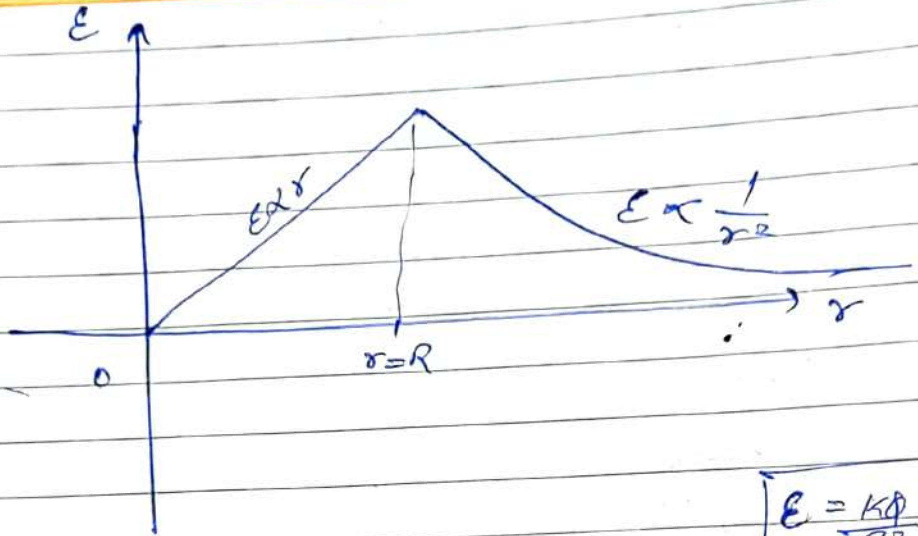
$\mathcal{E}_p$ inside the solid sphere



$$\mathcal{E}_p = \frac{K\left[\frac{q}{\frac{4}{3}\pi R^3}\right]\left[\frac{4}{3}\pi r^3\right]}{r^2}$$

$$\boxed{\mathcal{E}_p = \frac{Kq\vec{r}}{R^3}}$$

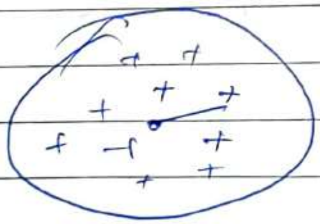
#



$\boxed{\mathcal{E} = \frac{K\rho}{R^3} r}$ inside

$\boxed{\mathcal{E} = \frac{K\rho}{r^2}}$ $R \geq$ outside

$\rho = \frac{Q}{4/3 \pi R^3}$



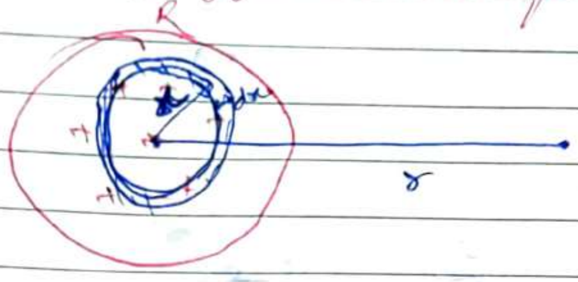
$\mathcal{E} = \frac{K\rho}{R^3} \vec{r}$

$\mathcal{E} = \frac{K\rho \frac{4}{3}\pi R^3}{R^3} r$

$\boxed{\vec{\mathcal{E}} = \frac{\rho}{3\varepsilon_0} \vec{r}}$

Question:

$\rho = \rho_0 x \rightarrow$ distance from centre.



find the Electric field at a distance r from the centre

i) $r > R$

ii) $r = R$

iii) $r < R$

Sol:

i) $\boxed{r > R}$ $\quad dE = \frac{Kq}{r^2}$

$\cancel{\int dE = \int \frac{K \rho_0 x \cdot \frac{4}{3}\pi R^3}{r^2}}$

$\cancel{E = \frac{K\rho_0 4 \pi R^3}{3\, r^2}}$

$\therefore dE = \int_0^R \frac{K(\rho_0 x)\, 4\pi x^2 dx}{r^2}$

$E = \frac{K\rho_0 4\pi}{r^2}\int_0^R x^3 dx$

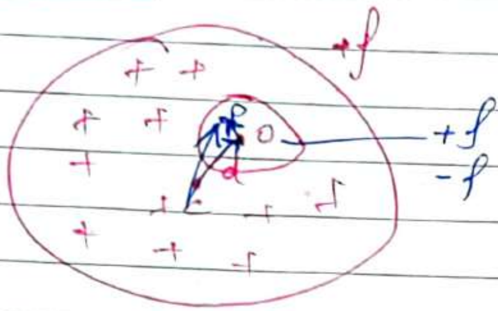
$E = \frac{K\rho_0 4\pi}{r^2}\frac{R^4}{4} = \frac{K\rho_0 R^4 \pi}{r^2}$

ii) $E = K\rho_0 R^2 \pi$

iii) $E_{r<R} = \int_0^r \frac{K(\rho_0 x)\, 4\pi x^2 dx}{r^2}$

$\frac{\rho}{3\varepsilon_0} d$

Question:



Solid sphere with spherical cavity

Find the electric field at any point inside the cavity (P).

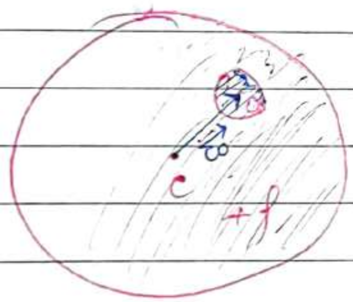
Sol:

$$E_P = \frac{\rho \vec{CP}}{3\varepsilon_0} + \frac{-\rho \vec{OP}}{3\varepsilon_0}$$

$$= \frac{\rho [\vec{CP} + \vec{OP}]}{3\varepsilon_0}$$

$$= \frac{\rho}{3\varepsilon_0} d$$

☆

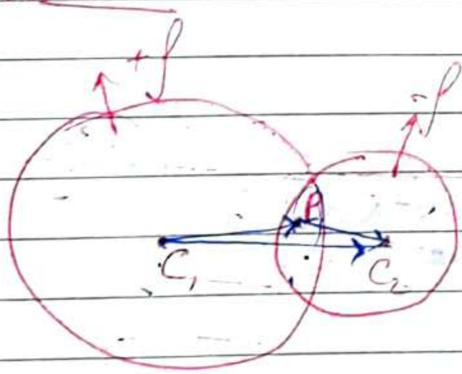


$$\boxed{E_{cavity} = \frac{\rho}{3\varepsilon_0} \vec{CO}}$$

↓

Uniform

Question:



Solid sphere Solid sphere

Find the electric field at point P.

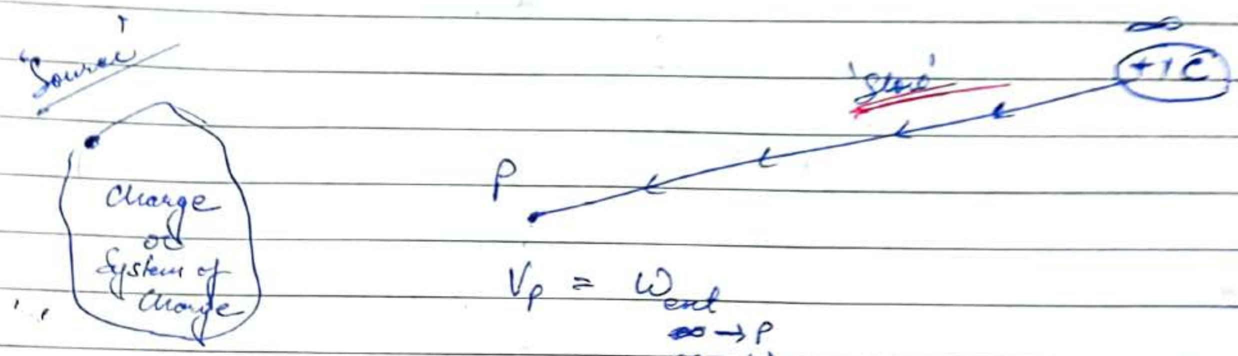
$$\vec{E_P} = \frac{\rho \vec{C_1P}}{3\varepsilon_0} + \frac{\rho \vec{C_2P}}{3\varepsilon_0}$$

$$= \boxed{\frac{\rho \vec{C_1C_2}}{3\varepsilon_0}}$$

↓

Uniform

Electric Potential :- [Scalar quantity] [V]



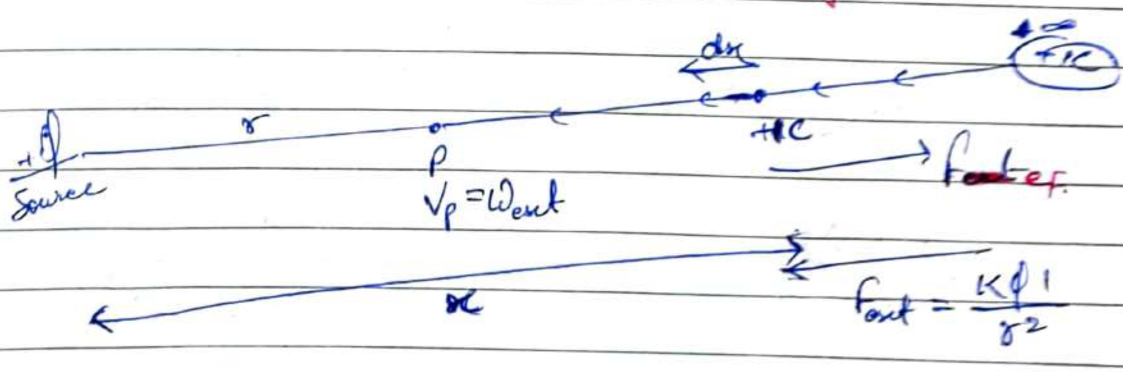
$V_P = W_{ext}$
$\infty \to P$
(slowly)
[on a +1C]

⇒ Workdone by external agent on a unit (+ve) charge [against electric field of source charge/charges] from ∞ to a point P in a slow process. ($F_{Res.} = 0$) is defined at Potential at point P.

→ Unit of potential is $\left[\frac{J}{C}\right]$ or Volt

→ Potential can be (−ve), or (+ve) or 0.

Potential Due to point Charge :-

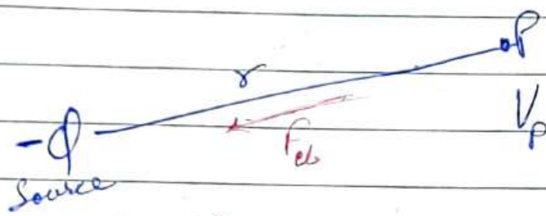


$$\therefore V_P = \int dW_{ext} = \int_{\infty}^{r} \left[\frac{Kq}{x^2}\right](-dx)\cos 0^\circ$$

→ dx is decreasing.

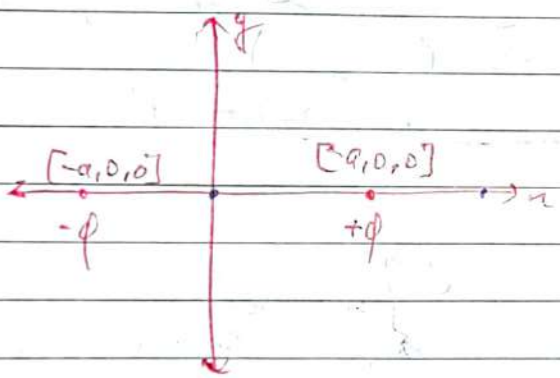
$$\Rightarrow \boxed{V_P = \frac{Kq}{r}}$$

#



$$V_P = \frac{-Kq}{r}$$

Question:-



Find the potential at following points.

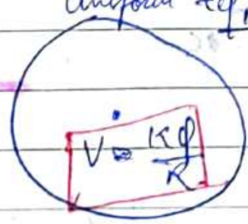
a) [0,0,0] = 0

b) [0,0,a] = 0

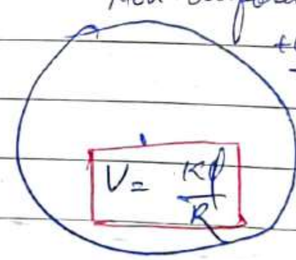
c) [2a,0,0] = $-\frac{2Kq}{3a}$

Potential due to Ring :-

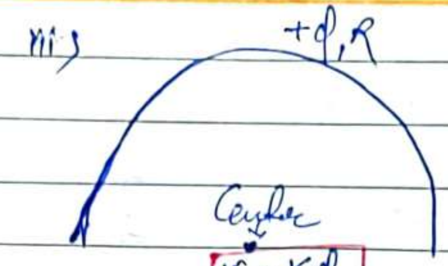
i) Uniform $+q, R$



ii) Non-uniform $+q, R$



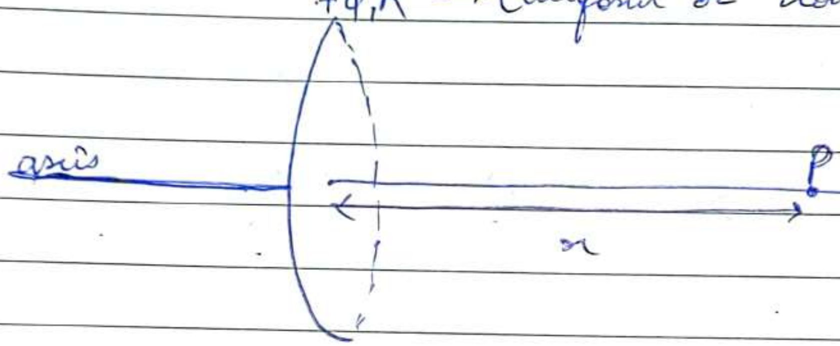
iii)



$$\boxed{V = \frac{Kq}{R}}$$

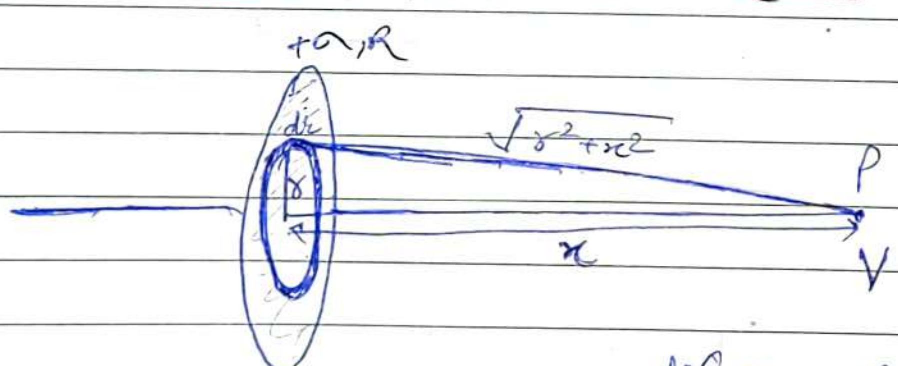
Potential on the axis of ring :-

+q, R → (uniform or non-uniform)



$$\boxed{V_P = \frac{Kq}{\sqrt{R^2+x^2}}}$$

Potential due to Disc :-



$$dV = \frac{K\,dq}{\sqrt{r^2+x^2}}$$

$r^2+x^2 = t^2$

$\cancel{2}r\,dr = \cancel{2}t\,dt$

$$V = \int_0^R \frac{K\sigma 2\pi r\,dr}{\sqrt{r^2+x^2}}$$

$$V = K\sigma 2\pi \int_0^R \frac{r\,dr}{\sqrt{r^2+x^2}}$$

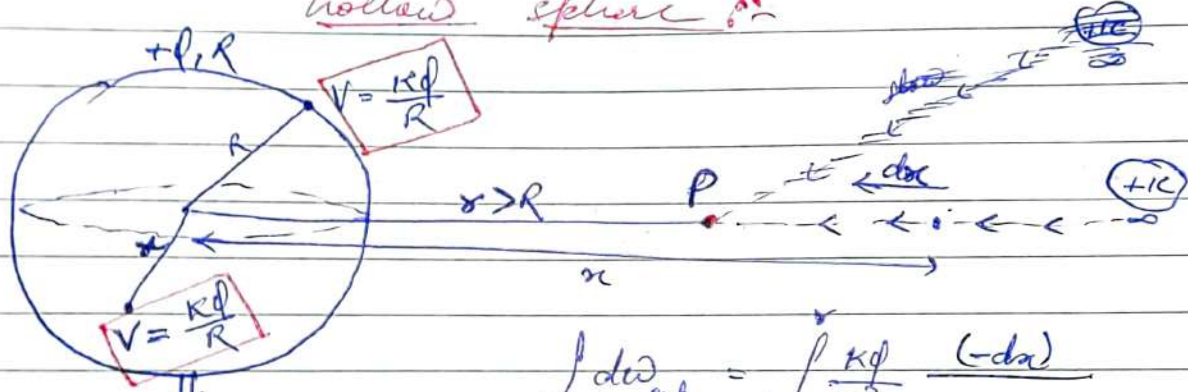
$$V = K\sigma 2\pi \int dt$$

$V = K\sigma 2\pi t$

$\Rightarrow V = K\sigma 2\pi \left[\sqrt{R^2+x^2}\right]_0^R$

$\Rightarrow \boxed{V = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+x^2} - R\right]}$

Potential due to uniformly charged hollow sphere :-



$\Downarrow$

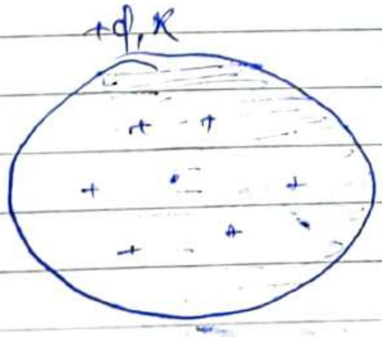
W_{ext} inside the sphere $= 0$

as, $E_p = 0$

$\int dW_{ext} = \int_{\infty}^{r} \frac{Kq}{x^2}(-dx)$

$\boxed{V_P = \frac{Kq}{r}}$

Potential due to Uniformly Charged Solid Sphere :-



i) at $r > R$

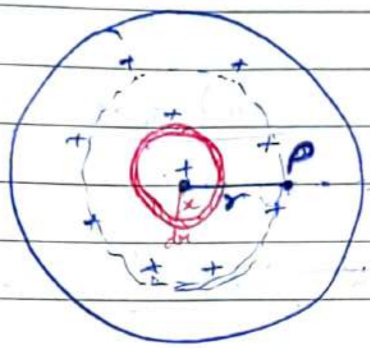
$\boxed{V = \frac{Kq}{r}}$

ii) at $r = R$

$\boxed{V = \frac{Kq}{R}}$

Potential inside Solid sphere :-

at $r < R$

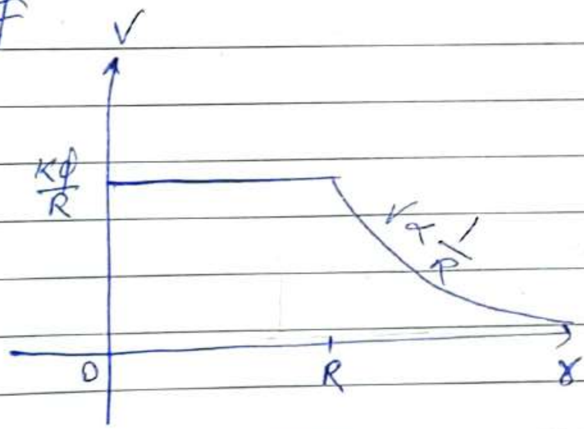


$$V_P = \int_0^r \frac{K\left[\frac{q}{\frac{4}{3}\pi R^3}\right]4\pi x^2\,dx}{r} + \int_r^R \frac{K\left[\frac{q}{\frac{4}{3}\pi R^3}\right]4\pi x^2\,dx}{x}$$

$$= \frac{\frac{3Kq}{R^3}\left[\frac{x^3}{3}\right]_0^r}{r} + \frac{3Kq}{R^3}\left[\frac{x^2}{2}\right]_r^R$$

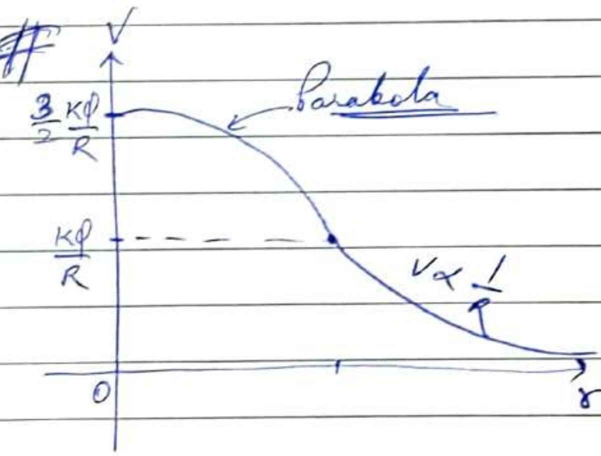
$$\boxed{V_P = \frac{Kq}{2R^3}\left[3R^2 - r^2\right]}$$

#



→ for hollow sphere

#



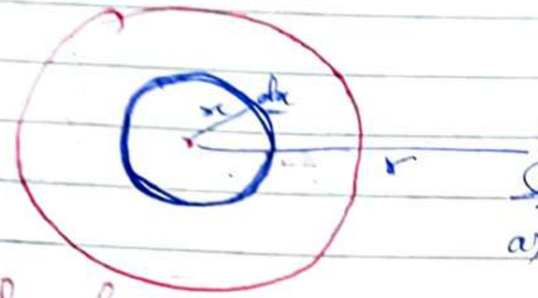
→ for solid sphere.

Question -

find the potential due to sphere at:

a) ~~small~~ $r > R$

b) $r < R$



$\rho = \rho_0 x$

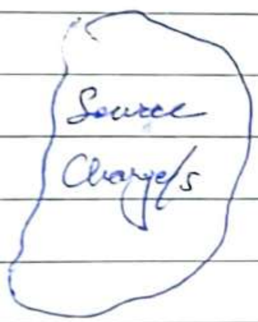
Sol:

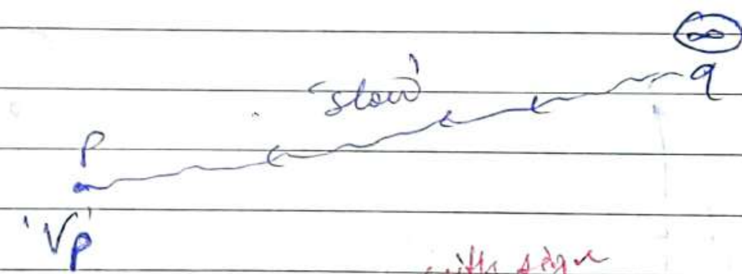
a) $$V_{P\,(r>R)} = \int_0^R \frac{K(\rho_0 x)\, 4\pi x^2\, dx}{r}$$

b) $$V_{P\,(r<R)} = \int_0^r \frac{K(\rho_0 x)\, 4\pi x^2\, dx}{r} + \int_r^R \frac{K(\rho_0 x)\, 4\pi x^2\, dx}{x}$$

★ Electrostatic field is a conservative field.

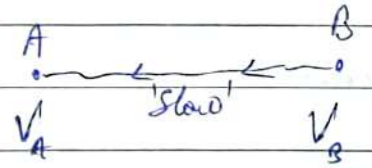
#





with sign

$$\boxed{W_{ext} = q\, V_P}$$

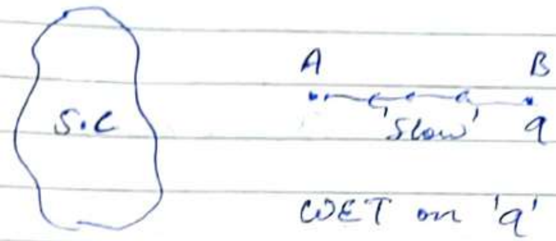


with sign

$$\boxed{W_{ext\,(B\to A)\,(slow)} = q\,(V_A - V_B)}$$

* $\underset{\substack{B\to A \\ \text{Slow}}}{W_{ext}} = q(V_A - V_B)$

#



WET on 'q'

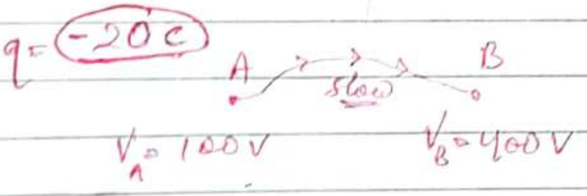
$W_{ext} + W_{E.F} = \Delta K \rightarrow 0$ (Slow)

$\Rightarrow W_{ext} = -W_{E.F}$

$\boxed{\underset{B\to A}{W_{E.F}} = q(V_B - V_A)}$

↳ Process → Slow ($a=0$)
→ $a \neq 0$

Question!-



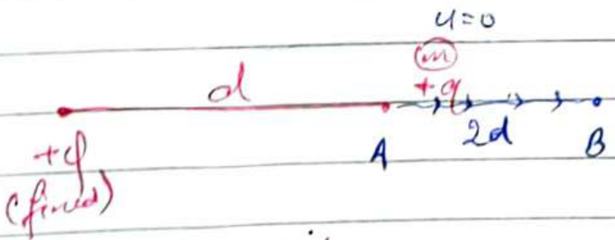
find workdone by external agent on q from A to B.

Sol:-

$\underset{\substack{A\to B \\ \text{Slow}}}{W_{ext}} = (-20)(300)\ J$

$= -6000\ J = -6\ kJ$

Question!-



No gravity

find the speed of small q charge when seperation becomes 3d.

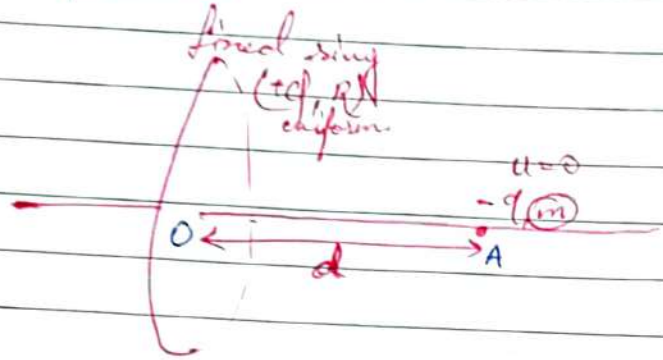
Soln.

$W_{all} = \Delta K$

$\Rightarrow q(V_A - V_B) = \Delta K$

$\Rightarrow q\left(\frac{KQ}{d} - \frac{KQ}{3d}\right) = \frac{1}{2}mV_0^2$

Question



find the max^m speed of $-q$ charge.

Sol'

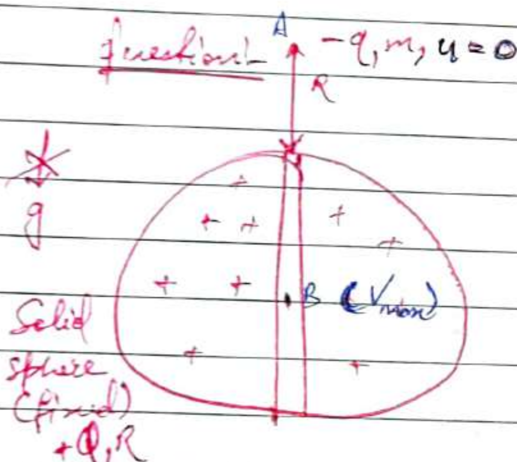
Applying WET.

$$\boxed{W_{E.F} = \Delta K}$$

$$W_{E.F\,A\to O} \Rightarrow -q(V_A - V_O) = \frac{1}{2}mV_{max}^2$$

$$\Rightarrow (-q)\left(\frac{KQ}{\sqrt{d^2+R^2}} - \frac{KQ}{R}\right) = \frac{1}{2}mV_{max}^2$$

Question



find the Max^m speed of $-q$ charge.

Sol!

Applying WET.

$$W_{E.F\,A\to B} = \Delta K$$

$$\Rightarrow -q(V_A - V_B) = \frac{1}{2}mV_{max}^2 - 0$$

$$\Rightarrow (-q)\left(\frac{KQ}{2R} - \frac{3}{2}\frac{KQ}{R}\right) = \frac{1}{2}mV_{max}^2$$

$$\Rightarrow \frac{2KQq}{\cancel{2}R} = \frac{1}{\cancel{2}}mV_{max}^2$$

$$\Rightarrow \sqrt{\frac{2KQq}{mR}} = V_{max}$$

Question:-

∞

$+q, m$ (Rest) $t=0$ $V_d \uparrow$

$+Q, M$ $\xleftarrow{\quad}$ u; $t=0$

No gravity. g

find the minimum seperation between the charges.

Sol:-

$\xleftarrow{v} +q, m$ $\qquad$ $\xleftarrow{v} +Q, M$

R_{min}

$$0 + Mu = (m+M)v \longleftarrow (1) \quad \text{(Momentum Conservation)}$$

$$0+0+\frac{1}{2}Mu^2 = \frac{1}{2}(m+M)v^2 + \frac{KQq}{r_{min}} \quad (E.C) \longleftarrow (II)$$

Electrostatic Potential Energy:-

1) Electrostatic P.E. of a point charge with source charge/s

$+q, P \longrightarrow U =$ Min. Work done by external agent on q from ∞ to P

'Source' System of charge/s

$\Rightarrow \boxed{U = qV_p}$ with sign

Interaction Energy of q with Source charges.

11) Potential Energy of two point charge system

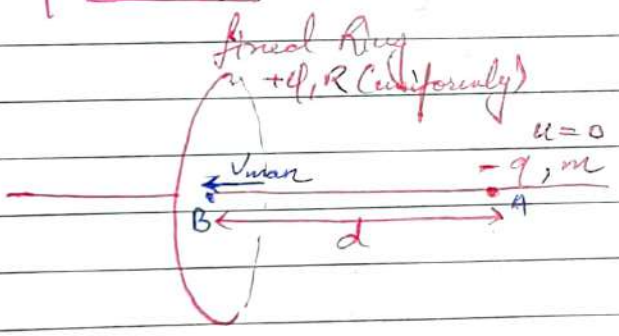
$$q_1 \overset{r}{\text{———}} q_2$$

U = Min work done by external agent to form system.

$$U = q_2\left(\frac{Kq_1}{r}\right)$$

$$\boxed{U = \frac{Kq_1 q_2}{r}} \rightarrow \text{with sign}$$

Question

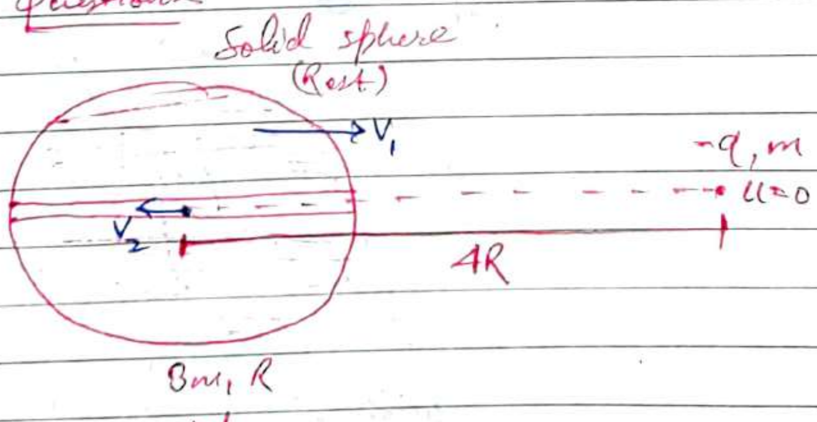


find the max^m speed of $-q$ charge.

Sol! From A to B, Apply E.C.,

$$0 + 0 + \frac{(-q)KQ}{\sqrt{R^2+d^2}} = \frac{1}{2}mv_{max}^2 + 0 + \frac{KQ(-q)}{R}$$

Question

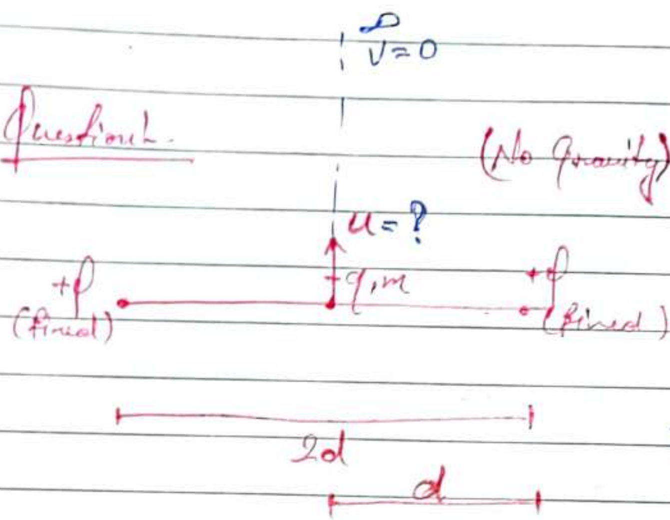


find the max^m speed of point charge.

Sol: (M.C) $0+0 = 3mv_1 - mv_2 \Rightarrow 3mv_1 = mv_2$ —— (I)

(E.C) $0+0+(-q)\frac{Kq}{4R} = \frac{1}{2}(3m)v_{1max}^2 + (-q)\frac{3}{2}\frac{Kq}{R} + \frac{1}{2}mv_2^2$ —— (II)

Question.



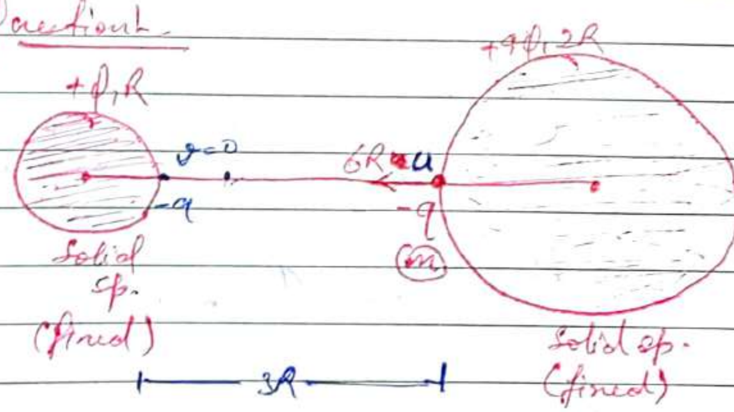
find the Escape velocity of −q charge

$\frac{1}{2}mu_{min}^2 + (-q)\frac{2Kq}{d} = 0+0$

$\Rightarrow \frac{1}{2}mu_{min}^2 = \frac{2Kq}{d}$

$\Rightarrow u_{min} = \sqrt{\frac{4Kq}{dm}}$

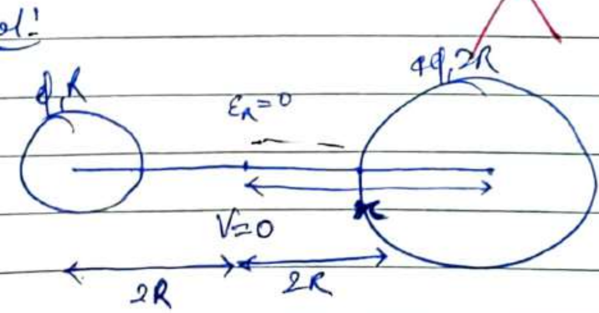
Question.



find the min^m value of u for which particle will hit small sphere

$\cancel{\frac{1}{2}mu_{min}^2 + (-q)\frac{K4q}{2R} + (-q)\frac{Kq}{4R} = 0 + (-q)\frac{Kq}{R} + (-q)\frac{4Kq}{5R}}$

Sol:



$\frac{K\cdot 4q}{x^2} = \frac{Kq}{(6R-x)^2}$

$\Rightarrow \boxed{x=4R}$

DPP-22

Sec-1 Q | Sec-1 Q Sec C 1,2,3,4,5,6,7,8, 9,10

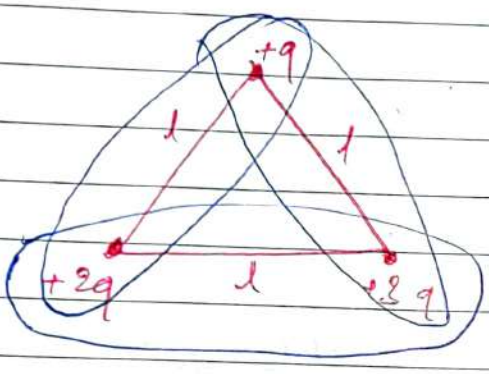
Date

Page

Sec D 01,3,4,5

$$\therefore \frac{1}{2}mu^2 + (-q)\left[\frac{4Kq}{2R} + \frac{Kq}{4R}\right] = 0 + (-q)\left[\frac{K4q}{4R} + \frac{Kq}{2R}\right]$$

Question:-



Find the Electrostatic P.E. of the system.

