A Theory of Cloud Bandwidth Pricing for Video-on-Demand Providers

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Abstract

Current-generation cloud computing is offered with usage-based pricing, with no bandwidth capacity guarantees, which is however unappealing to QoS-sensitive applications such as video-on-demand (VoD) or online gaming. With the emerging technology of bandwidth reservation in datacenter networks, we consider a new type of service where cloud tenants, such as Netflix and OnLive (a gaming company), can make reservations for bandwidth guarantees from the cloud at negotiable costs to support continuous media streaming. We argue that it is beneficial to coordinate requests and multiplex such bandwidth reservations using a profit-making *broker* in the market while controlling the performance risks. We ask the question—under what kind of economic conditions can a profit-driven broker really assist cloud bandwidth consolidation and benefit tenants by lowering the bandwidth price? We give such conditions and prove that the market has a unique Nash equilibrium where the unit bandwidth reservation cost for a tenant critically depends on its demand burstiness and demand correlation to the market. We propose a novel dynamic pricing scheme that determines the bandwidth price for each tenant based on online estimation of its demand statistics. The evaluation based on a real-world VoD workload dataset shows that our theory can lower the market price for cloud bandwidth reservation by around 50%.

Index Terms

Cloud computing, pricing, bandwidth reservation, broker, demand statistics, multimedia workload.

I. Introduction

Cloud computing is gradually changing the way that multimedia content providers operate their businesses, including video-on-demand (VoD) and online gaming companies. Traditionally, these companies invest in commodity servers as business grows and acquire monthly bandwidth deals from Internet Service Providers (ISPs). Nowadays, they can be freed from the complexity of hardware maintenance and network administration, relying on the cloud for service. One particular example is that Netflix [1] moved its streaming servers, encoding software, search engines, and huge data stores to Amazon Web Services (AWS), all in 2010 [2]. Moreover, many interactive gaming companies, e.g., OnLive [3], also depend on the cloud to stream their content to online players.

Since multimedia workload has stringent streaming rate requirements, each content provider needs a guarantee of server outgoing bandwidth to sustain continuous media delivery. However, as all tenants share the underlying network in a public cloud, the unpredictable bandwidth or network share is often one of the major causes of performance issues [4], [5]. The good news is that recent developments on datacenter engineering are enabling bandwidth reservation in virtual machines (VMs) [6] as well as providing the tenant with a virtual network connecting its computing instances with guaranteed bandwidth [7].

Although current cloud providers charge its tenants a bandwidth fee in a pay-as-you-go model based on the number of bytes transferred in each hour [5], we believe that, when multiple content providers (such as Netflix or OnLive) are on board to use cloud services from cloud providers (such as Amazon), there will be a *market* between content providers and cloud providers, and commodities to be traded in such a market consist of *bandwidth reservations*, so that content streaming performance can be guaranteed. As a buyer in such a market, each tenant in the cloud can periodically make reservations for bandwidth capacity to satisfy its demand projection. Specifically, each tenant should reserve capacity that accommodates both its expected demand and unexpected demand variation in the predictable future. The expectation and variance of its future demand can be estimated online using historical demand information, which is easily obtainable from cloud monitoring services (e.g., Amazon CloudWatch provides a free resource monitoring service to AWS customers at a 5-minute frequency [8]).

However, since demands at all tenants are unlikely to surge simultaneously, the aggregate reserved cloud bandwidth — that the tenants have paid for — are under-utilized most of the time. A classical way to improve utilization is to encourage statistical multiplexing and reserve for multiple tenants together to accommodate their combined peak load instead of each tenant's peak load individually. As diversification and anti-correlation come into effect, less cushion capacity needs to be reserved to guard against unexpected demand fluctuation. Apparently, improving utilization is a win-win measure: better utilization of bandwidth at cloud providers may lead to reduced pricing for bandwidth reservations in the market, eventually benefiting content providers. Given multiple cloud providers in the market, we ask the question — can we use a simple mechanism inherent in the market to optimally mix tenant demands, save total cloud bandwidth reservation for tenants and thus lower the market bandwidth price?

The answer is yes. Cloud brokerage has recently emerged as a cloud service intermediator connecting buyers and sellers of computing resources. For example, Zimory [9], as a spinoff of Deutsche Telekom, claims to be the first online marketplace for cloud computing. In this paper, we propose an entirely new type of brokerage that collects bandwidth demand statistics from content providers, selling them probabilistic performance guarantees, while reserving as few resources as possible from multiple cloud providers for them. The broker charges each VoD provider a fee for the service guarantee provided according to a certain pricing policy, while paying cloud providers their fees for bandwidth reservations. In a healthy market, the broker should be able to (1) make a profit, an incentive for all selfish entities, (2) save the total resource reservation by optimally mixing demands, and (3) lower bandwidth prices billed to content providers to attract business.

We present an in-depth analysis of bandwidth pricing policies in such a broker-assisted market, and make a number of original contributions with our analysis.

First, the broker needs to decide a load routing matrix from all tenants to all cloud providers. Load routing should be such that tenant demands are mixed based on anti-correlation to save the aggregate capacity reservation from all cloud providers. We formulate this problem as convex optimization and give a nearly close-form solution to the problem while drawing insights.

However, a selfish broker may not have the incentive to mix and direct demand optimally; it may even reject demand if accepting it is not lucrative — the broker is yet always driven by profit. Our second contribution is thus to study how to enforce a good pricing policy (to be adopted by the broker), such that a profit-maximizing broker behaves in the same way as an altruistic broker that saves the cloud bandwidth reservation through optimal load routing. We give a necessary and sufficient condition for all such "good" pricing policies. Such a pricing region characterization also helps us to understand the maximum price discount that each tenant can enjoy in a healthy market.

Third, we study a free market where the pricing policy are not enforced authoritatively. Instead, each tenant can submit a pricing strategy at its own will, while the broker makes admission decisions to each tenant based on all the submitted prices. We prove that the selfishness of tenants will inevitably drive their chosen pricing strategies into a unique Nash equilibrium, at which each tenant enjoys the lowest possible price in the "good" pricing region. As a highlight, we derive the equilibrium payment of each tenant in a close-form function of its expected demand, demand burstiness as well as its demand correlation to the market.

Finally, we conduct online bandwidth trading simulations driven by the real-world workload of more than 200 video channels from an operational VoD system over two 800-minute periods. We have developed a novel dynamic pricing scheme in a broker-assisted market that varies the price charged to each tenant periodically based on prediction of its demand statistics. Such demand-statistics-based pricing is different from the traditional usage-based pricing. We observe that with the help of a broker, our theory can lower the market price for cloud bandwidth reservations by around 50% on average, and save cloud resources by over 30%.

The remainder of this paper is organized as follows. We describe our system model in Sec. II and outline main contributions in Sec. III. In Sec. IV, we give an efficient solution to optimal cloud resource multiplexing. We study pricing in controlled and free markets in Sec. V and VI, respectively. We present simulation results in Sec. VII. Sec. VIII reviews related work, and Sec. IX concludes the paper.

II. System Model

With broker-assisted bandwidth reservation, the system operates periodically on a short-term basis. At the beginning of a short-term period, each tenant makes a bandwidth reservation at the broker to satisfy its future bandwidth demand. The broker attempts to statistically multiplex bandwidth demands from the tenants to save the total bandwidth reservation cost, while providing each tenant with a certain level of service guarantee. At the end of the short-term period, the broker charges each tenant a fee for the service guarantee.

From the perspective of tenants, having the ability to reserve bandwidth on a short-term basis offers a unique advantage. Many user-faced multimedia workloads, such as VoD bandwidth demand and online gaming, involve diurnal periodicity and are highly predictable even at a fine granularity of 10-minute intervals [10], [11]. Tenants running predictable workloads are able to dynamically reserve (and pay for) bandwidth that just suffices to accommodate the predicted short-term demand, leading to a cost-effective operation mode. From the perspective of cloud providers, offering users the ability to make bandwidth reservations constitutes an important performance guarantee, which is a desirable feature that makes the cloud provider more attractive to QoS-sensitive tenants.

A. The Cloud and Tenants

We now present our system model in more detail. We start from a description of cloud providers and tenants.

Cloud Providers. We assume there are S cloud providers in the system, each of which has an outgoing bandwidth capacity C_s, $s = 1, \ldots, S$. Let C_{sum} be the aggregate bandwidth capacity of all the cloud providers, i.e., $C_{\mathsf{sum}} = \sum_s C_s$. Throughout this paper, we assume that C_{sum} is sufficiently large to satisfy all the demands in the system. This assumption is justified by the "illusion of infinite capacity" [5] and the fact that costs of high-end routers are dropping more quickly than before.

