

Assignment 2

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latex-tikz codes from

<https://github.com/diya-goyal-29/AI1103/blob/main/Assignment%202/Assignment%202.tex>

Gate problem no. 42

Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to ...

Solution

Mean = $\mu = 0$

Variance = $\sigma = 1$

Gaussian Probability Distribution function

$$\begin{aligned} &= p(x) \\ &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \\ &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \end{aligned}$$

$$\phi(t) = \int_{-\infty}^{\infty} p(x) \exp(tx) dx \quad (0.0.1)$$

$$\phi'(t) = \int_{-\infty}^{\infty} p(x)x \exp(tx) dx \quad (0.0.2)$$

$$E[X] = \phi'(0) \quad (0.0.3)$$

$$= \int_{-\infty}^{\infty} |x|p(x) \quad (0.0.4)$$

$$= 2 \cdot \int_0^{\infty} xp(x)dx \quad (0.0.5)$$

$$= 2 \cdot \int_0^{\infty} x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \quad (0.0.6)$$

$$= \sqrt{\frac{2}{\pi}} \cdot \int_0^{\infty} x \exp\left(-\frac{x^2}{2}\right) dx \quad (0.0.7)$$

$$= \sqrt{\frac{2}{\pi}} \quad (0.0.8)$$

$$= 0.7978 \quad (0.0.9)$$