Sol:

Basic :-

$$0 + 3q\left[\frac{Kq}{l}\right] + 2q\left[\frac{Kq}{l} + \frac{K3q}{l}\right]$$

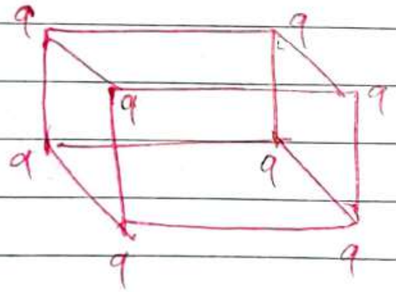
(or)

Interaction Energy

$$q\left[\frac{K2q}{l}\right] + 2q\left[\frac{K3q}{l}\right] + 3q\left[\frac{Kq}{l}\right]$$

Question:-

Repeat previous question!

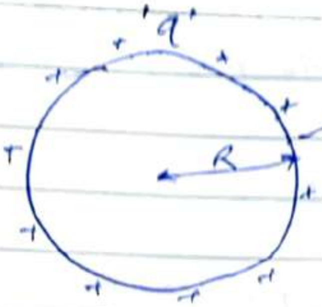


'Cube' (l)

$$U_{\text{symm. sys.}} = \frac{n}{2}\left[U_{\text{one charge with other}}\right]$$

$$U = \frac{8}{2}\left[\left[3\times\frac{Kq^2}{l}\right] + \left[3\times\frac{Kq^2}{\sqrt{2}l}\right] + \left[\frac{Kq^2}{\sqrt{3}l}\right]\right]$$

iii) Potential Energy of a uniformly charged hollow sphere / Self Energy of a hollow sphere.



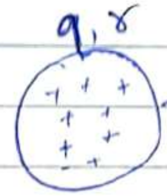
slow

dq

$$\int dW_{ext} = \int_0^Q (dq)\frac{Kq}{R}$$

$$\boxed{U_{self} = \frac{KQ^2}{2R}}$$ (hollow sphere)

iv) Self Energy of a uniformly charged Solid Sphere.



slow

dq

$$\int dW_{ext} = \int_0^R dq\left(\frac{Kq}{r}\right)$$

$$q = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

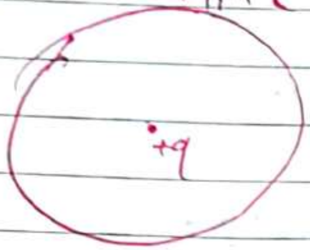
$$q = \frac{Qr^3}{R^3}$$

$$\frac{dq\,R^3}{Q} = 3r^2\,dr$$

$$\boxed{U_{self} = \frac{3}{5}\frac{KQ^2}{R}}$$ (Solid sphere)

Question:-

+Q, R (hollow sphere)



find the total P.E of the system

Sol:-

$$U_{system} = U_{q \leftrightarrow hollow\,sphere} + U_{self\,(H.S)}$$

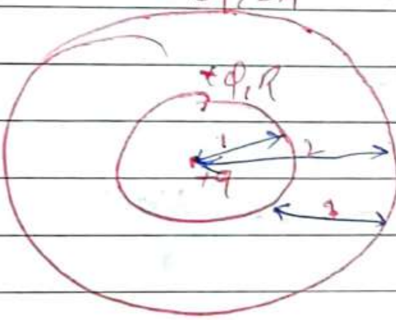
$$= \frac{KqQ}{R} + \frac{KQ^2}{2R}$$

$$= \frac{2KqQ + KQ^2}{2R}$$

$$= \frac{KQ(2q+Q)}{2R}$$

Question:-

-3Q, 2R



Repeat previous question.

Sol:-

$$U_{system} = \frac{-K(-3Q)^2}{4R} + \frac{K(Q)^2}{2R} + \underset{(1)}{\frac{KqQ}{R}} + \underset{(2)}{\frac{-Kq3Q}{2R}}$$

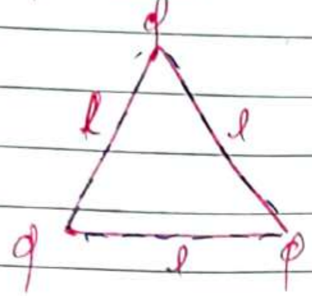
$$+ \frac{QK(-3Q)}{\underbrace{2R}_{\text{dep.}}} \quad (3)$$

$\boxed{W_{cons.} = -\Delta U}$

$\Rightarrow \boxed{W_{Elect} = -\Delta U}$

$\Rightarrow \boxed{W_{ext} = \Delta U}$ ↳ q (slow)

Question:-



i) If system is released from given position, then find W_{EF} on the system on the whole process.

Sol:-

$$W_{EF} = -\Delta U = U_i - U_f$$

$$= \frac{3kq^2}{l} - 0$$

ii) If mass of each particle is m, then find the final speed of each particle

WET:

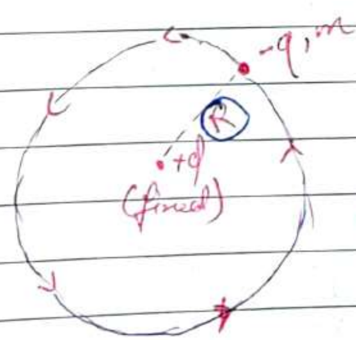
$$W_{all} = \Delta K$$

$$W_{EF} = \Delta K$$

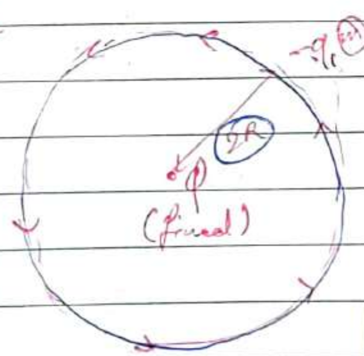
$$\Rightarrow \frac{3kq^2}{l} = \frac{1}{2}(3m)v^2$$

$$\Rightarrow v = \sqrt{\frac{2kq^2}{lm}}$$

Question:-



initial Condition



final Cond^n

find W_{ext} agent.

Sol:-

$$W_{ext} + W_{EF} = \Delta K$$

$$W_{ext} = \Delta K + \Delta U$$

$$W_{ext} = (K_f - K_i) + (U_f - U_i)$$

$$W_{ext} = (U_f + K_f) - (U_i + K_i)$$

DPP-23

Ex-1 ⇒ C13, C14, D2, E all

Ex-2 ⇒ 7, 9, 10, 11, 12 | ⇒ C1 – C7, 10, 11, 12, 13, 14

D1 – D2

E1 – E2

Sol:

$$W_{Ext} = -\left(\frac{-Kq\phi}{R} + \frac{1}{2}mV_1^2\right) + \left(\frac{-Kq\phi}{2R} + \frac{1}{2}mV_2^2\right)$$

(Initial) (final)

$$\frac{Kq\phi}{R^{\cancel{2}}} = \frac{mV_1^2}{\cancel{R}} \Rightarrow V_1 = \sqrt{\frac{Kq\phi}{m}}$$

$$\frac{Kq\phi}{\cancel{4}_2 R^{\cancel{2}}} = \frac{mV_2^2}{\cancel{2}\cancel{R}} \Rightarrow V_2 = \sqrt{\frac{K\phi q}{2m}}$$

Energy Density: $\left[J/m^3\right]$ $\boxed{= \frac{1}{2}\varepsilon_0 E^2}$

↳ air/vaccum

for other medium

$$\boxed{\frac{1}{2}\varepsilon_0 \varepsilon_r E^2}$$

Question:

$l <<<<<< r$

find Electrostatic P.E stored in the cube.

+Q

r

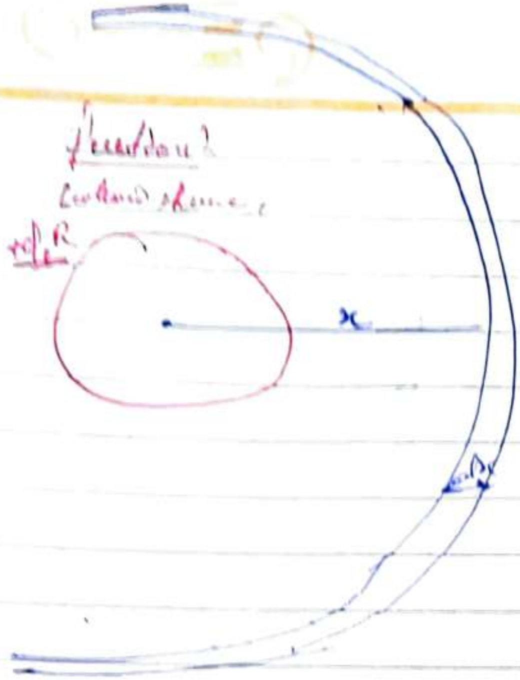
l

$$E.D = \frac{1}{2}\varepsilon_0\left(\frac{KQ}{r^2}\right)^2$$

$$\therefore P.E. = \left[\frac{1}{2}\varepsilon_0 \frac{K^2Q^2}{r^4}\right] \times l^3$$

Question!

Hollow sphere



Find the self energy of hollow sphere

Sol)

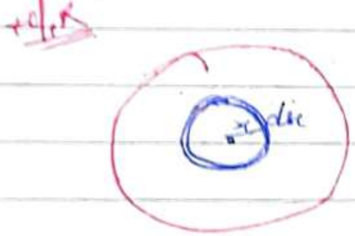
$$U_{inside} = \frac{1}{2}\varepsilon_0 (0)^2 \times \frac{4}{3}\pi R^3 = 0$$

$$U_{outside} = \int_R^\infty \frac{1}{2}\varepsilon_0 \left(\frac{Kq}{x^2}\right)^2 \times 4\pi x^2 dx$$

$$= \frac{4\pi}{2}\varepsilon_0 K^2 q^2 \frac{dx}{x^2}$$

$$\underset{(\text{hollow sphere})}{U_{self}} = \frac{1}{2}\frac{Kq^2}{R} \quad \text{——— (1)}$$

Question!-



Solid sphere

Find the self energy of solid sphere

$$U_{inside} = \int_0^R \frac{1}{2}\varepsilon_0 \left(\frac{Kq\,x}{R^3}\right)^2 4\pi x^2 dx$$

$$= \frac{4\pi}{2}\varepsilon_0 \frac{K^2q^2}{R^6}\int_0^R x^4 dx$$

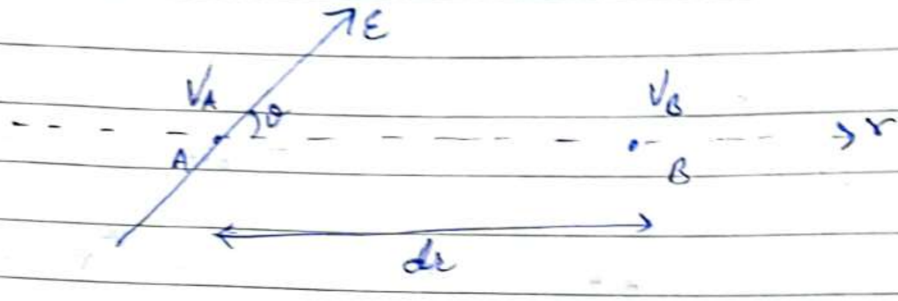
$$= \frac{4\pi}{2}\varepsilon_0 \frac{K^2q^2}{R^6}\frac{R^5}{5}$$

$$= \frac{1}{10}\frac{Kq^2}{R}$$

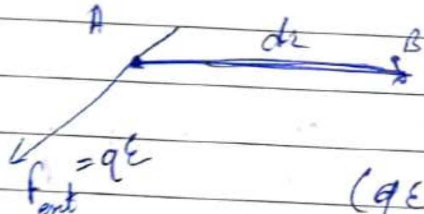
$$U_{outside} = \frac{1}{2}\frac{Kq^2}{R} \quad [\text{from } (1)]$$

$$\therefore \underset{(\text{solid sphere})}{U_{self}} = \frac{3}{5}\frac{Kq^2}{R}$$

Relation betn $\vec{E}$ and V :-



$$W_{\substack{ext \\ A\to B}} = q(V_B - V_A) = q\,dV$$



$$(qE)\,dr\cos(\pi-\theta) = q\,dV$$

$$\Rightarrow -(E\cos\theta)\,dr = dV$$

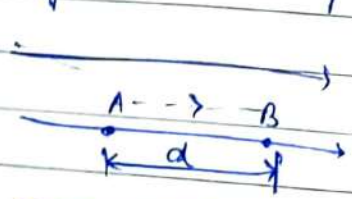
$$\Rightarrow \boxed{-\vec{E}\cdot\vec{dr} = dV}$$

$$\Rightarrow -\frac{dV}{dr} = E\cos\theta$$

$$\Rightarrow \boxed{-\frac{dV}{dr} = E_{along}}$$

* Special Cases :-

If $\vec{E}$ is uniform

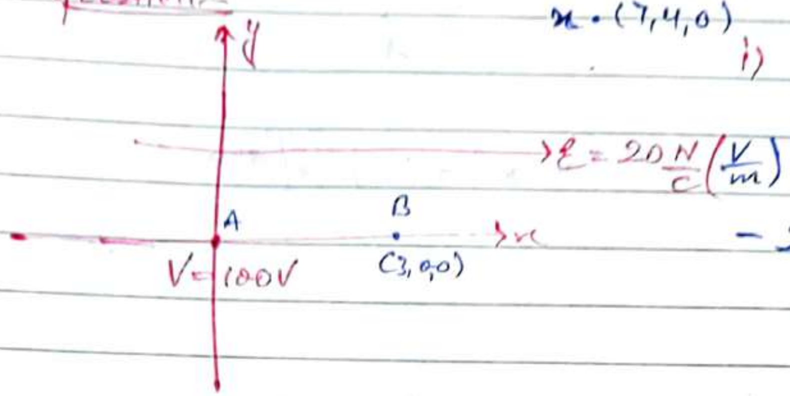


$$-Ed\cos 0^\circ = V_B - V_A$$

$$\Rightarrow \boxed{V_A - V_B = Ed}$$

* In the direcⁿ of Electric ~~flux~~ field, potential decreases.

Question:-



i) find the potential at (3,0,0)

$$-20 \times 3 \times (1) = V_B - 100$$

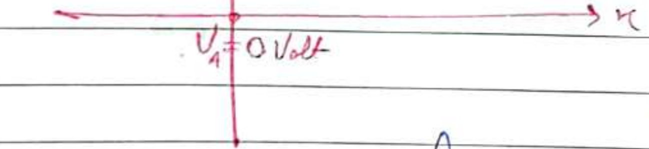
~~$V_B = 100$ Volt~~

$$V_B = 100 - 60 = 40\,V$$

ii) find the potential at (7,4,0)

$$V_x = -40\,V$$

Question:-



find the potential at (4,4,4)

Sol:

$$(-80 + 160 - 120) = -40\,V$$

or

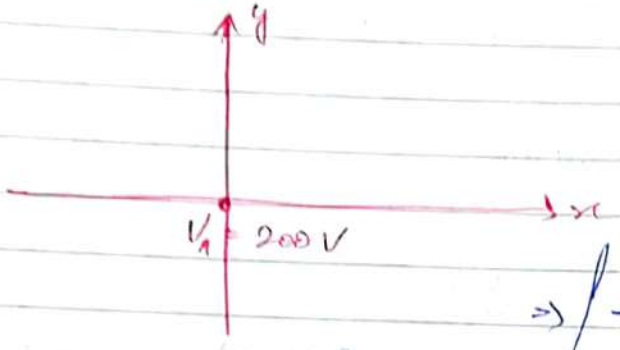
$$-\vec{E} \cdot \vec{dl} = dV$$

$$\int -[20\hat{i} - 40\hat{j} + 30\hat{k}] \cdot [dx\hat{i} + dy\hat{j} + dz\hat{k}] = \int dV$$

$$\int_0^4 -20\,dx + \int_0^4 +40\,dy - \int_0^4 80\hat{k} = \int_{0\,\text{volt}}^{V'} dV$$

Question 2 -

find the potential at 4, 4, 4.



$\vec{E}\ (20x\hat{i} + 20\hat{j})\,V$

Sol:

$$-\vec{E}\cdot\vec{dr} = dV$$

$$\Rightarrow \int -[20x\hat{i} + 20\hat{j}]\cdot[dx\hat{i} + dy\hat{j} + dz\hat{k}] = \int dV$$

$$\Rightarrow \int_0^4 -20x\,dx - \int_0^4 20\,dy = \int_{200}^{V'} dV$$

$$\Rightarrow \left[\frac{-20x^2}{2}\right]_0^4 - \left[20y\right]_0^4 = \left[V\right]_{200}^{V'}$$

$$\Rightarrow -160 - 80 = V' - 200$$

$$\Rightarrow -240 + 200 = V' \Rightarrow V' = -40\,V$$

* $-\int \vec{E}\cdot\vec{dr} = \int dV$

*

$$\boxed{\begin{aligned} V &= f(x, y, z) \\ &\text{(given)} \\ \vec{E} &= ? \\ E_x &= -\frac{\partial V}{\partial x} \\ E_y &= -\frac{\partial V}{\partial y} \\ E_z &= -\frac{\partial V}{\partial z} \end{aligned}}$$

Question:-

$V = (6x + 6y)\,V$

A charged particle of charge $q = 10\,C$ is released from $(10, 4)$. Find the time at which it will cross y-axis.

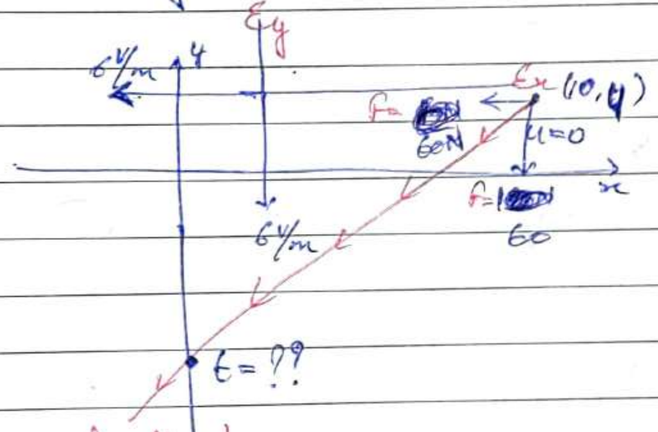
$[m = 2\,Kg]$

Sol:

$\vec{E}_x = -6$

$E_y = -6$

$\therefore \vec{E} = -6\hat{i} - 6\hat{j}$



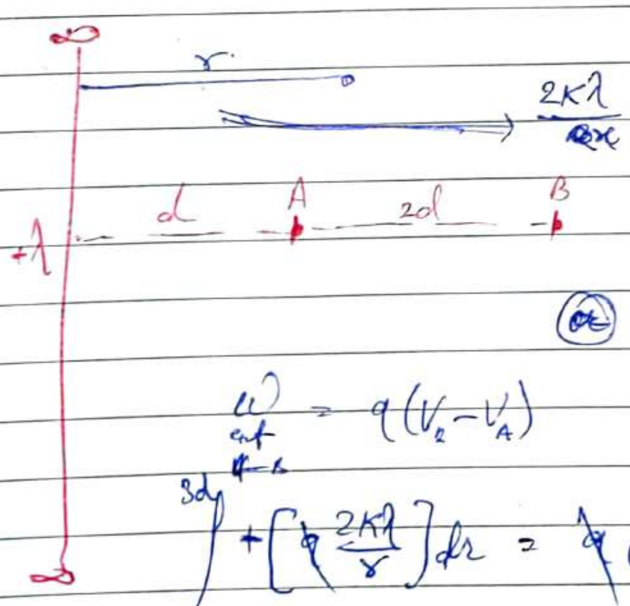
$\longleftarrow$

$10 = 0 + \frac{1}{2}(30)t^2$

$t = \sqrt{\frac{2}{3}}$ sec.

Question:-

Find the Potential diff. betn A and B. $(V_A - V_B)$



Sol: $\boxed{-dV = \vec{E} \cdot d\vec{r}}$

$\int (V_A - V_B) = \int \frac{2K\lambda}{x} dx$

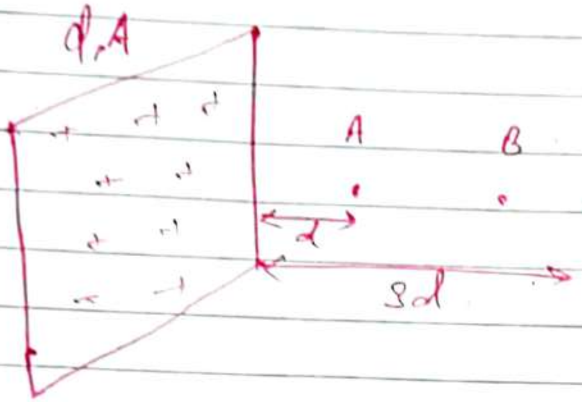
$= 2K\lambda \ln 3$

$= 2K\lambda \ln 3$

$W_{ext\, A \to B} = q(V_B - V_A)$

$\int_d^{3d} + \left[q \frac{2K\lambda}{r}\right] dr = q(V_A - V_B)$

Question :-



find $V_A - V_B$.

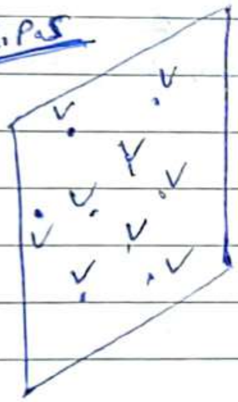
Sol:

$\Rightarrow \left(\cancel{q}\frac{\sigma}{2\varepsilon_0}\right)2d = \cancel{q}(V_A - V_B)$

$\frac{Qd}{A\varepsilon_0} = (V_A - V_B)$

Equipotential Surface :-

E.P.S

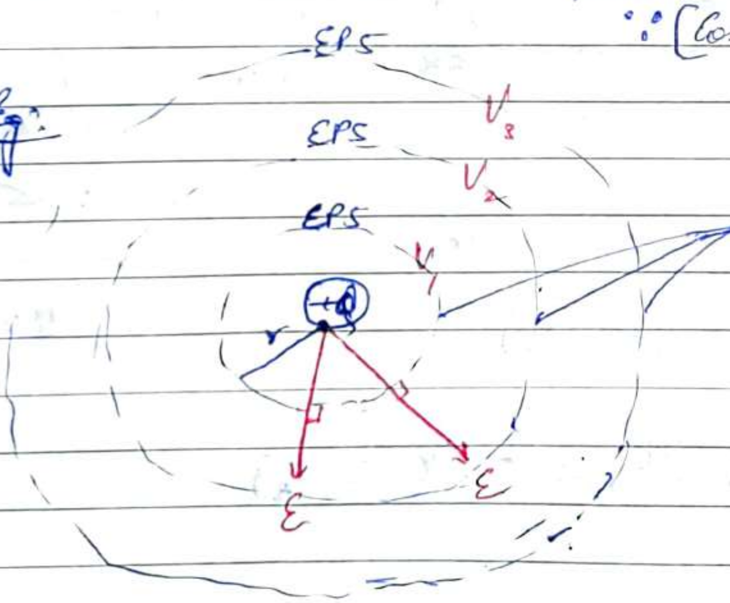


$\Rightarrow$ Electric field is $\perp$ to equipotential surface (or) $\vec{E} = 0$

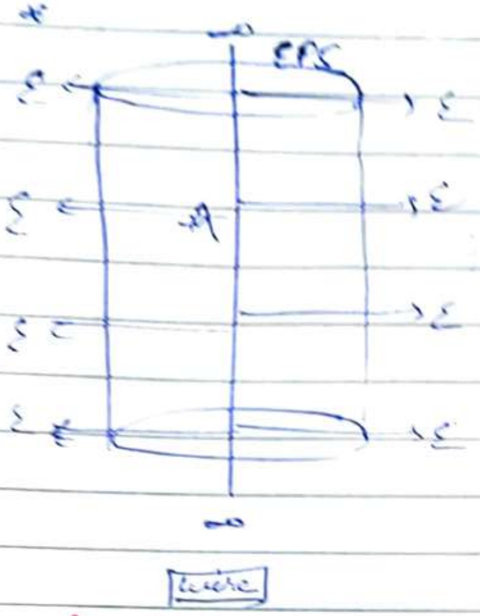
$\Rightarrow$ Workdone by External agent on the charge small q from one point to another point is 0. (in a slow process)

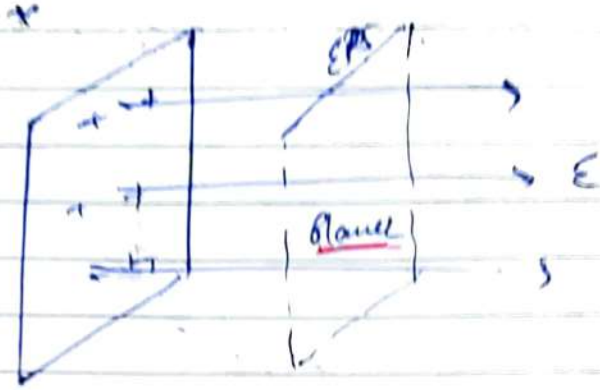
$\because [\cos\theta = 0]$

Eg:



Spherical ~~EP~~ EPS.

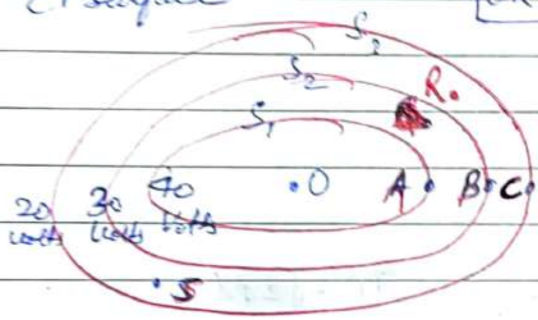




Question:-

EP surface

Given $[OR < OS]$



Q) Choose correct options.

a) $\vec{E}$ at B is towards right.

b) $\vec{E}$ at B is towards left

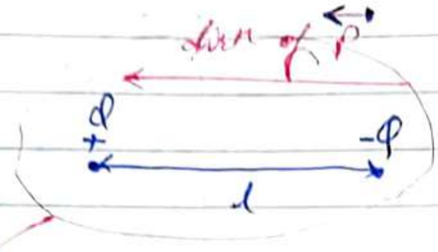
c) $|\vec{E}|$ at S = $|\vec{E}|$ at R

d) $|E|_S > |E|_R$

e) $|E|_S < |E|_R$

$$\boxed{E = \left|\frac{-dV}{dr}\right|}$$

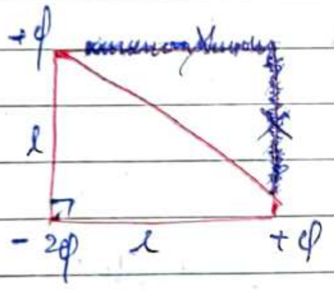
Electric dipole :-



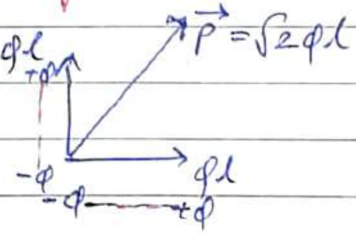
⇒ Electric Dipole moment :- [$\vec{P}$]

$$\boxed{P = |q|\, l}$$

Question :-



Find the $\vec{P}$ of the system.



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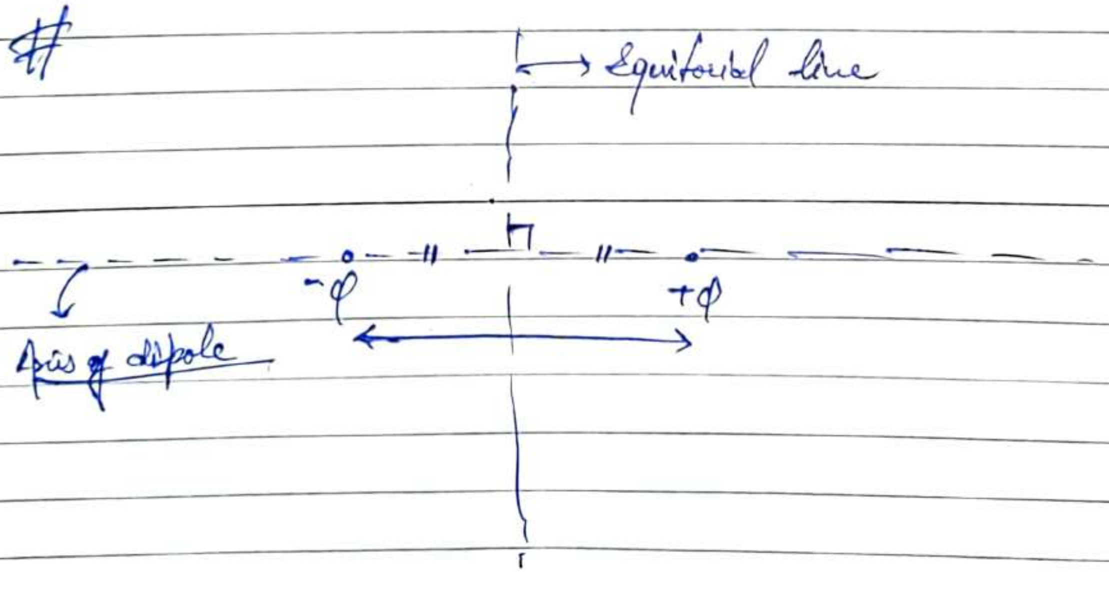


Electric field due to dipole :-

(i) $\vec{E}$ on the axis :

$$E_{net/axis} = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2}$$

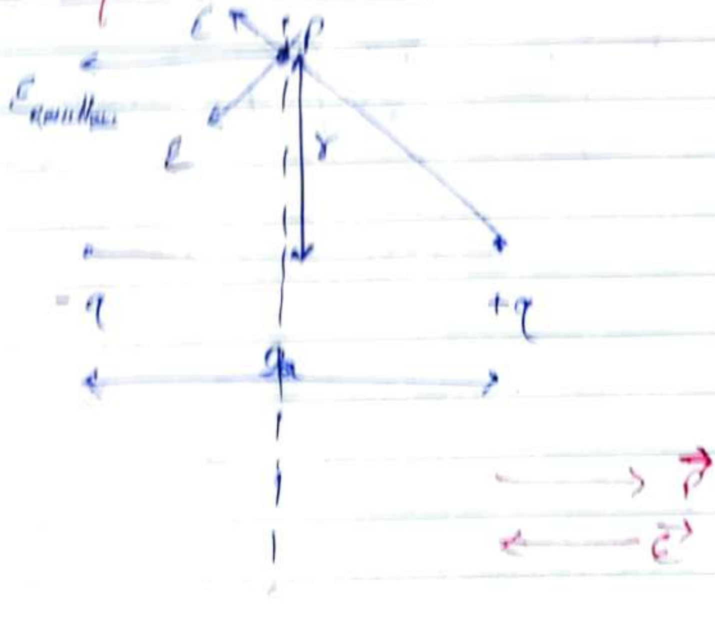
$$= \frac{Kq\,(4ar)}{(r^2-a^2)^2}$$

If $r >>> a$, then

$$E_{axis} = \frac{Kq\,2(2a)\,r}{r^4}$$

$$\boxed{\vec{E}_{axis} = \frac{2K\vec{p}}{r^3}}$$

(ii) $\vec{E}$ on Equatorial line :-

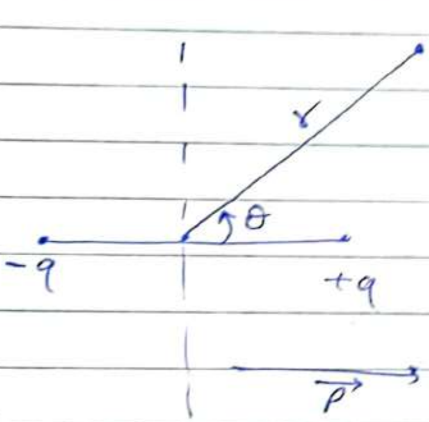


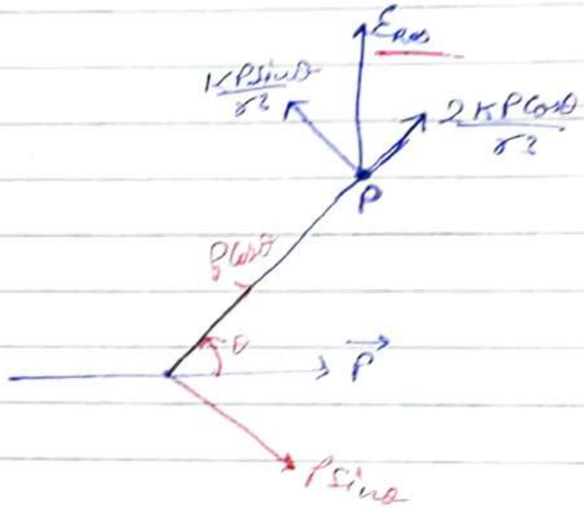
$$E_{Res} = 2E\cos\theta$$
$$= 2\frac{Kp}{r^2+a^2}\frac{a}{\sqrt{r^2+a^2}}$$

If $r >>> a$, then.

$$\boxed{E_{Res} = \frac{Kp}{r^3}} \quad or \quad \boxed{\vec{E}_{Res.} = \frac{-K\vec{P}}{r^3}}$$

③ $\vec{E}$ at (r,θ) :-

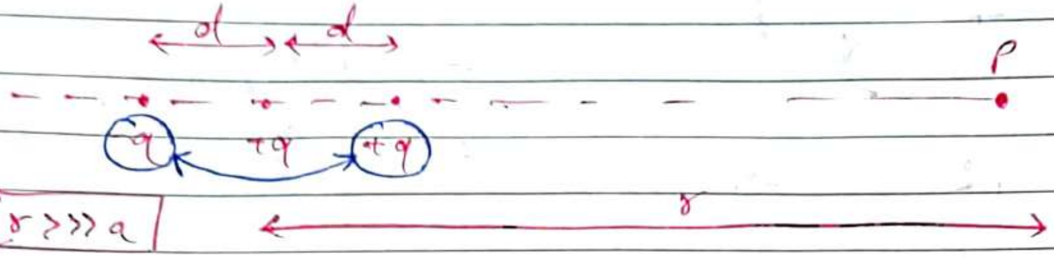




$$\therefore E_{Res} = \sqrt{\left(\frac{KP\sin\theta}{r^3}\right)^2 + \left(\frac{2KP\cos\theta}{r^3}\right)^2}$$

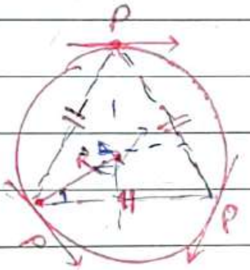
$$\boxed{E_{Res} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}}$$

Question:- Find the $\vec{E}$ at P.



$$\vec{E}_p = \frac{4Kqd}{r^3} + \frac{Kq}{r^2}$$

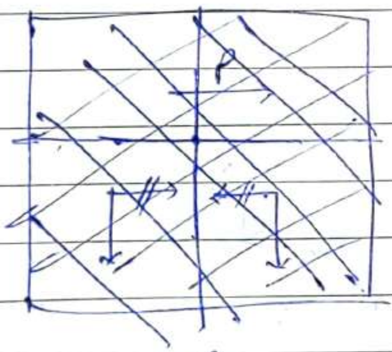
Question:- find the $\vec{E}_{Res.}$ at the centre

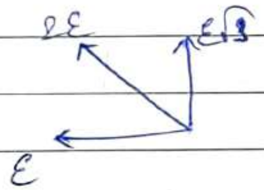


Sol:

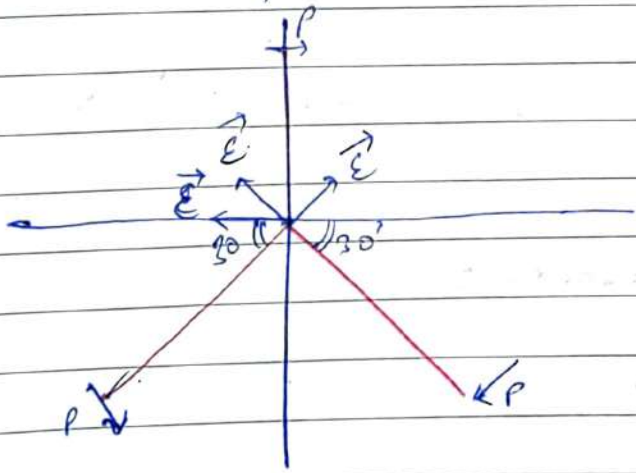
$$\cancel{E_{Res} = \frac{2KP}{r^2+a^2}\,\frac{a}{\sqrt{r^2+a^2}}}$$

$$E = \frac{+KP}{r^3} \quad \text{--- (1)}$$



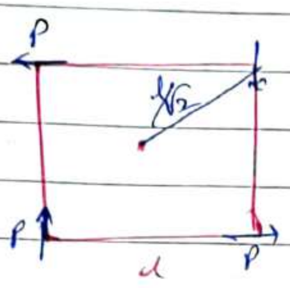


$$\therefore E_{Res} = \frac{2KP}{R^3}$$



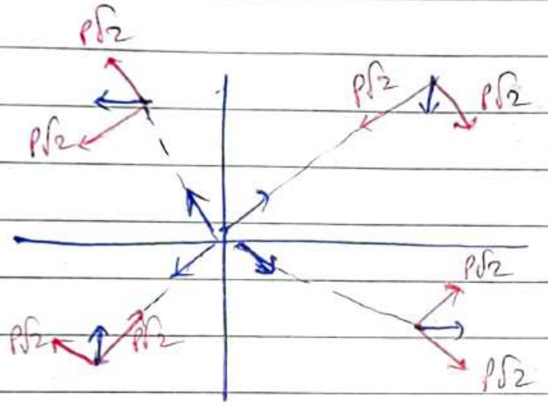
Question:-

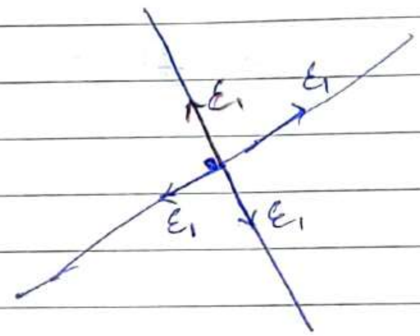
find the $\vec{E}_{Res}$ at Centre



$$\cancel{E = \frac{KP\sqrt{1+3\cos^2\theta}}{r^3} \quad r = \frac{l}{\sqrt{2}}}$$

$$E_1 = \frac{2K\,P/\sqrt{2}}{(l/\sqrt{2})^3}$$

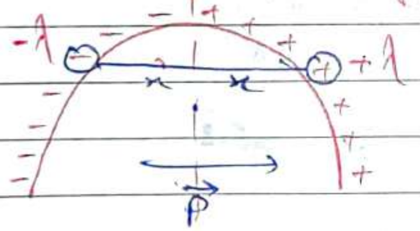




$$\boxed{E_{Res} = 0}$$

Question:-

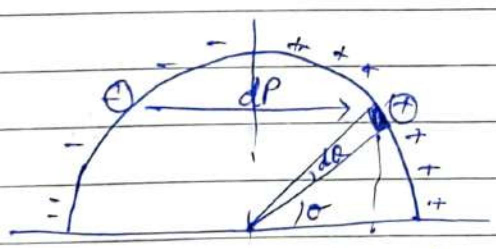
A semi circular ring (R) find the $\vec{P}$ of the system



$$\cancel{\int dP = \int_0^R dq\, 2x}$$

$$\cancel{P = \frac{\lambda(2\pi R)\cdot 2R^2}{2}}$$

$$\cancel{\vec{P} = 2\lambda\pi R^3}$$

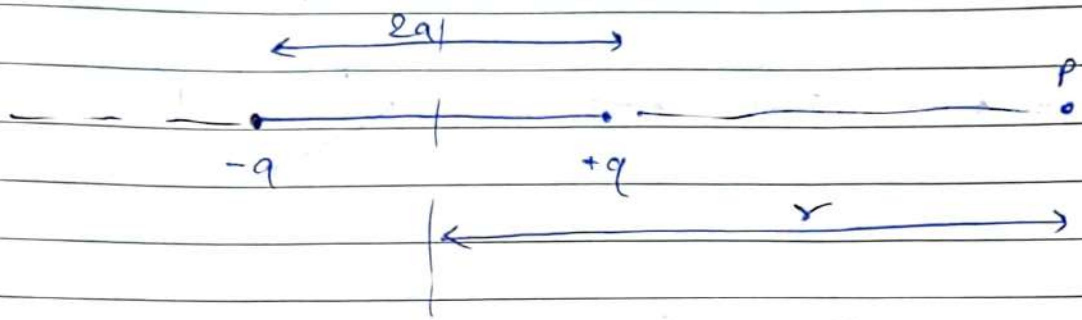


$$\int dP = \int_0^{\pi/2} [\lambda R\, d\theta]\cdot 2R\cos\theta$$

$$P = 2\lambda R^2$$

Potential due to dipole :-

(i) Potential on the axis :-



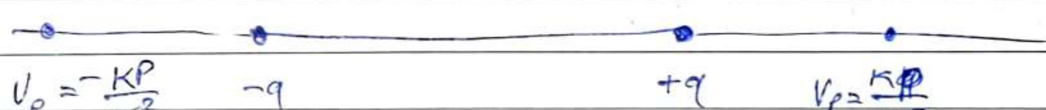
$$V_p = \frac{Kq}{(r-a)} - \frac{Kq}{(r+a)} = Kq\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{Kq(2a)}{r^2-a^2}$$

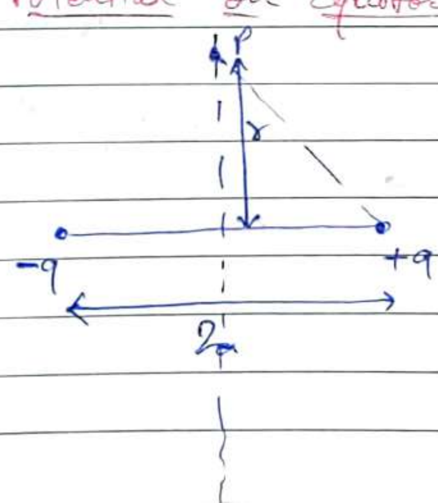
if, $r >>>>> a$

$$\boxed{V_{P(axis)} = \frac{KP}{r^2}}$$

#



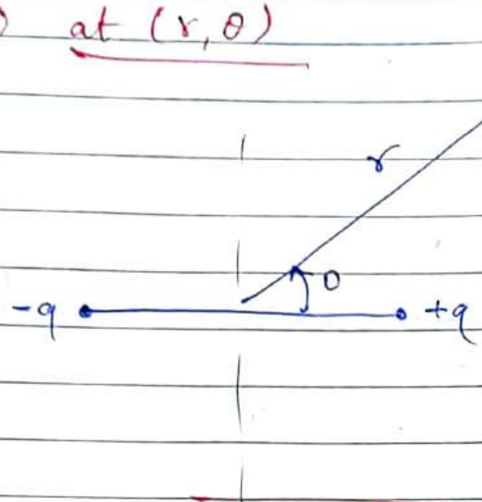
(ii) Potential on Equatorial line :-

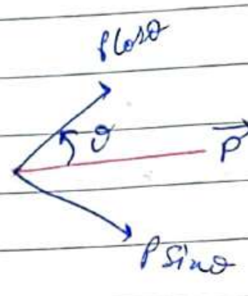


$$\boxed{V_p = 0}$$

Ssc-1 I Sec H (1,2,3,4)
II Sec H (1,2,8,6)

③ at (r, θ)

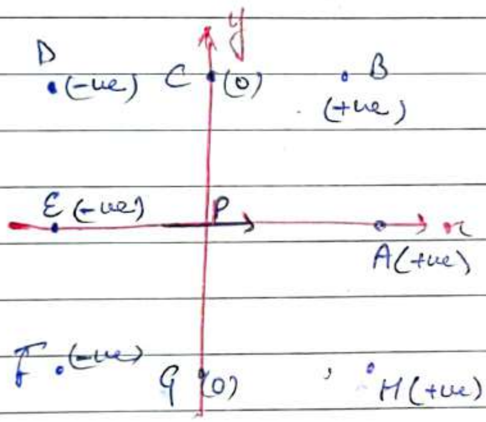




$$\boxed{V_P = \frac{KP\cos\theta}{r^2}}$$

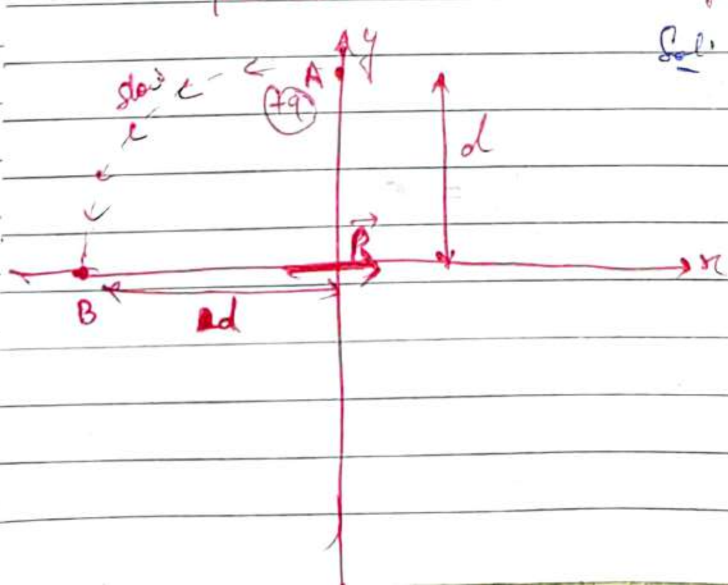
Question

Ⓠ Show the sign of potential.



Question

find W_{ext} in this process.

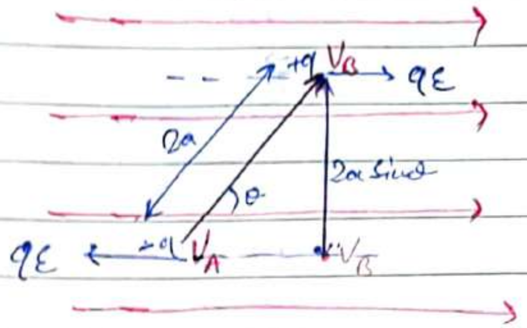


Sol:

$$W_{ext\,A\to B} = q(V_B - V_A)$$

$$= q\left(-\frac{KP}{d^2}\right)$$

Dipole in Uniform Electric field :-



* $\vec{F}_{Net} = 0$
* $\tau_R = (qE)2a\sin\theta$

$$\boxed{\tau_R = PE\sin\theta}$$

$$\boxed{\vec{\tau}_R = \vec{P}\times\vec{E}}$$

* Dipole iss tarah se rotate hota hai jis-se Ki $\vec{P}$ hamesa $\vec{E}$ ki disn me aane Ki Kosish Krta h.

*

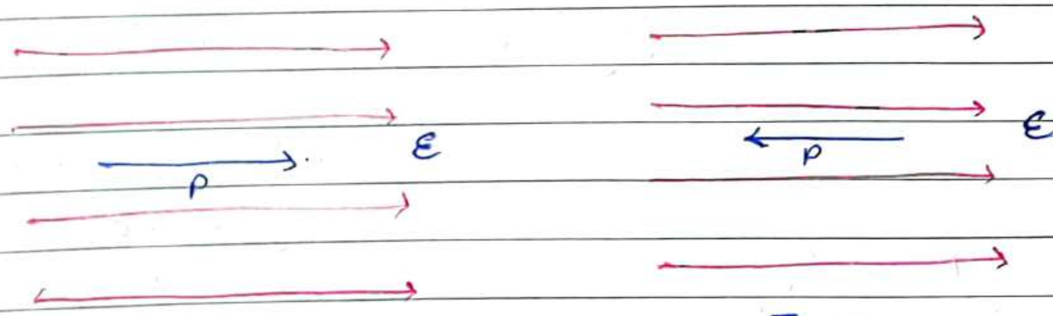
$$U_{\substack{dipole \\ (\text{due to } \vec{E})}} = (-q)V_A + q(V_B)$$

$$= -q(V_A - V_B) = -qE\,2a\cos\theta$$

$$\boxed{U = -PE\cos\theta}$$

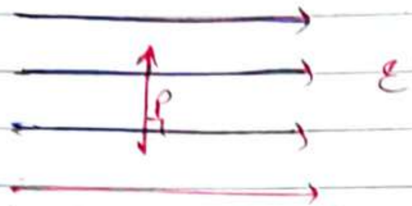
$$\boxed{U = -\vec{P}\cdot\vec{E}}$$

Special Cases :-



$\tau_R = 0$ 'Stable Eqllibrium'

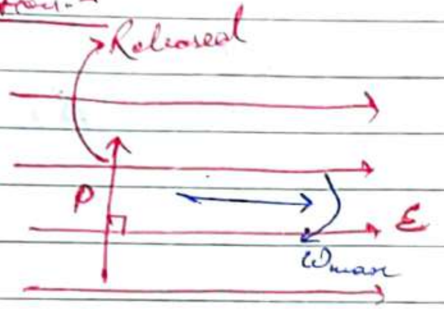
$\tau_R = 0$ 'Unstable Eqm'



No Eq^m

$\vec{\tau}_{max} = PE$

Question:-



find the max^m angular velocity of the dipole

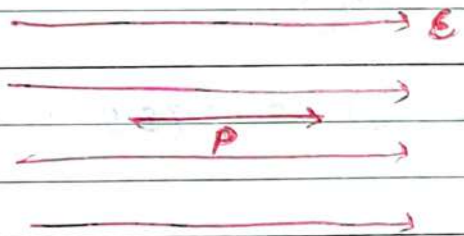
Sol!

$0 + (-PE\cos\theta) = \frac{1}{2} I_0 \omega_{max}^2 + -PE\cos 0°$

MOI of dipole about COM $= I_0$

$\Rightarrow \sqrt{\frac{2PE}{I_0}} = \omega_{max}$

Question:-

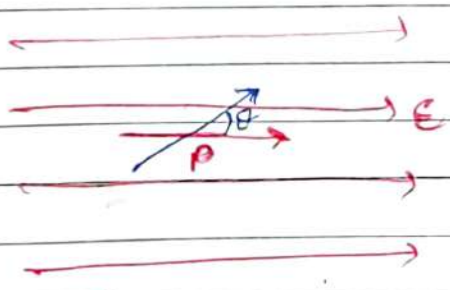


find $W_{ext.}$ to rotate the dipole by 180° in a slow process.

$W_{ext} = \Delta U$

$= PE + PE = 2PE$

Question:-



If dipole is rotated by any small angle and then released, then find its time period.

$\vec{\tau} = PE\sin\theta$

$\Rightarrow I_0\alpha = PE\theta$

$\Rightarrow \alpha = \underbrace{\frac{PE\theta}{I_0}}_{K\theta \to \omega^2\theta}$

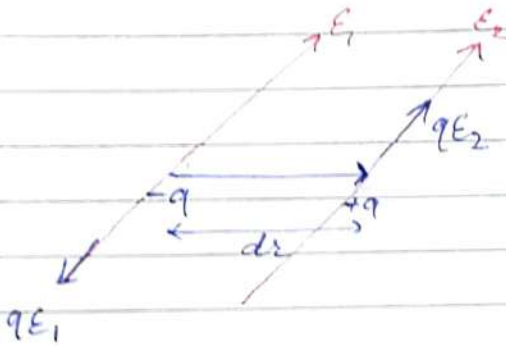
$\boxed{I_{com} = I_0}$

$$\boxed{\frac{q}{mc} = \frac{PE}{I_0}}$$

$\Rightarrow \omega = \sqrt{\frac{PE}{I_0}}$

$\therefore T = 2\pi\sqrt{\frac{I_0}{PE}}$

Force on dipole in Non-Uniform $\vec{E}$:-



(magnitude wise non-uniform $\vec{E}$)

$\vec{F}_{Res.} = q(E_2 - E_1)$

$= q(E_2 - E_1)\frac{dr}{dr}$

$$\boxed{F = P\frac{dE}{dr}}$$

Question!

find the force on dipole due to point charge.

$r >>>>$ length of dipole

Sol!

$F = P\frac{dE}{dr}$

$\int F = \int \frac{P}{dr}\left[\frac{KQ}{r^2}\right]$

$F = -\frac{2KQP}{r^3}$

$$\therefore \boxed{F = \frac{2KQP}{r^3}} \leftarrow$$

(or)

$$F_{on\ q} = q\,E_{due\ to\ P}$$

$$= \frac{2KP}{r^3}\,q$$

$$\therefore F_{\substack{on\ \vec{P} \\ due\ to \\ +q}} = -\frac{2KP}{r^3}\,q$$

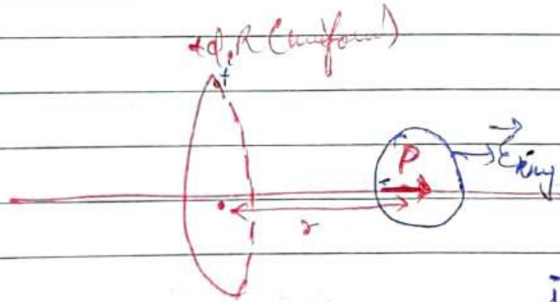
Question:- find the forces between the dipoles.

Sol^n

$$F_{\substack{P_2 \\ due\ to \\ P_1}} = P_2 \frac{d}{dr}\frac{2KP_1}{r^3}$$

$$\boxed{F_{P_2} = \frac{6KP_1P_2}{r^4}}$$

Question:- find the force on the ring due to dipole.

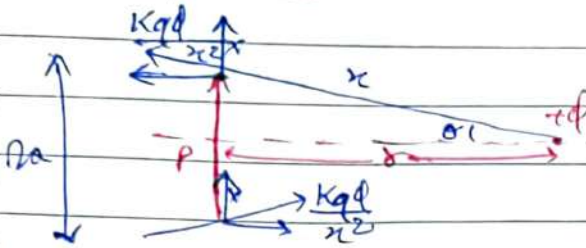


$$F_{ring} = \frac{Kq}{R}\frac{d}{dr}\frac{2KP}{r^3}$$

$$\boxed{F_{\substack{ring \\ due\ to \\ P}} = \frac{Kq}{R}\,\frac{6KP}{r^4}}$$

$$F_P = P\frac{d}{dr}\frac{Kqr}{\left(\sqrt{R^2+r^2}\right)^{3/2}}$$

Question:-



find the torque on dipole due to point charge.

Sol:-

$$\vec{\tau} = (F\cos\theta)\,2a$$

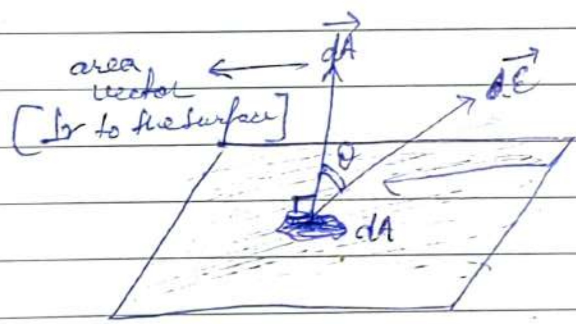
$$= \left(\frac{Kpq}{x^2}\right)\frac{r}{x}\,2a$$

$$= \frac{PKqr}{x^3} \approx \frac{PKq}{r^2}$$

or If r is very large $\rightarrow \vec{E}$ uniform

$$\vec{\tau} = P\left(\frac{Kq}{r^2}\right)\sin\theta$$

Electrostatic flux:- [ϕ] [Scaler]



Small flux passing through small area dA

$$\boxed{d\phi = \vec{E}\cdot\vec{dA}}$$

$$\boxed{d\phi = E\,dA\cos\theta}$$

Total flux:-

$$\boxed{\phi = \int E\,dA\cos\theta}$$

DPP-26

Ex-1 Dipole - all
Ex-1 III (2)
Ex-2 I 6, 7, 8, 9
III 11, 12

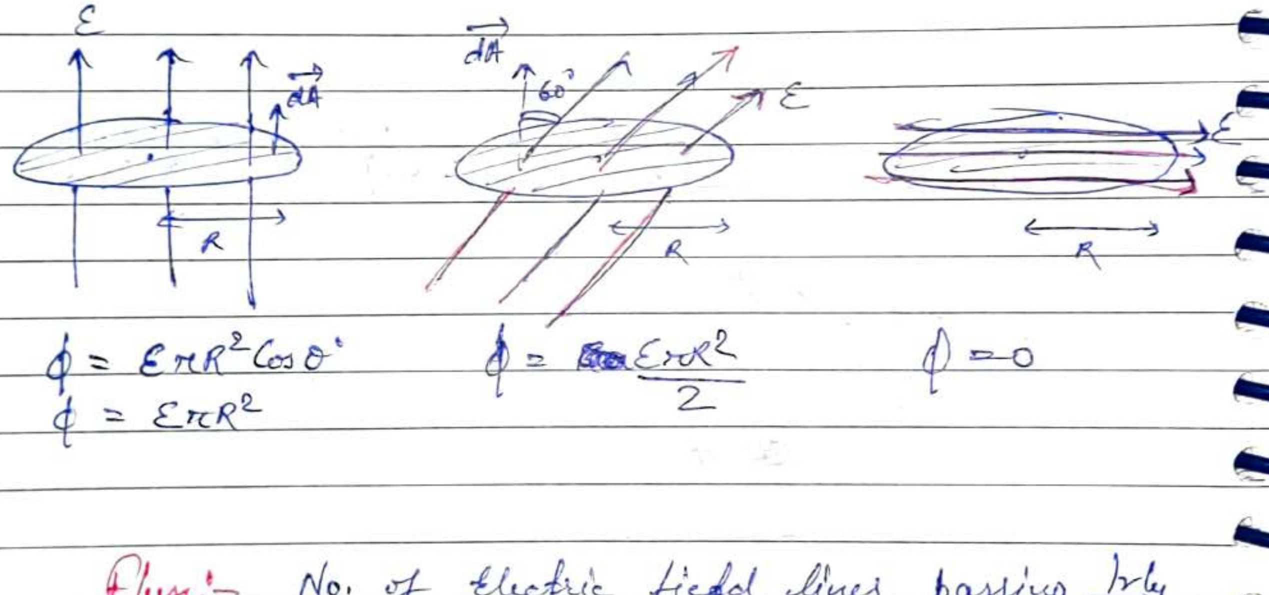
Ex-3 I 4, 5, 13, 16, 19, 21, 22, 23, 24, 25, 26, 27, 29, 30.

- Unit = $\frac{N}{C} \cdot m^2$ (gauss)

Special Case:-

If $\vec{E}$ is uniform & $\vec{\theta}$ is constant.

Total flux: $\boxed{\phi = EA\cos\theta}$



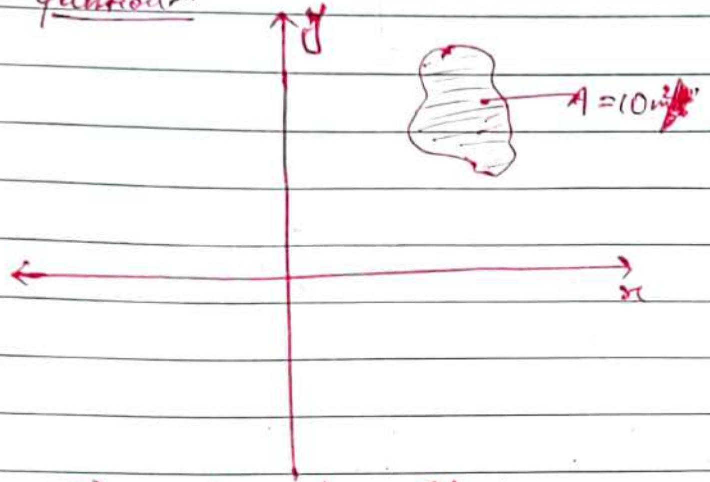
$\phi = E\pi R^2 \cos 0°$
$\phi = E\pi R^2$

$\phi = \frac{E\pi R^2}{2}$

$\phi = 0$

Flux:- No. of Electric field lines passing Ir'ly from a surface is called flux passing through that surface.

Question:-



$\vec{E} = (10\hat{i} + 20\hat{j} + 30\hat{k})\ \frac{N}{C}$

find the flux passing through given area.

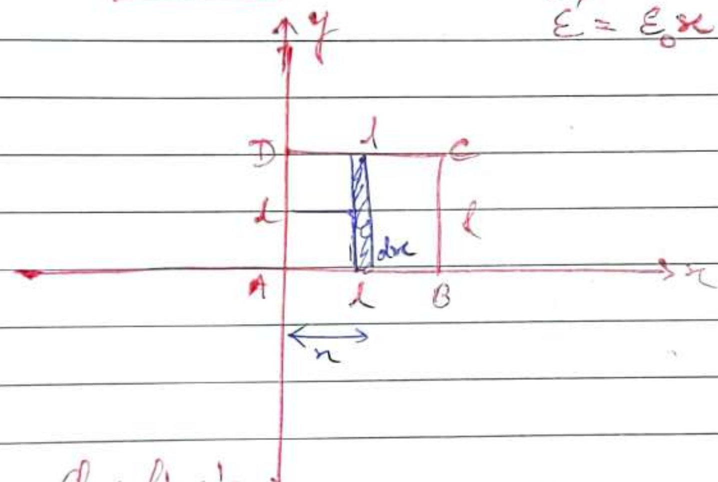
Sol:

$\phi = 30 \times 10 = 300$ gauss

Question:-

$\vec{E} = E_0 x \hat{k}$

find the flux through given square



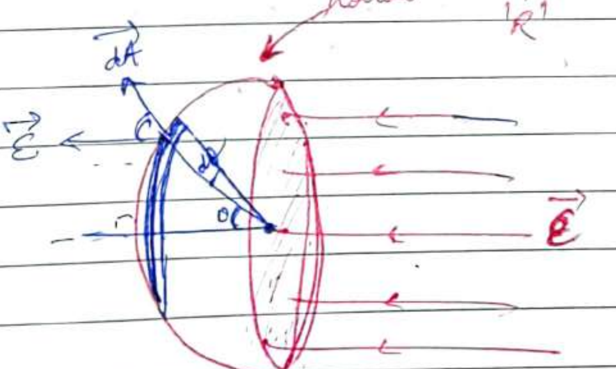
Sol:

$\phi = \int \vec{E} \cdot d\vec{A}$

$= \int_0^l E_0 x\, l\, dx \cos 0°$

$= \frac{E_0 l^3}{2}$

Question:-



find the flux passing through the curved surface of hemisphere

Sol: $\vec{E}$ is uniform &

$d\phi = E\, dA \cos\theta$

$\int d\phi = \int_0^{\pi/2} E\, 2\pi R \sin\theta\, R\, d\theta \cos\theta$

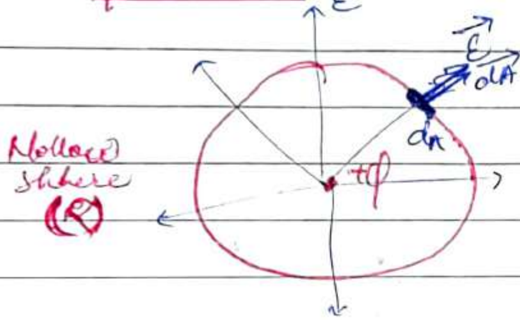
$\phi = E\pi R^2 \int_0^{\pi/2} \sin 2\theta$

2^{nd} method [By definition of flux]

$$\Rightarrow \phi = E \pi R^2$$

$\because$ flux passing through curved surface = flux passing through opening.

Question- find the flux passing through hollow sphere.



Sol^n ~~$\vec{E}$ is uniform & θ is constant~~

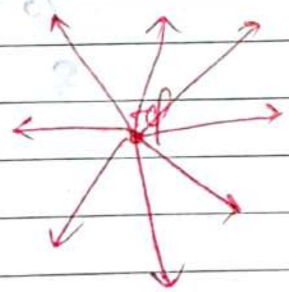
~~$\phi = E \cdot 4\pi R^2$~~

$$d\phi = \int \left[\frac{Kq}{R^2}\right] \cos 0^\circ \, dA$$

$$\phi = \frac{Kq}{R^2} 4\pi R^2$$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

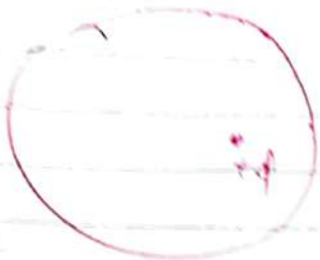
★ Imp :-



Total Electric field lines produced by a point +q charge is $\frac{q}{\varepsilon_0}$.

Gradient

Find the flux through hollow sphere.

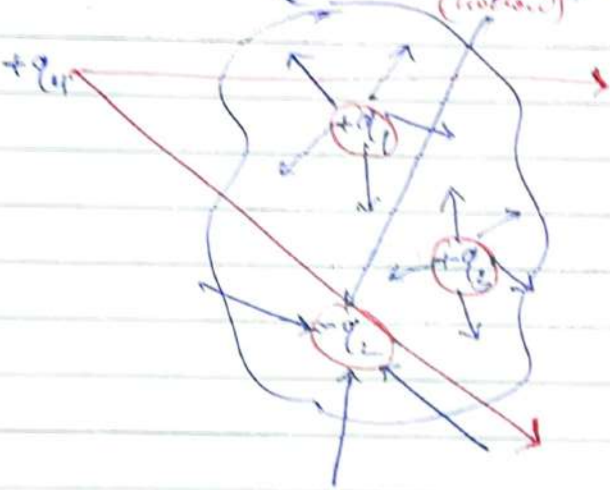


hollow sphere (R)

$$\phi = \frac{q}{\varepsilon_0}$$

Gauss law :-

(Closed surface) '3D' (hollow)



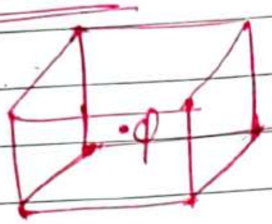
Outward flux = +ve
Inward flux = -ve

$$\boxed{\phi_{\text{from closed surface}} = \frac{\Sigma \text{ inside charge}}{\varepsilon_0}} = \frac{q_1 + q_3 - q_2}{\varepsilon_0}$$

Gauss law :-
Total flux passing through a closed surface depends only on the total charge present inside the closed surface.

Question

Hollow cube



i) find the total flux passing through the cube

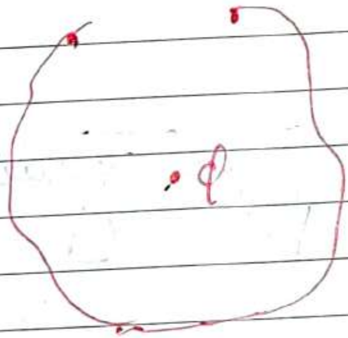
Sol $\phi = \frac{q}{\varepsilon_0}$

ii) find the flux from each face

$$\phi_{each\ face} = \left(\frac{q}{\varepsilon_0}\right)/6$$

Question

hollow surface



If flux passing through the surface is ϕ, then

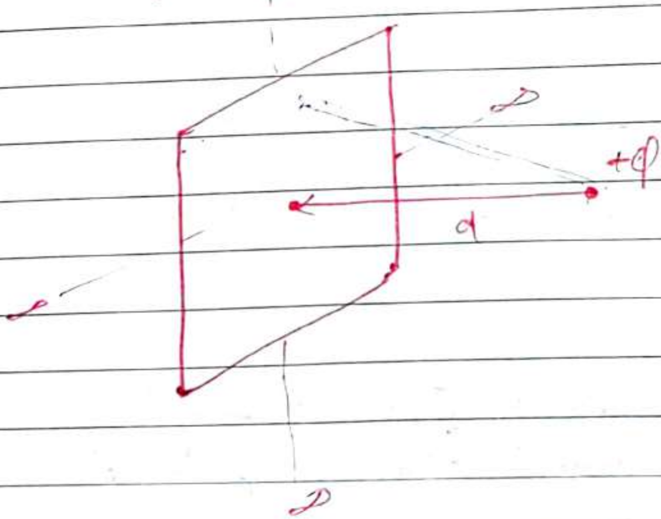
A) $\phi = \frac{q}{\varepsilon_0}$

B) $\phi > q/\varepsilon_0$

~~C)~~ $\phi < q/\varepsilon_0$

d) Can't say

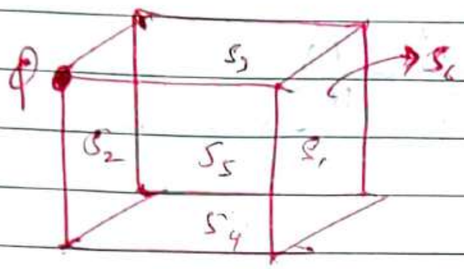
Question



Find the flux passing through given plate

$$\phi = \left(\frac{q}{\varepsilon_0}\right) \times \frac{1}{2}$$

Question:-



Cube

i) find the total flux passing through cube

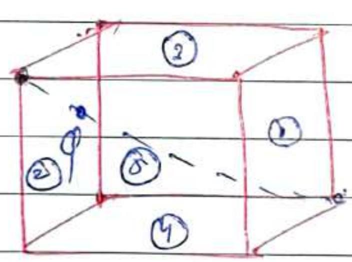
$$\phi_{cube} = \left(\frac{\phi}{8}\right) \times \frac{1}{\varepsilon_0}$$

ii) find the flux passing through each surface

$$\phi_{S_1} = \phi_{S_2} = \phi_{S_5} = 0$$

$$\phi_{S_4} = \phi_{S_1} = \phi_{S_6} = \left(\frac{\phi}{8\varepsilon_0}\right) \times \frac{1}{3}$$

Question:-



find $\phi_{S_1} + \phi_{S_3}$

$$\left(\frac{\phi}{\varepsilon_0} \times \frac{1}{6} \times 2\right) = \phi_{S_1} + \phi_{S_3}$$

(or)

$$\phi_2 = \phi_3 = \phi_5 = x$$

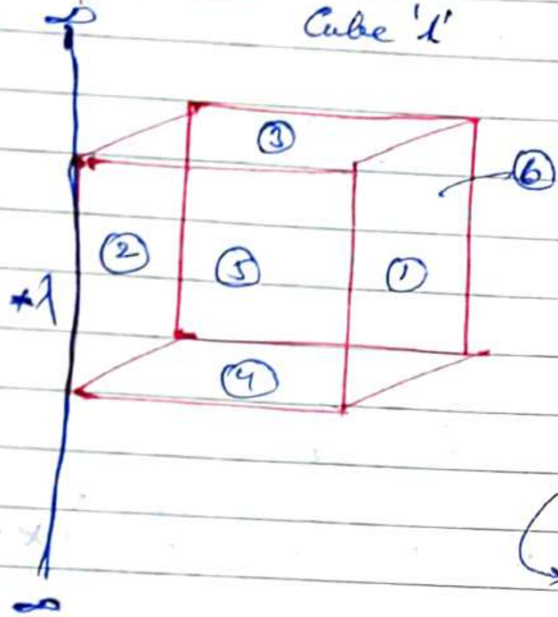
$$\phi_1 = \phi_4 = \phi_6 = y$$

$$\therefore \frac{\phi}{\varepsilon_0} = 3x + 3y$$

$$\frac{\phi}{\varepsilon_0 \times 3} = (x+y)$$

Question:-

Cube 'l'



i) Find the flux passing through the cube

$$\phi_{cube} = \frac{(\lambda l)}{\varepsilon_0} \times \left(\frac{1}{4}\right) \rightarrow \text{Solid state}$$

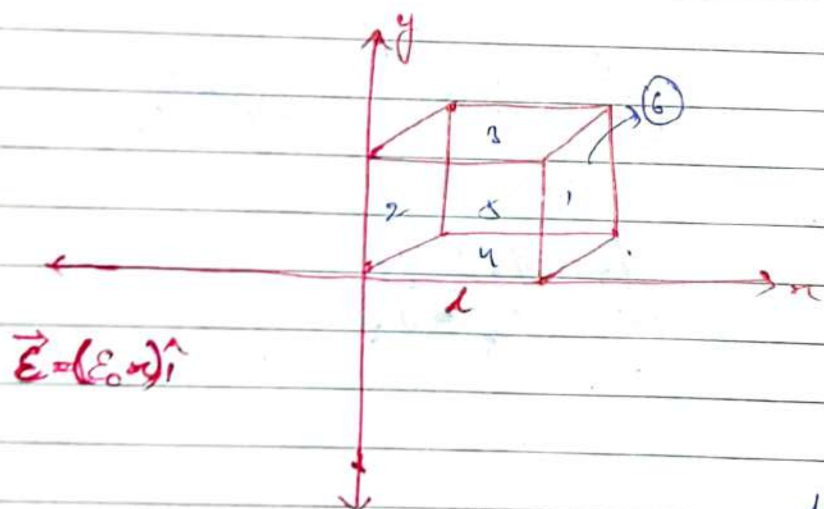
ii) find the flux passing through each surface

$$\phi_{②} = \phi_5 = 0$$

$$\phi_{③} = \phi_{④} = 0$$

$$\phi_1 = \phi_6 = \left(\frac{\lambda l/4}{\varepsilon_0}\right) \times \frac{1}{2}$$

Question-



find the total charge enclosed by cube

$$\phi_{cube} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$$

$$\boxed{d\phi = E\,dA\cos\theta}$$

$$\phi_{cube} = (\varepsilon_0 l)\,l^2 \cos 0^\circ + 0 + 0 + 0 + 0 + 0$$

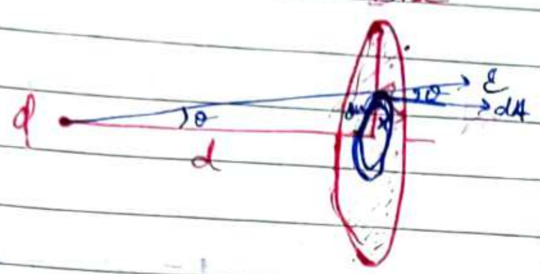
Also,

$$\phi_{cube} = \frac{q_{inside}}{\varepsilon_0}$$

$$\therefore \boxed{\varepsilon_0^2 l^3 = q_{inside}}$$

Question

Find the flux passing through the disc.