Tenants. We assume there are N tenants in the system, each of which reserves bandwidth on a short-term basis. Without loss of generality, we consider one such short-term period. Suppose that tenant i's bandwidth demand in this period is a random variable D_i with mean μ_i and variance σ_i^2. To ensure that its demand D_i is satisfied with a high probability $1-\epsilon$, tenant i needs to reserve bandwidth $B_i = f_\epsilon(D_i)$, which is $1-\epsilon$ percentile of the distribution of D_i, i.e., we have $\Pr(D_i > B_i) \leq \epsilon$, where ϵ is defined as the *risk factor*.

We next introduce several useful notations with regard to the random demands of the N tenants: $D_1, \ldots, D_N$. Let $\mathbf{D} = [D_1, \ldots, D_N]^\mathsf{T}$, $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_N]^\mathsf{T}$ and $\boldsymbol{\sigma} = [\sigma_1, \ldots, \sigma_N]^\mathsf{T}$. Note that the random demands $D_1, \ldots, D_N$ may be highly correlated due to the correlation between video genres, viewer preferences and video release times. Denote ρ_{ij} the correlation coefficient of D_i and D_j, with $\rho_{ii} \equiv 1$. Let $\boldsymbol{\Sigma} = [\sigma_{ij}]_{N\times N}$ be the $N \times N$ symmetric demand covariance matrix, with $\sigma_{ii} = \sigma_i^2$, and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j, \quad \text{for } i \neq j$.

For convenience, denote σ_M the standard deviation of the "market" demand $\sum_i D_i$, and σ_{iM} the covariance between D_i and the market demand $\sum_i D_i$. Clearly, we have

$$\sigma_{iM} = \mathbf{E}[(D_i - \mu_i)(\sum_j D_j - \sum_j \mu_j)] = \sum_{j=1}^{N} \sigma_{ij}, \tag{1}$$

$$\sigma_M = \sqrt{\mathbf{Var}[\sum_i D_i]} = \sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}} = \sqrt{\sum_{i,j} \sigma_{ij}}. \tag{2}$$

We further denote $\rho_{iM} = \sigma_{iM}/\sigma_i\sigma_M \in [-1, 1]$ the correlation coefficient between D_i and the "market" demand $\sum_i D_i$.

B. The Broker and Load Routing Matrix

As shown in Fig. 1, the essence of the broker is a *load routing weight matrix* $\mathbf{W} = [w_{si}]_{S\times N}$, where w_{si} represents the proportion of tenant i's demand D_i served by cloud provider s, for

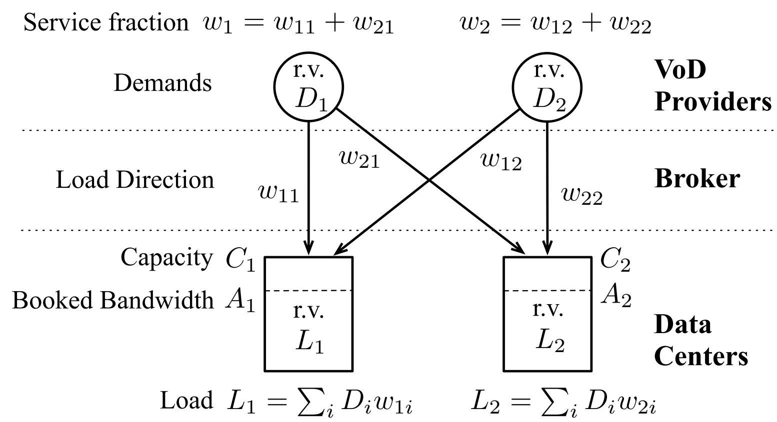


Fig. 1. A system of 2 cloud providers and 2 tenants. Random variables are labeled with "r.v."

$s = 1, \ldots, S$ and $i = 1, \ldots, N$, with $0 \le w_{si} \le 1$. At the cloud provider side, given $[w_{si}]$, the bandwidth load imposed on cloud provider s is a random variable $L_s = \sum_i w_{si} D_i$.

Similar to $B_i = f_\epsilon(D_i)$, we define $A_s = f_\epsilon(L_s)$ ($A_s \le C_s$) as the amount of bandwidth reserved from cloud provider s to ensure that the load L_s is satisfied with a high probability $1 - \epsilon$. Clearly, the reserved bandwidth A_s must not exceed the capacity of cloud provider s, i.e., $A_s \le C_s$, or equivalently,

$$\Pr(L_s > C_s) \le \epsilon, \quad \forall s. \tag{3}$$

For convenience, let $\mathbf{L} = [L_1, \ldots, L_S]^\mathsf{T}$ and let $\mathbf{w}_s = [w_{s1}, \ldots, w_{sN}]^\mathsf{T}$. Note that $\mathbf{w}_s$ can be interpreted as the *workload portfolio* of cloud provider s. From tenant i's point of view, given $[w_{si}]$, the total fraction of its demand D_i served by all cloud providers is $w_i = \sum_s w_{si} \le 1$. For convenience, let $\mathbf{w} = [w_1, \ldots, w_N]$. Clearly, we have $\mathbf{w} = \sum_s \mathbf{w}_s$.

The broker takes demand estimates $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ and cloud provider capacities $C_1, \ldots, C_S$ as inputs, and outputs a load routing weight matrix $\mathbf{W}^* = [w_{si}^*]$ to achieve a certain goal in favor of cloud providers, tenants, or the broker, or all of them. The output $\mathbf{W}^*$ corresponds to a way that N inter-correlated random demands $D_1, \ldots, D_N$ are packed into S cloud providers of limited capacities. For convenience, let $w_i^* = \sum_s w_{si}^*$ and let $\mathbf{w}_s^* = [w_{s1}^*, \ldots, w_{sN}^*]$. A "good" broker should satisfy the demand of tenants while reserving as little bandwidth resource as possible from cloud providers. In other words, the routing matrix $\mathbf{W}^*$ should be chosen such that 1) $w_i^* = 1$, for all tenants $i = 1, \ldots, N$; 2) the total bandwidth reservation $\sum_s A_s$ is minimized.

C. Bandwidth Pricing in the Presence of a Broker

Without loss of generality, we assume that each cloud provider charges a unit bandwidth fee of \$1 for every unit bandwidth reserved in each short-term period. The broker needs to pay $\$1 \cdot \sum_s A_s$ in total to the cloud providers for booking a total amount of $\sum_s A_s$ bandwidth. On the other hand, it charges tenant i a fee of $P_i(w_i)$ for accommodating w_i portion of its bandwidth demand D_i according to some pricing strategy $P_i(\cdot)$. Under a given $\mathbf{W}$, the broker profit is thus

$$R(\mathbf{W}) = \sum_i P_i(w_i) - \sum_s A_s. \tag{4}$$

We make minimum assumptions on the pricing strategy $P_i(\cdot)$ adopted by the broker as follows:

Definition 1: A *bandwidth pricing strategy* $P_i(\cdot)$ relative to tenant i is a concave function $P_i(w_i)$ of the service portion $w_i \in [0, 1]$, with $P_i(0) = 0$. The collection of bandwidth pricing strategies $\{P_i(\cdot) : i = 1, \ldots, N\}$ forms a *pricing policy*.

We observe that these assumptions on $P_i(\cdot)$ are quite reasonable. $P_i(\cdot)$ is concave because in real life wholesale is cheaper than retail, i.e., the more a user buys some goods, the lower the unit price $P(w_i)/w_i$ she has to pay. $P_i(0) = 0$ because a tenant should pay nothing when receiving no service. In our system model, the pricing policy $\{P_i(\cdot)\}$ can be authoritatively enforced in a *controlled market*, or negotiated between the broker and each tenant in a *free market*. As a special case, the broker may adopt the cloud pricing policy $\{P_i^0(\cdot)\}$:

$$P_i^0(w_i) = f_\epsilon(D_i w_i) \cdot 1, \tag{5}$$

with $P_i^0(1) = f_\epsilon(D_i) = B_i,\ \forall i$. In other words, the broker charges the same amount of money as cloud providers do. Note that a "good" pricing policy $\{P_i(\cdot)\}$ should help tenants to save money, i.e., $P_i(w_i) \leq P_i^0(w_i), \quad \forall w_i \in [0, 1], \quad \forall i \in 1, \ldots, N$. Otherwise, tenants will have little incentive to reserve bandwidth through a broker instead of from cloud providers directly.

III. Main Contributions

In this section, we highlight our three main contributions in the paper.

Cloud Bandwidth Saving. From the cloud and social wellness point of view, a basic goal is to enhance resource efficiency in the cloud. When aggregate capacity supply is sufficient, this means to serve all given demands $\mathbf{D}$ using as little bandwidth resource $\sum_s A_s$ as possible, i.e.,

$$\min_{\mathbf{W}} \sum_s A_s \tag{6}$$

$$\text{s.t.} \quad A_s \leq C_s,\ \forall s, \quad w_i = 1,\ \forall i. \tag{7}$$

This leads to the first question we ask in this paper:

(Q1) How to efficiently determine the load routing matrix $\mathbf{W}^*$ to minimize the aggregate bandwidth reservation from all cloud providers?

