Smart Brain-Computer Interface and Decoding Methods

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Abstract

Neural Engineering & Smart System Lab. is BCI-focused lab where they utilized machine learning, and deep learning to process brain EEG signals for health monitoring and diagnosis.

There are several steps such as signal acquisition, feature extraction, and classification, before utilizing the EEG signal. In this semester, I learn how to use Spectral Power for EEG feature extraction and utilize KNN, KFD as well as Fisher Criterion for the classification.

Here is a graph for better understanding of the general EEG signal acquisition method

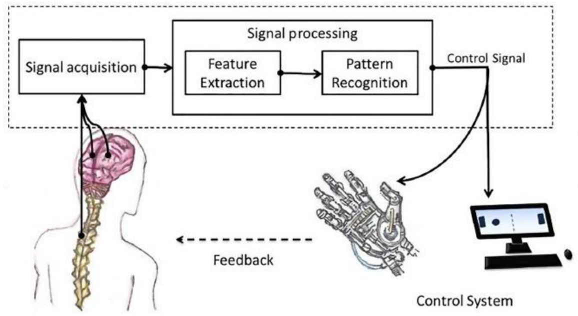

Figure 1. *EEG Signal Acquisition Method. Graph from Shiv Kumar Mudgal, Suresh K Sharma, Jitender Chaturvedi, Anil Sharma, Brain computer interface advancement in neurosciences: Applications and issues*

Introduction

Neural Engineering & Smart System Lab. is led by prof Yi-Hung Liu, who focused his research on Brain-Computer Interface (BCI), Electroencephalography-based Computer-Aided Diagnosis (EEG CADx), Translational Neuro engineering, Machine Learning, Deep Learning, Intelligent Prognosis and health Monitoring for Industrial Process and Equipment.

There are several steps before utilizing the processed EEG signal such as:

1. Signal acquisition
2. Feature Extraction
3. Classification/ Pattern recognition
4. Utilize the signal for different purposes

Motivation

Growing up I was always fascinated by the human brain as a lot of sci-fiction movies and books stated that there is much more our brain can do than what we use now. Then, I stumbled upon prof Yi-Hung Liu's research around brain and machine learning, which really spark my interest as I believe BCI will be the next generation of technology. There is a lot of research regarding EEG signal processing that has helped a lot of people, such as early detection of dementia, automated prosthetics limbs that can be moved only by the thought of moving, and tons of amazing research that I can't mention.

Description of Research Work

The brain is the most important, crucial, and unique to a human being. Brain cells communicate through electrical impulses (neural electric activity) that can be recorded as EEG signals and they vary between people.

Electroencephalogram (EEG)

In 1875, Caton discovered that the human brain could produce electrical activity. From 1929 to 1932, a work by Berger set the pioneer of recording human brain activity by placing electrodes positioned upon the intact skull.

The appearance of the EEG signal depends on the location of electrodes on the skull and the subject's state of vigilance. Whereas the factor that influenced the character of the signal can be summarized as follows [1]:

1. Age of subject
2. The mental state of the subject (degree of wakefulness, level of vigilance)
3. Region of the brain
4. Hereditary factors
5. Influence on the brain (injuries, functional disturbances, diseases, stimuli, chemical influence, drugs, etc.)
6. Disturbances (artifacts), can be technical (i.e., caused by the recording equipment or surroundings) or biological (i.e., produced by the subject, such as scalp muscle activity, arterial pulsations, eye movements, the signal from the heart).

The international 10-20 electrodes placement is the most use method of positioning the scalp electrodes. The system is based on the relationship between location of electrode and area of cerebral cortex. The number '10' and '20' refers to the distance between adjacent electrodes are either 10% or 20% of total front-back or right-left distance of the skull.