$d\phi = E\,dA\cos\theta$

$\cos\theta = \dfrac{d}{\sqrt{x^2+d^2}}$

$d\phi = \dfrac{Kq}{d^2+x^2}(2\pi x)dx\cos\theta$

$\int_0^{\phi} d\phi = \int_0^{R} \dfrac{Kq\,2\pi\, x\,dx}{(d^2+x^2)^{3/2}}$

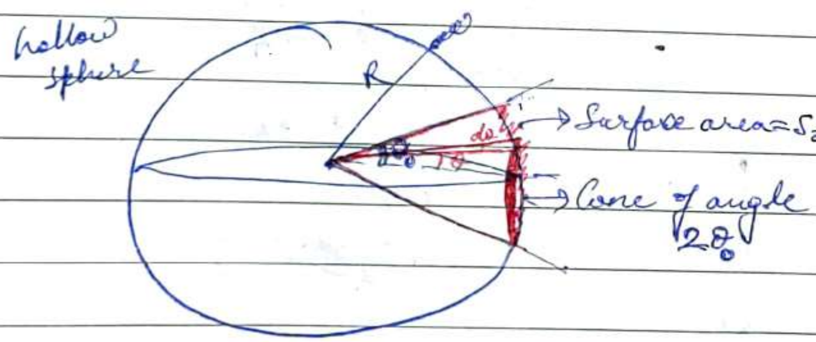
$x^2+d^2 = t^2$

$2x\,dx = 2t\,dt$

$\phi = Kq\,2\pi \dfrac{t\,dt}{t^3}$

$= Kq\,2\pi\left[\dfrac{-1}{t}\right]_0^R$

Solid Angle :-



$\int dS = \int_0^{\theta_0} 2\pi(R\sin\theta)R\,d\theta$

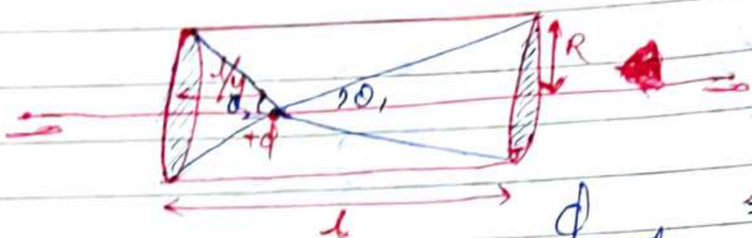
$\boxed{S_0 = 2\pi R^2[1-\cos\theta]}$

$\therefore \phi_{cone} = \dfrac{\frac{q}{\varepsilon_0}}{4\pi R^2} \times S_0$

$= \dfrac{q}{\varepsilon_0}\left[\dfrac{1-\cos\theta}{2}\right]$

Question:-

Find the flux passing through the curved surface of the cylinder.



$$\phi_{opening} = \frac{q}{\varepsilon_0}\left[\frac{1-\cos\theta_1}{2} + \frac{1-\cos\theta_2}{2}\right]$$

$$\cos\theta_1 = \frac{3l}{4} \times \sqrt{R^2+\left(\frac{3l}{4}\right)^2}$$

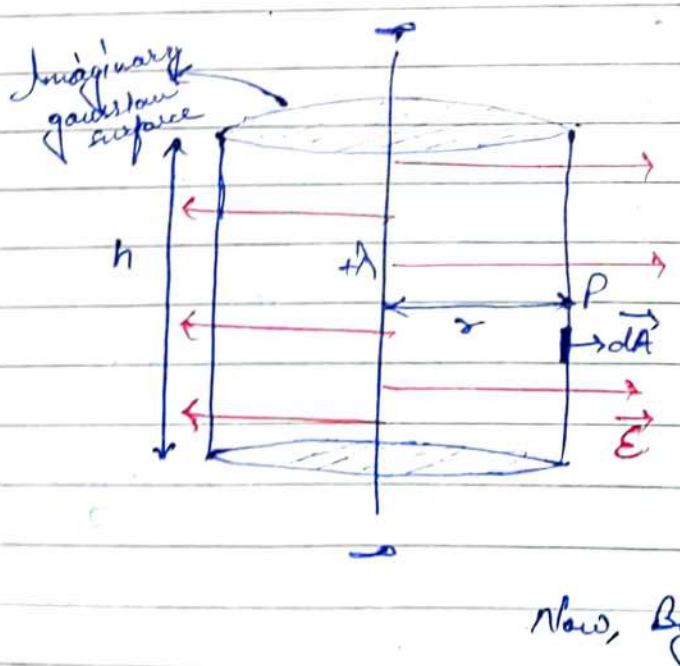
$$\cos\theta_2 = \frac{l}{4} \times \sqrt{R^2+\frac{l^2}{16}}$$

$$\therefore \phi_{surface} = \phi_{Total} - \phi_{opening}$$

$$= \frac{q}{\varepsilon_0} - \frac{q}{\varepsilon_0}\left[\frac{2-\cos\theta_1-\cos\theta_2}{2}\right]$$

Applications of Gauss law:-

i) To find Electric field ($\vec{E}$) due to infinite wire.



$$\phi_{g.s} = \int \vec{E}\cdot\vec{dA}$$

$$= \cancel{\int_{upper} \vec{E}\cdot\vec{dA}} \rightarrow 0$$

$$= \cancel{\int_{lower} \vec{E}\cdot\vec{dA}} \rightarrow 0$$

$$= \int_{curved} \vec{E}\cdot\vec{dA}$$

$$\phi_{g.s} = E\,2\pi r h \quad \text{--- (i)}$$

Now, By Gauss law,

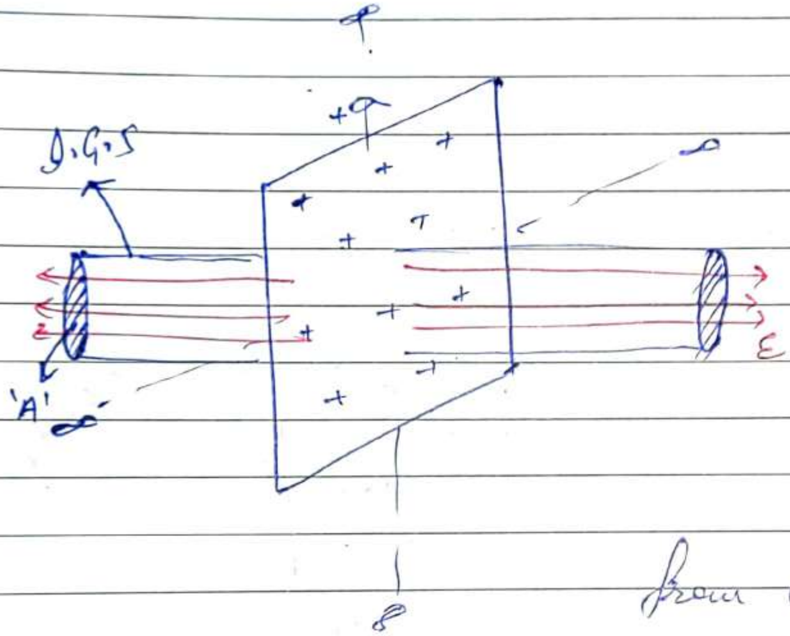
$$\phi_{g.s} = \frac{\lambda h}{\varepsilon_0} \quad \text{--- (ii)}$$

from (i) and (ii),

$$E 2\pi r \cancel{h} = \frac{\lambda \cancel{h}}{\varepsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2K\lambda}{r}}$$

ii) To find $\vec{E}$ due to a plate :-



$$\phi_{G.S} = \int \vec{E} \cdot \vec{dA}$$

$$= \int_{left} + \int_{right} + \cancel{\int_{curved}} \rightarrow 0$$

$$= 2(EA) \quad \text{——— (i)}$$

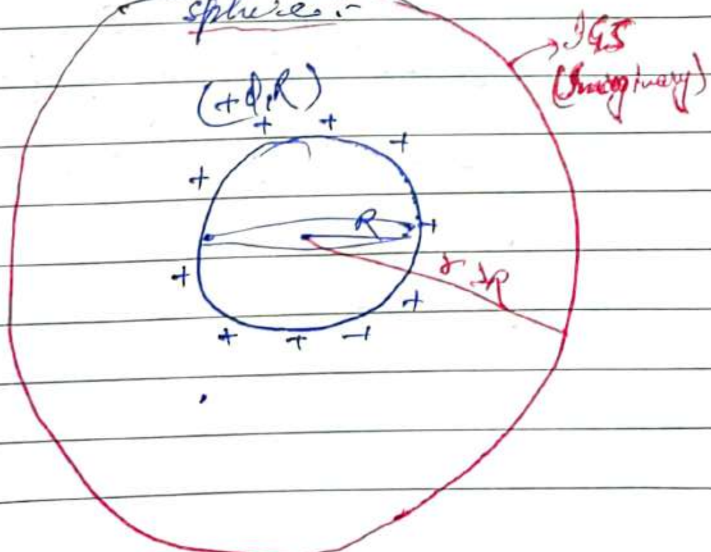
Also, by Gauss law,

$$\phi_{G.S} = \frac{\sigma A}{\varepsilon_0} \quad \text{——— (ii)}$$

from (i) & (ii),

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

iii) To find $\vec{E}$ due to uniformly charge hollow sphere :-



$$\phi_{G.S} = \int \vec{E} \cdot \vec{dA}$$

$$\phi_{G.S} = E 4\pi r^2$$

$\phi_{G.S} = \frac{q}{\varepsilon_0}$ [by Gauss law]

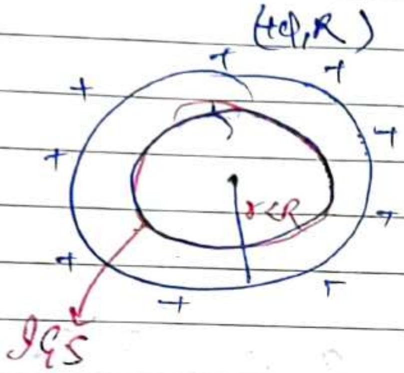
$$\Rightarrow E 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{Kq}{r^2}}$$

Case I :- $r > R$ and $r = R$

$$\boxed{E = \frac{Kq}{r^2}}$$

Case (ii) :- $r < R$

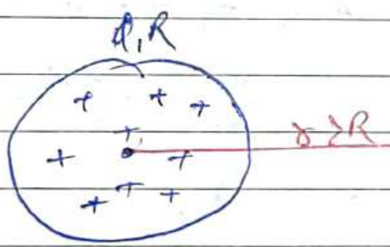


$\phi_{G.S} = E\, 4\pi r^2$ | $\phi_{G.S}$ (by Gauss law) $= \frac{0}{\varepsilon_0}$

$\therefore \boxed{\vec{E} = 0}$

iv) To find $\vec{E}$ due to Solid sphere :-

Case I :- If $r > R$ and $r = R$



$\phi_{G.S} = \int \vec{E} \cdot \vec{dA}$

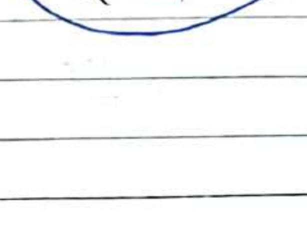
$\phi_{G.S} = E\, 4\pi r^2$ — (i)

$\phi_{G.S}$ (by Gauss law) $= \frac{q}{\varepsilon_0}$ — (ii)

from (i) & (ii)

$$\boxed{E = \frac{Kq}{r^2}}$$

Case II :- If $r < R$, then



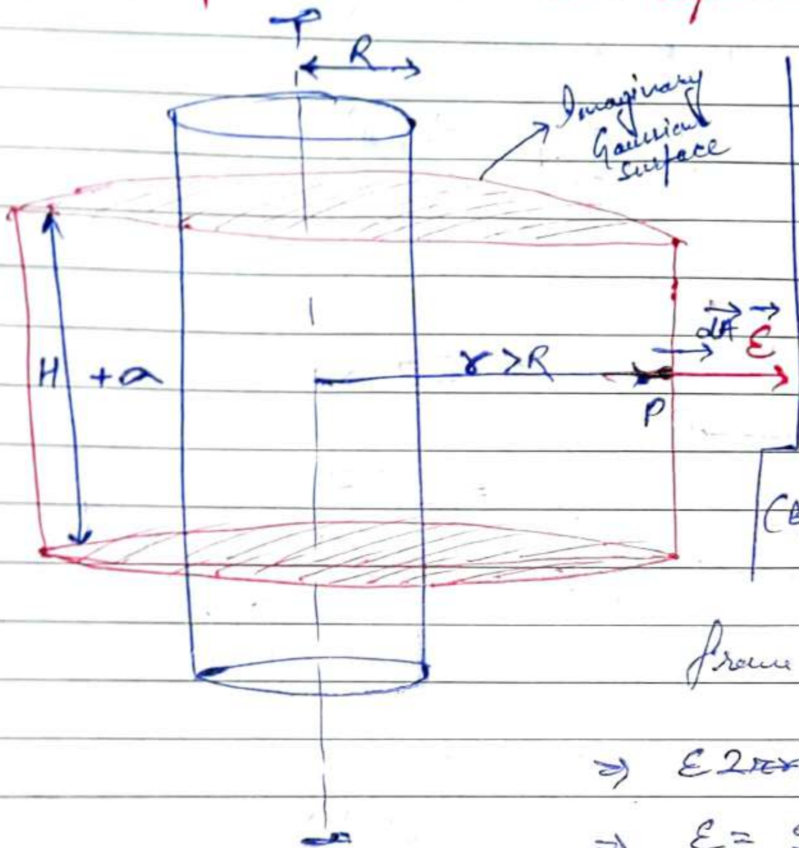
$\phi_{G.S} = E\, 4\pi R^2$ — (i)

$\phi_{G.S}$ (by Gauss law) $= \dfrac{\left(\frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3\right)}{\varepsilon_0}$ — (ii)

$$\boxed{E = \frac{Kq}{R^3}\, r}$$

$\frac{\rho}{3\varepsilon_0} \vec{r}$

v) To find $\vec{E}$ due to infinite hollow cylinder:-



$$\phi_{g.s} = \cancel{Top} + \cancel{bottom} + Curved$$

$$\phi_{g.s} = \int \vec{E} \cdot \vec{dA}$$

$$= E\, 2\pi r H \quad \text{--- (I)}$$

$$\phi_{g.s} = \frac{\sigma 2\pi R H}{\varepsilon_0} \quad \text{--- (II)}$$

(By Gauss law)

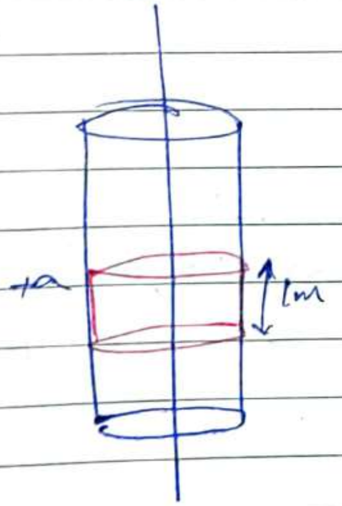
from (I) & (II),

$$\Rightarrow E\, 2\pi r H = \frac{\sigma\, 2\pi R H}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma R}{r \varepsilon_0}$$

$$\boxed{E = \frac{R}{r} \frac{\sigma}{\varepsilon_0}}$$

#



$$\lambda \left[\begin{matrix}\text{Charge} \\ \text{per unit} \\ \text{length}\end{matrix}\right] = \sigma 2\pi r \cdot 1$$

$$\sigma R = \frac{\lambda}{2\pi}$$

$$\boxed{E = \frac{2K\lambda}{r}}$$

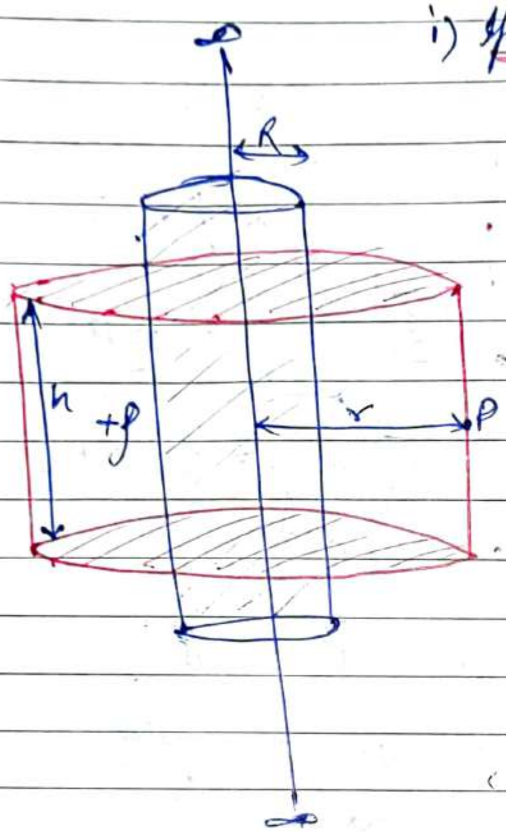
ii) at $r = R$

$$\boxed{E = \frac{2K\lambda}{R}}$$

iii) at $r < R$

$$\boxed{E = 0}$$

ii) To find $\vec{E}$ due to infinite solid cylinder:-



i) If $r > R$

$$\phi_{G.S} = \oint \vec{E} \cdot \vec{dA}$$

$$= \cancel{\int_{Top}}^{0} + \cancel{\int_{bottom}}^{0} + \int_{curved}$$

$$= E\,2\pi r h \quad \text{—— (i)}$$

$$\phi_{G.S} = \frac{\rho \pi R^2 h}{\varepsilon_0} \quad \text{—— (ii)}$$

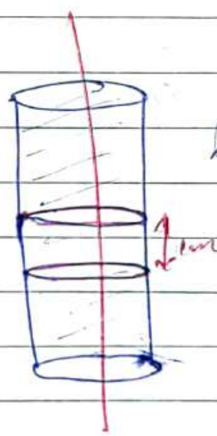
(by Gauss law)

from (i) and (ii).

$$E\,2\pi r h = \frac{\rho \pi R^2 h}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\rho R^2}{\varepsilon_0 2 r}}$$

#



$$\lambda = \rho \pi R^2 \times 1$$

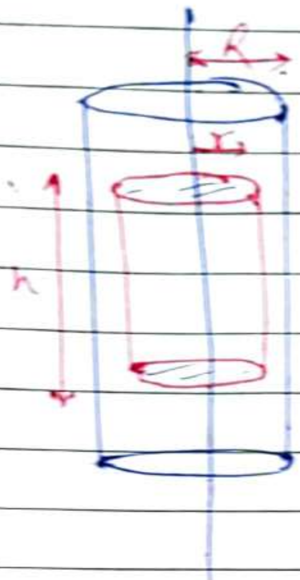
$$\Rightarrow \rho R^2 = \frac{\lambda}{\pi}$$

$$\boxed{E = \frac{\lambda}{\varepsilon_0 2\pi r}} \Rightarrow \boxed{E = \frac{2K\lambda}{r}}$$

ii) at $r = R$

$$\boxed{E = \frac{2K\lambda}{R}}$$

iii) at $r < R$:-



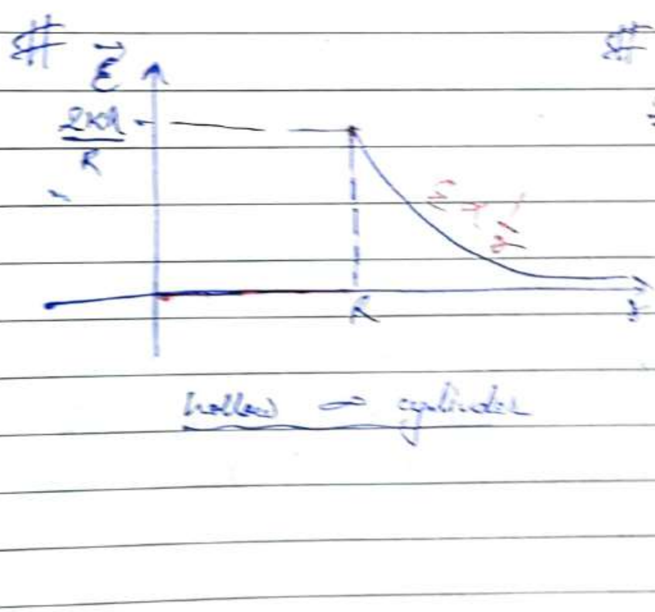
* $\oint_{\text{GS}} = E 2\pi r h$ —— (i)

* $\phi_{\text{GS}} = \dfrac{\rho \pi r^2 h}{\varepsilon_0}$ —— (ii)
(Gauss law)

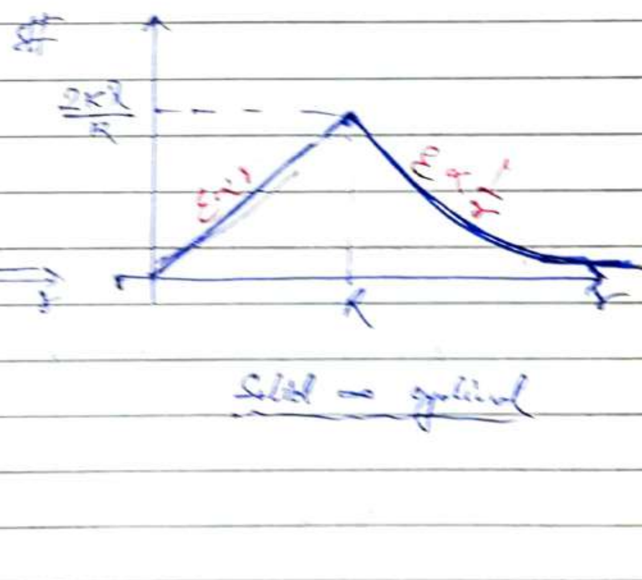
from (i) and (ii)

$\Rightarrow E 2\pi r h = \dfrac{\rho \pi r^2 h}{\varepsilon_0}$

$\Rightarrow \boxed{E = \dfrac{\rho r}{2\varepsilon_0}}$



hollow — cylinder

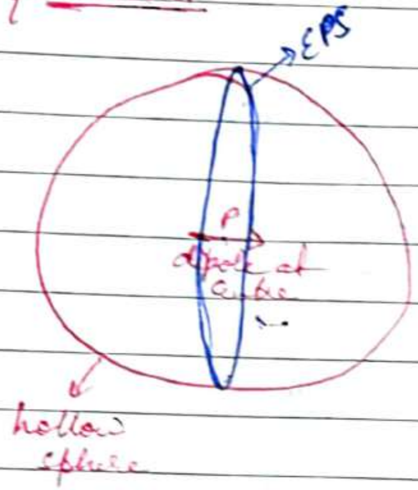


Solid — cylinder

DPP-29
Ex-1 I II
II I 1,5,6,7,8
9,10,14
Ex-2 I 10,20
II 16,15,17,18

$$\frac{Kq}{R^3}\sqrt{[1+3\cos^2\theta]}$$

Questions:-

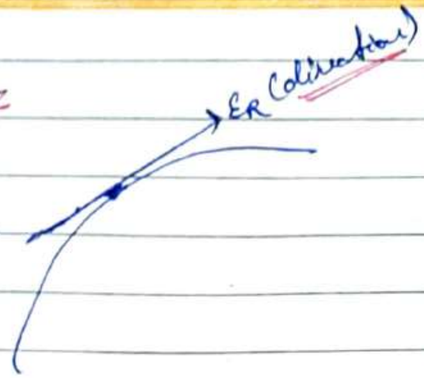


A) Electric field at each point on the sphere is 0.
(B) Sphere ki surface pr kam se kam 2 points aise hai jaha $\vec{E}$ 0 hai.
c) $\vec{E}$ is non-zero at every point ~~not~~ on the sphere
d) Total flux through the sphere is 0.
e) There are ∞ point on sphere at which potential is zero.
f) There is no such points on the sphere at which potential is 0.

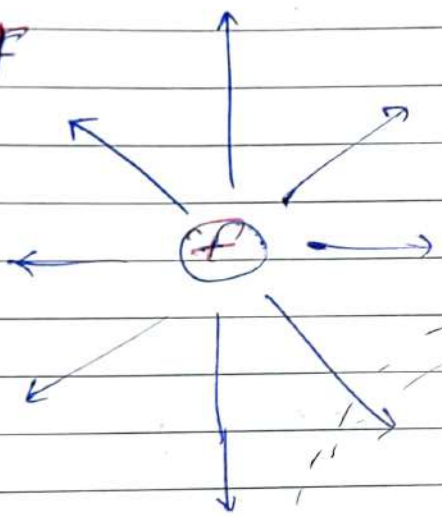
Electric lines of force:-

An imaginary line, the tangent to which at a point gives the dirn of resultant electric field at that point is called electric line of force.

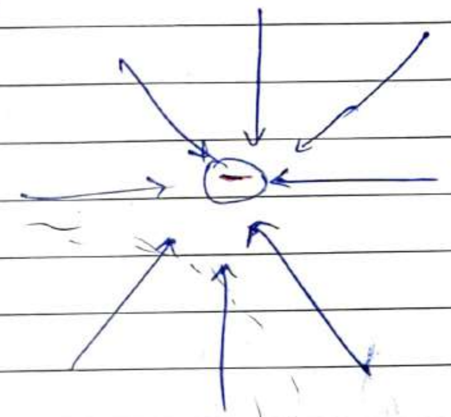
#


E_R (Colination)

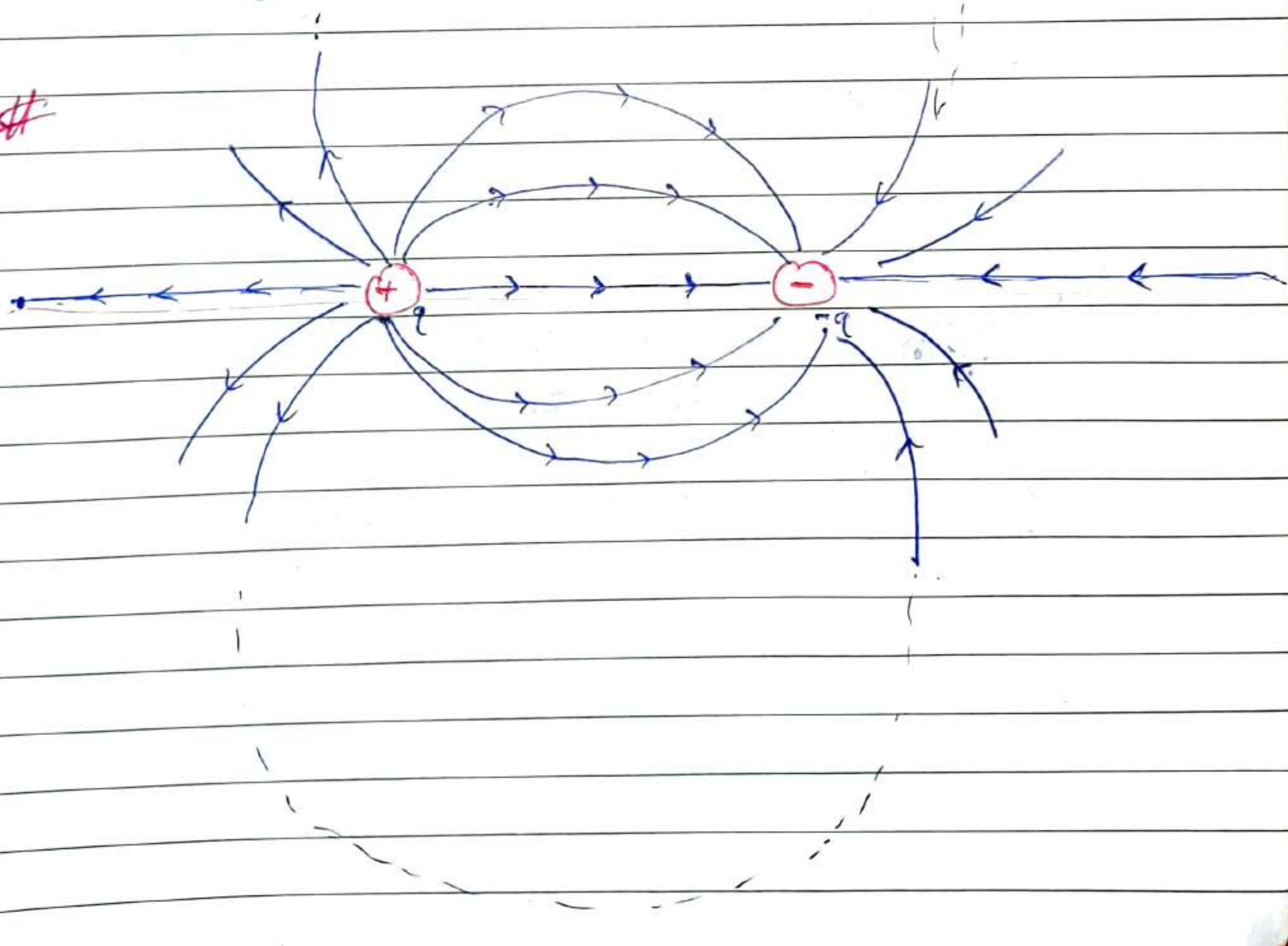
#


+

#


−

#

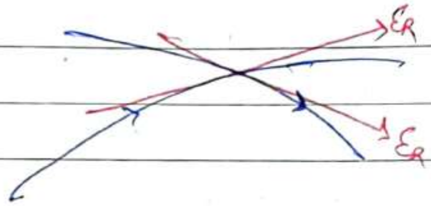

+
q
−
-q

★ discontinuous

NOT POSSIBLE

Electric lines cannot be discontinuous because at discontinuous points its shows many dirn of $\vec{E}$ which is not possible

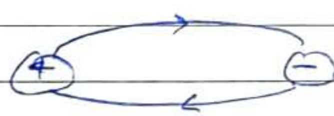
★



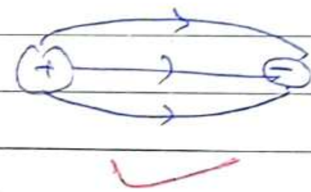
Electric lines cannot intersect each other.

NOT POSSIBLE

★ Electric lines kabhi bhi closed loop nhi banati hai.



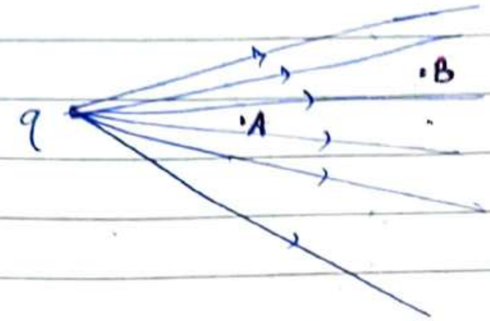
✗ NOT POSSIBLE



Proof:-

$W_{elect} \neq 0$
(Cons. field loop)

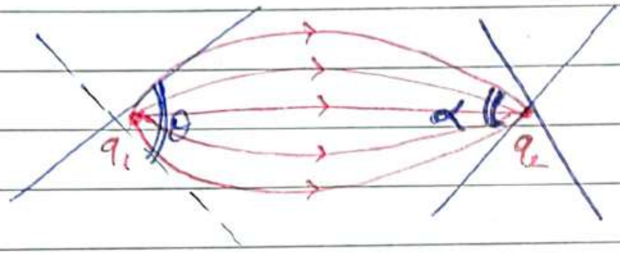
In a closed path, $W_{cons. force} = 0$



$\Rightarrow$ Density of Electric line $\propto |\vec{E}|$

$$\boxed{\Rightarrow |\vec{E_A}| > |\vec{E_B}|}$$

Question:-

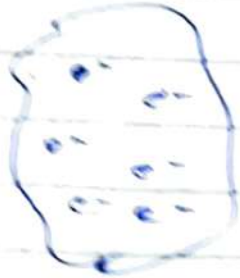


find $\frac{q_1}{q_2}$ in terms of α & θ.

Sol!

$$\frac{q_1}{\varepsilon_0}\left[\frac{1-\cos\theta/2}{2}\right] = \frac{q_2}{\varepsilon_0}\left[\frac{1-\cos\alpha/2}{2}\right]$$

$$\Rightarrow \frac{q_1}{q_2} = \left[\frac{1-\cos\alpha/2}{1-\cos\theta/2}\right]$$



Conductor

(e^- → free to move)

[Total charge = 0]

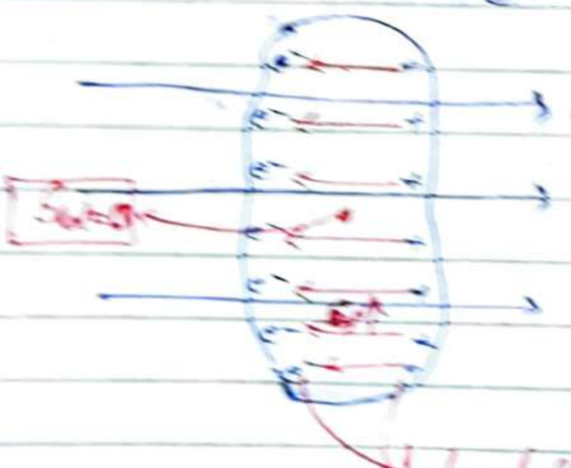
Insulator

(e^- → fixed)

⇒ Properties of Conductors:- (under electrostatic conditions)
→ charges at rest

1) Resultant electric field inside conducting material is always zero.

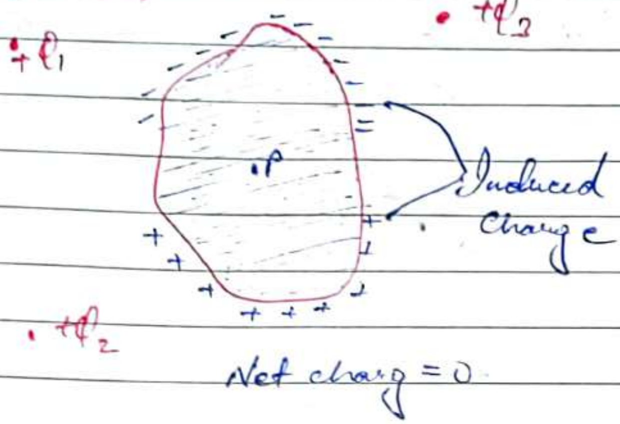
Conductor (Net Charge = 0)



Induced Charges

⇒ Total induced charges = 0

Question:



A) Electric field at P due to q_1 is 0

B) Electric field at P due to q_2 is 0

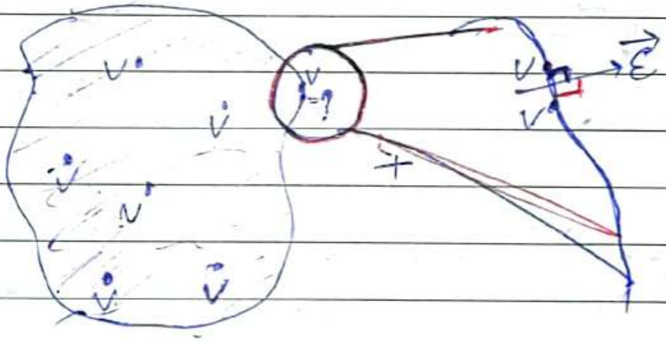
C) ——— ——— q_3 is 0

d) ——— ——— due to induced charge is 0

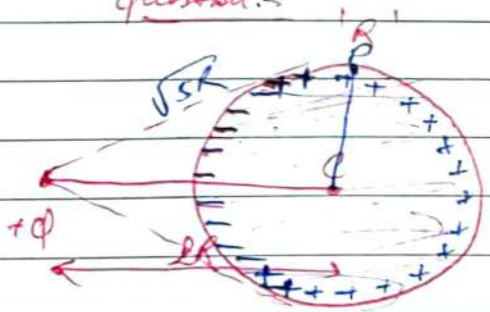
e) NOT (resultant $\vec{E}$ at P = 0)

→ Properties

ii) Potential (net V_P) at each point of a conducting material is same.



Question:



Conducting sphere [Net charge = 0]

find the potential at point P. due to induced charges.