Intuitively, statistical anti-correlation between tenants needs to be utilized to optimally consolidate workload. In Sec. IV, we provide a simple criterion to fast check if all given demands can be served or not, without having to solve (6). If yes, we give nearly closed-form solutions to (6).

Pricing in Controlled Markets. Note that objective (6) requires the broker to be altruistic to the cloud and tenants. However, a selfish broker has no obligation to accommodate all demands for service, i.e., it may output $w_i^* < 1$ for some i. Neither will it necessarily optimize load routing to maximize cloud bandwidth saving. Instead, the broker may simply decide $\mathbf{W}^*$ by maximizing its profit $R(\mathbf{W})$ as follows

$$\max_{\mathbf{W}} R(\mathbf{W}) = \textstyle\sum_i P_i(w_i) - \sum_s A_s \tag{8}$$

$$\text{s.t.} \quad A_s \le C_s, \quad \forall s. \tag{9}$$

We refer to such a broker as a *profit-driven* broker. We observe that even a profit-driven broker is willing to participate in the controlled market, since $R(\mathbf{W}^*) \geq R(\mathbf{W})\big|_{w_{si}=0,\forall s,i} = 0$ for all possible pricing policies. Although there is always some incentive for the broker to operate regardless of the pricing policy $\{P_i(\cdot)\}$, the key question is

(Q2) Under what pricing policy $\{P_i(\cdot)\}$ a profit-driven broker is exactly an altruistic broker that maximizes cloud bandwidth saving while accommodating all demands for service, i.e., problems (8) and (6) are equivalent?

An answer to this question is significant, in that it enables the optimal allocation of cloud resources simply through a money-driven broker, by enforcing a good pricing policy $\{P_i(\cdot)\}$ to the market. In Sec. V, we give a necessary and sufficient condition for such good pricing policies. As has been mentioned, broker pricing $\{P_i(\cdot)\}$ should be upper-bounded by the cloud pricing $\{P_i^0(\cdot)\}$. Since the broker can turn a profit from bandwidth saving through *arbitrage* (out of no investment), tenants may expect to obtain price discounts. In Sec. V, we also find the fair discount each tenant can enjoy, without hurting the interests of the cloud and other tenants.

Pricing in Free Markets. The good pricing policies found by answering question (Q2) are enforced in the market by a supervisory agency other than the broker or tenants. However, in a free market, a selfish tenant has the freedom to negotiate with the broker on the price it should pay. We assume each tenant i can submit to the broker its own pricing strategy $P_i(\cdot)$. In return, based on all the submitted pricing strategies $\{P_i(\cdot)\}$, the broker decides a service fraction w_i^* for each i by maximizing its own profit. The submitted prices can affect all tenants' utilities. We assume each tenant expects to be fully served (i.e., $w_i^* = 1$, $\forall i$), though striving to lower its payment. Intuitively, if $P_i(\cdot)$ is too low, the broker will not serve D_i completely out of profit concerns. Ironically, if $P_i(\cdot)$ is sufficiently high, D_i may not get fully served either, depending on the prices other tenants submit. We ask the question:

(Q3) In a free market where each selfish tenant competes for service by submitting its own pricing strategy $P_i(\cdot)$, will $\{P_i(\cdot)\}$ reach an equilibrium point?

In Sec. VI, we use game theory to show that $\{P_i(\cdot)\}$ will converge to a **unique Nash equilibrium** $\{P_i^*(\cdot)\}$. It turns out $\{P_i^*(\cdot)\}$ forms the lower border of the good pricing region, which means that without intervention, the free market itself will lead to the optimal load routing, where the tenants receive the maximum price discounts. A key finding of this paper is the following theorem:

Theorem 1: In equilibrium market, tenant i pays

$$\$\ P_i^*(w_i^*) = \mu_i + [f_\epsilon(D_i) - \mu_i]\rho_{iM} = \mu_i + \theta \cdot \sigma_i \rho_{iM} \tag{10}$$

for bandwidth reservation, where $\rho_{iM} \in [-1, 1]$ is the correlation coefficient between D_i and $\sum_i D_i$.

In Sec. VI, we discuss the connections between demand statistics and bandwidth pricing.

IV. Optimal Bandwidth Saving

In this section, we answer question (Q1) — what are the load routing weights $\mathbf{W}^*$ that achieve the most bandwidth saving? For simplicity, we assume each random demand D_i is Gaussian-distributed. This assumption will be verified by real-world traces in Sec. VII. Recall that for a Gaussian random variable X, we have

$$f_\epsilon(X) = \mathbf{E}[X] + \theta\sqrt{\mathbf{Var}[X]}, \quad \theta = F^{-1}(1-\epsilon), \tag{11}$$

where $F(\cdot)$ is the CDF of normal distribution $\mathcal{N}(0,1)$. For example, when $\epsilon = 2\%$, we have $\theta = 2.05$. Note that if each D_i is Gaussian-distributed, so is $L_s = \sum_i w_{si} D_i$, and we have

$$\begin{cases} \mathbf{E}[L_s] = \mu_1 w_{s1} + \ldots + \mu_N w_{sN} = \boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s, \\ \mathbf{Var}[L_s] = \sum_{i,j} \rho_{ij}\sigma_i\sigma_j w_{si} w_{sj} = \mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s. \end{cases}$$

It follows that

$$B_i = f_\epsilon(D_i) = \mu_i + \theta\sigma_i,$$
$$A_s = f_\epsilon(L_s) = \boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s + \theta\sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s}.$$

Therefore, cloud resource minimization (6) has the following form under Gaussian demands:

$$\min_{\mathbf{W}} \textstyle\sum_s(\boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s + \theta\sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s}), \tag{12}$$

$$\text{s.t.} \quad \boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s + \theta\sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s} \le C_s, \quad \forall s, \tag{13}$$

$$\mathbf{w} = \textstyle\sum_s \mathbf{w}_s = \mathbf{1}, \tag{14}$$

$$\mathbf{0} \preceq \mathbf{w}_s \preceq \mathbf{1}, \quad \forall s, \tag{15}$$

where $\mathbf{1} = [1, \ldots, 1]^\mathsf{T}$ and $\mathbf{0} = [0, \ldots, 0]^\mathsf{T}$ are N-dimensional column vectors. Constraint (13) is equivalent to $A_s \le C_s$ and thus to (3).

Although problem (12) is convex optimization, it has coupled objectives and constraints, and takes numerical solvers a long time to converge for a large S. However, in the following, we show that when capacity supply is sufficient, nearly closed-form solutions to problem (12) can be derived by exploiting its unique structure.

Theorem 2: When $C_{\mathsf{sum}} \ge \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}$, an optimal solution $[w_{si}^*]$ is given by

$$w_{si}^* = \alpha_s, \quad \forall i, \quad s = 1, \ldots, S, \tag{16}$$

where $\alpha_1, \ldots, \alpha_S$ can be any solution to

$$\sum_s \alpha_s = 1, \; 0 \le \alpha_s \le \min\left\{1, \frac{C_s}{\boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}}\right\}, \; \forall s. \tag{17}$$

When $C_{\mathsf{sum}} < \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}$, there is no feasible solution that satisfies constraints (13) to (15).

Proof: The function $f(\mathbf{w}_s) = \sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s}$ is a cone and a convex function. We have

$$f\left(\frac{\mathbf{w}_1 + \mathbf{w}_2}{2}\right) \le \frac{f(\mathbf{w}_1) + f(\mathbf{w}_2)}{2}, \tag{18}$$

or equivalently,

$$\sqrt{(\mathbf{w}_1 + \mathbf{w}_2)^\mathsf{T}\boldsymbol{\Sigma}(\mathbf{w}_1 + \mathbf{w}_2)} \le \sqrt{\mathbf{w}_1^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_1} + \sqrt{\mathbf{w}_2^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_2}.$$

Applying the above inequality iteratively, we can prove

$$\sum_s \sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s} \ge \sqrt{(\sum_s \mathbf{w}_s^\mathsf{T})\boldsymbol{\Sigma}(\sum_s \mathbf{w}_s)} = \sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}. \tag{19}$$

If $\mathbf{w} = \mathbf{1}$ is feasible, by (13) and (19) we have

$$\sum_s C_s \geq \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}} = \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}. \tag{20}$$

If $\sum_s C_s \geq \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}$, it is easy to verify (17) is feasible. When $w^*_{si} = \alpha_s$ given by (16), we find (13), (14) and (15) are all satisfied. Hence, (16) is a feasible solution and $\mathbf{w} = \mathbf{1}$ is feasible. By (19), the objective function (12) satisfies

$$\sum_s (\boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s + \theta\sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s}) \geq \boldsymbol{\mu}^\mathsf{T}\mathbf{w} + \theta\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}} = \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}. \tag{21}$$

We find that $[w^*_{si}]$ given by (16) achieves the above inequality with equality, and thus is also an optimal solution to (12). ■

A. *Discussions*

Using Theorem 2, the broker can fast check if all demands can be served, simply by comparing the total cloud capacity C_{sum} with total bandwidth required for all demands combined:

$$f_\epsilon(\sum_i D_i) = \mathbf{E}[\sum_i D_i] + \theta\sqrt{\mathbf{Var}[\sum_i D_i]} = \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}.$$

If $C_{\mathsf{sum}} < \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}$, it is infeasible to pack all demands into the limited resources at a risk factor of ϵ. Otherwise, all demands can be accommodated.