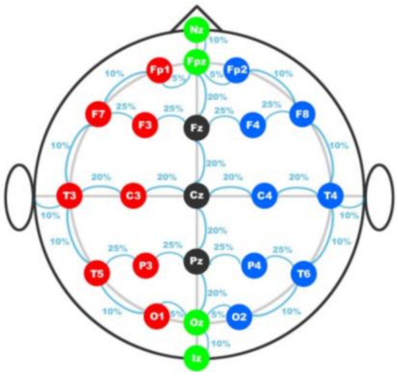

Figure 2. *10/20 System Electrode Distances. Graph from 10/20 System Positioning. Trans Cranial Technologies ltd. 2012*

Electrode	Lobe
F	Frontal
T	Temporal
C	Central *
P	Parietal
O	Occipital

Table 1. *Electrode "Code" and Its Corresponding Lobe. Graph from 10/20 System Positioning. Trans Cranial Technologies ltd. 2012*

The students who participated in this lab were grouped, taught new concepts, and given homework weekly by the seniors. Therefore, here are several concepts we have learned:

1. **Spectral Power**

Power Spectral Density (PSD) was used as a feature extraction method for EEG wave. PSD distributes the signal power over frequency and shows the power as a function of frequency. The signals coming from our brain are noisy, non-stationary, complex, and high - dimensional. EEG PSD is computed based on the parametric autoregressive (AR) model which provides information on signal power from each relatively narrow frequency sub-band (Brain wave) that gives better frequency resolution [2][3].

$$\mathbf{PSD}[n] = |\mathbf{F}[n]|^2, n = 0,1,2,\dots,N-1$$

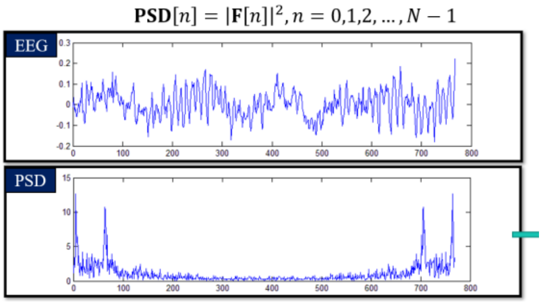

Figure 3. *Comparison Between EEG Signal and EEG PSD Signal*

EEG contains different specific frequency bands, and the feature in sub-bands are important to characterize different brain states. Here are 5 widely recognized sub-bands of human's EEF frequency waves [4]:

Frequency band	Frequency	Brain states
Gamma (γ)	>35 Hz	Concentration
Beta (β)	12–35 Hz	Anxiety dominant, active, external attention, relaxed
Alpha (α)	8–12 Hz	Very relaxed, passive attention
Theta (θ)	4–8 Hz	Deeply relaxed, inward focused
Delta (δ)	0.5–4 Hz	Sleep

Table 2. *Characteristic of The Five Basic Brain Waves. Table from Priyanka A. Abhang, Bharti W. Gawali, Suresh C. Mehrotra, Chapter 2 - Technological Basics of EEG Recording and Operation of Apparatus*

Power Spectral Density (PSD)

Characteristic of a power spectral density function, G: (1) must be in a continuous function, (2) $G(f_x)$ is proportional to the power in the sinusoidal signals with frequency f_x, and (3) the sum of power in the constituent waveform equals the total average power of the signal.

The average power of a signal for an interval of [0, T] can be defined as:

$$\bar{P}(T) = \frac{1}{T}\int_0^T |x(t)|^2\, dt$$

signal x substitute with the Fourier series into:

$$\bar{P}(T) = |a_0|^2 + 0.5\sum_{i=1}^{\infty}|a_i|^2 + |b_i|^2$$

$$= \sum_{i=-\infty}^{\infty}\frac{|X(T, if_0)|^2}{T^2}$$

the associated signal frequency of if_0 Hz, namely $a_i\cos(2\pi i f_0 t)$ and is given by

$$|c_{-i}|^2 + |c_i|^2 = (|a_i|^2 + |b_i|^2)/2$$

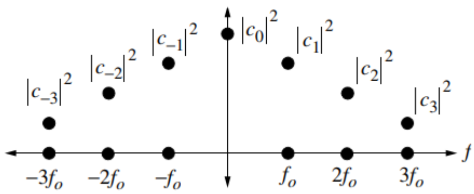

Figure 4. *Display of Power in Sinusoidal Components of A Signal. Graph from Roy M. Howard, "The Power Spectral Density," in* Principles of Random Signal Analysis and Low Noise Design: The Power Spectral Density and its Applications *, IEEE, 2002, pp.59-91, doi: 10.1002/0471439207.ch3.*

based on the relationship:

$$c_i = X(T, if_0)/T$$

and Figure.1 can be constructed as:

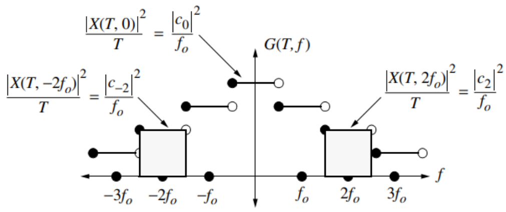

Figure 5. *A Power Spectral Density Function. The shaded areas equal the power associated with sinusoidal components that have a frequency of 2fₒ Hz. Graph from Roy M. Howard, "The Power Spectral Density," in* Principles of Random Signal Analysis and Low Noise Design: The Power Spectral Density and its Applications *, IEEE, 2002, pp.59-91, doi: 10.1002/0471439207.ch3.*

and by Parseval's relationship:

$$\int_0^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(T,f)|^2 df$$

scaling both integrals by T yields results in:

$$\bar{P}(T) = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} \frac{|X(T,f|^2}{T} df$$

$$= \int_{-\infty}^{\infty} G(T,f) df$$

So, signal power spectral density x on interval [0, T] is defined according to:

$$G(T,f) = \frac{|X(T,f)|^2}{T}$$

and was known as sample spectral density (Jenkins, 1968 p.211; Parzen, 1962 p.109) [5].

***Assignment**

The given assignment provided 2 data, Alzheimer's Disease (AD) and Healthy Control (HC) with each data consisting of 30 channels, 45000 data length, and the sampling rate of 500 Hz. The purpose of this task is feature extraction for both AD and HC data into 6 sub-bands, consisting of delta, theta, alpha, beta low, beta high, and gamma with the frequency band of interest:

δ	band :	1~4Hz
θ	band :	4~8Hz
α	band :	8~13Hz
β_{low}	band :	13~20Hz
β_{high}	band :	20~30Hz
γ	band :	30~45Hz

Table 3. *Frequency Band of Interest*

Methodology

Load the data of both AD and HC, and cut the data from 30x45000 into 30x4500. Apply PSD by Fast Fourier

Transform (FFT). Then obtain the Spectral Power by summation of PSD in the same bands (delta, theta, alpha, beta low, beta high, and gamma) by:

$$\boldsymbol{SP}_{w^*} = \sum_{n \in w^*} \boldsymbol{PSD}[n]$$

Where: w^* : Frequency band of interest

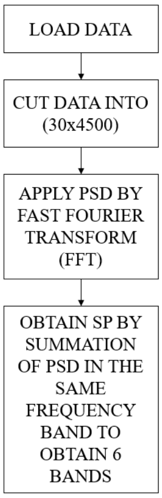

Figure 6. *Spectral Power Methodology Graph*

The result can be seen in *Appendix I*

2. K-Nearest Neighbor (K-NN)

The K-nearest neighbor (KNN) is a supervised machine learning algorithm that is used for classification purposes because of its high adaptive and easy-to-understand design.

K-NN Algorithm

The algorithm consists of parameter *k*, which indicates the number of nearest neighbor(s) from the training dataset. The nearest neighbor(s) is determined by the shortest distance from the test point to the training dataset, then a majority vote is conducted to determine which class the test point belongs to [6].