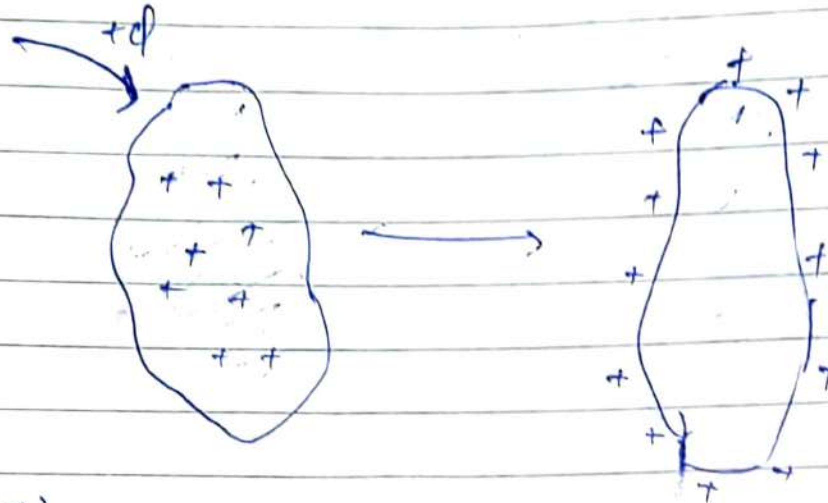
$$V_{C\,(Net)} = V_{\text{due to } q} + V_{\text{due to induced}}$$

$$\Rightarrow V_{C\,(Net)} = \frac{Kq}{2R} + 0$$

$$\therefore V_{P\,(Net)} = \frac{Kq}{2R} = V_{\text{due to } q} + V_{\text{due to induced I.C}}$$

$$\Rightarrow \frac{Kq}{2R} = \frac{Kq}{\sqrt{5}R} + V_{I.C}$$

iii) Kisi conductor ko diya gya charge ya induced charge ya induced charge/s us conductor ki Outermost surface pr aa jata hai.

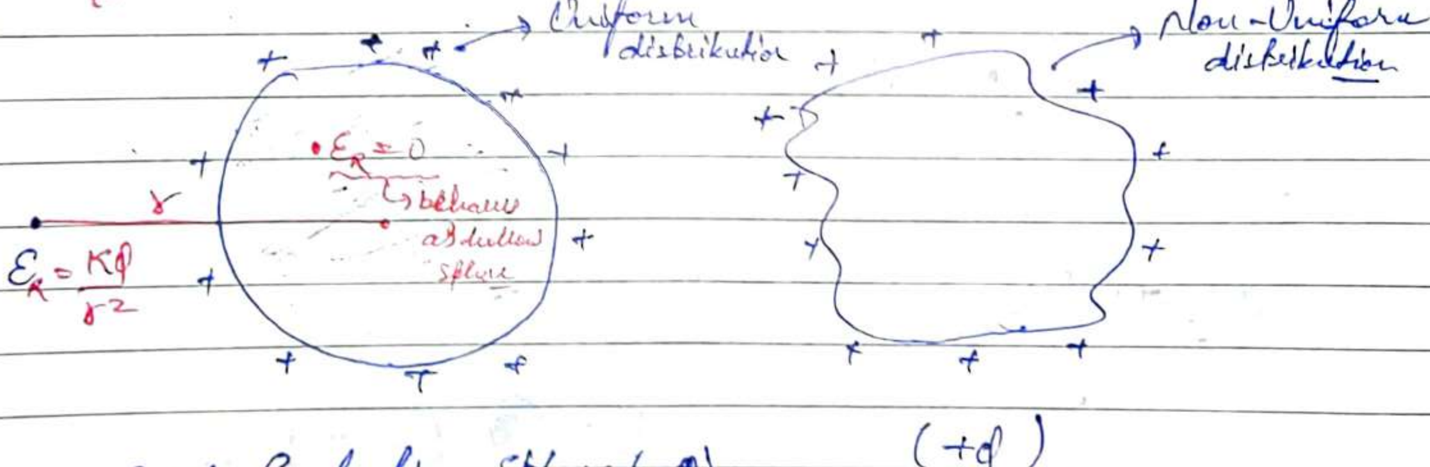


Proof:

$\phi = \oint \vec{E}\cdot d\vec{A} = 0$

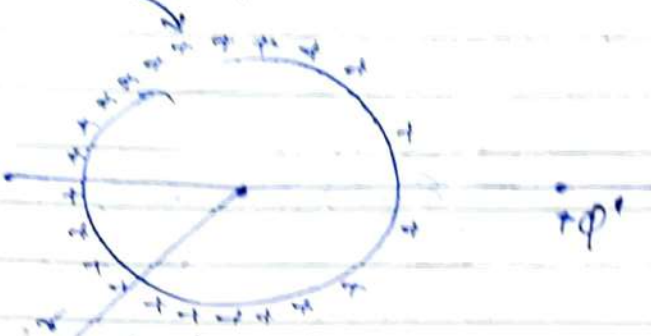
$\phi = \dfrac{q}{\varepsilon_0}$

$\boxed{q = 0}$



Solid Conducting Sphere (+q)   (+q)

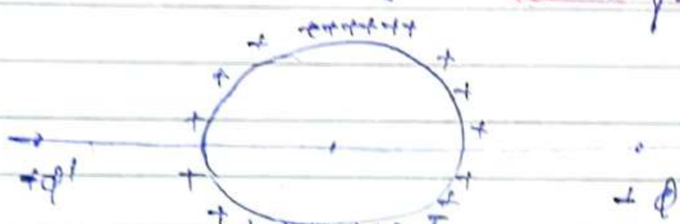
Non-Uniform



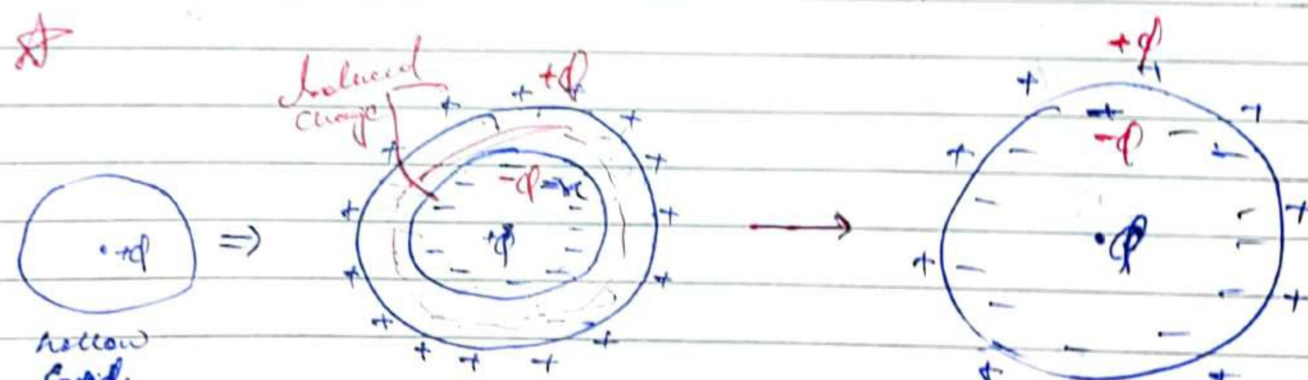
Solid Cond. Sphere
(+q)

$\boxed{E_x \neq \frac{Kq}{r^2}}$

Non Uniform



Solid Cond. Sphere
(+q)

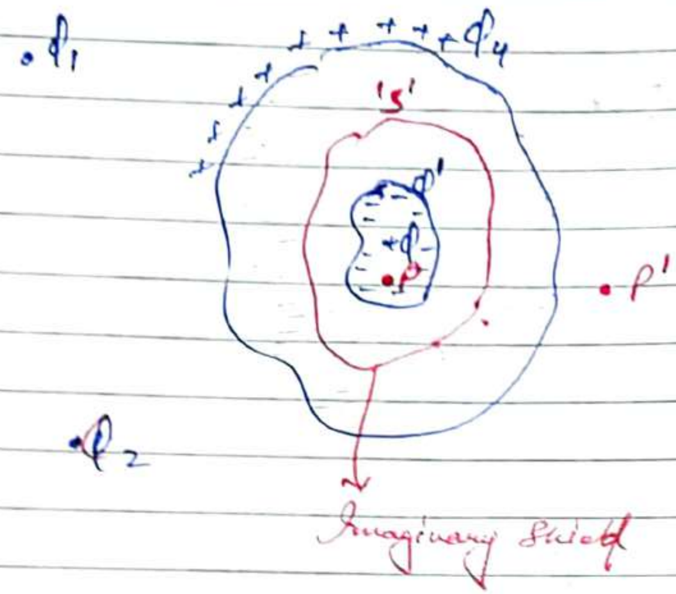


$q = \oint \vec{E} \cdot dA = 0$

$\frac{q+x}{\varepsilon_0} = 0$

$\Rightarrow x = -q$

iv) Electrostatic shielding :-

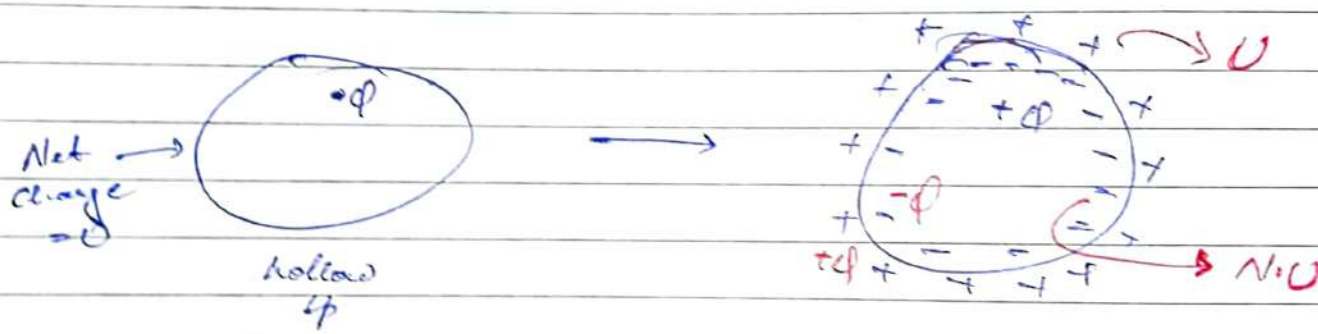


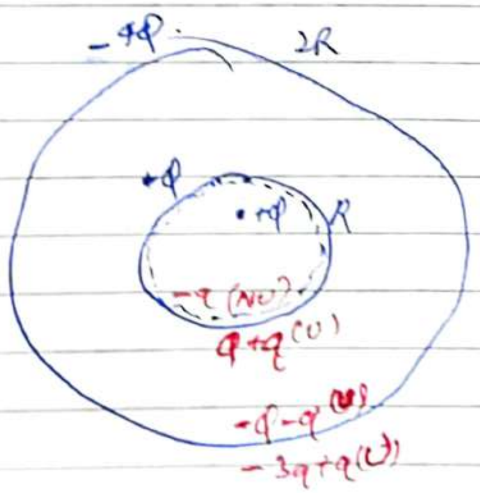
$\vec{E}_{(Net)}$ at point P due to outside charges of S $= 0$

or

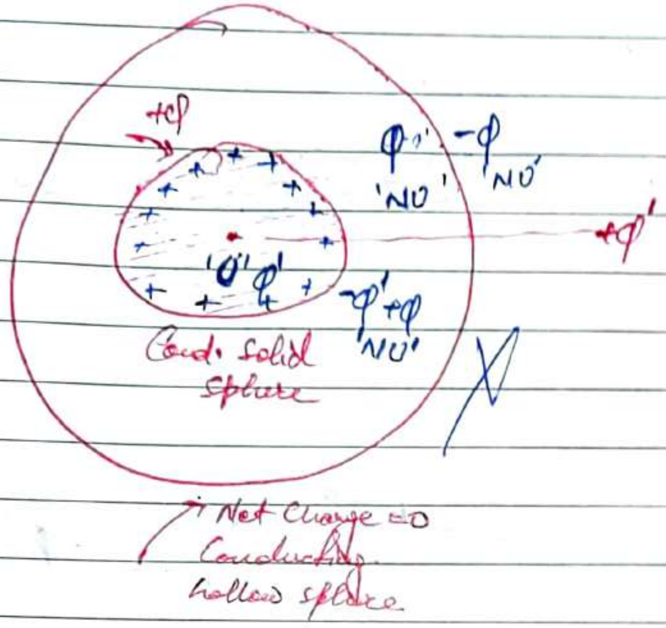
$E_{(Net)}$ at P' due to inside charge of S $= 0$

No Explanation

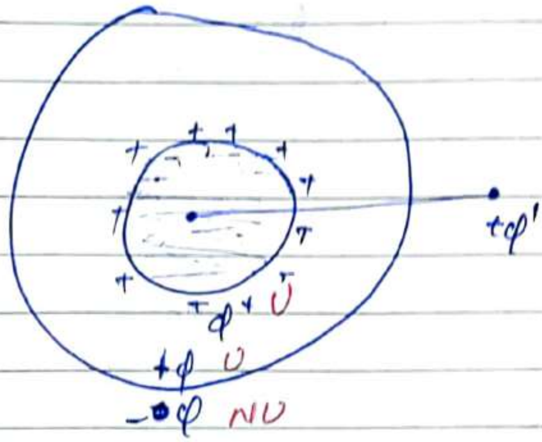




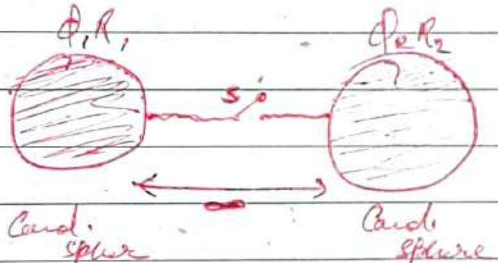
Question:-

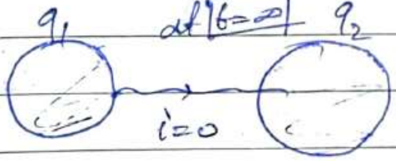


Show the charge on each surface and comment on its distribution.



Question:





i) If switch is closed then find final charge on each sphere.

$$q_1 + q_2 = q_1 + q_2 \quad \text{---} (1)$$

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \quad \text{---} (2)$$

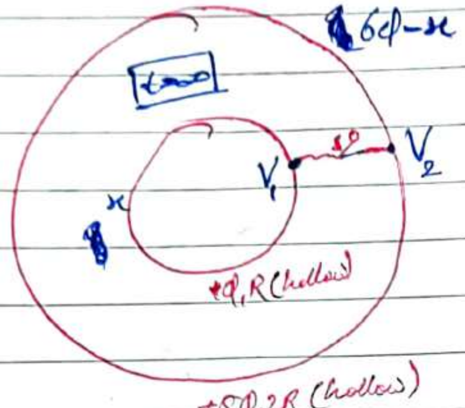
ii) find the heat produced in this process.

$$U_i = \frac{Kq_1^2}{2R_1} + \frac{K}{2}\frac{q_2^2}{R_2} + 0$$

$$U_f = \frac{Kq_1^2}{2R_1} + \frac{Kq_2^2}{2R_2} + 0$$

$\therefore$ Heat produced $= U_i - U_f$

Question:- Repeat previous question

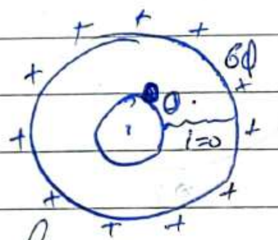


i) $V_1 = V_2$

$$\frac{Kx}{R} + \cancel{\frac{K(6Q-x)}{2R}} = \cancel{\frac{K(6Q-x)}{2R}} + \frac{Kx}{2R}$$

$$\boxed{x = 0}$$

ii)

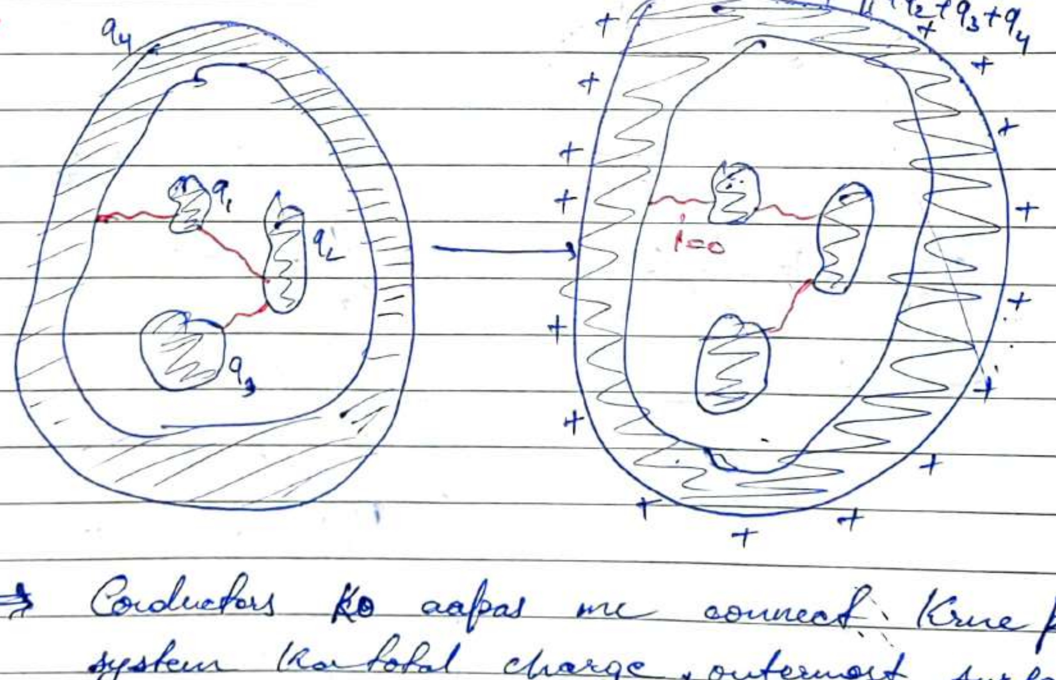


final Condition

$$U_i = \frac{KQ^2}{2R} + \frac{K(5Q)^2}{2(2R)} + Q\,\frac{K5Q}{2R}$$

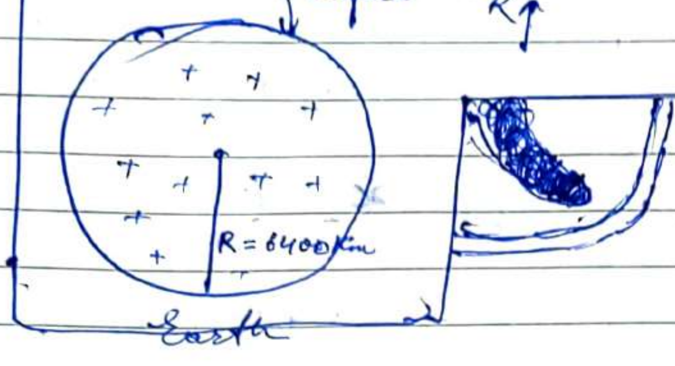
$$U_f = \frac{K(6Q^2)}{2(2R)}$$

∴ Heat produced $= U_i - U_f$

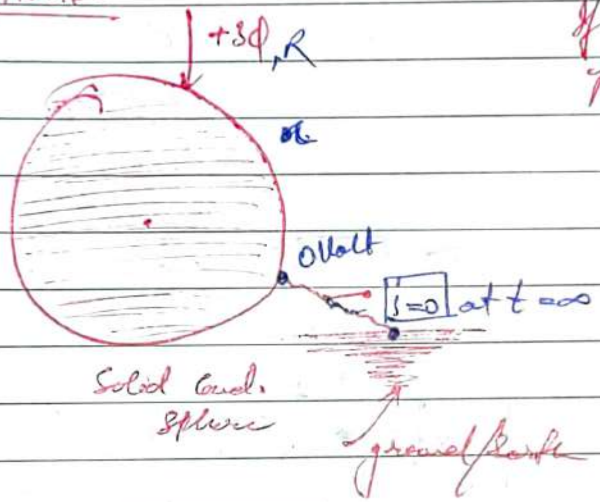


⇒ Conductors ko aapas me connect krne pr, system ka total charge, outermost surface pr aa jata hai.

$$V_{surface} = \frac{Kq}{R\uparrow} \approx m\,Volt \text{ or } \mu\,Volt \approx 0\,Volt$$



Question:-



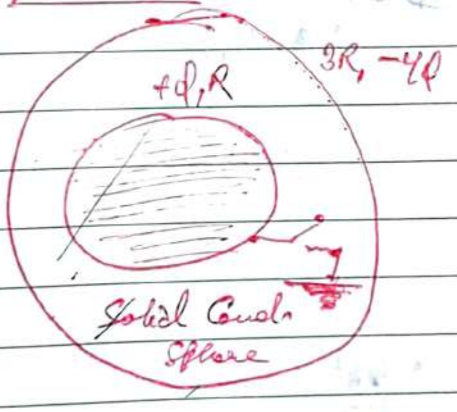
If switch is closed then find the final charge on the sphere

Sol:

$$\frac{Kx}{R} = 0$$

$$\boxed{x = 0\,C}$$

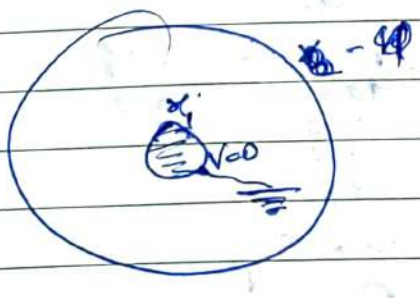
Question:-



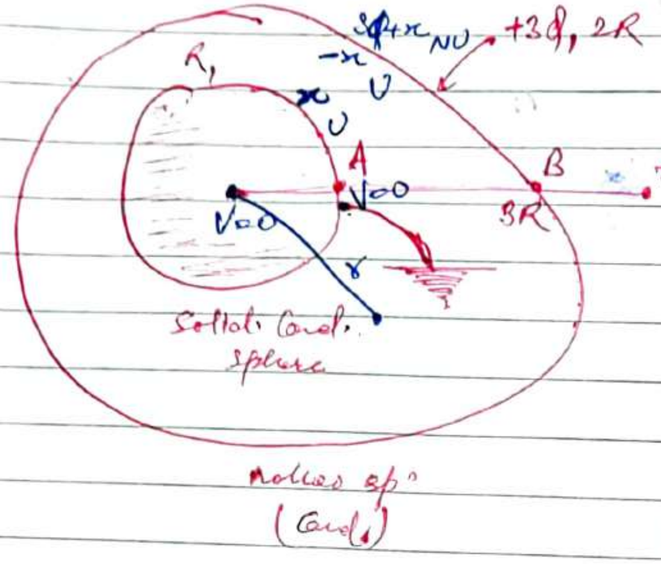
If switch is closed then find the final charge on each sphere

$$\frac{Kx_1}{R} + \frac{K(-4q)}{3R} = 0$$

$$\boxed{x = \frac{4q}{3}}$$



Question:-



i) find the final charge on solid sphere.

$$V_c = \frac{Kx}{R} + \frac{K3\phi}{2R} + \frac{K\phi}{3R} = 0$$

$$\Rightarrow \frac{6Kx + 9K\phi + 2K\phi}{6R} = 0$$

$$\Rightarrow 6x = -11\phi$$

$$\Rightarrow x = -\frac{11\phi}{6}$$

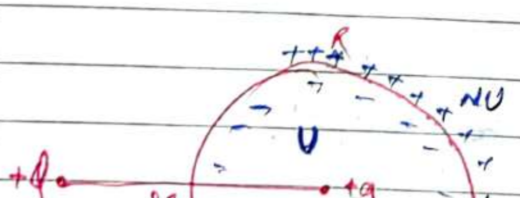
ii) find the V_{diff} between A & B

$$dV = -\int E \cdot dr$$

$$\left|\int dV\right| = \left|-\int_R^{3R} \frac{K\left(\frac{11\phi}{6}\right)}{r^2}\, dr\right|$$

$$V_{diff} = \left[\frac{-3K\,11\phi}{6(3r)}\right]_R^{3R}$$

Question:-



i) find the force on $+\phi$ charge due to conducting shell.

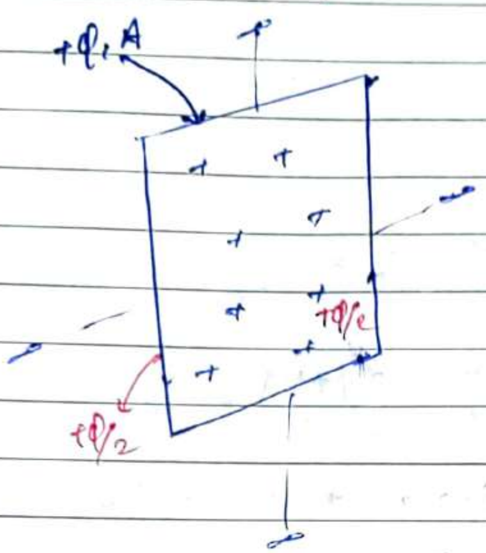
Sol:-

$$F_{+\phi} = F_{+q \text{ of the shell on the } +q} = \frac{Kq\phi}{9R^2}$$

ii) find resultant force on small q.

Ans - 0

Electric field due to conducting infinite sheet.



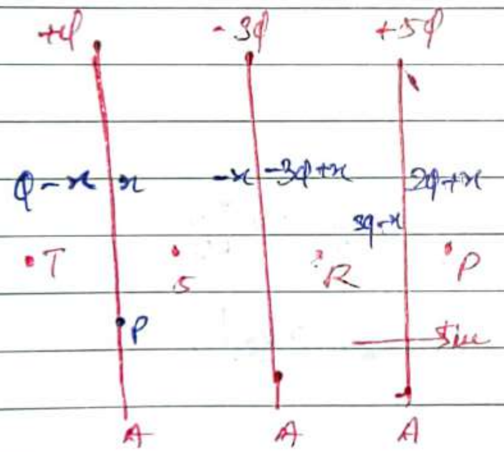
$$E_P = \frac{q/2}{2A\epsilon_0} + \frac{q/2}{2A\epsilon_0} = \frac{q}{2A\epsilon_0}$$

Same as insulting sheet.

$$\boxed{\sigma' = \frac{q}{2A} \Rightarrow \frac{\sigma}{\epsilon_0}}$$

Question:-

i) find E_{net} at the given points.



$$E_P = \frac{5q - 3q + q}{2A\epsilon_0}$$

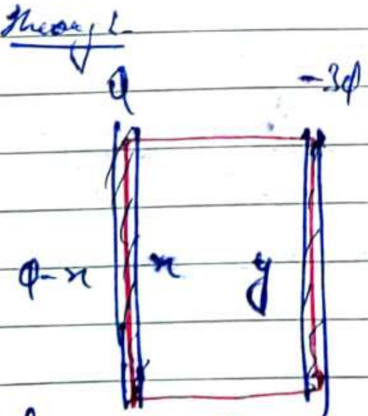
$$E_R = \frac{-5q - 3q + q}{2A\epsilon_0}$$

$$E_S = \frac{-5q + 3q + q}{2A\epsilon_0}$$

$$E_T = \frac{-q + 3q - 5q}{2A\epsilon_0}$$

ii) find the charge on each plate.

Theory:-



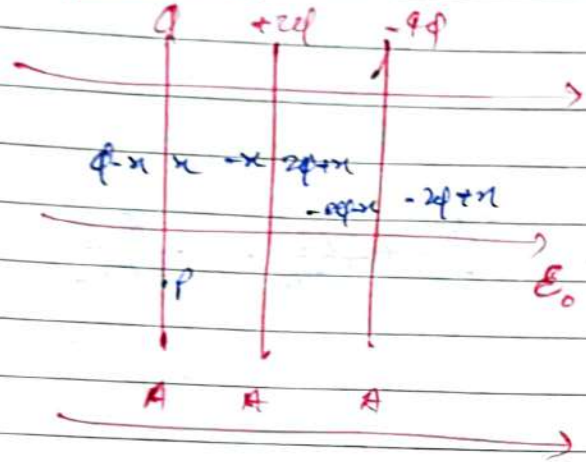
$$q = \frac{x+y}{\epsilon_0} = \oint \vec{E} \cdot \vec{dA} = 0$$

$$\boxed{x = -y}$$

$$E_{Ross\,(P)} = 0$$

$$\Rightarrow \frac{q - x}{2A\epsilon_0} = \frac{2q + x}{2A\epsilon_0} \Rightarrow \boxed{x = \frac{-q}{2}}$$

Question!-



find the charge on each side of each plate.

$E_{Res.}$ at P $=0$

$\Rightarrow \frac{q-x}{2A\epsilon_0}+E_0=\frac{-2q+x}{2A\epsilon_0}$

$\Rightarrow \frac{q-x}{2A\epsilon_0}-\left(\frac{2q+x}{-2A\epsilon_0}\right)=-E_0$

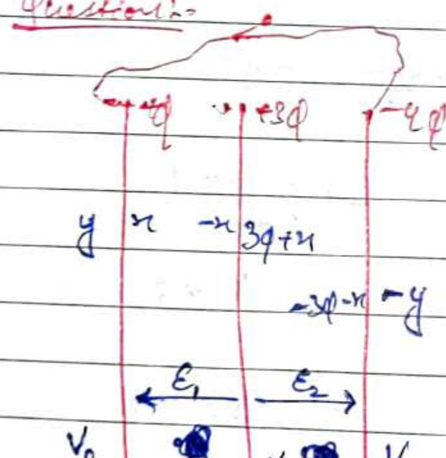
$\Rightarrow q-x+2q-x=-\epsilon_0^2 2A$

$\Rightarrow 3q-2x=\epsilon_0^2 2A$

$\Rightarrow 2x=3q-\epsilon_0^2 A\times 2$

$\Rightarrow x=\frac{3q}{2}-\epsilon_0^2 A$

Question!-



find charge on each plate

i) $y=-y \Rightarrow \boxed{y=0}$

ii) let $\boxed{V>V_0}$,

$E_1 d=E_2 2d$

$\frac{E_1}{}=2E_2$

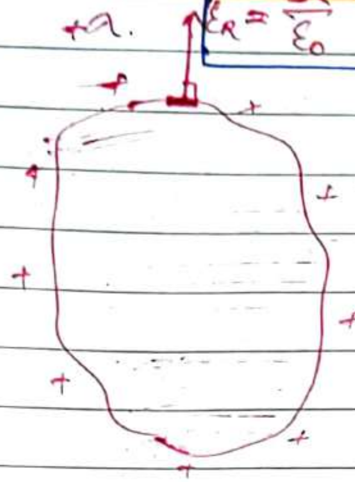
$\frac{-x}{2A\epsilon_0}-\frac{x}{2A\epsilon_0}=2\left(\frac{3q+x}{2A\epsilon_0}-\frac{(-3q-x)}{2A\epsilon_0}\right)$

$\Rightarrow -\cancel{2}x=\cancel{2}(6q+2x)$

$\Rightarrow -6q=-3x$

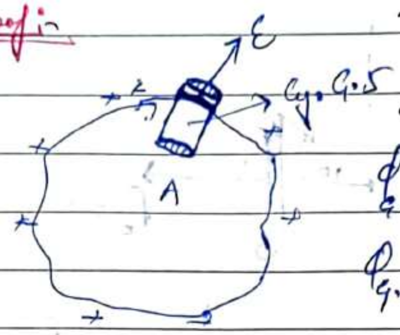
$\Rightarrow \boxed{x=-2q}$

#



$$E_R = \frac{\sigma}{\varepsilon_0}$$

Proof :-



$$\phi_{G.S} = \frac{\sigma A}{\varepsilon_0} \quad \text{(i)}$$

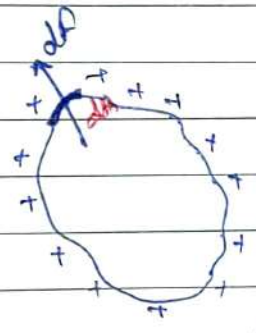
$$\phi_{G.S} = \int \vec{E} \cdot dA$$

$$= E \cdot A \quad \text{(ii)}$$

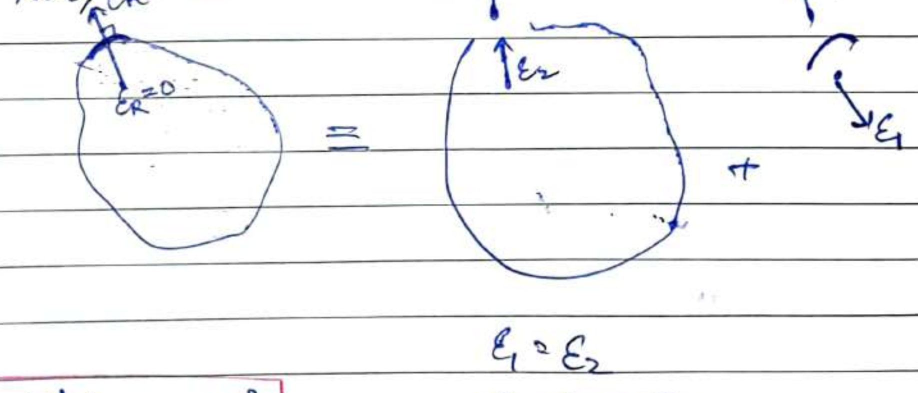
from (i) & (ii),

$$E_R = \frac{\sigma}{\varepsilon_0}$$

Electrostatic Pressure :-



$$P = \frac{dF}{dA}$$ → 'Baki' bache charges ke karan element pe lagne wala force.



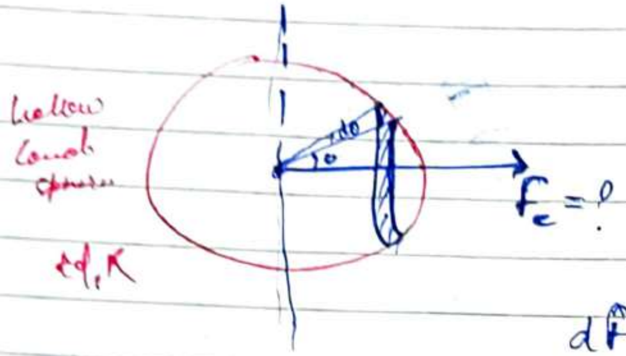
$$E_1 = E_2$$

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$$

$$2E_2 = \frac{\sigma}{\varepsilon_0} \Rightarrow E_2 = \frac{\sigma}{2\varepsilon_0}$$

$$\therefore \boxed{P = \frac{(\sigma dA)E_2}{dA} = \frac{\sigma^2}{2\varepsilon_0}}$$

Question 2



Find the Electrostatic force on the left half due to Right half.

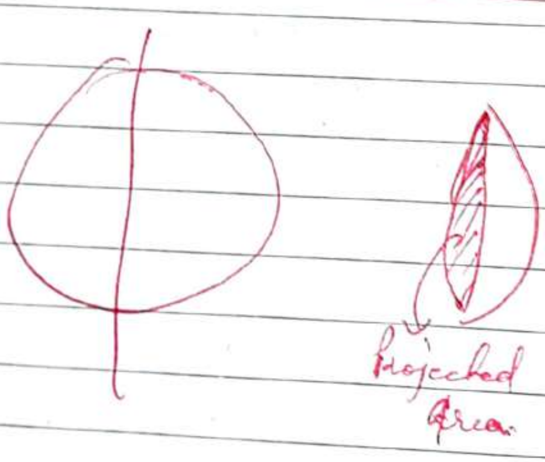
$d\vec{F} = P\,dA$

$$F = \int_0^{\pi/2} \left(\frac{\sigma^2}{2\epsilon_0}\, 2\pi R\sin\theta\, R\,d\theta\right)\cos\theta \times 2$$

$$\sigma = \frac{q}{4\pi R^2}$$

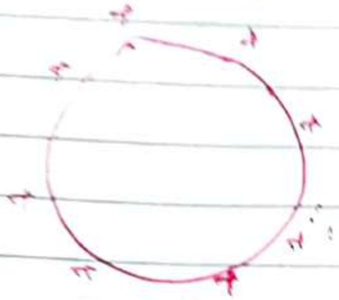
$$F = \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$$

☆ $\boxed{F_{elec} = \text{Elec Pressure} \times \text{Projected Area}}$



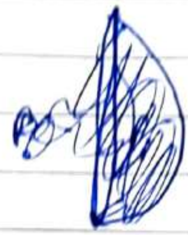
Ex-1 all
Ex-2 I II III IV all

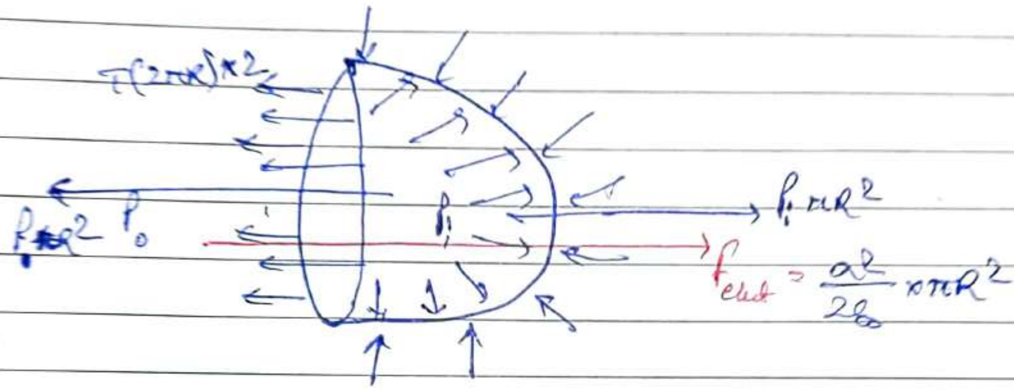
Question:- q, R, T



Find the excess pressure
(Pressure inside − Pressure outside)

Sol^n





$$\therefore P_1 \pi R^2 + \frac{\sigma^2}{2\varepsilon_0} \pi R^2 = T(2\pi R) \times 2 \times P_0 \pi R^2$$

$$\boxed{P_1 - P_0 = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}}$$

Electrostatic Potential and Capacitance

Electric potential difference: The amount of W.D. in moving a unit +ve ~~ch~~. Test charge from one point to the other in against of electrostatic force without any $\vec{acc}$.