When supply is sufficient, Theorem 2 implies that in the optimal solution, each tenant should direct its workload into S cloud providers following the same weights $\alpha_1, \ldots, \alpha_S$, with $\sum_s \alpha_s = 1$. An altruistic broker can efficiently find a feasible set of $\alpha_1, \ldots, \alpha_S$ subject to linear constraints (17), and promise each tenant i to serve all its demand, under the same agreement that α_s of D_i will be directed to cloud provider s.

The maximum bandwidth saving from joint bandwidth booking as compared to booking bandwidth individually for each tenant is

$$\begin{aligned}\Delta B(\mathbf{W}^*) &= \textstyle\sum_i B_i - \sum_s A_s = \sum_i(\mu_i + \theta\sigma_i) - \sum_s(\boldsymbol{\mu}^\mathsf{T}\mathbf{w}^*{}_s + \theta\sqrt{\mathbf{w}^*{}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}^*{}_s}) \\ &= \theta(\boldsymbol{\sigma}^\mathsf{T}\mathbf{1} - \sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}) = \theta(\textstyle\sum_i \sigma_i - \sigma_M),\end{aligned} \tag{22}$$

which is θ times the gap between the sum of all demand standard deviations and the standard deviation of all demands combined. Furthermore, the number of cloud providers S does not affect bandwidth saving: theoretically, there is no loss of resource efficiency if a big cloud provider is substituted by numerous small cloud providers with the same aggregate capacity.

V. PRICING REGION IN CONTROLLED MARKETS

In this section, we answer question (Q2)—if the broker is profit-driven, what pricing policies need be enforced in the market in order to benefit cloud resource efficiency and the interests of tenants? Theorem 2 provides a necessary and sufficient condition for supply to exceed demand. We thus assume $\Sigma_s C_s \geq \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}$ throughout the paper.

We assume a selfish broker determines $\mathbf{W}^*$ by maximizing its own profit via (8). Similarly, under Gaussian demands, we can translate (8) into the following problem:

$$\max_{\mathbf{W}} \textstyle\sum_i P_i(w_i) - \sum_s (\boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s + \theta\sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s}), \tag{23}$$

$$\text{s.t.} \quad \boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s + \theta\sqrt{\mathbf{w}_s^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}_s} \leq C_s, \quad \forall s, \tag{24}$$

$$\mathbf{w} = \textstyle\sum_s \mathbf{w}_s \preceq \mathbf{1}, \tag{25}$$

$$\mathbf{0} \preceq \mathbf{w}_s \preceq \mathbf{1}, \quad \forall s, \tag{26}$$

A selfish broker does not only have a different objective (23) than (12), but also has a constraint (25) different from (14), in that a profit-driven broker has no obligation to accommodate all demands for service, if it cannot gain a higher profit by doing so. Instead, the broker decides the service fraction $w_i^* \geq 1$ for each tenant depending on the pricing policy $\{P_i(\cdot)\}$. We aim to find all pricing policies such that (23) is equivalent to (12).

The following theorem gives a necessary and sufficient condition for a good pricing policy:

Theorem 3: Broker profit maximization (23) and cloud bandwidth resource minimization (12) have the same optimal solution (16), if and only if

$$P_i'(1) \geq \mu_i + \theta \cdot \frac{\sigma_{iM}}{\sigma_M}, \quad \forall i, \tag{27}$$

where σ_{iM} is the covariance between D_i and $\sum_i D_i$ given by (1) and σ_M is the standard deviation of $\sum_i D_i$ given by (2). Furthermore, if $P_i'(1) < \mu_i + \theta\sigma_{iM}/\sigma_M$ for some i, then $\mathbf{w}^* \neq \mathbf{1}$.

Proof: We first show that if $P_i'(1) \geq \mu_i + \theta\sigma_{iM}/\sigma_M$, $\forall i$, (16) is also an optimal solution to (23). Using the bound (19), problem (23) can be relaxed to the following concave problem:

$$\max_{\mathbf{w}} R_u(\mathbf{w}) := \textstyle\sum_i P_i(w_i) - (\boldsymbol{\mu}^\mathsf{T}\mathbf{w} + \theta\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}), \tag{28}$$

$$\text{s.t.} \quad \boldsymbol{\mu}^\mathsf{T}\mathbf{w} + \theta\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}} \leq \textstyle\sum_s C_s, \tag{29}$$

$$\mathbf{w} \preceq \mathbf{1}, \tag{30}$$

where $R_u(\mathbf{w}) \geq R(\mathbf{W})$ is a concave function, and (29) is a relaxation of constraints (24). Since $\boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}} \leq \sum_s C_s$, $\mathbf{w} = \mathbf{1}$ is a feasible solution to (28).

The derivative of $R_u(\mathbf{w})$ with respect to w_i is given by

$$\begin{aligned}\frac{\partial R_u(\mathbf{w})}{\partial w_i}\bigg|_{\mathbf{w}=1} =& P_i'(w_i)\bigg|_{w_i=1} - \mu_i - \frac{\theta \sum_{j=1}^N \sigma_{ij} w_j}{\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}}\bigg|_{\mathbf{w}=1} \\ =& P_i'(1) - \mu_i - \frac{\theta \mathbf{Cov}[D_i, \sum_j D_j]}{\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}}} \\ =& P_i'(1) - \mu_i - \frac{\sigma_{1M}}{\sigma_M} \end{aligned} \tag{31}$$

If (27) is satisfied, then $\partial R_u(\mathbf{w})/\partial w_i\big|_{\mathbf{w}=1} \geq 0$, $\forall i$. According to the gradient ascent algorithm for concave maximization, $\mathbf{w} = \mathbf{1}$ is the optimal solution to (28). If $w_{si} = \alpha_s$ given by (16), we have the following 3 facts:

- All constraints (24)-(26) are satisfied;
- $R_u(\mathbf{w}) = R(\mathbf{W})$;
- $R_u(\mathbf{w}) = R_u(\mathbf{1})$ in problem (28) reaches optimality.

Therefore, (16) is an optimal solution to (23).

Now we prove the converse. If there exists an i such that

$$P_i'(1) < \mu_i - \frac{\sigma_{1M}}{\sigma_M}, \tag{32}$$

the optimal solution $\mathbf{w}^*$ to the relaxed problem (28) must satisfy $w_i^* < 1$. Then the original problem (23) has an optimal solution $w_{si}^* = \beta_s$, $\forall i, \forall s$, where β_s solves

$$\begin{cases} 0 \leq \beta_s \leq \min\{1, \frac{C_s}{\boldsymbol{\mu}^\mathsf{T}\mathbf{w}^* + \theta\sqrt{\mathbf{w}^{*\mathsf{T}}\boldsymbol{\Sigma}\mathbf{w}^*}}\},\ \forall s, \\ \sum_s \beta_s = 1. \end{cases}$$

Let $w_{si}^{**} = \alpha_s$, $\forall i, \forall s$ be given by (16). Since

$$R\big([w_{si}^{**}]_{S\times N}\big) = R_u(\mathbf{1}) < R_u(\mathbf{w}^*) = R\big([w_{si}^*]_{S\times N}\big),$$

we have proved that w_{si}^{**} given by (16) is not an optimal solution to (23). ■

We depict the region of good pricing policies in Corollary 1, which relies on the following lemma.

Lemma 1: Let $f(x)$ be a concave function on $[0, 1]$ with $f(0) = 0$. Let k be a (constant) real number. If $f'(1) \geq k$, then for each $x \in (0, 1]$, $f(x) \geq kx$. In this case, if $f(x') = kx'$ for some $x' \in (0, 1]$, then $f(x) \equiv kx$, $\forall x \in [0, 1]$.

Proof: Note that the function $f(x)$ is bounded by

$$f'(1)x \leq f(x) \leq f'(1)(x - 1) + f(1),\ \forall x \in (0, 1], \tag{33}$$

where the inequality on the left comes from the fact

$$f(x) = \int_0^x f'(t)dt + f(0) \geq \int_0^x f'(1)dt = f'(1)x \tag{34}$$

and the inequality on the right comes from the concavity.