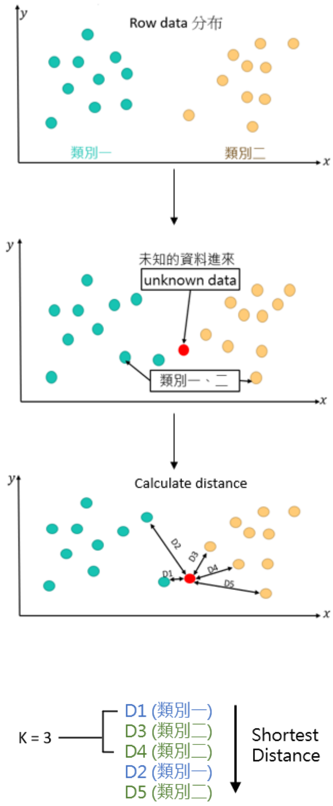

Figure 7. *K-NN Algorithm Visualization*

Leave-One-Out Cross-Validation (LOO-CV)

Cross-validation is a resampling method to separate data into test and train models on different iterations to estimate the accuracy of the classification or regression models.

This cross-validation technique takes the number of folds equal to the number of iteration of datasets. Meaning the learning algorithm is applied once for each dataset, making all other datasets becomes training sets and leaving one as the test (Efron, 1982).

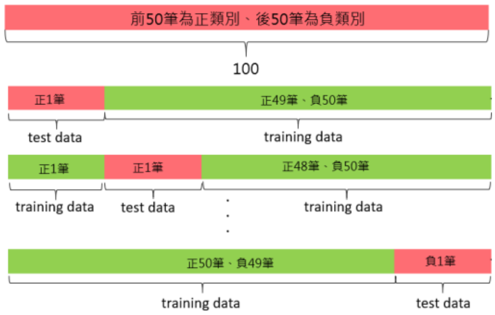

Figure 8. *LOO-CV Visualization*

***Assignment**

The given assignment provided an Iris dataset with 150 types of iris data and 4 features consist of sepal length, sepal width, petal length, petal width, and 3 labels. The purpose of this task is to apply the K-NN algorithm in LOO-CV.

Methodology

Load the iris dataset. Separate the iris dataset into: (1) X = feature (features of sepal length, sepal width, petal length, petal width) and (2) y = labels (1,2,3). Apply LOO-CV, split the train, and test, in each iteration leaving 1 data for test and the rest to train. Apply K-NN in each iteration, set the value of *k,* find the shortest distance from the test point in the training dataset, sort from the closest one to the farthest one and take the first until *kth* data, take the label of the data, and apply majority vote for the most common data will be the predicted label of the test data. Finally, find the accuracy by comparing the predicted label with the label. Take the score of each iteration and take the average for the accuracy of the model.

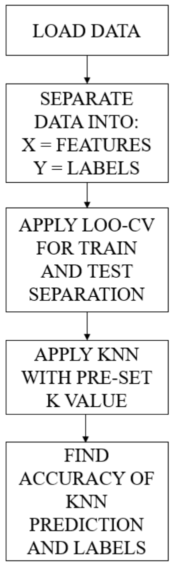

Figure 9. *K-NN Methodology Graph*

The result can be seen in *Appendix II*

3. Fractal Dimension

Measurement of how "complicated" a self-similar figure is over a certain time interval, and can be expressed with:

$$D = \frac{\log(L)}{\log(d)}$$

Where:

L = sum(dist(i,i+1))

d = max(dist(1,i))

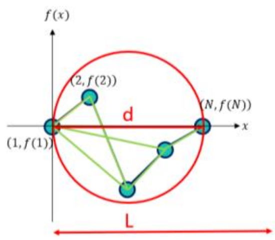

Figure 10. *Katz Fractal Dimension*

***Assignment**

The given assignment provided a dataset of 30x1500. The purpose of this task is to apply Katz's Fractal Dimension.

Methodology

Using equation:

$$D = \frac{\log(L)}{\log(d)}$$

and normalization became:

$$D = \frac{\log\left(\frac{L}{a}\right)}{\log\left(\frac{d}{a}\right)}\ , where\ n = \frac{L}{a}$$

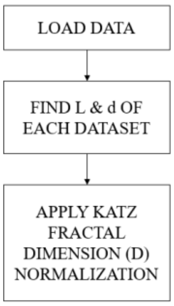

Figure 11. *KFD Methodology Graph*

The result can be seen in *Appendix III*

4. **Fisher Criterion**

Fisher criterion is a discriminant criterion function that is defined by the ratio between-class scatter to within-class scatter. By optimizing this, one can obtain an optimal discriminant projection axis. When being projected onto this projection axis, the within-class scatter is minimized whereas the between-class scatter is maximized [7].