W external agent

q_0 (+ve. test charge)

A B

$$\boxed{V_A - V_B = \frac{W_{external\ B \to A}}{q_0}}$$

* The test charge should be moved very slowly (No change in speed or K.E of q_0)

* ~~ka~~ It is a Scalar quantity and it's unit is J/C.

Electrical potential: The amount of work done in bringing a unit +ve test charge from Infinity to that point, in against of electrostatic force without any $\vec{acc}$.

$$\text{Electrostatic Potential } (V) = \frac{\text{Work done } (W)}{\text{charge } (q_0)}$$

* It's a Scalar quantity and it S.I unit is Volt, $\boxed{1V = 1J/C}$
* let $(V_\infty = 0)$

q_0 (+ve Test charge)

A $B\infty$

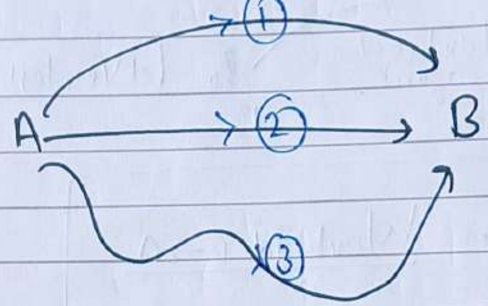
$$V_A - V_\infty = \frac{W_{\infty \to A}}{q_0}$$

$$\boxed{\text{Potential at A } (V_A) = \frac{W_{\infty \to A}}{q_0}}$$

Some Imp. points :-

i) $W_{external}$ agent Shall be Considered
(This work is Against electric force)

ii) This Work is path Independent.
(Reason : Electrostatic force is Conservative.)



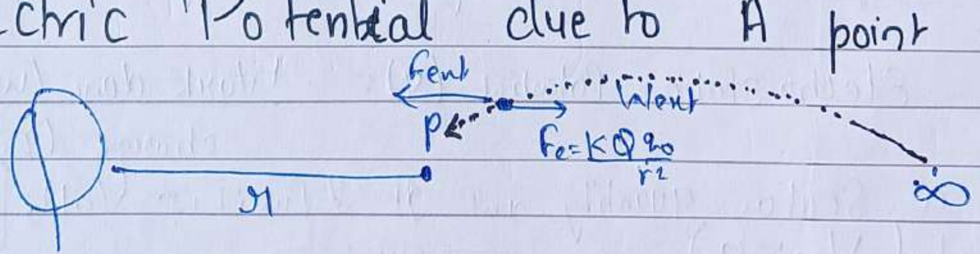
$$W_1 = W_2 = W_3$$

* Work done by the Conservative force in a round trip is Zero.

$$\boxed{W_{A \to B \to A} = 0}$$

iii) Test charge should be Very Small.
(So that it has no electric field.)

Electric Potential due to A point charge Q



We will Calculate. $W_{ext}\ P \to \infty$

$$\therefore \quad W_{ext\ \infty \to P} = -W_{P \to \infty}$$

$$F_{ext} = F_{electric} = \frac{kQq_0}{r^2}$$

$$W.D_{p\to\infty} = \int F \cdot dr = \int \frac{k \cdot Q q_0}{r^2} dr \cos 180$$

$$= -kQq_0 \int_r^{\infty} \frac{1}{r^2} dr$$

$$= -kQq_0 \left[\frac{-1}{r}\right]_r^{\infty} = -kQq_0 \left[\cancel{\frac{-1}{\infty}}^{0} + \frac{1}{r}\right]$$

$$= -\frac{kQq_0}{r}$$

$$W_{\infty\to p} = -W_{p\to\infty}$$

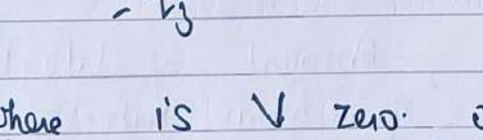
$$= -\left(-\frac{kQq_0}{r}\right) = \frac{kQq_0}{r}$$

$$V_p = \frac{kQq_0}{r \cdot q_0} = \frac{kQ}{r}$$

$$\boxed{V_p = \frac{kQ}{r}}$$

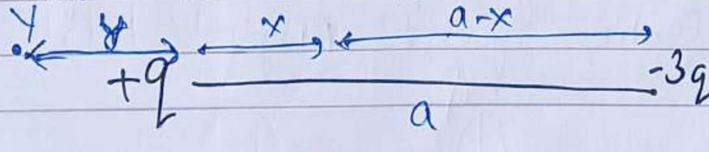
* $\boxed{\text{charge } Q \text{ is taken with sign}}$

if



$$\boxed{V_p = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} - \frac{kq_3}{r_3}}$$

(Q) where is V zero on the line joining two charges



$$V_x = \frac{kq}{x} - \frac{3kq}{(a-x)}$$

$$\boxed{V_x = 0}$$

$$\frac{3kq}{a-x} = \frac{kq}{x}$$

$$3x = a - x$$

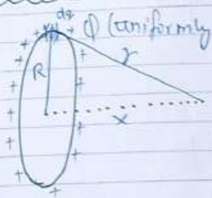
$$\boxed{x = a/4}$$

$$V_y = \frac{kq}{y} - \frac{3kq}{(a+y)}$$

$$\frac{3kq}{a+y} = \frac{kq}{y} \qquad \boxed{V_y = 0}$$

$$3y = a+y$$

$$\boxed{y = \frac{a}{2}}$$



Q (uniformly distributed on a Ring)

$$r = \sqrt{R^2+x^2}$$

dq is a point charge

$$dV = \frac{k\,dq}{r}$$

$$\int dV = \int \frac{k}{\sqrt{R^2+x^2}}\,dq$$

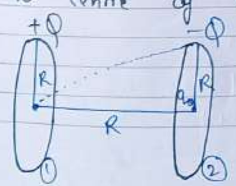
$$V = \frac{k}{\sqrt{R^2+x^2}}\int dq = \frac{kQ}{\sqrt{R^2+x^2}}$$

$$\boxed{V_{axis} = \frac{kQ}{\sqrt{R^2+x^2}}}$$

$$\boxed{V_{centre} = \frac{kQ}{R}}$$ put $x=0$

* potential is maximum at Centre.

Q3 Given $q_0 << Q$, Amount of Work done in moving q_0 from Centre of 2nd Ring to Centre of 1st Ring.



$$V_1 = \frac{kQ}{R} - \frac{kQ}{\sqrt{R^2+R^2}} = \frac{kQ}{R} - \frac{kQ}{\sqrt{2}R}$$

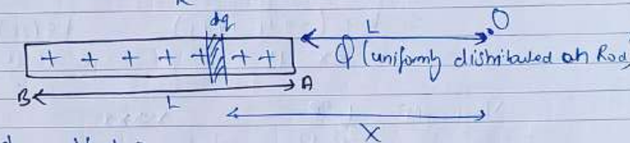
$$V_2 = \frac{-kQ}{R} + \frac{kQ}{\sqrt{2}R} = \frac{-kQ}{R} + \frac{kQ}{\sqrt{2}R}$$

$$V_1 - V_2 = \frac{W.D_{2\to1}}{q_0}$$

$$\frac{2kQ}{R} - \frac{2kQ}{\sqrt{2}R} = \frac{WD_{2\to1}}{q_0}$$

$$(2-\sqrt{2})\frac{kQq_0}{R} = W_{2\to1}$$

Q11T



Q (uniformly distributed on Rod)

Find V at O.

$$dV = \frac{k\,dq}{x}$$

$$V = k\int_{L}^{2L} \frac{dq}{x}$$

$$V = k\int_{L}^{2L} \frac{\lambda\,dx}{x}$$

$$V = k\lambda\left[\log|x|\right]_{L}^{2L}$$

$$V = \frac{Q}{L}k\left(\log 2L - \log L\right)$$

$$V = \frac{kQ}{L}\log 2$$

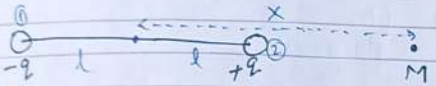
$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\log 2}$$

$$\lambda = \frac{dq}{dx}$$

$$\boxed{\lambda = \frac{Q}{L}}$$

Potential due to an Electric Dipole

i) on the axis of dipole {distance x from centre of dipole}



$$V_{net\ on\ M} = V_1 + V_2$$
$$= \frac{-Kq}{(x+l)} + \frac{Kq}{(x-l)}$$
$$= \frac{Kq(-x+l+x+l)}{x^2-l^2} = \frac{Kq(2l)}{x^2-l^2}$$

$$V_{net\ on\ M} = \frac{KP}{x^2-l^2}$$

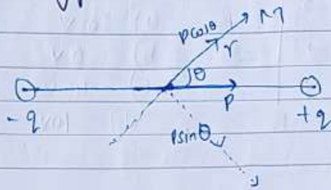
Generally $x >> l$ $\therefore V_{net} = \frac{KP}{x^2}$

ii) on the ⊥ bisector of dipole (dist 'x' from Centre)

$$V_M = V_1 + V_2$$
$$= \frac{-Kq}{\sqrt{x^2+l^2}} + \frac{Kq}{\sqrt{x^2+l^2}} = 0$$

$$\boxed{V_{\perp bisector} = 0}$$

(iii) at any point (r, θ)

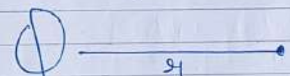


$$V_M = \underset{axis}{V_{p\cos\theta}} + \underset{\perp bisector}{V_{p\sin\theta}}$$
$$= \frac{KP\cos\theta}{r^2} + 0$$

$$\boxed{V_M = \frac{1}{4\pi\epsilon_0}\frac{P\cos\theta}{r^2}}$$

Relation b/w Electric field & Potential.

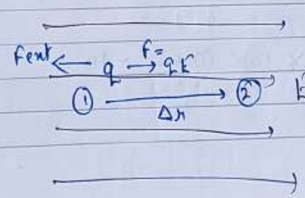
$$\left[\Delta V_{1\to 2} = -\vec{E}\cdot\vec{\Delta r}\right]$$



$$\vec{E} = \frac{KQ}{r^2} \quad (\text{vector})$$
$$V = \frac{KQ}{r} \quad (\text{Scalar})$$

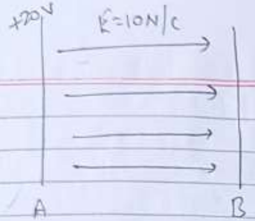
" Electric Potential Always decreases in the direction of E. Field"

Case 1: when $\vec{E}$ is uniform



$$F_{ext} = -q\vec{E}$$
$$W_{12} = -q\vec{E}\cdot\vec{\Delta r}$$
$$V_2 - V_1 = -\vec{E}\cdot\vec{\Delta r}$$
$$\boxed{|\Delta V| = \vec{E}\cdot\vec{\Delta r}}$$

Q



$V_B = ?$

a) 20 V
b) 0 V
c) 40 V
d) 10 V

$$|\Delta V| = E \cdot \Delta r$$
$$= 10 \times 2 \times \cos 0^\circ$$
$$|\Delta V| = 20\,V$$

∴ but potential always decreases in the direction of electric field.

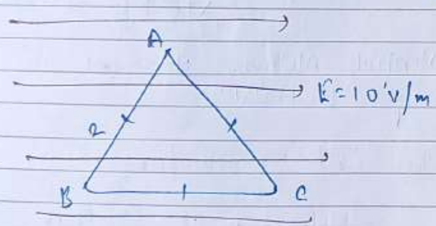
Hence $V_B = 0$

Q

$E = 20\,V/m$

i) $V_B - V_A = -20\,V$
ii) $V_A - V_D = 0\,V$
iii) $V_D - V_C = 20\,V$
iv) $V_C - V_A = -\sqrt{2} \times 20 \times \cos 45^\circ = -\frac{20}{\sqrt{2}} = -10\sqrt{2}\,V$

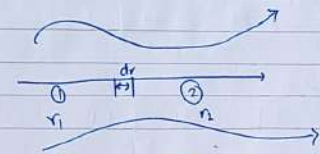
Q



i) $V_A - V_B = |\Delta V| = 2 \times 10 \times \cos 120^\circ = -10$

ii) $V_B - V_C = 20\,V$

Case: II when $\vec{E}$ field is non-uniform



$$dV = -\vec{E} \cdot d\vec{r}$$

$$\Delta V = \int_{r_1}^{r_2} dV = -\int_{r_1}^{r_2} \vec{E} \cdot d\vec{r}$$

$$E = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$
$$r = x\hat{i} + y\hat{j} + z\hat{k}$$
$$dr = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$dV = -(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$dV = -E_x\,dx - E_y\,dy - E_z\,dz$$

partially diff. wrt x ($y, z \rightarrow$ Constant hence $dy = 0$, $dz = 0$)

$$\frac{\partial V}{\partial x} = -E_x$$

Similarly $\frac{\partial V}{\partial y} = -E_y$, $\frac{\partial V}{\partial z} = -E_z$

$$\therefore \vec{E} = \left(-\frac{\partial V}{\partial x}\right)\hat{i} - \left(\frac{\partial V}{\partial y}\right)\hat{j} - \left(\frac{\partial V}{\partial z}\right)\hat{k}$$

$$|\vec{E}| = \sqrt{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2}$$

Q $V = 2x - y + z$

Find $\vec{E}$ at $(1,0,1)$ & also $|\vec{E}|$

$E_x = -\frac{\partial V}{\partial x} = 2$

$E_y = -\frac{\partial V}{\partial y} = -1 \qquad \vec{E} = 2\hat{i} - \hat{j} + \hat{k}$

$E_z = -\frac{\partial V}{\partial z} = 1 \qquad |\vec{E}| = \sqrt{6}$

Q $V = x^2y + y^2z$

Find $\vec{E}$ at $(1,1,1)$ & also $|\vec{E}|$

$E_x = 2xy = \left(-\frac{\partial V}{\partial x}\right) \qquad E_y = \left(-\frac{\partial V}{\partial y}\right) = x^2 + 2yz$

$E_z = y^2 = \left(-\frac{\partial V}{\partial z}\right)$

$\vec{E} = (2xy)\hat{i} + (x^2+2yz)\hat{j} + (y^2)\hat{k}$

at $(1,1,1)$

$\vec{E} = 2\hat{i} + 3\hat{j} + \hat{k}$

$|\vec{E}| = \sqrt{4+9+1} = \sqrt{14}$

Q $V = x^2y + y^2z + z^2x$

Find $\vec{E}$ at $(1,0,1)$ & $|E|$

$E_x = 2xy + z^2 \qquad E_y = x^2 + 2yz$

$E_z = y^2 + 2zx$

$\vec{E} = (2xy+z^2)\hat{i} + (x^2+2yz)\hat{j} + (y^2+2zx)\hat{k}$

$\vec{E} = \hat{i} + \hat{j} + \hat{k} \qquad |E| = \sqrt{3}$

Q4 $\vec{E} = x\hat{i} - 2y\hat{j} + z\hat{k}$

find ΔV b/w $A(0,2,4)$ & $B(2,1,0)$

$\Delta V = -\int_A^B \vec{E}\cdot d\vec{r}$

$\Delta V = -\int_A^B (x\hat{i} - 2y\hat{j} + z\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$\Delta V = \int_A^B (x\,dx - 2y\,dy + z\,dz\,\hat{k})$

$\Delta V = -\int_A^B x\,dx + 2\int_A^B y\,dy + -\int_A^B z\,dz$

$\Delta V = -\left[\frac{x^2}{2} - \frac{2y^2}{2} + \frac{z^2}{2}\right]_{(0,2,4)}^{(2,1,0)}$

$\Delta V = \left(\frac{4}{2} - 1 + 0\right) - (0 - 4 + 8)$

$\Delta V = -(1-4) = +3V$

$V_B - V_A = 3V$

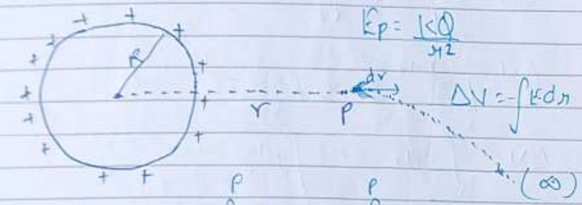
Electrostatic Potential due to charged sphere

Spheres
- Hollow sphere (Thin shell)
  - Conducting (Metallic)
  - Non Conducting (Dielectric / Insulating)
- Solid sphere
  - Conducting
  - Non-Conducting → 2nd Case: Charge in whole Vol.

1st Case: Charge Resides on Surface

Case 1: Potential due to hollow (Conducting + Non Conducting) or Solid conducting Sphere at 'r' dist. from Centre of Sphere. [Outside & inside]

1) outside $(r > R)$



$E_p = \frac{KQ}{r^2}$

$\Delta V = -\int E\,dr$

$V_p - V_\infty = -\int_\infty^P E \cdot dr = -\int_\infty^P \frac{KQ}{r^2}\,dr \cos 0$

$V_p = -KQ \int_\infty^r \frac{dr}{r^2}$

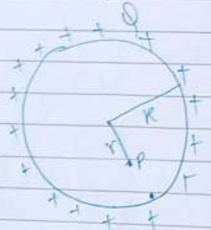
$V_p = +KQ \left[\frac{1}{r}\right]_\infty^r$

$\boxed{V_p = \frac{KQ}{r}}$ $\boxed{E_p = \frac{KQ}{r^2}}$

$\boxed{V_{surface} = \frac{KQ}{R}}$

ii) inside:

$r < R$



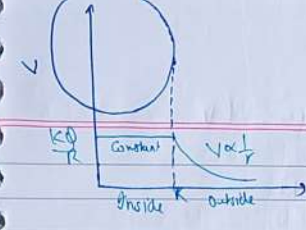
$\boxed{\vec{E_p} = 0}$

$\Delta V =$

$V_p - V_{surface} = -\int E\,dr$

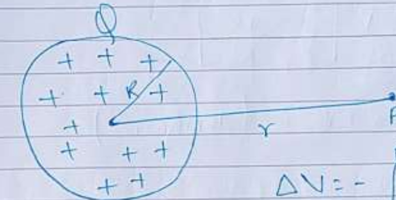
$V_p - V_{surface} = 0$

$\boxed{V_p = \frac{KQ}{R}}$



Case 2: Potential due to Solid Non conducting / Dielectric Sphere (R) at distance 'r' from Centre of Sphere (outside & inside).

1) Outside: $r > R$



$E_p = \frac{KQ}{r^2}$

$\Delta V = -\int_\infty^P E \cdot dr$

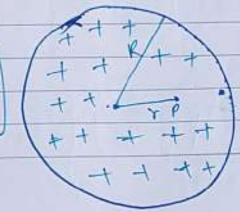
$\Delta V = -\int_\infty^r \frac{KQ}{r^2}\,dr \cos 0$

$\Delta V_p = -KQ \int_\infty^r \frac{1}{r^2}\,dr = -KQ\left[-\frac{1}{r}\right]_\infty^r$

$V_p - \cancel{V_\infty}^{0} = \boxed{\Delta V_p = \frac{KQ}{r}}$ $\therefore \boxed{V_p = \frac{KQ}{r}}$

ii) Inside: $r < R$

$\boxed{E_p = \frac{KQr}{R^3}}$



$\Delta V = -\int_S^P E\,dr$

$V_p - V_S = -\int_S^P \frac{KQ}{R^3} r\,dr \cos 0$

$V_p - V_S = -\frac{KQ}{R^3}\left[\frac{r^2}{2}\right]_R^r$

$$V_p - \frac{KQ}{R} = \frac{-KQ}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$V_p - \frac{KQ}{R} = \frac{-KQ r^2}{2R^3} + \frac{KQ R^2}{2R^3}$$

$$V_p = \frac{KQ}{R} + \frac{KQ}{2R} - \frac{KQ}{2R\cdot R^2} r^2$$

$$V_p = \frac{3KQ}{2R} - \frac{KQ}{2R}\cdot\frac{r^2}{R^2}$$

$$V_p = \frac{KQ}{2R}\left(\frac{3R^2 - r^2}{R^2}\right)$$

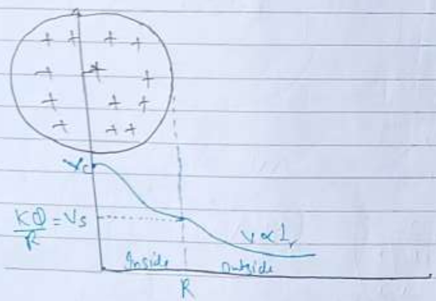
$$\boxed{V_p = \frac{KQ}{R}\left[\frac{3}{2} - \frac{r^2}{R^2}\right]}$$

$V_p \propto -r^2$ (decreases)

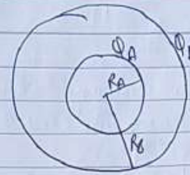
V at Centre $(r=0)$?

$$V_c = \frac{KQ}{R}\left[\frac{3}{2}\right] = \frac{3}{2}\frac{KQ}{R}$$

$$\therefore \boxed{V_c = \frac{3}{2}V_s}$$



Q1 Two Concentric Shells



Find Potential at the Surface of shell A & shell B.

$V_{A_s} = ?$ $\quad V_{B_s} = ?$

$$V_{A_s} = \underset{\substack{\text{due to } Q_A \\ \text{(Surface point)}}}{V_A} + \underset{\substack{\text{due to } Q_B \\ \text{(inside point)}}}{V_A}$$

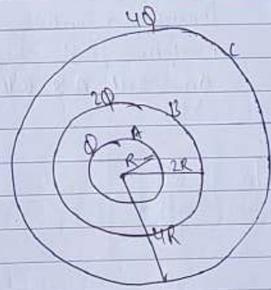
$$\boxed{V_{A_s} = \frac{K\cdot Q_A}{R_A} + \frac{KQ_B}{R_B}}$$

$$V_{B_s} = \underset{\substack{\text{due to } Q_B \\ \text{(at Surface)}}}{V_B} + \underset{\substack{\text{due to } Q_A \\ \text{(outside point)}}}{V_B}$$

$$\boxed{V_{B_s} = \frac{KQ_B}{R_B} + \frac{KQ_A}{R_B}}$$

$$\boxed{V_{A_s} > V_{B_s}}$$

Q2



Find potential on Surface of Shell B & C

$$V_{A\text{ on }s} = \frac{KQ}{R} + \frac{K2Q}{2R} + \frac{K4Q}{4R} = \frac{3KQ}{R}$$

$$V_{B\text{ on }s} = \frac{KQ}{2R} + \frac{K2Q}{2R} + \frac{K4Q}{4R}$$

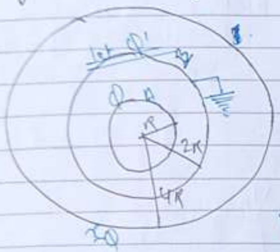
$$V_{B\text{ on }s} = \frac{3KQ}{2R} + \frac{KQ}{R} = \frac{5KQ}{2R}$$

$$V_{C\text{ on }s} = \frac{4KQ}{4R} + \frac{K2Q}{4R} + \frac{KQ}{4R} = \frac{7KQ}{4R}$$

+ve charge हमेशा higher potential से lower potential mai jata hai or -ve charge lower se higher

Earthing: Connect any Conductor to Earth. ($V_{earth} = 0$)

Q3



After earthing shell B, find charge on shell B

$\boxed{V_B = 0}$

$$V_B = \frac{KQ}{2R} + \frac{KQ'}{2R} + \frac{K2Q}{4R}$$

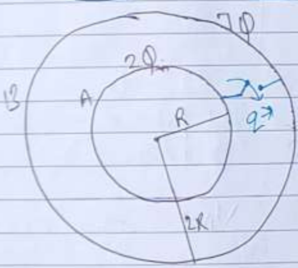
$$0 = \frac{KQ}{R} + \frac{KQ'}{2R}$$

$\boxed{Q' = -2Q}$

∴ change on shell B is $-2Q$

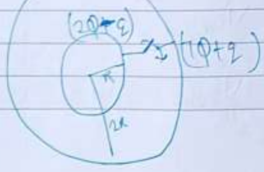
* jha earthing hogi waha potential zero hoga. zaruri nhi hai charge bhi zero ho.

Q4



Two shells are connected through a conducting wire. Find the final charge on shell A & shell B

$$V_{A\,(surface)} = V_{B\,(surface)}$$



$$\frac{K(2Q-q)}{R} + \frac{K(7Q+q)}{2R} = \frac{K(2Q-q)}{2R} + \frac{K(7Q+q)}{2R}$$

$$\frac{2(2Q-q) + (7Q+q)}{\cancel{2R}} = \frac{2Q+7Q}{\cancel{2R}}$$

$$4Q - 2q + 7Q + q = 9Q$$

$$-q = -2Q$$

$$\boxed{q = 2Q}$$

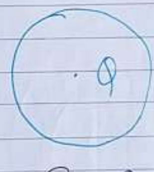
∴ Charge on sphere A = 0 and on B = 9Q

* Sphere's ko connect karne se vo single shell ban gya. or pura charge outer sphere (surface) par aa jaaega.

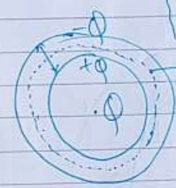
* Whenever we connect two sphere/shells, potential of both shells become equal

Conducting Shells:

* Electric field inside a conductor is zero under electrostatic condition

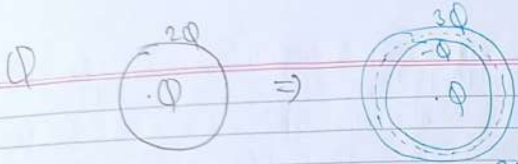


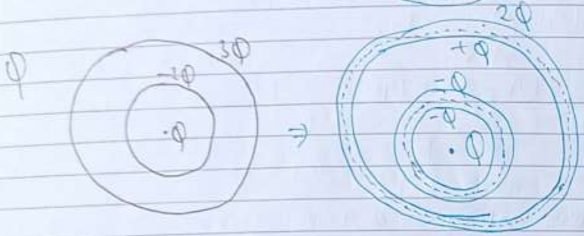
Thin Conducting shell

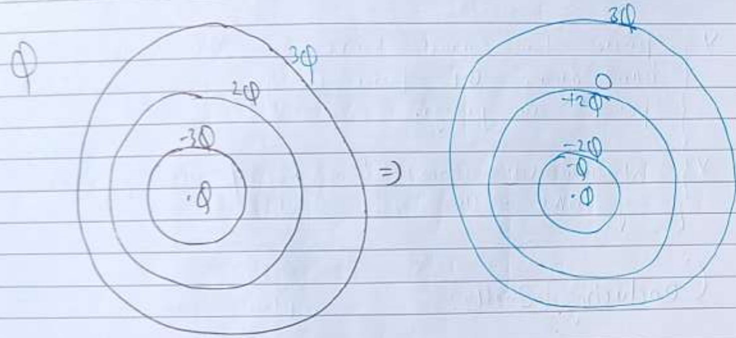


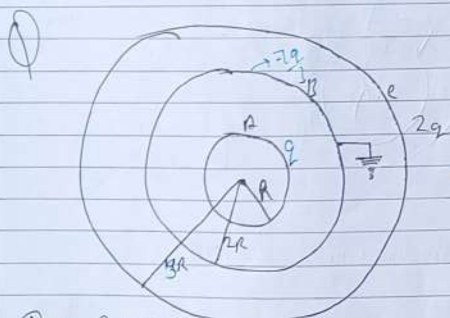
Gauss Law: $E = 0$, $q_{inside} = 0$

* Charge distribution on inner and outer surface.









Three Concentric Conducting Shells
B is earthed show charge distribution
on Surface of each shell.

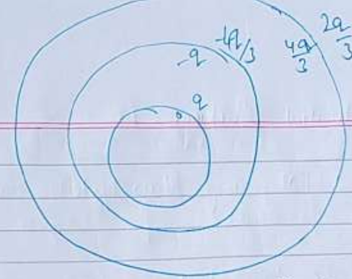
$$V_B = \frac{Kq}{2R} + \frac{Kq'}{2R} + \frac{K2q}{3R}$$

$$0$$

$$-\left(\frac{3Kq+4Kq}{6R}\right) = \frac{Kq'}{2}$$

$$-\left(\frac{7q}{6} = \frac{q'}{2}\right)$$

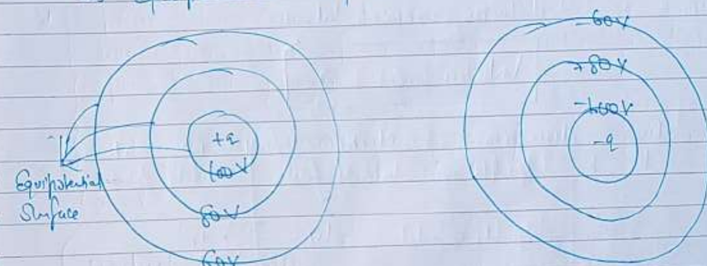
$$\boxed{q' = -\frac{7q}{3}}$$



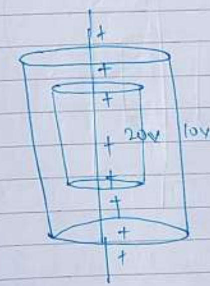
Equipotential Surface : Surface on which Potential is equal (Same) Every where. (3-D)

Ex- for a point charge

* Concentric spheres with charge as Centre of sphere as Equipotential Surface.



Ex: linear charge



* Concentric cylinders with linear charge as Axis of Cylinder are Equipotential Surface

Properties of Equipotential Surface.

i) The potential diff b/w any two points on Equipotential Surface is Zero.

imp ii) No. Work is Done by electric force in moving a charge on Equipotential Surface

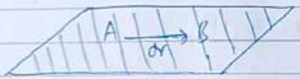
$$V_B - V_A = \frac{W_{external\ B \to A}}{q}$$

$$0 = W_{external}$$

$\therefore W_{electric\ force} = -W_{ext}$

$$\boxed{W_{elec.\ force} = 0}$$

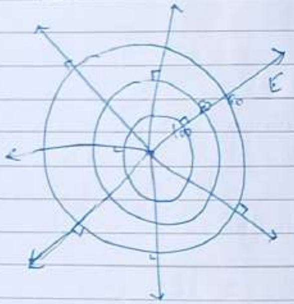
iii) The direction of electric field is always $\perp$ to equipotential Surface

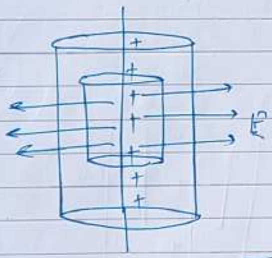


$$dV = -\vec{E} \cdot \vec{dr}$$

$$0 = \vec{E} \cdot \vec{dr}$$

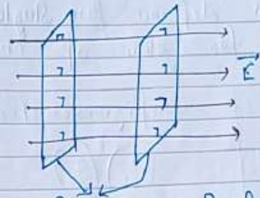
$\therefore \vec{E} \perp dr$





* Potential always decreases in the direction of $\vec{E}$ field

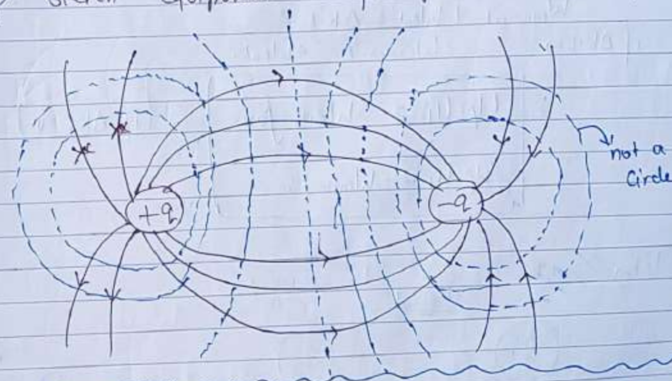
Q Draw Equipotential Surface for uniform $\vec{E}$ field as shown



Equipotential Surface / plane

iv) Two Equipotential Surface will never intersect each other

Q Sketch Equipotential Surface for an Electric Dipole



(v) A charged Conductor of any shape under Equilibrium (Electrostatic Condition) is an Equipotential Surface

(vi) Equipotential Surface are Crowded in strong $\vec{E}$ field region

Electrostatic Potential Energy (U)

* Electric P.E is not defined for single charge it is only defined for system of charges.

Config. Initial	Configuration Final
q_1 $\quad$ q_2	q_1 q_2

change in P.E $(U_f - U_i) = -W_{\text{conservative force}}\ (F_E)$

according to Work Energy theorem

$$W_{\text{cons.f}} + W_{\text{ext}} = \Delta K.E$$

if $\Delta K.E = 0$ $\quad W_{\text{conser.f}} = -W_{\text{ext}}$.

$$\therefore \boxed{\begin{aligned} U_f - U_i &= W_{\text{ext force}} \\ U_f - U_i &= -W_{\text{cons. force}} \end{aligned}} \quad (\text{if } \Delta KE = 0)$$

Electro static P.E of two charges (final)

q_1 ———— q_2

Initial

q_1 $\quad$ q_2
∞
$\boxed{U_\infty = 0}$

$$U_f - U_i = +W_{\text{ext agent}}$$

$$U_f - \cancel{U_\infty}^{0} = W_{\text{ext agent}}$$

$$U_f = \underset{\text{in bringing } q_1}{W_{\text{ext agent}}} + \underset{\text{in bringing } q_2}{W_{\text{ext agent}}}$$

q_1 ——r—— q_2 $\quad$ P $\quad\quad$ q_2 $\quad$ ∞

$$F = \frac{-Kq_1q_2}{r^2}$$

$$V_P - \cancel{V_\infty}^{0} = \frac{W_{\text{ext agent}}}{q_2}$$

$$\frac{Kq_1}{r} = \frac{W_{\text{ext agent}}}{q_2}$$

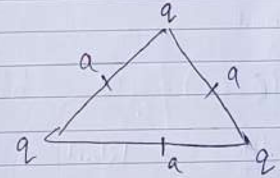
$$\frac{Kq_1q_2}{r} = W_{\text{ext agent}}.$$

$W_{\text{ext agent}}$ in Bringing $q_1 = 0$ $\quad\quad$ $\therefore W_{\text{ext agent}}$ in Bringing $q_2 = \frac{Kq_1q_2}{r}$

$$U_f = 0 + \frac{Kq_1q_2}{r}$$

P.E of System of charges $\quad \boxed{U_f = \frac{Kq_1q_2}{r}}$

Q1

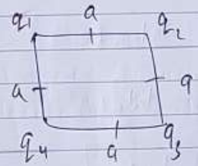


Find $U_{\text{system}} = ?$

$$\therefore U_{\text{system}} = \frac{Kqq}{a} + \frac{Kqq}{a} + \frac{Kqq}{a}$$

$$= 3\frac{Kq^2}{a}$$

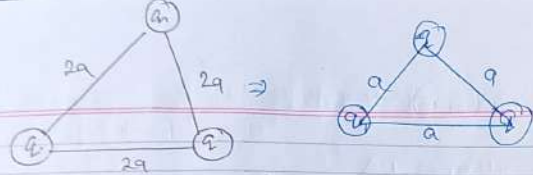
Q



Find U_{system}

$$U_{\text{system}} = \frac{Kq_1q_2}{a} + \frac{Kq_2q_3}{a} + \frac{Kq_3q_4}{a} + \frac{Kq_1q_4}{a} + \frac{Kq_2q_4}{\sqrt{2}a} + \frac{Kq_1q_3}{\sqrt{2}a}$$

$$= \frac{K}{a}\left(q_1q_2 + q_2q_3 + q_3q_4 + q_1q_4 + \frac{q_1q_3}{\sqrt{2}} + \frac{q_1q_2}{\sqrt{2}}\right)$$



Q3

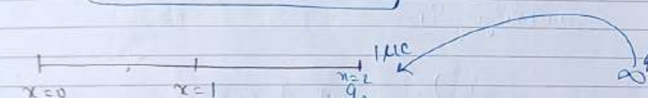
Find work done by external force to change config

$U_f - U_i = W_{ext\ force}$

$\frac{3kq^2}{a} - \frac{3kq^2}{2a} = W_{ext\ force}$

$\boxed{\frac{3kq^2}{2a} = W_{ext\ Force}}$

Q4.