Hence, if $f'(1) \geq k$, then we have

$$f(x) \geq f'(1)x \geq kx, \ \forall x \in (0, 1].$$

In this case, if $f(x') = kx'$ for some $x' \in (0, 1]$, then we have

$$kx' \geq f'(1)x' \geq kx'.$$

It follows that $f'(1) = k$. Further, note that

$$f(x') = \int_0^{x'} f'(x)dx \geq \int_0^{x'} kdx \geq kx',$$

with equality holds if and only if $f'(x) = k, \ \forall x \in (0, x']$. Thus, we have $f'(x) = k$, for all $x \in (0, 1]$. That is, $f(x) \equiv kx$, $\forall x \in [0, 1]$. ■

Corollary 1: In a good pricing policy $\{P_i(\cdot) : i = 1, \ldots, N\}$, each $P_i(\cdot)$ must satisfy $\forall w_i \in [0, 1]$,

$$(\mu_i + \theta \cdot \frac{\sigma_{iM}}{\sigma_M})w_i \leq P_i(w_i) \leq (\mu_i + \theta\sigma_i)w_i. \tag{35}$$

Proof: In a good pricing policy, each $P_i(\cdot)$ cannot exceed the cloud provider pricing $P_i^0(\cdot)$, otherwise, the tenants would have booked bandwidth from cloud providers directly. Therefore, $P_i^0(w_i) = (\mu_i + \theta\sigma_i)w_i$ is the upper bound for $P_i(\cdot)$. Consider another pricing policy $\{P_i^*(\cdot)\}$, where

$$P_i^*(w_i) = (\mu_i + \theta \cdot \frac{\sigma_{iM}}{\sigma_M})w_i, \quad \forall i. \tag{36}$$

By Theorem 3 and Lemma 1, in a good pricing policy, for each i, we have

$$P_i(w_i) \geq (\mu_i + \frac{\theta\sigma_{iM}}{\sigma_M}) \cdot w_i, \quad \forall w_i \in (0, 1],$$

with equality achieved only if $P_i(\cdot) \equiv P_i^*(\cdot)$. ■

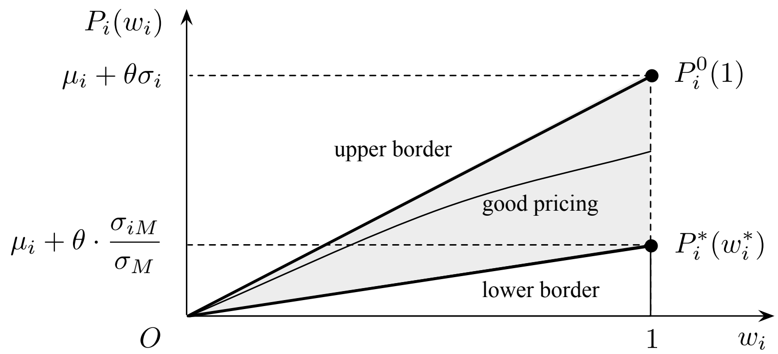


Fig. 2. The region of $P_i(\cdot)$ in a good pricing policy $\{P_i(\cdot)\}$. $P_i(\cdot)$ is between $P_i^*(\cdot)$ and $P_i^0(\cdot)$, and satisfies $P_i'(1) \geq P_i^{*'}(1)$.

A. Broker Profit and Price Discounts in the Pricing Region

Fig. 2 illustrates the region of $P_i(\cdot)$ in a good pricing policy. Each good $P_i(\cdot)$ is a concave function between $P_i^*(\cdot)$ and $P_i^0(\cdot)$ (inclusive) with $P_i'(1) \geq \mu_i + \theta\sigma_{iM}/\sigma_M$. In the good pricing region, a selfish broker behaves like an altruistic broker that optimizes cloud bandwidth saving while serving all demands. When pricing policy falls out of this region, the broker will reject a part of demands to chase a profit, i.e., there exists a j such that $w_j^* < 1$, which is undesirable when supply is sufficient. Note that $P_i(\cdot)$ affects not only w_i^* but also w_j^* for all j. This phenomenon will be understood in Sec. VI.

We further analyze the broker profit $R(\mathbf{W}^*)$ in the pricing region. Under any good pricing policy, the optimal solution to (23) is $w_{si}^* = \alpha_s$ given by (16) and $w_i^* = 1$. Thus, the broker's payment to cloud providers is

$$\sum_s A_s = \sum_s (\boldsymbol{\mu}^\mathsf{T}\mathbf{w}_s^* + \theta\sqrt{\mathbf{w}_s^{*\mathsf{T}}\boldsymbol{\Sigma}\mathbf{w}_s^*}) = \boldsymbol{\mu}^\mathsf{T}\mathbf{1} + \theta\sqrt{\mathbf{1}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{1}} = \sum_i \mu_i + \theta\sigma_M, \tag{37}$$

Using Corollary 1, we obtain

$$R(\mathbf{W}^*) \leq \sum_i (\mu_i + \theta\sigma_i)w_i^* - \sum_s A_s = \theta(\sum_i \sigma_i - \sigma_M) = \Delta B(\mathbf{W}^*), \tag{38}$$

with equality achieved when $P_i(\cdot) \equiv P_i^0(\cdot)$, and

$$R(\mathbf{W}^*) \geq \sum_i (\mu_i + \frac{\theta\sigma_{iM}}{\sigma_M})w_i^* - \sum_s A_s = \theta \cdot \frac{\sum_i \sigma_{iM}}{\sigma_M} - \theta\sigma_M = 0 \tag{39}$$

with equality achieved when $P_i(\cdot) \equiv P_i^*(\cdot)$.

The above bounds show that the maximum broker profit is essentially the maximum achievable bandwidth saving $\Delta B(\mathbf{W}^*)$ from joint bandwidth booking given by (22), which depends on the gap between the sum of all demand standard deviations and the standard deviation of market demand. Moreover, the maximum profit is achieved when the broker adopts the same pricing policy $\{P_i^0(\cdot)\}$ as cloud providers do. On the other hand, when $P_i(\cdot) \equiv P_i^*(\cdot)$, the broker profit is zero, which means all the profit made from cloud bandwidth multiplexing has been rewarded to tenants as price discounts.

In a controlled market, the lowest price that tenant i should be charged for having all its demand served is

$$P_i^*(1) = \mu_i + \theta \cdot \frac{\sigma_{iM}}{\sigma_M}. \tag{40}$$

If any tenant i is charged less than $P_i^*(1)$, the broker will deny a part of all demands to turn a higher profit.

VI. Pricing in Free Markets: A Game Theoretical Analysis

In Sec. V, good pricing is enforced as a policy by a third party other than the broker and tenants, which is difficult to implement in reality. To closely model a free market, we assume each selfish tenant can freely bargain with the broker to settle down a bandwidth price. Each tenant i simply submits to the broker any pricing strategy $P_i(\cdot)$ it prefers and accepts the service fraction w_i^* returned by the broker. Based on $\{P_i(\cdot)\}$ collected from all tenants, the broker determines load routing $\mathbf{W}^*$ and thus the service fraction w_i^* for each i by maximizing its own profit. We answer question (Q3) — what will the prices eventually look like in such a free market of selfish players?

We define a game played by all tenants, each with an independent strategy $P_i(\cdot)$, i.e., the price it submits. Under a set of submitted strategies $\{P_i(\cdot)\}$, we define a utility function associated with each tenant i as

$$U_i\big[P_1(\cdot), \ldots, P_N(\cdot)\big] = \begin{cases} -P_i(w_i^*), & \text{if } w_i^* = 1, \\ -\infty, & \text{if } w_i^* < 1, \end{cases} \tag{41}$$

where the broker determines w_i^* by solving (23).

The rationale behind (41) is that in a system of sufficient capacity, a selfish tenant always 1) expects to get fully served, and 2) tries to reduce its price if condition 1) is met. In other words, unlike the case of scarce metal (e.g., gold), it is not profitable for a tenant to deliberately deny user requests to trade for a lower unit bandwidth cost p_i. In reality, popularity matters more to a tenant than instantaneous profit per user. In a market of sufficient supply, if a tenant is found sacrificing parts of user requests to chase for cheaper bandwidth deals, it will lose reputation and revenue in the long run.

To find the equilibrium market price, we just need to find the Nash equilibrium in the above game, where no tenant i can get a better utility by unilaterally changing $P_i(\cdot)$. Intuitively, tenant i cannot submit too low a $P_i(\cdot)$, beyond which the broker will be unable to guarantee $w_i^* = 1$ via (23), leading to $U_i = -\infty$. From the analysis in Sec. V, one may conjecture that $\{P_i^*(\cdot)\}$ is a Nash equilibrium. However, is this the unique Nash equilibrium? Could there be another equilibrium point where each tenant submits a very low price and gets a utility of $-\infty$ without being able to better off by changing its own pricing? In other words, can market collapse happen?