Fisher score can be obtained by:

$$Fisher\ score(k) = \frac{{S_B}^{(k)}}{{S_W}^{(k)}}$$

$$(kth\ diagonal\ value)$$

Whereas:

S_B is between-class scatter, and can be obtained from:

$$S_B = \sum_{i=1}^{C} P_i\,(m_i - m)^T (m_i - m)$$

S_W is within-class scatter, and can be obtained from:

$$S_w = \sum_{i=1}^{C} P_i\, S_i\,, (where\ P_i\ = \frac{n_i}{\sum_{i=1}^{C} n_i})$$

and S_i is sample covariance matrix for class *i*:

$$S_i = \frac{1}{n_i}\sum_{j=1}^{n_i}(x_{ij} - m_i)^T (x_{ij} - m_i)$$

***Assignment**

The given assignment provides 7 EEG data of 3 AD and 4 HC with the data set of 30x45000. The purpose of this assignment is to find the Fisher Score (*k*).

Methodology

Load the given data 3 AD and 4 HC with the size of 30x45000 and cut the data into 30x1500. Take the SP of each class and categorize it into 6 bands (delta, theta, alpha, beta low, beta high, and gamma). Find the fisher score of the 180 features and sorts them from highest to low.

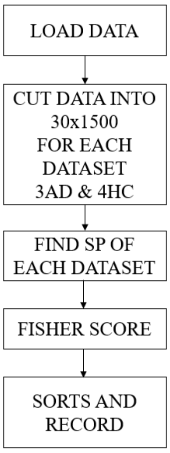

Figure 12. *Fisher Criterion Method Graph*

The targeted results can be seen in *Appendix IV*

Conclusion

I learn a lot throughout this 1 semester. It's not easy but I would love to learn more deeply about the process and data manipulation as a whole. Prof Yi-Hung Liu took into consideration how the student will learn in his lab without effectively without feeling burdened. Indirectly, these given assignments introduced me to the lab research and helped me understand the step-by-step process. I would love to explore further and deeper about smart brain-computer interface and decoding methods, and would be honored if given the chance of helping research around this topic area.

Throughout this class, I learn that the most common feature extraction of EEG signal is through SP, and then the classification process can use any machine learning algorithm best suits the purpose of research as in this, we utilized KNN algorithm, and Fisher Criterion.

Lastly, I also want to apologize for my slow learning process. All of the seniors in the lab are really nice as they are always ready to help and discuss, but with the pandemic that happened during this semester, it gets really hard and uncomfortable to go to the laboratory and risk others' health, thus I mostly conduct everything by researching online and translate the material taught by seniors, but overal I really enjoy the process that I found myself late at night reading, analyzing, writing the algorithms and debug it.

Reference

[1] A. Isaksson, A. Wennberg and L. H. Zetterberg, "Computer analysis of EEG signals with parametric models," in Proceedings of the IEEE, vol. 69, no. 4, pp. 451-461, April 1981, doi: 10.1109/PROC.1981.11988.

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[3] Wei Bin Ng et al 2019 IOP Conf. Ser.: Mater. Sci. Eng. 557 012032

[4] Priyanka A. Abhang, Bharti W. Gawali,

Suresh C. Mehrotra, Chapter 2 - Technological Basics of EEG Recording and Operation of Apparatus

[5] Roy M. Howard, "The Power Spectral Density," in Principles of Random Signal Analysis and Low Noise Design: The Power Spectral Density and its Applications , IEEE, 2002, pp.59-91, doi: 10.1002/0471439207.ch3.