$q_1 = -1\mu C$ $q_2 = +1\mu C$

A mind charge $q_3 = 1\mu C$ is brought from ∞ to x=2 find W.D by external agent

$U_f - U_i = W_{ext\ agent}$

$\cancel{\frac{kq_1q_2}{1}} + \frac{kq_2q_3}{1} + \frac{kq_1q_3}{2} - \cancel{\frac{kq_1q_2}{1}} = W_{external}$

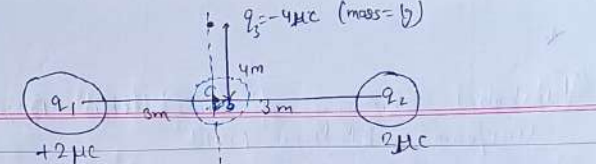
$\frac{k1^2\mu C}{1} - \frac{k1^2\mu C}{2} = W_{ext}$

$\frac{k}{2}1\mu C = W_{ext}$

$\frac{9\times10^9\cdot(1\times10^{-6})^2}{2} = W_{ext}$

$\boxed{4.5\times10^{-3} = W_{ext}}$

Q5



if q_3 is left from Rest. The speed of q_3 when it reaches O is nearly — ?

There is no Non Conservative force.

∴ $M.E_i = M.E_f$

$U_i + K_i = U_f + K_f$

$\cancel{\frac{kq_1q_2}{6}} + 0 = \cancel{\frac{kq_1q_2}{6}} + \frac{kq_1q_3}{3} + \frac{kq_2q_3}{3} + \frac{1}{2}mv^2$

$+ \frac{kq_1q_3}{5} + \frac{kq_2q_3}{5}$

$-\frac{1}{2}mv^2 = \frac{kq_1q_3}{3} + \frac{kq_2q_3}{3} - \frac{kq_1q_3}{5} - \frac{kq_2q_3}{5}$

$-\frac{1}{2}mv^2 = \left(-\frac{8k}{3} + -\frac{8k}{3} + \frac{8k}{5} + \frac{8k}{5}\right)\times10^{-12}$

$-mv^2 = 2\times10^{-12}k\left(-\frac{16}{3} + \frac{16}{5}\right)$

$-mv^2 = 2\times10^{-12}k \times -\frac{32}{15} =$

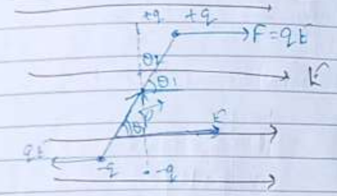
$mv^2 = \frac{+64\cdot9\times10^{9}\times10^{-12}}{15}$

$10^{-3}v^2 = \frac{64\times9\times10^{-3}}{15}$

$v = \sqrt{\frac{64\times9}{15}} = \frac{8\times3}{4} \approx 6$ m/s

★

Potential Energy of an Electric Dipole in a uniform Electric Field



$\Delta U = U_f - U_i$
$= -W_{\text{electric force}}$
$= W_{\text{external agent}}$

$W = \vec{F} \cdot \vec{disp} = \vec{\tau} \cdot \vec{\theta}$

$\vec{\tau} = \vec{P} \times \vec{E}$
$= PE\sin\theta$ clockwise

Rotation $\theta_1 \longrightarrow \theta_2$ (anticlockwise)

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta \cos 180^\circ = -\int_{\theta_1}^{\theta_2} PE\sin\theta \, d\theta$$

$$W = -PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$W = PE(\cos\theta)_{\theta_1}^{\theta_2}$$

$$\boxed{W = PE(\cos\theta_2 - \cos\theta_1)}$$

$U_f - U_i = -W_{\text{electric force}}$

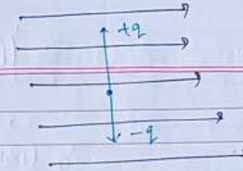
$U_{\theta_2} - U_{\theta_1} = -PE(\cos\theta_2 - \cos\theta_1)$

$\boxed{\text{Let } \theta_1 = 90^\circ \quad \& \quad U_{90} = 0^\circ}$ Assume

$U_\theta = -PE\cos\theta$

$$\boxed{U_\theta = -\vec{P} \cdot \vec{E}}$$

Configuration I:
$\theta = 90^\circ$

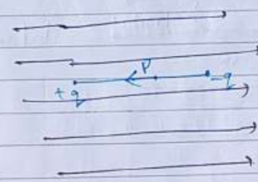


$F_{net} = 0$
$\tau_{net} = PE\sin\theta$
$= PE_{max}$
$U_{90} = -P.E\cos\theta$
$= 0^\circ$

Imp
Config. II:
$\theta = 0^\circ$

$F_{net} = 0$
$\tau_{net} = 0$
$U_0 = -PE\cos 0^\circ$
$= -PE$ min
(Stable Equilibrium)

Imp
Config. III.
$\theta = 180^\circ$



$F_{net} = 0$
$\tau_{net} = 0$
$U_{180} = -PE\cos 180^\circ$
$= PE$ (max)
Unstable Equilibrium

Work Done in Rotating an Electric Dipole in a Uniform Electric Field.

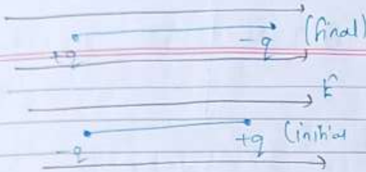
$\Delta U = U_{\theta_2} - U_{\theta_1} = -W_{\text{electric force}} = W_{\text{external force}}$.

* $W_{\text{electric force}} = U_{\theta_1} - U_{\theta_2} = -\Delta U$

* $W_{\text{external force}} = U_{\theta_2} - \theta_{\theta_1} = \Delta U$

$$\boxed{U_\theta = -PE\cos\theta}$$

Q1

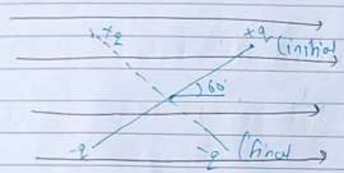


Find $W_{ext\ agent}$

Rotate $\theta = 0° \longrightarrow \theta = 180°$
(Stable Eq.) (unstable Eq)

$W_{ext\ agent} = U_f - U_i = U_{\theta_2} - U_{\theta_1}$
$= U_{180} - U_{0°}$
$= -PE\cos 180 - (-PE\cos 0°)$
$= PE + PE = 2PE$

Q2

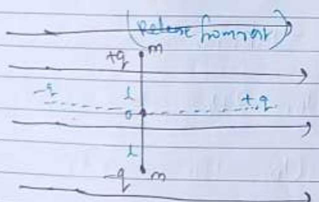


$W_{ext\ agent}$

$W_{ext\ agent} = U_{\theta_2} - U_{\theta_1}$
$= -PE\cos 120 + PE\cos 60°$
$= \frac{PE}{2} + \frac{PE}{2} = PE$

$\therefore W_{elect.\ force} = -\Delta U = -PE$

Q3

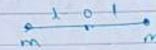


Find angular Velocity of diple when it passes mean position? ($\theta = 0°$)

Conservation of Mechanical Energy

$U_f + K_f = U_i + K_i$
$-PE + \frac{1}{2} I\omega^2 = 0 + 0$
$I\omega^2 = 2PE$
$\omega^2 = \frac{\cancel{2}(2l \cdot q)E}{\cancel{2}ml \cdot l}$

$$\boxed{\omega = \sqrt{\frac{2qE}{ml}}}$$



$I = ml^2 + ml^2 = 2ml^2$

Capacitor: "Stores Electrical Energy & Supply it at once".

(or)

"Capacitor is a conductor which stores electrical charge".

Capacitance:

Capacity / Ability To hold (Electrical Energy) Electrical charge.

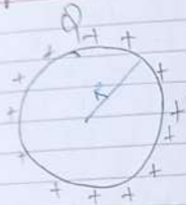
$$\boxed{C = \frac{Q}{V}}$$

* C is independent of 'Q' & 'V'
* C depends on dimensions of conductor & property of medium.
* S.I. unit of $C = \frac{Q}{V} = \frac{C}{V}$ = Farad = F

$C = 2F = \frac{2C}{V}$

matlab Conductor mai 2 Coulomb ka Charge rakhne se 1 Volt badhega

Spherical Capacitor / Conductor



$V_S = \frac{KQ}{R}$

$C = \frac{Q}{V}$

$C = \frac{Q}{\frac{KQ}{R}}$

$\boxed{C = \frac{1}{4\pi\varepsilon_0} \times R}$ ... $C = \frac{R}{\frac{1}{4\pi\varepsilon_0}}$

$\therefore \boxed{C = 4\pi\varepsilon_0 R}$

* depends upone dimension (R)
* depends upon medium (ε_0)

Q1. if R = 1m find it's Capacitance

$C = \frac{R}{K} = \frac{1}{9\times10^9} = \frac{10}{9\times10^{10}} \approx 1.1\times10^{-10}\ F$

$\boxed{C = 1.1\times10^{-4}\ \mu F}$

Q2. find the Radius of a Capacitor, C = 1F

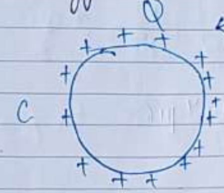
$C = 1F$

$\frac{R}{K} = 1$

$\boxed{R = 9\times10^9\ m}$

* Sphere Capacitor ka Capacitance bahot kmm hota hai

Energy stored in a Capacitor:



dq, dq charge lekar aae

dq ∞

At $t=0,\ Q=0,\ V=0$

$\boxed{t=t,\ Q=q\ \ V=\frac{q}{C}}$

$t=\infty,\ Q=Q,\ V=Q/C$

* Work done by external agent to bring each dq element from ∞ to this sphere is stored in the form of Potential Energy

Work done to bring dq charge from ∞ to sphere (q, V, C)

$V - \cancel{V_\infty}^{0} = \frac{dW}{dq}$

$dW = V\,dq$

$W_{ext} = \int dW = \int V\,dq = \int_0^Q \frac{q}{C}\,dq$

$W_{ext} = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$

$U_f - \cancel{U_i}^{0} = \frac{Q^2}{2C}$

$\boxed{U = \frac{Q^2}{2C}}$ $\boxed{C = \frac{Q}{V}}$

In terms of V & C

$\boxed{U = \frac{CV^2}{2}}$

In terms of Q & V

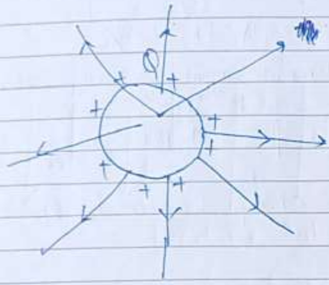
$\boxed{U = \frac{QV}{2}}$

Problem with Spherical Capacitor

i) Very low Capacitance

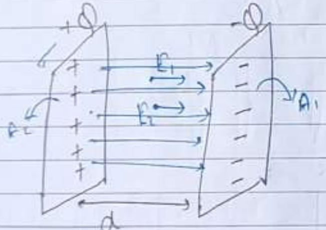
$R = 1m \qquad C = 1.1\times10^{-10} F = 1.1\times10^{-4}\,\mu F$

ii)



* Energy is not Bonded. (distributed Energy from $R \longrightarrow \infty$)

Parallel Plate Capacitor



(Very Small)

$$\boxed{\sigma = \frac{Q}{A}}$$

* This plate behave like ∞ plate.

$A_1 = A_2$

Conditions:

i) $d <<< \text{Area}$

ii) $A_1 = A_2$

Here, Capacitance of a Capacitor is

$$\boxed{C = \frac{Q}{\Delta V}}$$

$Q \longrightarrow$ charge on Capacitor

$\Delta V \longrightarrow$ P.d b/w plates

$E_1 = \frac{\sigma}{2\epsilon_0} \qquad E_2 = \frac{\sigma}{2\epsilon_0}$

$E_{net} = \vec{E_1} + \vec{E_2} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$dV = -E \cdot dr$

$|dV| = E \cdot dr$

$\Delta V = E \cdot d \cdot \cos 0^\circ$

$$\boxed{\Delta V = \frac{Qd}{A\epsilon_0}}$$

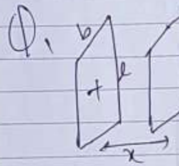
$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\epsilon_0}} = \frac{A\epsilon_0}{d}$$

* Capacitance depend upon dimensions and medium.

$$\boxed{C = \frac{A\epsilon_0}{d}}$$

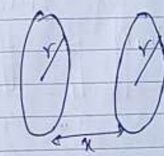
$d <<< A, \quad C\uparrow\uparrow\uparrow$

Q_1



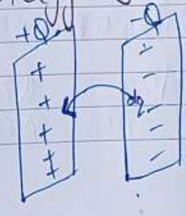
Find C = ?

Q_2



$C = \frac{lb\epsilon_0}{x} \qquad C = \frac{\pi r^2 \epsilon_0}{x}$

Energy Stored in a Charged Capacitor.



$t=0, \quad Q=0, \quad \Delta V=0, \quad C.$

$t=t \quad Q=q, \quad \Delta V = V = \frac{q}{C}$

$t=\infty \quad Q=Q \quad \Delta V = \frac{Q}{C}$

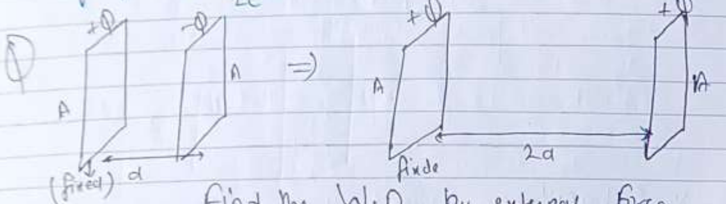
* $\boxed{C = \frac{Q}{\Delta V}}$

$$\Delta V = \frac{dW_{ext}}{dq}$$

$$dW_{ext} = dq\,\Delta V = dq\,\frac{q}{c}$$

$$W_{ext\,agent} = \int dw = \int_0^Q \frac{q\,dq}{c} = \frac{1}{c}\int_0^q q\,dq = \frac{1}{c}\frac{Q^2}{2}$$

* work done by ext agent is stored in the form of P.E $= \frac{Q^2}{2C}$

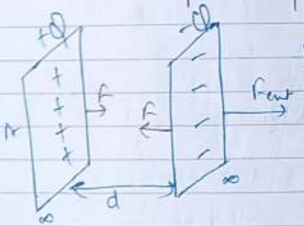


Find the W.D by external force

$$W_{ext} = \Delta U = U_f - U_i$$
$$= \frac{Q^2}{2C_f} - \frac{Q^2}{2C_i}$$
$$= \frac{Q^2\,2d}{2A\epsilon_0} - \frac{Q^2 d}{2A\epsilon_0}$$

$$\boxed{W_{ext} = \frac{Q^2 d}{2A\epsilon_0}}$$

Force on one plate of Capacitor due to other plate



$E_{21} = E$ at (2) due to (1) $= \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$

Force on (2) due to this $E = QE$

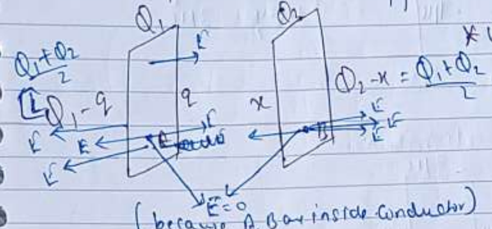
$$= \frac{Q \cdot Q}{2A\epsilon_0} = \frac{Q^2}{2A\epsilon_0}$$

$F_{ext} = F_{electric} = \frac{Q^2}{2A\epsilon_0}$

$W_{ext} = F_{ext} \times disp = \frac{Q^2 d}{2A\epsilon_0}$

What if we have different charge on each plate

* What is the charge distribution on each face of each plate



(because A, B are inside conductor)

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

$$E_A = \frac{Q_1 - q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} - \frac{x}{2A\epsilon_0} - \frac{(Q_2 - x)}{2A\epsilon_0}$$

$$0 = \frac{1}{2A\epsilon_0}\left(Q_1 - q - q - \cancel{x} - Q_2 + \cancel{x}\right)$$

$$Q_1 - Q_2 - 2q = 0 \quad \text{—(1)}$$

$$E_B = \frac{Q_2 - x}{2A\epsilon_0} - \frac{x}{2A\epsilon_0} - \frac{(Q_1 - q)}{2A\epsilon_0} - \frac{q}{2A\epsilon_0}$$

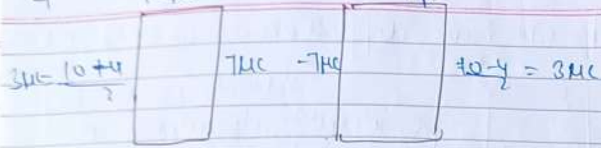
$$0 = \frac{1}{2A\epsilon_0}\left(Q_2 - Q_1 - 2x\right)$$

$$Q_2 - Q_1 - 2x = 0$$

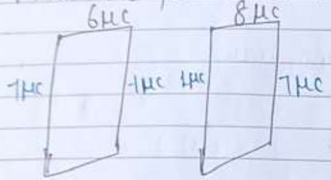
$$\therefore \boxed{q = \frac{Q_1 - Q_2}{2}} \qquad \boxed{x = \left(\frac{Q_2 - Q_1}{2}\right)}$$

$$\boxed{C = \frac{A\epsilon_0}{d}}$$

Q find charge distribution?

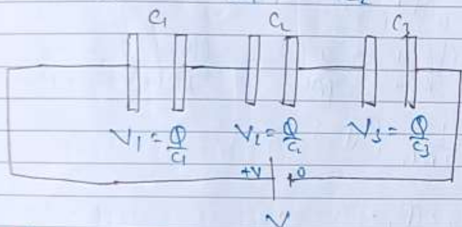


Q Final charge distribution



Combination of Capacitor.

Series : $\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

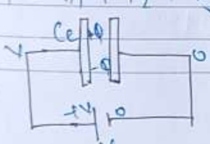


→ Q is same in each Capacitor.

→ V is different in each Capacitor.

$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

→ $V = V_1 + V_2 + V_3$



$V = \frac{Q}{C_e}$

$V = V_1 + V_2 + V_3$

$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

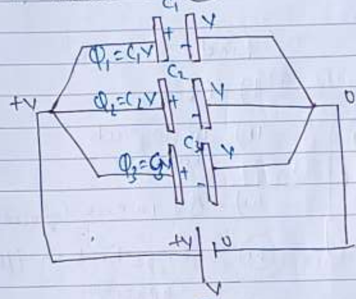
* $\boxed{\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

* for 2 C $\boxed{C_e = \frac{C_1 C_2}{C_1 + C_2}}$

* if 2 capacitor $C_1 = C_2 = C$

then $\boxed{C_e = \frac{C}{2}}$

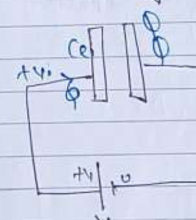
Parallel Combinations



→ V is same in each Capacitor

→ Q is diff

$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

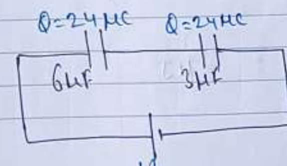


$Q = Q_1 + Q_2 + Q_3$

$C_eV = C_1V + C_2V_2 + C_3V$

$\boxed{C_e = C_1 + C_2 + C_3}$

Q



i) C_e

ii) Q on each Capacitor

iii) ΔV on each Capacitor

iii) Energy in 3μF capacitor

Sol^n $\frac{1}{C_e} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

$\boxed{C_e = 2}$

$Q = C_e \cdot V$
$= 2 \times 12 = 24 \mu C$

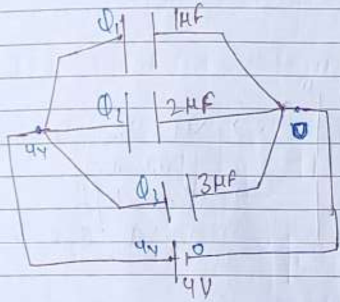
ii) $V_1 = \frac{Q}{C_1} = \frac{24}{6} = 4V$

$V_2 = \frac{Q}{C_2} = \frac{24}{3} = 8V$

iv) $U = \frac{1}{2} C V_2^2 = \frac{3 \times 10^{-6}}{2} \times 8 \times 8 = 96 \times 10^{-6} J$

$= 96 \mu J$

Q



Find
1) C_e
ii) Q in each Capacitor
iii) ΔV in each Capacitor
(iv) Energy stored in 1μF Capacitor

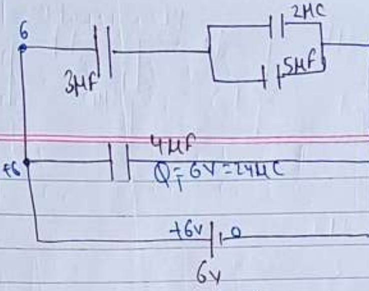
1) $C_e = C_1 + C_2 + C_3 = 6 \mu F$

ii) $\Delta V = 4V$

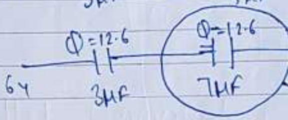
iii) $Q_1 = C_1 V = 4 \mu C$
$Q_2 = C_2 V = 8 \mu C$
$Q_3 = C_3 V = 12 \mu C$

iv) $U = \frac{Q^2}{2C} = \frac{4 \times 4}{2 \times 1} = 8 \mu J$

Q_3



Find the ratio of charge on 5μF capacitor to 4μF capacitor



$V = \frac{Q}{C} = \frac{12.6}{7} = 1.8V$

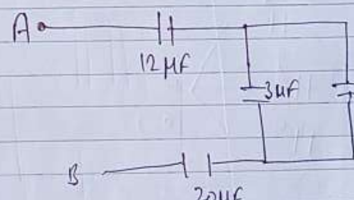
$C_e = \frac{21}{10}$ $\therefore Q = CV = \frac{21}{10} \times 6 = \frac{126}{10} = 12.6 \mu C$

$Q = 5 \times V =$

$Q_2 = 5 \times 1.8 = 9$

$\therefore Q_1 : Q_2 = 9 : 24 = 3 : 8$

Q_4



Find C_e b/w A & B

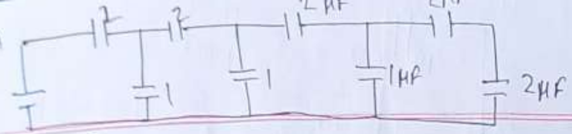
$C_{e1} = 3 + 2 = 5$

$\frac{1}{C_e} = \frac{1}{12} + \frac{1}{5} + \frac{1}{20}$

$\frac{1}{C_e} = \frac{5 + 12 + 3}{60}$

$\boxed{C_e = 3 \mu F}$

Qs



Fe Ce b/w A & B

$C_{e_1} = 1$

$C_{e_2} = 2$

$C_{e_3} = 1$

$C_{e_4} = 2$

$C_{e_5} = 1$

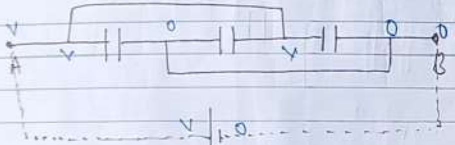
$C_{e_6} = 2$

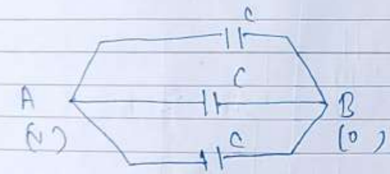
$\boxed{C_e = 1}$

Type 2: Wire Connection Problem

Find Equivalent Capacitor b/w A & B

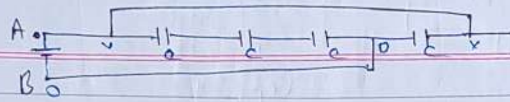
(1)

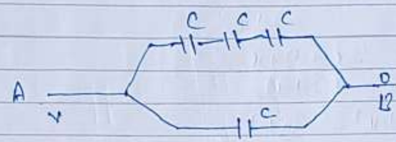




$C_e = C + C + C = 3C$

(2)

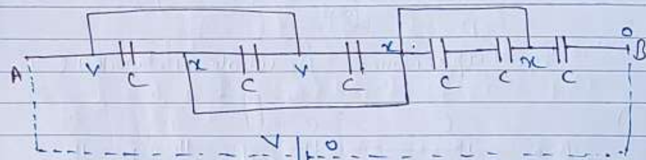




$C_{e_1} = \frac{C}{3}$

$C_e = \frac{C}{3} + C = \frac{4C}{3}$

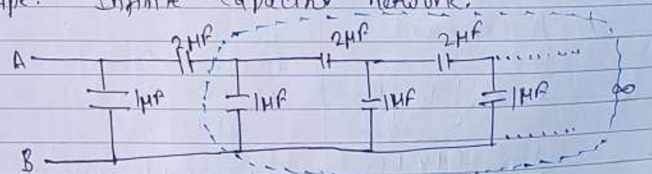
(3)



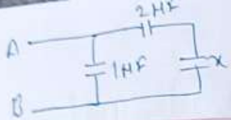
$C_{e_1} = 3C$

$C_e = \frac{C \cdot 3C}{3C + C} = \frac{3C}{4}$

Type: Infinite Capacitor network.



Let Equivalent Capacitance of whole network be 'x'



$\frac{1}{C_{e_1}} = \frac{1}{2} + \frac{1}{x}$

$\frac{1}{C_{e_1}} = \frac{2+x}{2x}$

$C_{e_1} = \frac{2x}{2+x}$

∴ $C_e = 1 + \frac{2x}{2+x}$

$x = \frac{2x+2+x}{(2+x)}$

$2x + x^2 = 3x + 2$

$x^2 - x - 2 = 0$

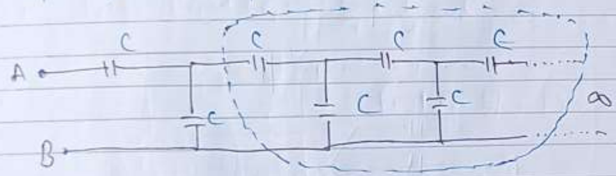
$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

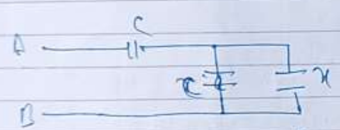
$\cancel{x = -1}$ $\boxed{x = 2}$

∴ Eq. Capacitance for whole network is 2.

(2)



let Eq. Capacitance be x



$C_e = x + C$

$\frac{1}{C_e} = \frac{1}{C} + \frac{1}{C+x}$

$\frac{1}{x} = \frac{C+x+C}{C^2+Cx}$

$C^2 + Cx = x(2C+x)$

$C^2 + Cx = 2Cx + x^2$

$x^2 + Cx - C^2 = 0$

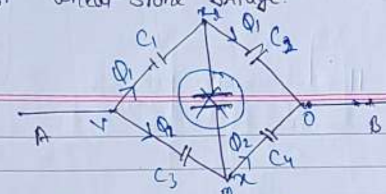
$x = \frac{-C \pm \sqrt{C^2+4C^2}}{2}$

$x = \frac{-C \pm \sqrt{5}C}{2}$

∴ $\boxed{x = \frac{-C + \sqrt{5}C}{2}}$

Type 3: Wheat stone Bridge:

(1)



let the potential at l & m is Equal

$V_l = V_m = x$

∴ $V - x = \frac{Q_1}{C_1}$ $V - x = \frac{Q_2}{C_3}$

$x - 0 = \frac{Q_1}{C_2}$ $x - 0 = \frac{Q_2}{C_4}$

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ | $\frac{Q_1}{C_2} = \frac{Q_2}{C_4}$

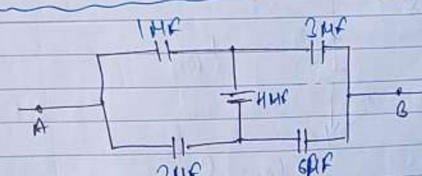
$\frac{Q_1}{Q_2} = \frac{C_1}{C_3}$ | $\frac{Q_1}{C_2} = \frac{Q_2}{C_4}$

∴ $\boxed{\frac{C_1}{C_3} = \frac{C_2}{C_4}}$

* The potential at l & m is Equal iff $\frac{C_1}{C_3} = \frac{C_2}{C_4}$

Hence, जो capacitor l, m se lga hoga vo useless hoga

(1)



Find C_e b/w A & B

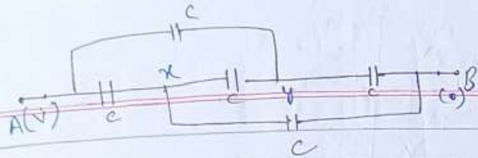
∴ $\frac{C_1}{C_3} = \frac{C_2}{C_4} = \frac{1}{2}$

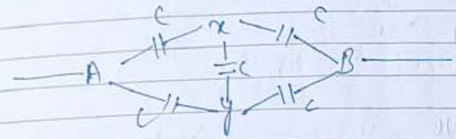
Hence. 4μF Capacitor is useless

∴ $C_{e_1} = \frac{3}{4}$ $C_{e_2} = \frac{3}{2}$

∴ $C_e = C_{e_1} + C_{e_2} = \frac{3}{4} + \frac{3}{2} = \frac{9}{4}$ μF

(2)





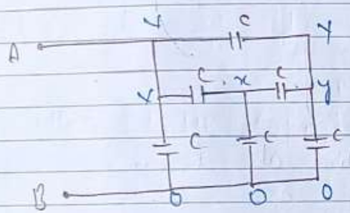
$\therefore \frac{C_1}{C_3} = 1 \qquad \frac{C_2}{C_4} = 1$

$\therefore$ Capacitor b/w x & y is useless.

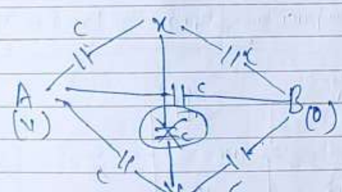
$\therefore C_{e_1} = \frac{C}{2}$, $C_{e_2} = \frac{C}{2}$

$\therefore C_e = C_{e_1} + C_{e_2} = \frac{C}{2} + \frac{C}{2} = C$

(3)



Find C_e b/w A & B



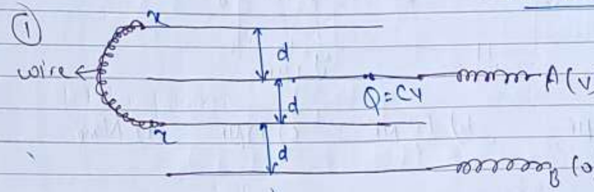
$C_{e_1} = \frac{C}{2} \qquad C_{e_2} = \frac{C}{2}$

$\therefore$ Ce b/w A & B $= \frac{C}{2} + \frac{C}{2} + C = 2C$

Type 4: Parallel plate problem

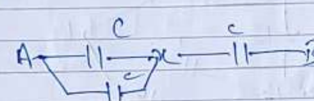
Fe. Ce. b/w A & B

(1)



$\boxed{C = \frac{\varepsilon_0 A}{d}}$

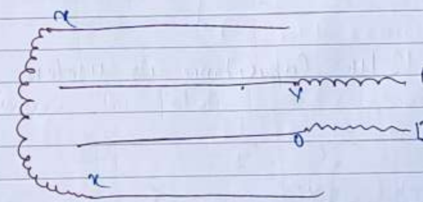
Area of all plate is Same = A

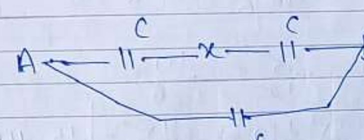


$C_{e_1} = 2C$

$\therefore \frac{1}{C_e} = \frac{1}{C_{e_1}} + \frac{1}{C} = \frac{1}{2C} + \frac{1}{C}$, $C_e = \frac{2C}{3} = \frac{2}{3}\frac{\varepsilon_0 A}{d}$

Q2

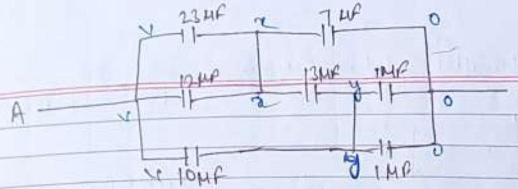




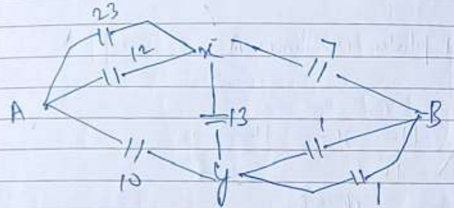
$C_{e_1} = \frac{C}{2}$

$\therefore C_e = \frac{C}{2} + C = \frac{3C}{2} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$

Q



a) $\frac{28}{3}$ μF b) $\frac{15}{2}$ μF c) 15 μF d) None



$C_{e_1} = 23 + 12 = 35$

$\frac{C_1}{C_3} = \frac{35}{10} = \frac{7}{2}$ | $\frac{C_2}{C_4} = \frac{7}{2}$

∴ Hence 13 μF Capacitance is useless

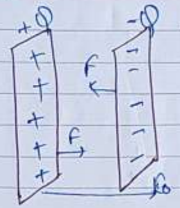
$\therefore \frac{1}{C_e} = \frac{1}{35} + \frac{1}{7} =$

$C_{e_3} = \frac{35 \times \cancel{7}}{\cancel{42}\, 6} = \frac{35}{6}$

$C_{e_4} = \frac{\cancel{10}}{\cancel{12}} = \frac{10}{6}$

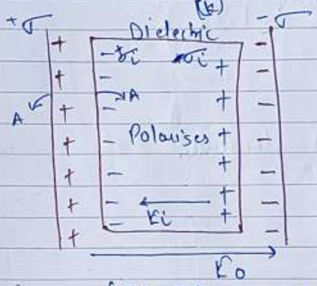
$\therefore C_e = \frac{35}{6} + \frac{10}{6} = \frac{45}{6} = \boxed{\frac{15}{2}\ \mu F}$ Ans

Dielectric in a Capacitor



$F = \frac{Q^2}{2\epsilon_0 A}$

Dielectric: Those material which do not allow Current but allow development of $\vec{E}$ field.
Dielectric are neither Conductor nor insulator and nor Semiconductors. They are other Class of material (Non Conductors)



(Ext. $\vec{E}$ due to plate)

* Charges are Induced in dielectric because of Ext $\vec{E}$.

* Induced charge density: σ_i

* $\vec{E_i}$ opposes E_0

E_{net} b/w plate $= E_0 - E_i$

* K → dielectric Constant (property of Material)

$\boxed{E_{net} = \frac{E_0}{K}}$

* The no. of times it decreases the electric field as compared to the electric field in Air/Vaccum is Called dielectric Constant (K).

$\therefore E_{net} = \frac{E_0}{K} = E_0 - E_i$

$E_0 = \frac{\sigma}{\epsilon_0}$ $\qquad$ $E_i = \frac{\sigma_i}{\epsilon_0}$

$$E_{net} = \frac{E_0}{k} = E_0 - E_i$$

$$\frac{\sigma}{\cancel{\epsilon_0} k} = \frac{\sigma}{\cancel{\epsilon_0}} - \frac{\sigma_i}{\cancel{\epsilon_0}}$$

Induced charge density = $\boxed{\sigma_i = \sigma\left(1 - \frac{1}{k}\right)}$

Potential difference

$V_0 = E_0 \times d$ (without dielectric)

$$V = \frac{E_0}{k} \times d = \frac{V_0}{k}$$

$\boxed{V = \frac{V_0}{k}}$ P.D also decrease.

Capacitance

$C_0 = \frac{Q}{V_0}$ (without dielectric)

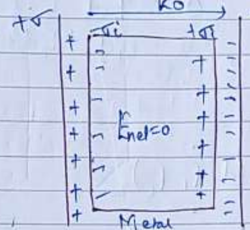
$$C = \frac{Q}{V_0/k} = \frac{Q}{V_0} \times k = kC_0$$

$$\therefore \boxed{C = kC_0}$$

Advantages of dielectric

1. Provides stability to plates
2. Prevents plate from sticking to each other
3. Increases Capacitance
4. Can prevent Breakdown of medium

Q What is the value of k for Metal/Conductor?



$E_{net} = 0$

$0 = \frac{E_0}{k}$

$k = \infty$

$\sigma_i = ?$

$\sigma_i = \sigma\left(1 - \frac{1}{k}\right)$

$\sigma_i = \sigma(1 - 0)$

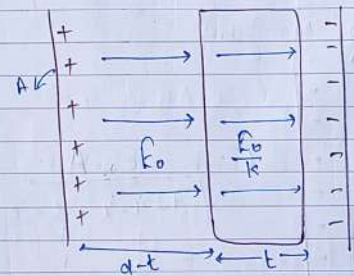
$\boxed{\sigma_i = \sigma}$

$E_0 = E_i$

$\boxed{E_{net} = 0}$

* K for Vaccume/Air = 1

Capacitor with partially filled Dielectric.