A. Nash Equilibrium: Existence and Uniqueness

Theorem 4: If tenants have utility (41) and the broker decides $\mathbf{W}^*$ by maximizing its profit through (23), then $\{P_i(\cdot)\}$ will converge to a **unique Nash equilibrium** $\{P_i^*(\cdot)\}$, where

$$P_i^*(w_i) = (\mu_i + \theta \cdot \frac{\sigma_{iM}}{\sigma_M})w_i, \quad 0 \le w_i \le 1, \tag{42}$$

where σ_{iM} and σ_M are given by (1) and (2), respectively.

Let $P_{-i}(\cdot)$ represent the pricing strategies of all tenants except for tenant i. The proof of Theorem 4 relies on the following two lemmas and Theorem 3 that characterize the solution structure of (23) given $\{P_i(\cdot)\}$.

Lemma 2: If $P_i'(1) < \mu_i + \theta\sigma_{iM}/\sigma_M$, then $w_i^* < 1$ regardless of $P_{-i}(\cdot)$.

Proof: It suffices to show that

$$\left.\frac{\partial R_u(\mathbf{w})}{\partial w_i}\right|_{w_i=1} < 0 \text{ for all } \mathbf{w} \text{ with } w_i = 1,$$

since in this case we have $w_i^* = 1$ regardless of $\{w_j\}_{j\neq i}$ by the gradient ascent algorithm.

Recall that

$$\frac{\partial R_u(\mathbf{w})}{\partial w_i} = P_i'(w_i) - \mu_i - \frac{\theta \sum_{j=1}^N \sigma_{ij} w_j}{\sqrt{\mathbf{w}^\mathsf{T}\mathbf{\Sigma}\mathbf{w}}}.$$

If $P_i'(w_i) < \mu_i + \theta\sigma_{iM}/\sigma_M$, then

$$\left.\frac{\partial R_u(\mathbf{w})}{\partial w_i}\right|_{w_i=1} < \theta \left.\left(\frac{\sigma_{iM}}{\sigma_M} - \frac{\sum_{j=1}^N \sigma_{ij} w_j}{\sqrt{\mathbf{w}^\mathsf{T}\mathbf{\Sigma}\mathbf{w}}}\right)\right|_{w_i=1}.$$

Now construct a function $g(w_1, \ldots, w_N)$ as

$$g(w_1, \ldots, w_N) = \frac{\sum_{j=1}^N \sigma_{ij} w_j}{\sqrt{\mathbf{w}^\mathsf{T}\mathbf{\Sigma}\mathbf{w}}}.$$

It is easy to see that $g(1,\ldots,1)=\sigma_{iM}/\sigma_M$. Hence, it suffices to show that

$$g(w_1,\ldots,w_N)\geq g(1,\ldots,1) \text{ for all } \mathbf{w} \text{ with } w_i=1.$$

Without loss of generality, we shall need only to prove the case when $N=2$ and $i=1$, that is,

$$\frac{\sigma_1^2+\rho_{12}\sigma_1\sigma_2 w_2}{\sqrt{\sigma_1^2+\sigma_2^2 w_2^2+2\rho_{12}\sigma_1\sigma_2 w_2}}\geq\frac{\sigma_1(\sigma_1+\rho_{12}\sigma_2)}{\sqrt{\sigma_1^2+\sigma_2^2+2\rho_{12}\sigma_1\sigma_2}}.$$

It is easy to check that the above inequality indeed holds. Hence, $\partial R_u(\mathbf{w})/\partial w_i|_{w_i=1}<0$ for all $\mathbf{w}$ with $w_i=1$, completing the proof. ■

Lemma 3: If $P_i(w_i)=P_i^0(w_i)=(\mu_i+\theta\sigma_i)w_i$ for $w_i\in[0,1]$, then $w_i^*=1$, regardless of $P_{-i}(\cdot)$.

Proof: It suffices to show that

$$\left.\frac{\partial R_u(\mathbf{w})}{\partial w_i}\right|_{w_i=1}\geq 0 \text{ for all } \mathbf{w} \text{ with } w_i=1,$$

since in this case we have $w_i^*=1$ regardless of $\{w_j\}_{j\neq i}$.

Recall that

$$\frac{\partial R_u(\mathbf{w})}{\partial w_i}=P_i'(w_i)-\mu_i-\frac{\theta\sum_{j=1}^N\sigma_{ij}w_j}{\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}}.$$

If $P_i(w_i)=(\mu_i+\theta\sigma_i)w_i$, then $P_i'(w_i)=\mu_i+\theta\sigma_i$. Thus,

$$\left.\frac{\partial R_u(\mathbf{w})}{\partial w_i}\right|_{w_i=1}=\theta\left.\left(\sigma_i-\frac{\sum_{j=1}^N\sigma_{ij}w_j}{\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}}\right)\right|_{w_i=1}.$$

Now construct a random variable X as $X=\sum_{j\neq i}w_jD_j+D_i$. By the Cauchy-Schwarz inequality, we have

$$\mathbf{E}^2\big[(D_i-\mu_i)(X-\mathbf{E}[X])\big]\leq\mathbf{E}\big[(D_i-\mu_i)^2\big]\cdot\mathbf{E}\big[(X-\mathbf{E}[X])^2\big]. \tag{43}$$

That is,

$$\left(\sigma_i^2+\sum_{j\neq i}\sigma_{ij}w_j\right)^2\leq\sigma_i^2\left.\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}\right|_{w_i=1}.$$

It follows immediately that

$$\sigma_i\geq\left.\frac{\sum_{j=1}^N\sigma_{ij}w_j}{\sqrt{\mathbf{w}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{w}}}\right|_{w_i=1}.$$

Hence, $\partial R_u(\mathbf{w})/\partial w_i|_{w_i=1}\geq 0$ for all $\mathbf{w}$ with $w_i=1$. ■

Proof of Theorem 4: We first show that $\{P_i^*(\cdot)\}$ is indeed a Nash equilibrium. We need to show that at $\{P_i^*(\cdot)\}$, any tenant i cannot increase its utility U_i by unilaterally changing $P_i(\cdot)$, i.e.,

$$U_i\big[P_i^*(\cdot), P_{-i}^*(\cdot)\big] \geq U_i\big[P_i(\cdot), P_{-i}^*(\cdot)\big] \quad \forall i. \tag{44}$$

We exhaust the possibilities of $P_i(\cdot)$ by considering the range of $P_i'(1)$. First, if

$$P_i'(1) < \mu_i + \theta\sigma_{iM}/\sigma_M,$$

by Lemma 2, $w_i^* < 1$, and by (41), we have

$$U_i\big[P_i(\cdot), P_{-i}^*(\cdot)\big] = -\infty < -P_i^*(1) = U_i\big[P_i^*(\cdot), P_{-i}^*(\cdot)\big].$$

Second, if $P_i'(1) \geq \mu_i + \theta\sigma_{iM}/\sigma_M$, while $P_{-i}(\cdot) \equiv P_{-i}^*(\cdot)$, according to Theorem 3, we have $w_i^* = \sum_s w_{si}^* = \sum_s \alpha_s = 1$. Hence, we have

$$U_i\big[P_i(\cdot), P_{-i}^*(\cdot)\big] = -P_i(1) \leq -P_i^*(1) = U_i\big[P_i^*(\cdot), P_{-i}^*(\cdot)\big].$$

The inequality above is due to Lemma 1 and the fact that $P_i^*(w_i)$ is a linear function in w_i that passes $(0, 0)$. We have thus proved (44). Therefore, $\{P_i^*(\cdot)\}$ is indeed a Nash equilibrium.

Now we show that $\{P_i^*(\cdot)\}$ is the ***unique*** Nash equilibrium, i.e., if $\{P_i(\cdot)\}$ is a Nash equilibrium, then $P_i(\cdot) \equiv P_i^*(\cdot)$. First of all, let us show that the following inequality holds:

$$P_i'(1) \geq \mu_i + \frac{\theta\sigma_{iM}}{\sigma_M}, \quad \forall i. \tag{45}$$

We prove (45) by contradiction. Assume there exists an i such that $P_i' < \mu_i + \theta\sigma_{iM}/\sigma_M$. By Lemma 2, $w_i^* < 1$ and thus $U_i = -\infty$. If tenant i uses another strategy $P_i^0(w_i) = (\mu_i + \theta\sigma_i)w_i$, then by Lemma 3, we have $w_i^* = 1$, regardless of $P_{-i}(\cdot)$. Hence,

$$U_i\big[P_i^0(\cdot), P_{-i}(\cdot)\big] = -P_i^0(1) > -\infty = U_i\big[P_i(\cdot), P_{-i}(\cdot)\big],$$

contradicting with the definition of Nash equilibrium that unilaterally changing $P_i(\cdot)$ cannot increase U_i. Therefore, (45) must hold.