[6] Uddin, S., Haque, I., Lu, H. et al. Comparative performance analysis of K-nearest neighbour (KNN) algorithm and its different variants for disease prediction. Sci Rep 12, 6256 (2022). https://doi.org/10.1038/s41598-022-10358-x

[7] (2009) Fisher Criterion. In: Li S.Z., Jain A. (eds) Encyclopedia of Biometrics. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-73003-5_585

Code can be accessed via Github: https://github.com/Madibas227/Special-Topic.git

Appendix I

AD Spectral Power

	Delta	Theta	Alpha	Beta_low	Beta_high	Gamma
1	4.87E+07	2.15E+07	1.36E+07	1.64E+07	1.78E+07	2.10E+07
2	5.29E+07	2.81E+07	2.96E+07	3.08E+07	4.31E+07	5.93E+07
3	4.86E+07	2.10E+07	1.36E+07	1.55E+07	1.39E+07	1.35E+07
4	3.81E+07	1.61E+07	1.65E+07	1.22E+07	8.69E+06	1.01E+07
5	1.41E+07	7.16E+06	5.22E+06	6.04E+06	4.86E+06	5.12E+06
6	3.53E+07	1.53E+07	1.27E+07	9.39E+06	9.46E+06	1.17E+07
7	6.18E+07	2.24E+07	1.44E+07	1.85E+07	1.59E+07	1.42E+07
8	4.83E+07	1.46E+07	2.01E+07	1.29E+07	8.31E+06	8.90E+06
9	1.81E+07	1.06E+07	1.58E+07	1.97E+07	3.39E+07	4.98E+07
10	3.10E+07	1.02E+07	1.52E+07	9.36E+06	8.10E+06	1.07E+07
11	5.47E+07	2.35E+07	1.55E+07	2.06E+07	2.08E+07	2.31E+07
12	4.85E+07	1.75E+07	2.35E+07	1.68E+07	1.89E+07	2.69E+07
13	1.88E+07	1.22E+07	1.29E+07	1.60E+07	1.90E+07	2.21E+07
14	2.43E+07	1.08E+07	2.48E+07	1.24E+07	1.82E+07	2.25E+07
15	7.98E+07	2.40E+07	1.69E+07	1.51E+07	1.44E+07	8.20E+06
16	4.52E+07	1.55E+07	2.55E+07	1.29E+07	1.15E+07	1.31E+07
17	2.31E+07	1.41E+07	1.66E+07	2.58E+07	2.12E+07	3.60E+07
18	2.49E+07	1.76E+07	2.38E+07	1.48E+07	1.99E+07	2.14E+07
19	5.04E+07	2.23E+07	1.71E+07	1.04E+07	1.07E+07	8.47E+06
20	5.16E+07	1.78E+07	2.22E+07	1.05E+07	1.10E+07	1.03E+07
21	2.70E+07	1.00E+07	1.34E+07	9.12E+06	1.07E+07	1.43E+07
22	2.50E+07	7.93E+06	1.22E+07	5.55E+06	7.19E+06	9.05E+06
23	3.29E+07	1.54E+07	1.95E+07	1.74E+07	3.04E+07	3.62E+07
24	3.07E+07	9.20E+06	6.28E+06	5.88E+06	5.26E+06	4.36E+06
25	8.06E+07	2.45E+07	2.03E+07	1.70E+07	1.33E+07	1.34E+07
26	1.03E+08	2.85E+07	2.51E+07	1.83E+07	1.59E+07	1.18E+07
27	1.04E+08	3.10E+07	2.52E+07	2.05E+07	1.89E+07	1.15E+07
28	1.09E+08	3.36E+07	2.52E+07	1.74E+07	1.77E+07	1.09E+07
29	7.63E+07	3.18E+07	2.19E+07	1.40E+07	1.33E+07	8.30E+06
30	3.06E+07	1.10E+07	8.51E+06	8.15E+06	9.63E+06	1.10E+07