$E = \frac{\sigma}{\epsilon_0}$

$E = \frac{Q}{A\epsilon_0}$

M-2 Series $\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$

$C_1 = \frac{\epsilon_0 A}{d-t}$ $\qquad$ $C_2 = \frac{k\epsilon_0 A}{t}$

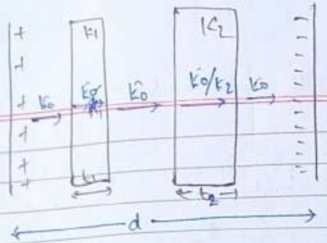
$\frac{1}{C_e} = \frac{d-t}{\epsilon_0 A} + \frac{t/k}{\epsilon_0 A}$

$C_e = \frac{\epsilon_0 A}{d-t+t/k}$

$$\Delta V = Ed$$
$$= E_1 d_1 + E_2 d_1$$
$$= E_0(d-t) + \frac{E_0}{k} t$$

$$\Delta V = E\left(d - t + \frac{t}{k}\right) = \frac{Q}{A\epsilon_0}\left(d - t + \frac{t}{k}\right)$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d-t+\frac{t}{k}}}$$



Find C_e b/w two plates?

$\Delta V = Ed$

$= E_1 d_1 + E_2 d_2 + E_3 d_3$

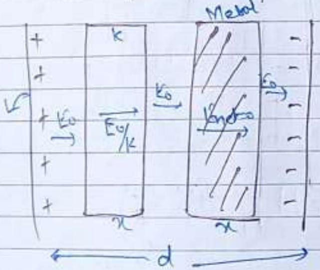
$$\Delta V = \frac{E_0}{K_1}(t_1) + \frac{E_0}{K_2}(t_2) + E_0(d - t_1 - t_2)$$

$$\Delta V = E_0\left[\frac{t_1}{K_1} + \frac{t_2}{K_2} + d - t_1 - t_2\right]$$

$$\Delta V = \frac{Q}{A\epsilon_0}\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + d - t_1 - t_2\right)$$

$$\boxed{C = \frac{\epsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K} + \frac{t_2}{K}} = \frac{Q}{\Delta V}}$$

Q. Find Capacitance of this System.

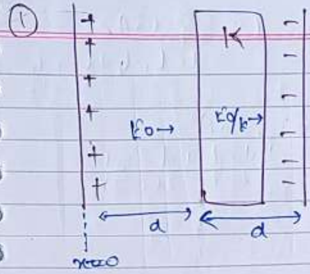


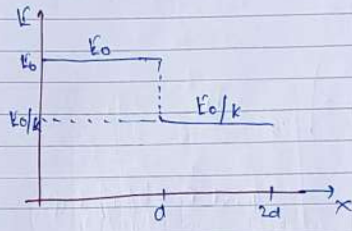
$$C = \frac{\epsilon_0 A}{\left(d - x - x + \frac{x}{k} + \frac{x}{\infty}\right)}$$

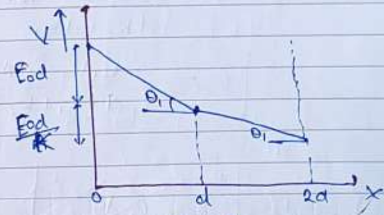
$$C = \frac{\epsilon_0 A}{\left(d - 2x + \frac{x}{K}\right)}$$

Graph of E v/s x & V v/s x

(1)

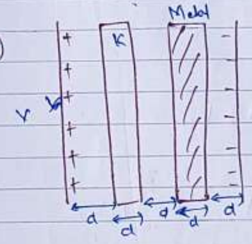


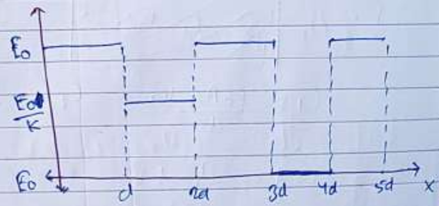


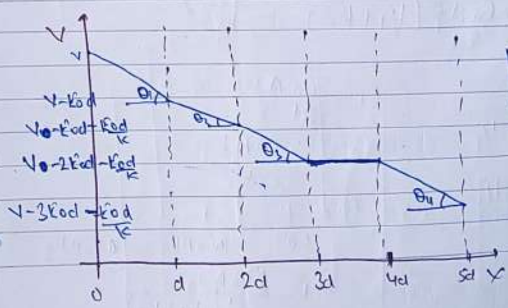


$\theta_1 > \theta_2$

(2)

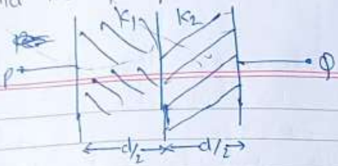






$$\boxed{\theta_1 = \theta_3 = \theta_4 > \theta_2}$$

(1) Find the Eq. capacitance b/w P & Q



$$C = \frac{\epsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{k_1} + \frac{t_2}{k_2}}$$

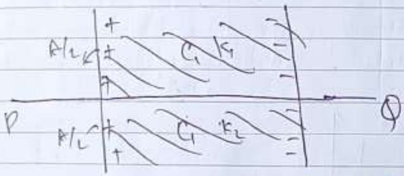
$$C = \frac{\epsilon_0 A}{d - \frac{d}{2} - \frac{d}{2} + \frac{d}{2k_1} + \frac{d}{2k_2}}$$

$$\therefore C = \frac{\epsilon_0 A}{\frac{d}{2k_1} + \frac{d}{2k_2}}$$

$$C = \frac{2\epsilon_0 A\, k_1 k_2}{dk_2 + dk_1}$$

$$\boxed{C = \frac{2k_1 k_2 \epsilon_0 A}{d(k_1 + k_2)}}$$

(2) Find C_e b/w p & Q

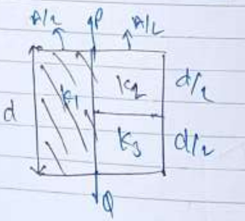


$$C_1 = \frac{\epsilon_0 A/2\, k_1}{d} \qquad C_2 = \frac{\epsilon_0 A/2\, k_2}{d}$$

$$C_{eq} = C_1 + C_2 = \frac{\epsilon_0 A k_1}{2d} + \frac{\epsilon_0 A k_2}{2d}$$

$$C_e = \frac{\epsilon_0 A}{2d}(k_1 + k_2)$$

(3)



$$C_1 = \frac{2\epsilon_0 A k_2}{2d}$$

$$C_2 = \frac{2\epsilon_0 A k_3}{2d}$$

$$\frac{1}{C_e} = \frac{d}{\epsilon_0 A k_2} + \frac{d}{\epsilon_0 A k_3}$$

$$C_{e_1} = \frac{\epsilon_0 A k_3 k_2}{d(k_2 + k_3)}$$

$$\therefore C_3 = \frac{\epsilon_0 A/2\, k_1}{d} = \frac{\epsilon_0 A k_1}{2d}$$

$$\therefore C_e = C_{e_1} + C_3$$

$$C_e = \frac{\epsilon_0 A k_3 k_2}{d(k_2 + k_3)} + \frac{\epsilon_0 A k_1}{2d}$$

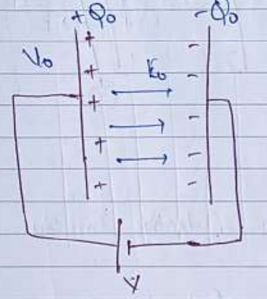
$$C_e = \frac{\epsilon_0 A}{d}\left[\frac{k_3 k_2}{k_2 + k_3} + \frac{k}{2}\right]$$

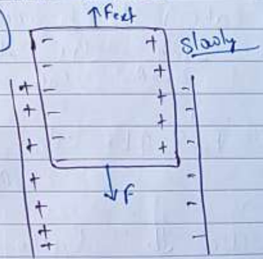
$$\boxed{C_e = \frac{\epsilon_0 A}{d}\left[\frac{2k_3 k_2 + k_1 k_2 + k_1 k_3}{2(k_2 + k_3)}\right]}$$

Battery Removed | Connected & Dielectric Inserted.

Case I: Battery Removed & Dielectric Inserted.

$\boxed{Q = \text{Constant}}$





Before Insertion of Dielectric	After insertion of Dielectric
$Q_i = Q_0$	$Q_f = Q_0$
$V_i = V_0$ $\quad V_0 = \frac{Q_0}{C_0}$	$V_f = \frac{V_0}{k}$
$E_i = E_0$	$E_f = \frac{E_0}{k}$
$C_i = C_0$	$C_f = kC_0$
$U_i = \frac{Q_0^2}{2C_0}$	$U_f = \frac{Q_0^2}{2kC_0}$

Work done by Ext agent

$= F_{ext}(-x)$

$= -ve$

$\therefore$ Work is obtained

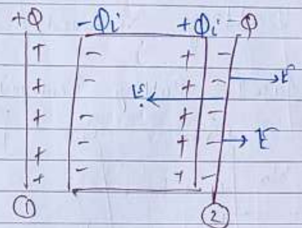
* decrease in Energy is taken by Ext agent in the form of work obtained.

Before

$\vec{E}$ due to (1) at (2) $= \dfrac{\sigma}{2\epsilon_0} = \dfrac{Q_0}{2\epsilon_0 A}$

$\vec{F_{21}} = qE = \dfrac{Q_0 \times Q_0}{2\epsilon_0 A} = \dfrac{Q_0^2}{2\epsilon_0 A}$

$\boxed{\vec{F_i} = \dfrac{Q_0^2}{2\epsilon_0 A}}$

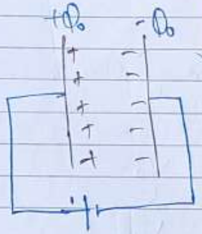


force on (2) due to all charges :

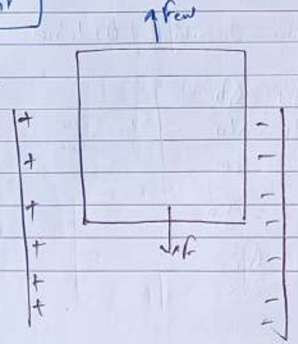
$\vec{E}$ at (2) due to all charge $= \dfrac{Q_0}{2\epsilon_0 A} - \cancel{\dfrac{Q_i}{2\epsilon_0 A}} + \cancel{\dfrac{Q_i}{2\epsilon_0 A}}$

$\therefore \vec{F_{21}} = qE = \dfrac{Q_0^2}{2\epsilon_0 A}$

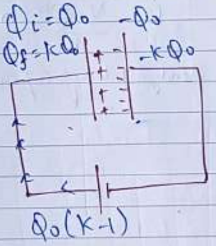
Case II : Battery Connected & Dielectric Inserted



$\boxed{V = \text{Constant}}$



Before	After
$C_i = C_0$	$C_f = KC_0$
$V_i = V_0$	$V_f = V_0$ Constant
$Q_i = Q_0$ $\quad Q_0 = C_0V_0$	$Q_f = KC_0V_0 = KQ_0$
$E_i = E_0$	$E_f = E_0$ (constant)
$E_i = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q_0}{A\epsilon_0}$	
$E_f = \dfrac{\sigma}{K\epsilon_0} = \dfrac{\cancel{K}Q_0}{\cancel{K}A\epsilon_0}$	
$U_i = \dfrac{1}{2}C_0V_0^2$	$U_f = \dfrac{1}{2}KC_0V_0^2$



charged supplied thru. battery $= Q_0(K-1)$

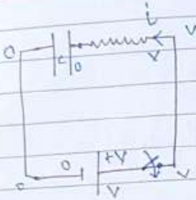
W.D by battery $= qV$

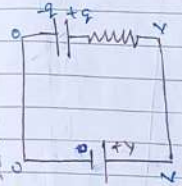
Energy supplied by battery $\left\{ \begin{aligned} &= Q_0(K-1)V_0 \\ &= Q_0V_0(K-1) \\ &= C_0V^2(K-1) \end{aligned} \right.$

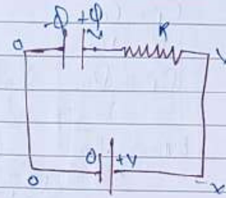
$\Delta U = U_f - U_i = \dfrac{1}{2}C_0V_0^2(K-1)$

$W_{ext} = -\dfrac{1}{2}C_0V_0^2(K-1)$

Circuits with Capacitor & Resistor







Switch is closed initially $t=0$

$\rightarrow q=0$ for Capacitor

$\Delta V = \frac{q}{C} = 0$ for Cap.

$$\boxed{i = \frac{V}{R}}$$

$t=t$

After some time

$\Delta V = \frac{q}{C}$

$i = \frac{V - q/C}{R}$

$t=\infty$

$Q = CV$

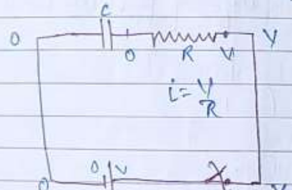
$i = \frac{V - q/C}{R}$

$i = \frac{V - \frac{CV}{C}}{R} = 0$

Steady state of Capacitor

Two states of Capacitor

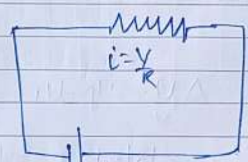
1) initial $t=0$, $i = \frac{V}{R}$



$t=0$

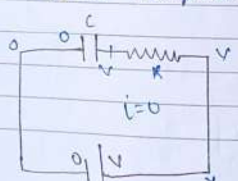
There is no Capacitor

$\Rightarrow$

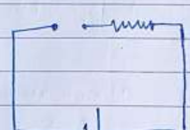


(Short Circuit Capacitor)

ii)

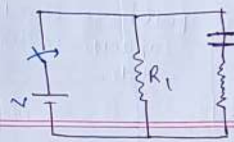


$\Rightarrow$

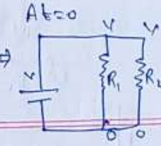


open Circuit

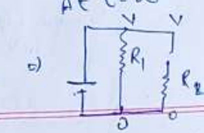
Q1



At $t=0$



At $t=\infty$



Find Current i_1 & i_2 in each resistor at

1) $t=0$

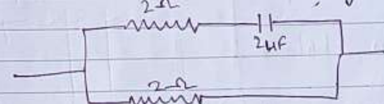
ii) Steady state

Sol^n 1) $i_1 = \frac{V}{R_1}$, $i = \frac{V}{R_2}$

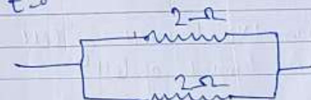
ii) Steady state. $i_1 = \frac{V}{R_1}$, $i_2 = 0$

Q2 Find Equivalent resistance at

1) $t=0$ ii) Steady State



1) $t=0$



$t=\infty$

$\frac{1}{R_e} = \frac{1}{2} + \frac{1}{2}$

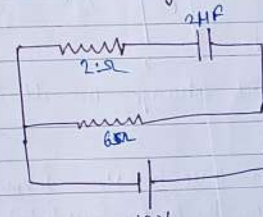
$$\boxed{R_e = 1\Omega}$$

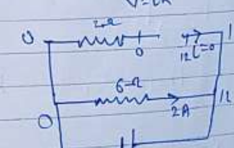
$R_e = 2\Omega$

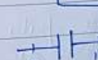
Q3 Find the charge stored on Capacitor in steady state

$V = iR$

(1)



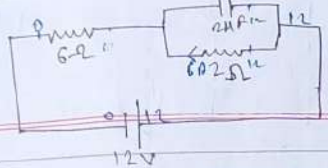




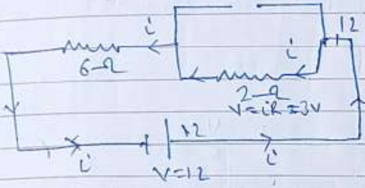
$Q = CV = 2 \times 12$

$= 24 \mu C$

Q_2

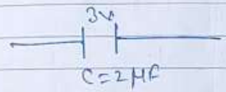


Find Charge stored in Capacitor in steady state.



$R_e = 6\Omega + 2\Omega$
$= 8\Omega$

$i = \frac{12}{8} = \frac{3}{2}A$



$Q = CV = 2 \times 3 = 6\mu C$

To find C_e b/w two points

① Connect an imaginary cell across two terminals

② Take out '+q' charge from +ve terminal of cell & '-ve' charge from -ve terminal of cell

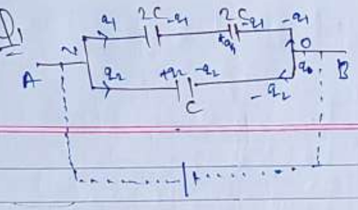
③ ~~Kia~~ ~~Klas~~ Distribute charge on each Capacitor

④ For closed loop $\sum \Delta V = 0$

use these eqn's to find charge on all capacitors

⑤ Choose any one path from +ve to -ve terminal of cell & Calculate $\sum \Delta V$ along the path. Equate this $\sum \Delta V = +\frac{q}{C_e}$

Q_1



Find C_e b/w A & B

$C_{e_1} = \frac{2C}{2} = C$

$C_e = C + C = 2C$

$q_1 + q_2 = q \quad \text{---(i)}$

Closed loop. $\sum \Delta V = 0 = \frac{-q_1}{2C} - \frac{q_1}{2C} + \frac{q_2}{C} = 0$

$\frac{-q_1 - q_1 + 2q_2}{2C} = 0$

$-q_1 + q_2 = 0$

$\boxed{q_1 = q_2}$

$\therefore \boxed{q_1 = q_2 = \frac{q}{2}}$

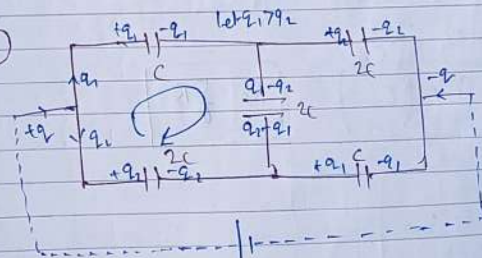
* let take upper path

$-\frac{q}{2(2C)} - \frac{q}{2\cdot(2C)} = -\frac{q}{C_e}$

$\frac{\cancel{2}q}{\cancel{4}C} = \frac{\cancel{+}q}{C_e}$

$\boxed{C_e = 2C}$

②



$q_1 + q_2 = q \quad ①$

Closed loop $\sum \Delta V = 0$

$-\frac{q_1}{C} - \frac{(q_1 - q_2)}{2C} + \frac{q_2}{2C} = 0$

$\frac{-2q_1 - q_1 + q_2 + q_2}{2C} = 0$

$-3q_1 + 2q_2 = 0$

$2q_2 = 3q_1$

$\boxed{q_2 = \frac{3}{2}q_1}$

$q_1 + q_2 = 5$

$\boxed{q_1 = \frac{2}{5}q, \; q_2 = \frac{3}{5}q}$

Let take upper path

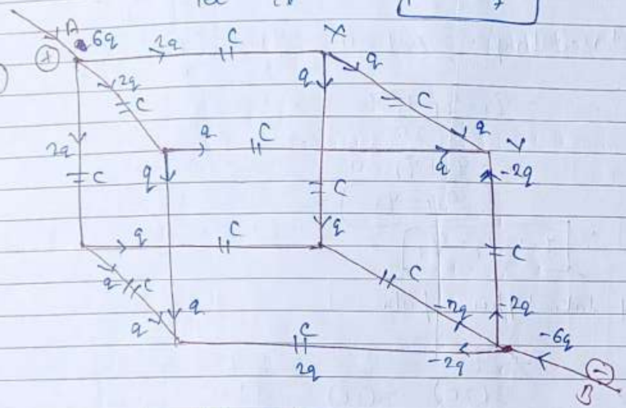
$$-\frac{q_1}{C} - \frac{q_2}{2C} = -\frac{q}{C_e}$$

$$\frac{-2q}{5C} - \frac{3q}{5(2C)} = -\frac{q}{C_e}$$

$$\frac{4q+3q}{10C} = \frac{q}{C_e}$$

$$\frac{7}{10C} = \frac{1}{C_e} \Rightarrow \boxed{C_e = \frac{10C}{7}}$$

3



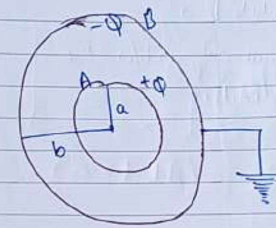
path A X Y B $\Sigma \Delta V$

$$-\frac{2q}{C} - \frac{q}{C} - \frac{2q}{C} = -\frac{6q}{C}$$

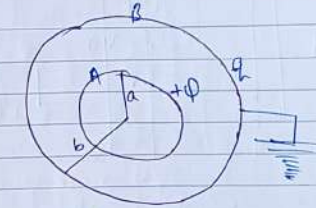
$$-\frac{5q}{C} = -\frac{6q}{C_e}$$

$$\boxed{C_e = \frac{6}{5}C} \qquad \boxed{R_e = \frac{5}{6}R}$$

Spherical Capacitor



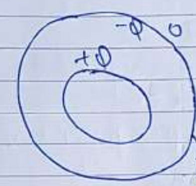
$V_B = 0$



$V_B = 0$

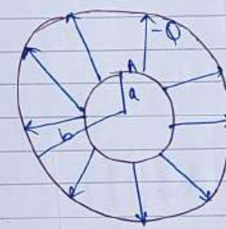
$$\frac{KQ}{b} + \frac{Kq}{b} = 0$$

$$q = -Q$$



$E_{inside\ conductor} = 0$

$$\oint E \cdot dA = \frac{q_{inside}}{\varepsilon_0} \Rightarrow \boxed{q_{in} = 0}$$



$\vec{E}$ is only b/w two spheres.

$V_B = 0$

$E_{inside\ a} = 0$, $E_{outside\ b} = 0$

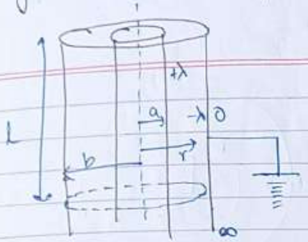
$$V_A = \frac{KQ}{a} - \frac{KQ}{b}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{V_A - V_B} = \frac{Q}{V_A}$$

$$C = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}} = \frac{1}{K\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$\boxed{C = \frac{4\pi\varepsilon_0 ab}{(b-a)}}$$

Cylindrical capacitor (∞ length) λ = charge/length



$E_{inside\ a} = 0$
$E_{outside\ b} = 0$

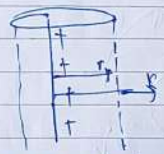
$\boxed{\text{E exist only in b/w the cylinders}}$

$$C = \frac{Q}{\Delta V} = \frac{\lambda l}{\Delta V}$$

$$C_0 = \frac{C}{l} = \frac{\lambda}{\Delta V} = \text{Capacitance per unit length}$$

$$|\Delta V| = \int E \cdot dr$$

$$= \int \frac{\lambda}{2\pi \epsilon_0 r} dr$$



$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \int \frac{1}{r}$$

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \left[\log|r|\right]_a^b$$

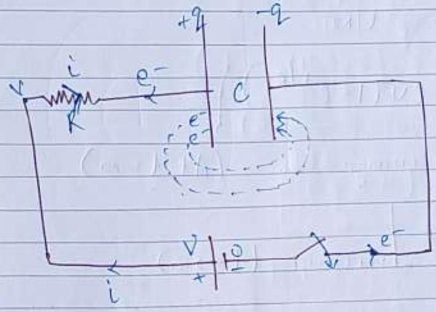
$$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \log\frac{b}{a}$$

$$C_0 = \frac{C}{l} = \frac{\cancel{\lambda}}{\frac{\cancel{\lambda}}{2\pi\epsilon_0}\log\frac{b}{a}} = \frac{2\pi\epsilon_0}{\log\frac{b}{a}}$$

$$\boxed{C_0 = \frac{C}{l} = \frac{2\pi\epsilon_0}{\log_e \frac{b}{a}}}$$

Charging & Discharging of Capacitor

Charging: Growth of charge.



* jaise jaise time charge badhta jaaega isme current kam hota jaaega

at $t = 0$
$q = 0$ on Capacitor
ΔV across Capacitor $= \frac{q}{c} = 0$

$\boxed{i = \frac{V}{R}}$ max charge

at some time 't'
ΔV across Capacitor $= \frac{q}{c}$

$$i = \frac{V - q/c}{R}$$

$\boxed{\text{charge} \uparrow \text{ and current decrease} \downarrow \text{, with time}}$

$$i = \frac{V - q/c}{R} = \frac{dq}{dt} = \frac{CV - q}{RC}$$

$$\int_0^q \frac{dq}{CV - q} = \int_0^t \frac{dt}{RC}$$

$$\left[\frac{\ln(CV - q)}{(-1)}\right]_0^q = \frac{1}{RC}(t)_0^t$$

$$\ln\left(\frac{CV-q}{CV}\right) = \frac{-t}{RC} \qquad \log_e x = y$$

$$x = e^y$$

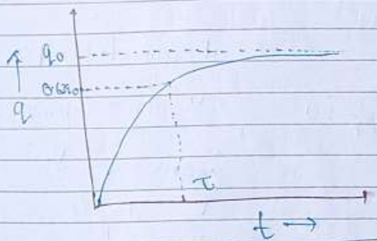
$$\frac{CV-q}{CV} = e^{-t/RC}$$

$$1 - \frac{q}{CV} = e^{-t/RC}$$

* RC = Time Constant 'τ' $\quad \boxed{q = CV(1-e^{-t/RC})}$

CV = max charge $= q_0$ (At $t=\infty$)

$\boxed{q = CV}$ max charge ($t \to \infty$)



$$\boxed{q = q_0\left(1 - e^{-t/RC}\right)}$$

q_0 = max charge = $CV \qquad \tau = RC$

when time $t = \tau$

$\boxed{\frac{1}{e} = 0.37}$

$$q = q_0(1 - e^{-\tau/\tau}) = q_0\left(1 - \frac{1}{e}\right)$$

$$q = q_0(1 - 0.37) =$$

$$\boxed{q = 0.63\, q_0}$$

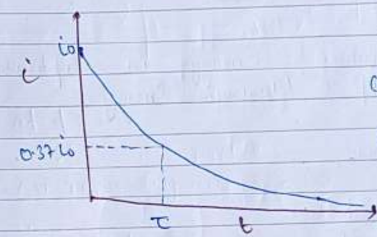
* In time τ, Capacitor is 63% charged

Current: $i = \frac{dq}{dt}$

$$= q_0\left(0 - e^{-t/RC}\right) \times \left(\frac{-1}{RC}\right)$$

$$i = \frac{q_0}{RC} e^{-t/RC} = \frac{CV}{RC} e^{-t/RC} = \frac{V}{R} e^{-t/RC}$$

$\boxed{i = i_0 e^{-t/RC}}$ At $t=0$, $i = \frac{V}{R}$ Max current $= i_0$

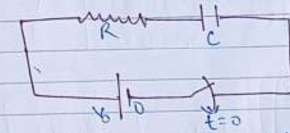


at $t = \tau$

$$i = i_0 e^{-\tau/\tau}$$

$$i = i_0\left(\frac{1}{e}\right) = 0.37\, i_0$$

Q1



Find the time in which Capacitor is 10% charged

$$q = \frac{q_0 \times 10}{100} = \frac{q_0}{10}$$

$$q = q_0(1 - e^{-t/RC})$$

$$\frac{q_0}{10} = q_0 (1 - e^{-t/RC})$$

$$\frac{1}{10} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = \frac{9}{10}$$

$$\ln e^{-t/RC} = \ln\left(\frac{9}{10}\right)$$

$$-\frac{t}{RC} \ln e = \ln(9/10)$$

$$-t/RC \times 1 = 2.303 \log_{10}(9/10)$$

$$\frac{-t}{RC} = 2.303\left(\log_{10} 9 - \log_{10} 10\right)$$

$$\frac{-t}{RC} = 2.303\left(2\log_{10} 3 - 1\right)$$

$\boxed{\log_{10} 3 = 0.47}$

$$\frac{-t}{RC} = 2.303\left(2 \cdot 0.47 - 1\right)$$

$$\frac{-t}{RC} = 2.303\left(-0.06\right)$$

$$\boxed{t = RC\,(0.138)}$$

Q Find the time in which Energy in Capacitor grows to Half of Maximum Energy?

$$E = \frac{1}{2} E_{max}$$

$$\frac{q^2}{2C} = \frac{1}{2}\frac{q_0^2}{2C}$$

$$q^2 = \frac{q_0^2}{2}$$

$$\boxed{q = \frac{q_0}{\sqrt{2}}}$$

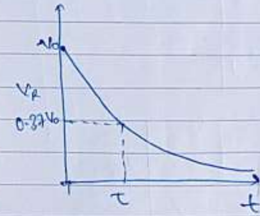
$$\boxed{\frac{q_0}{2} = q_0\left(1 - e^{-t/RC}\right)}$$

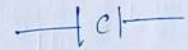
from this Eqn we can Calculate t

Q Draw the plots for potential drop in Resistor & Capacitor v/s time.

$$V_R = iR$$
$$= i_0 e^{-t/RC} \times R$$
$$= \frac{V_0}{R} \times e^{-t/RC} \times R$$

$$\boxed{V_R = V_0 e^{-t/\tau}}$$

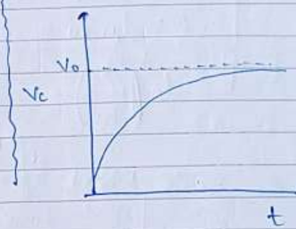




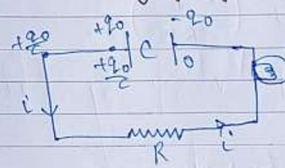
$$V_C = \frac{q}{C} = \frac{q_0}{C}\left(1 - e^{-t/RC}\right)$$

$$V_C = \frac{V_0 C}{C}\left(1 - e^{-t/RC}\right)$$

$$\boxed{V_C = V_0\left(1 - e^{-t/\tau}\right)}$$



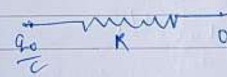
Discharging of Capacitor : Decay of charge



initially at $t = 0$

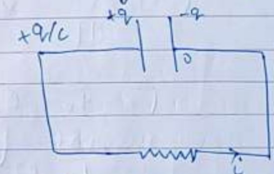
$q = q_0$ on Capacitor (max charge)

ΔV across Capacitor $= \frac{q}{C} = \frac{q_0}{C}$



$$i = \frac{q_0/C}{R} = \frac{q_0}{RC}$$

$$\boxed{i = \frac{q_0}{RC} = \text{max Current} = i_0}$$



After time 't'

$q = q$

$$i = \frac{q/C - 0}{R}$$

$$i = \frac{q}{RC}$$

Here $i = \frac{-dq}{dt}$

$\therefore -\frac{dq}{dt} = \frac{q}{RC}$

$\int_{q_0}^{q} \frac{dq}{q} = \int_0^t \frac{-dt}{RC}$

$[\ln q]_{q_0}^{q} = \frac{-1}{RC}(t)_0^t$

$\ln q - \ln q_0 = \frac{-1}{RC}(t)$

$\ln\left(\frac{q}{q_0}\right) = -\frac{t}{RC}$

$\frac{q}{q_0} = e^{-t/RC}$

$$\boxed{q = q_0 e^{-t/RC}}$$

$i = \frac{-dq}{dt} = -q_0 e^{-t/RC}\left(\frac{-1}{RC}\right)$

$i = \frac{q_0}{RC} e^{-t/RC}$

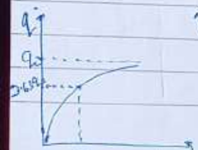
$$\boxed{i = i_0 e^{-t/RC}}$$

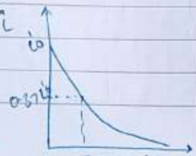
Charging

$q\uparrow \quad i\downarrow$

$q = q_0(1-e^{-t/\tau})$

$i = i_0 e^{-t/\tau} \quad \boxed{\tau = RC}$



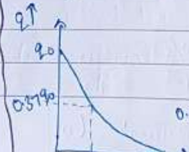


Discharging

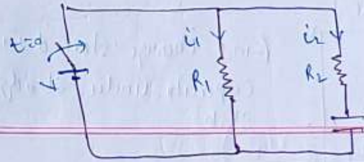
$q\downarrow \quad i\downarrow$

$q = q_0 e^{-t/\tau}$

$i = i_0 e^{-t/\tau}$

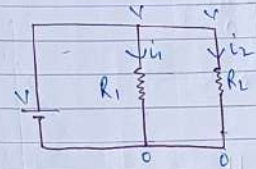


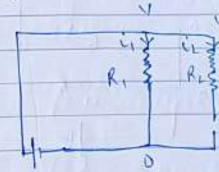
Q1



Find i_1 & i_2

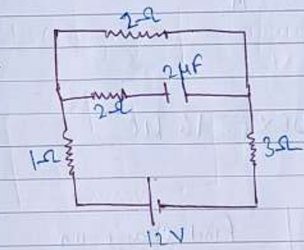
i) $t=0$ ii) At steady state



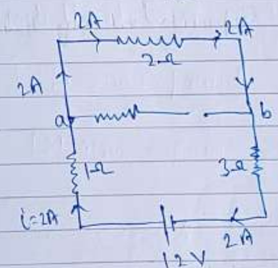


$i_1 = \frac{V}{R_1}, \; i_2 = \frac{V}{R_2}$ $\qquad$ $i_1 = \frac{V}{R_1} \qquad i_2 = 0$

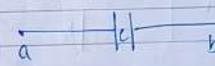
Q2



Find the charge stored in capacitor under steady state.

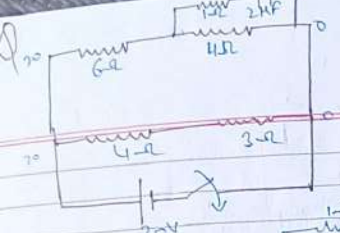


$i = \frac{V}{R_e} = \frac{12}{6} = 2\,A$



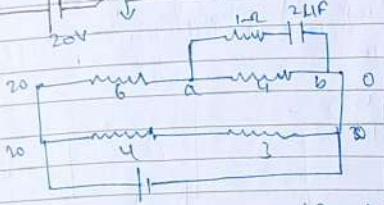
$V_{ab} = iR$
$= 2 \cdot 2\Omega$
$= 4\,V$

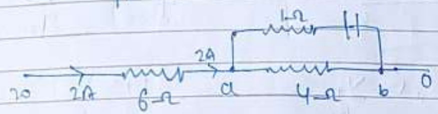
$q = CV$
$= 2\mu F \cdot 4 = 8\,\mu C$



Find charge stored in capacitor under steady state.

Sol^n



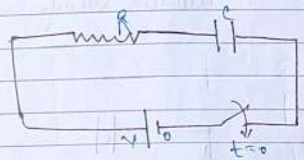


$$i = \frac{V}{R_e} = \frac{20}{10} = 2\,A$$

$$V_{ab} = 8\,V$$

$$q = CV = 2\,\mu F \times 8 = 16\,\mu C$$

Q4

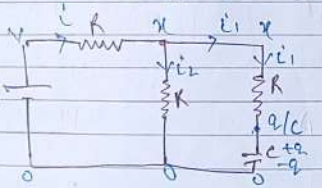


Find charge on Capacitor at any time 't'?

$$q = q_0(1 - e^{-t/RC})$$

$$q = CV(1 - e^{-t/RC})$$

Q5



Find the charge on Capacitor at any time 't'?

at $t = t$ $q = q$

ΔV across capacitor $= \frac{q}{C}$

$$i = i_1 + i_2$$

$$\frac{V-x}{R} = \frac{x - q/C}{R} + \frac{x-0}{R}$$

$$V - x = x - \frac{q}{C} + x$$

$$\frac{CV+q}{C} = 3x \implies x = \frac{CV+q}{3C}$$

$$i_1 = \frac{\frac{CV+q}{3C} - \frac{q}{C}}{R} = \frac{CV-2q}{3CR}$$

$$i_1 = \frac{dq}{dt} = \frac{CV-2q}{3RC}$$

$$\int \frac{dq}{CV-2q} = \int \frac{dt}{3RC}$$

$$\left[\frac{\ln(CV-2q)}{-2}\right]_0^q = \frac{1}{3RC}(t)_0^t$$

$$\ln(CV-2q) - \ln(CV) = \frac{-2t}{3RC}$$

$$\ln\left(\frac{CV-2q}{CV}\right) = \frac{-2t}{3RC}$$

$$\frac{CV-2q}{CV} = e^{-2t/3RC}$$

$$2q = CV(1 - e^{-2t/3RC})$$

$$q = \frac{CV}{2}(1 - e^{-2t/3RC})$$