If (45) holds, by Theorem 3, we have $w_i^* = \sum_s w_{si}^* = \sum_s \alpha_s = 1, \quad \forall i$, and thus $U_i\big[P_i(\cdot), P_{-i}(\cdot)\big] = -P_i(1), \quad \forall i$. If (45) holds, by Lemma 1, $P_i(1) \geq (\mu_i + \frac{\theta\sigma_{iM}}{\sigma_M}) \cdot 1 = P_i^*(1)$, with equality achieved *only if* $P_i(w_i) \equiv (\mu_i + \frac{\theta\sigma_{iM}}{\sigma_M})w_i \equiv P_i^*(w_i)$ for $w_i \in [0, 1]$. In other words, if $P_i(\cdot)$ is not $P_i^*(\cdot)$,

$$U_i\big[P_i(\cdot), P_{-i}(\cdot)\big] = -P_i(1) < -P_i^*(1) = U_i\big[P_i^*(\cdot), P_{-i}(\cdot)\big],$$

contradicting with the fact that $\{P_i(\cdot)\}$ is a Nash equilibrium. Therefore, $P_i(\cdot)$ must be $P_i^*(\cdot)$. In other words, $\{P_i^*(\cdot)\}$ is the unique Nash equilibrium. ■

B. Market Properties and Equilibrium Bandwidth Costs

Theorem 4 shows that even without a supervisory party, the selfishness of tenants necessarily drives $\{P_i(\cdot)\}$ into an equilibrium $\{P_i^*(\cdot)\}$, which is exactly the lower border of all good pricing policies. Therefore, in a free market, a profit-driven broker is naturally a workload consolidator that optimizes the cloud resource efficiency. In equilibrium market, we have $P_i(\cdot) \equiv P_i^*(\cdot)$ and broker profit $R(\mathbf{W}^*) = 0$ according to (39). This is reasonable since in the presence of competition, the broker would be unable to exploit the profit on no investment gained from bandwidth multiplexing. Yet the broker can turn a positive profit through agent fees, membership fees, ads and other means. In addition, the free market automatically guarantees a fairness on bandwidth costs across different tenants.

Let us now analyze the equilibrium bandwidth cost for each tenant. At market equilibrium, $w_i^* = 1$ and $P_i^*(w_i^*) = P_i^*(1)$ given by (40). Rewriting (40), we have

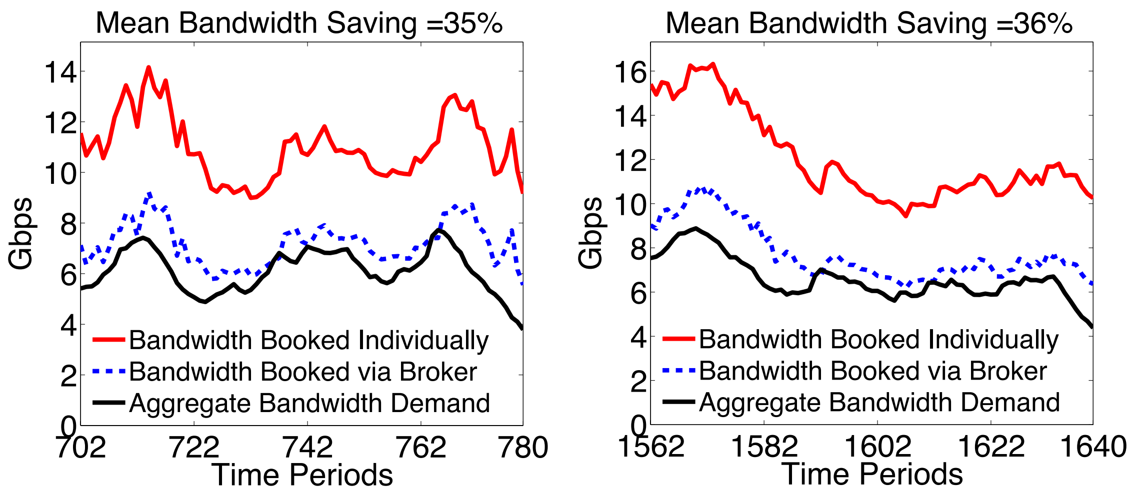
$$\$\ P_i^*(w_i^*) = \mu_i + \theta \cdot \sigma_i \rho_{iM} = \mu_i + [f_\epsilon(D_i) - \mu_i] \cdot \rho_{iM},$$

proving Theorem 1.

The payment consists of two parts: \$1 per unit bandwidth for mean demand μ_i, and \$$\rho_{iM}$ per unit bandwidth for reservation exceeding the expected demand μ_i. Apparently, the lower the ρ_{iM}, the more discount tenant i receives. In particular, if $\rho_{iM} < 0$, D_i is negatively correlated with market demand. Surprisingly, except for a charge on mean demand, now tenant i earns a bonus of $-\theta\sigma_i\rho_{iM} > 0$ to be in the system. The reason is that tenant i serves as a risk neutralizer: whenever market demand has a random increase, D_i will decrease to release occupied resources to accommodate the market surge. This helps the broker save the bandwidth reservation and hedge under-provision risks.

VII. Trace-Driven Simulations

In this section, we simulate a broker-assisted cloud bandwidth trading system, evaluate its benefit quantitatively through trace-driven simulations, and verify assumptions. Our workload traces come from UUSee [12], an operational large-scale VoD service based in China. The dataset contains the bandwidth demand in UUSee video channels sampled every 10 minutes during 2008 Summer Olympics. We consider two time spans in the traces, time periods 702—780 and 1562—1640, containing 91 and 176 concurrent video channels, respectively. We let each video channel i represent tenant i, with D_i equal to the channel's demand, assuming the same workload (or demand) is served by the clouds in a broker-assisted market.



(a) $\epsilon = 5\%$, $\epsilon' = 5.06\%$

(b) $\epsilon = 1\%$, $\epsilon' = 1.27\%$

Fig. 3. The aggregate bandwidth $\sum_s A_s(t)$ booked by the broker, compared to the aggregate bandwidth $\sum_i B_i(t)$ needed if each channel books individually and the real aggregate demand $\sum_i D_i(t)$.

In our system, bandwidth reservation and trading are carried out online every $\Delta t = 10$ minutes. (The same methodology can be applied to any other values of Δt, e.g., 1 hour.) Before time t, the broker should have obtained estimates $\boldsymbol{\mu}_t = [\mu_{1t}, \ldots, \mu_{Nt}]$ and $\boldsymbol{\Sigma}_t = [\sigma_{ijt}]$ for all tenants in the coming period $[t, t + \Delta t)$. Such statistics can be predicted accurately based on historical demand data, since VoD demand follows repeated daily patterns and is highly predictable [10], [11], [13]. Some established time series forecasting tools in econometrics (e.g., [14], [15]) may be used for prediction. In particular, we implement an online version of seasonal ARIMA and GARCH models introduced in [10], [11] to predict $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$. In practice, historical demand data can be easily obtained from cloud providers through their logging services or from cloud providers themselves. Once $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$ are obtained, the broker maximizes its profit by solving (23), making load routing decision $\mathbf{W}^*$ and bandwidth reservation A_s from each cloud provider s. The broker serves as a gateway that directs w^*_{si} fraction of tenant i's incoming video requests to cloud provider s for service during $[t, t + \Delta t)$. The above process is repeated for the next period $[t + \Delta t, t + 2\Delta t)$.

To evaluate the benefit of such a system, we quantify the equilibrium point in the market. Fig. 3 evaluates the saving on cloud bandwidth resources in equilibrium market. Due to multiplexing, the aggregate bandwidth $\sum_s A_{st}$ booked by the broker is less than the aggregate bandwidth $\sum_i B_{it}$ needed if each channel books individually. We set $\epsilon = 5\%$ and $\epsilon = 1\%$ for time periods 702—780 and 1562—1640, respectively, and result into an actual under-provision probability of $\epsilon' = 5.06\%$ and $\epsilon' = 1.27\%$ in Fig. 3(a) and Fig. 3(b), respectively. Although $\sum_i B_{it}$ always exceeds the real aggregate demand $\sum_i D_{it}$, it does not mean there is no outage when booking individually: each individual channel is still subject to an under-provision probability ϵ. The broker can save bandwidth reservation by more than 30% on average with controllable quality risks.