HC Spectral Power

	Delta	Theta	Alpha	Beta_low	Beta_high	Gamma
1	1.59E+08	3.32E+07	3.78E+07	3.69E+07	5.52E+07	6.79E+07
2	1.44E+08	3.31E+07	3.01E+07	3.32E+07	4.53E+07	5.59E+07
3	1.01E+08	4.61E+07	3.46E+07	3.50E+07	4.23E+07	4.72E+07
4	9.95E+07	4.90E+07	3.91E+07	4.12E+07	4.86E+07	4.83E+07
5	8.48E+07	2.19E+07	2.22E+07	2.62E+07	3.55E+07	4.72E+07
6	6.98E+07	2.34E+07	2.96E+07	3.82E+07	4.79E+07	6.11E+07
7	8.29E+07	4.26E+07	3.90E+07	3.53E+07	4.86E+07	4.86E+07
8	8.00E+07	4.92E+07	3.54E+07	4.34E+07	5.24E+07	4.83E+07
9	5.45E+07	1.39E+07	2.02E+07	2.65E+07	4.32E+07	6.14E+07
10	3.46E+07	1.87E+07	1.97E+07	2.98E+07	4.03E+07	4.79E+07
11	7.52E+07	3.05E+07	2.53E+07	4.43E+07	4.58E+07	5.15E+07
12	7.57E+07	3.33E+07	3.03E+07	4.23E+07	5.20E+07	4.58E+07
13	1.97E+08	7.53E+07	6.13E+07	2.00E+08	2.14E+08	3.01E+08
14	6.74E+07	1.77E+07	2.81E+07	8.97E+07	1.11E+08	2.15E+08
15	4.37E+07	2.41E+07	2.74E+07	4.46E+07	4.92E+07	5.42E+07
16	6.55E+07	3.17E+07	3.08E+07	4.88E+07	5.47E+07	4.48E+07
17	3.46E+07	1.41E+07	1.76E+07	2.94E+07	3.85E+07	4.53E+07
18	2.80E+07	1.69E+07	2.56E+07	5.28E+07	7.00E+07	1.32E+08
19	4.41E+07	1.71E+07	2.44E+07	3.76E+07	4.18E+07	4.79E+07
20	5.66E+07	2.23E+07	2.82E+07	4.35E+07	4.97E+07	4.95E+07
21	3.34E+07	1.29E+07	1.69E+07	3.46E+07	4.33E+07	6.67E+07
22	1.02E+08	1.47E+07	2.22E+07	3.00E+07	4.89E+07	5.52E+07
23	3.78E+07	1.28E+07	1.89E+07	3.66E+07	6.32E+07	9.48E+07
24	4.39E+07	1.61E+07	1.75E+07	2.99E+07	5.51E+07	6.93E+07
25	9.89E+07	7.71E+07	5.43E+07	4.19E+07	4.93E+07	4.56E+07
26	9.39E+07	5.79E+07	4.22E+07	5.04E+07	4.86E+07	4.84E+07
27	8.73E+07	5.94E+07	4.10E+07	5.08E+07	5.59E+07	4.70E+07
28	7.69E+07	4.25E+07	3.92E+07	5.69E+07	5.95E+07	5.01E+07
29	6.47E+07	3.35E+07	3.30E+07	5.27E+07	5.98E+07	5.13E+07
30	7.55E+07	1.75E+07	1.84E+07	3.55E+07	5.95E+07	6.97E+07

Appendix II

k=1	
Feature	Accuracy
1	67.33%
2	43.33%
3	96.00%
4	94.66%
1234	93.33%

k=3	
Feature	Accuracy
1	77.33%
2	35.33%
3	96.00%
4	94.66%
1234	94.00%

Appendix III

KFD	
1	1.04189
2	1.0912
3	1.03291
4	1.02495
5	1.01587
6	1.03093
7	1.03768
8	1.02313
9	1.07097
10	1.02809
11	1.0602
12	1.04279
13	1.04049
14	1.0383
15	1.02845
16	1.03203
17	1.05296
18	1.04617
19	1.02906
20	1.02981
21	1.0361
22	1.02259
23	1.07196
24	1.01482
25	1.03499
26	1.03798
27	1.03035
28	1.03478
29	1.03019
30	1.03429