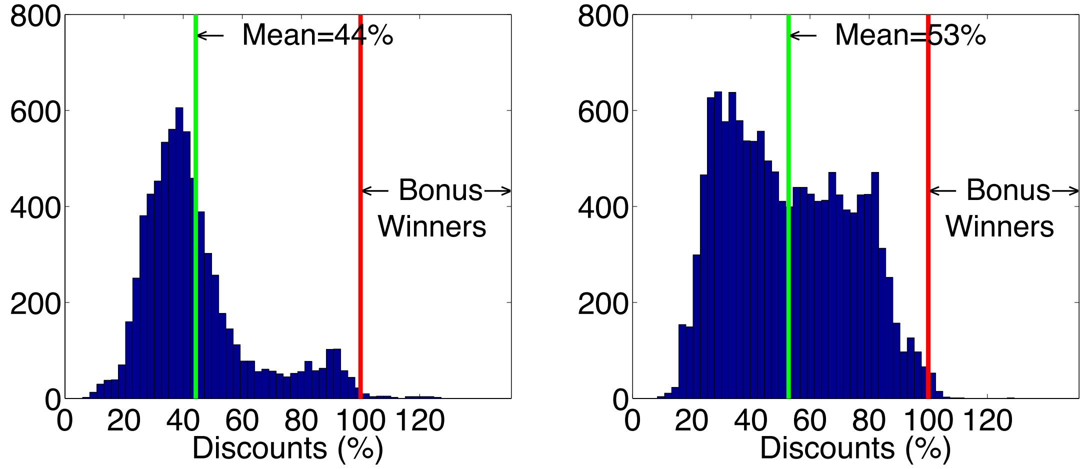
We further evaluate the price discount each tenant enjoys. At time t, tenant i pays \$ $P^0_{it}(1) = \mu_{it} + \theta\sigma_{it}$ if reserving bandwidth individually, but pays \$ $P^*_{it}(1) = \mu_{it} + \theta\sigma_{it}\rho_{iMt}$ in the equilibrium market with a broker. The discount it receives at time t is $1 - P^*_{it}(1)/P^0_{it}(1)$. Fig. 4 plots the histogram of discounts $1 - P^*_{it}(1)/P^0_{it}(1)$ for all i and t, showing that the mean price discount depends on demand statistics and even exceeds 50% during the second time span.

An interesting finding is that discount is over 100% for some rare cases, which means that the payment of some tenant i, $P^*_{it}(1) = \mu_{it} + \theta\sigma_{it}\rho_{iMt} < 0$, at some time t. We call such tenants "bonus winners," since the broker would even not mind paying to have them in the system. As pointed out in Sec. VI, this is because "bonus winners" have demands negatively correlated to the market and thus are risk neutralizers: whenever there is an increase in market demand, the demand of "bonus winners" drops to make resources available for other tenants in the market.

Finally, we validate the assumption that D_{it} is Gaussian. For each channel i, the prediction innovation is $D_{it} - \mu_{it}$ at time t. Fig. 5(a) shows the QQ plot of $D_{it} - \mu_{it}$ for all $t = 0, \Delta t, 2\Delta t, \ldots$ of a typical channel $i = 121$, which indicates $\{D_{it} - \mu_{it}\}$ sampled at Δt-intervals is a Gaussian process. Thus, it is reasonable to assume D_{it} follows the same Gaussian distribution within $[t, t + \Delta t)$ as long as Δt is small. Fig. 5(b) shows the QQ plot of $\sum_i (D_{it} - \mu_{it})$, implying that the aggregated demand $\sum_i D_{it}$ tends to Gaussian even if D_{jt} does not for some channel j. Since the load at each cloud provider L_s is aggregated from many demands, it is reasonable to assume L_s is Gaussian.

VIII. Related Work

Cloud computing is usually offered with usage-based pricing (pay-as-you-go), which requires a customer to pay just for the cloud resources consumed in the usage period [5], [16]. Amazon's S3 and EC2 use this pricing model. Pay-as-you-go is different from reserving a resource. The latter involves paying a negotiated cost to have the resource over a time period, whether or not the resource is used. Although suitable for delay-insensitive applications, pay-as-you-go is insufficient as an operation model for bandwidth-intensive and quality-stringent applications like VoD or gaming, since no performance guarantees are provided in general.



(a) t =702—780, 91 channels

(b) t =1562—1640, 176 channels

Fig. 4. The histogram of payment discounts of all channels at all times in equilibrium, i.e., the histogram of $1-P_i^*(1,t)/P_i^0(1,t)$ for all i and t.

However, cloud bandwidth reservation is becoming technically feasible. Virtualization techniques for supporting cloud-based IPTV services has been researched by major U.S. VoD providers like AT&T [17]. There have also been proposals on data center traffic engineering to offer bandwidth guarantees for egress traffic from virtual machines (VMs) [6]. Furthermore, video demand forecasting techniques have been proposed, such as the non-stationary time series models introduced in [10], [11], and video access pattern extraction via principal component analysis in [13].

In contrast to pay-as-you-go, we propose a new cloud pricing model that allows multimedia content providers such as VoD or online gaming companies to reserve bandwidth resources from the cloud at a negotiable cost. In particular, we use a broker to perform bandwidth multiplexing while controlling quality risks. Although the engineering benefits and implementation of cloud brokerage to interconnect clouds into a global cloud market has been discussed in [18], the highlight of this paper is the analysis of the economic properties of such broker-assisted bandwidth markets, using optimization theory and game theory.

The idea of statistical multiplexing and resource overbooking has been empirically evaluated for a shared hosting platform in [19]. VM consolidation with independent dynamic bandwidth demands has been considered in [20]. In contrast to these works, our work exploits the characteristics of tenant demand statistics in order to provide theoretical performance guarantees. *First*, we leverage the fact that the workload of user-faced multimedia applications such as VoD or gaming can be *fractionally* split into video requests. Therefore, optimal workload consolidation is no more than a set of load routing weights between tenants and cloud providers. *Second,* we formulate quality-assured resource minimization using Value at Risk (VaR), a popular risk measure in financial risk management [21], leading to provable performance guarantees.

IX. Concluding Remarks

In this paper, we consider the scenario that multiple tenants running user-faced multimedia workload can make reservations for bandwidth guarantees from cloud service providers in order to support continuous media streaming. Such tenants (content providers) attract inter-correlated random demands that can be directed to multiple cloud providers, each with a bandwidth capacity. We introduce a profit-making *broker* that statistically mixes demands based on anti-correlation while controlling the resource shortage risks. We study the behavior of the broker and how each tenant should be charged in such a broker-assisted market.

The region of all good pricing policies is characterized, such that making a profit at the broker is equivalent to maximizing cloud resource utilization through optimal load routing from tenants to the cloud providers. In a free market where tenants can negotiate and bargain for bandwidth cost, we prove that the bandwidth prices charged to the tenants will converge to a unique Nash equilibrium, which forms the lower bound of the good pricing region. Furthermore, the equilibrium bandwidth price billed to a tenant critically depends on its demand expectation, burstiness and its demand correlation to the market. Real-world traces verify that the presence of a broker can lower the market price for cloud bandwidth reservation by around 50% on average and save cloud bandwidth resources by over 30%.

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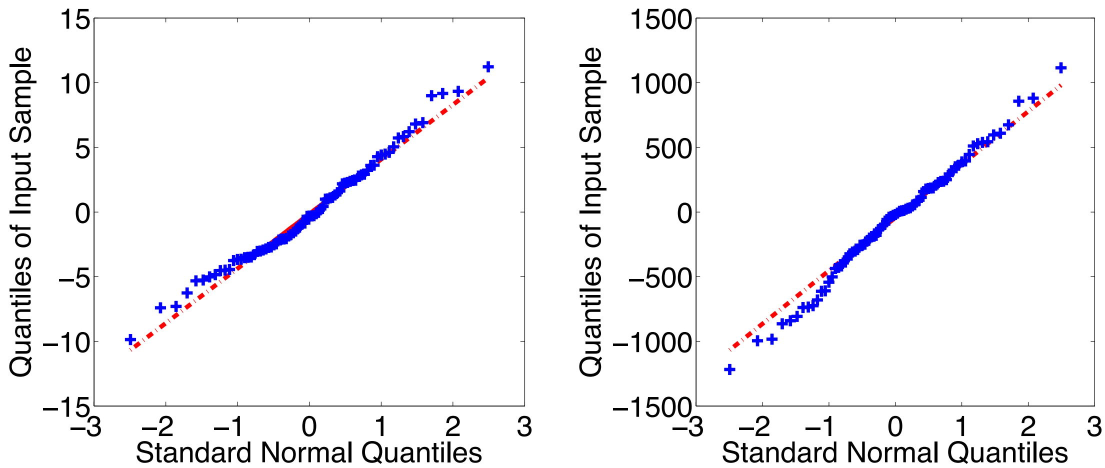

(a) A typical channel

(b) All channels combined

Fig. 5. QQ plot of prediction errors for t =1562—1640 versus standard normal distribution.

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