Appendix IV

排名	F_score	特徵	排名	F_score	排名	F_score	排名	F_score	排名	F_score	排名	F_score
1	3.276957593	ch6_gamma	31	0.492049222	61	0.316716442	91	0.164762548	121	0.06083105	151	0.014034102
2	2.783473982	ch10_gamma	32	0.48828667	62	0.293639891	92	0.157593937	122	0.058345512	152	0.013646657
3	1.9310432	ch30_gamma	33	0.483205593	63	0.286498026	93	0.152974207	123	0.058264433	153	0.013027966
4	1.586621049	ch24_gamma	34	0.480031513	64	0.285065162	94	0.151964108	124	0.055169316	154	0.012820114
5	1.413206701	ch22_gamma	35	0.479658732	65	0.284801209	95	0.147153013	125	0.054133283	155	0.011623198
6	1.373359103	ch14_delta	36	0.461356318	66	0.276421002	96	0.146377612	126	0.053666832	156	0.011495273
7	1.02113283		37	0.459753348	67	0.267610571	97	0.144707212	127	0.049668007	157	0.011355602
8	0.919925106		38	0.45931999	68	0.266881582	98	0.138638511	128	0.048844339	158	0.011350332
9	0.860921103		39	0.453262262	69	0.258168042	99	0.135125245	129	0.048216357	159	0.009338124
10	0.857779034		40	0.442241521	70	0.248463568	100	0.133804698	130	0.048014084	160	0.008394615
11	0.843843591		41	0.43706112	71	0.247977907	101	0.13156988	131	0.043587256	161	0.008081214
12	0.797622519		42	0.429872732	72	0.244977231	102	0.119322928	132	0.040921045	162	0.007671902
13	0.75716213		43	0.422137478	73	0.244305541	103	0.119309802	133	0.039282872	163	0.007107412
14	0.718710946		44	0.412379	74	0.239940111	104	0.117171851	134	0.035905969	164	0.006973864
15	0.702377385		45	0.404935226	75	0.233899688	105	0.113905444	135	0.033915972	165	0.006867406
16	0.701217573		46	0.397661235	76	0.229184199	106	0.111833115	136	0.03264081	166	0.005851896
17	0.680354686		47	0.396763788	77	0.22234373	107	0.107350334	137	0.030977808	167	0.004655588
18	0.649992034		48	0.385242847	78	0.216366086	108	0.099169633	138	0.028443338	168	0.002813945
19	0.633150238		49	0.370140652	79	0.21521637	109	0.099097692	139	0.027437788	169	0.002674542
20	0.591034604		50	0.369960774	80	0.211388632	110	0.095799065	140	0.025696647	170	0.001837318
21	0.580558185		51	0.369566947	81	0.203929299	111	0.092645001	141	0.023695418	171	0.00165562
22	0.550941022		52	0.364769981	82	0.203003885	112	0.086107573	142	0.023141518	172	0.001245614
23	0.550547064		53	0.359938286	83	0.202484935	113	0.081971981	143	0.022685835	173	0.001075458
24	0.528950101		54	0.359508313	84	0.193965986	114	0.077047746	144	0.020437053	174	0.000938987
25	0.521365077		55	0.334829513	85	0.18638645	115	0.076561055	145	0.020332816	175	0.000785684
26	0.514111705		56	0.332551157	86	0.177171409	116	0.068989751	146	0.020150735	176	0.00052595
27	0.510885322		57	0.331193904	87	0.176117101	117	0.066503869	147	0.019562703	177	0.000352956
28	0.510724162		58	0.323009791	88	0.174220155	118	0.065772028	148	0.018755323	178	3.88E-05
29	0.507818143		59	0.322608232	89	0.166395459	119	0.065511284	149	0.018470326	179	3.25E-05
30	0.498251948		60	0.31940463	90	0.16613006	120	0.062051212	150	0.017501902	180	2.08E-05

**For this assignment my result can be found in my Github, as I need extra time to finish this and the deadline for the report is already due.