Assignment 2

Diya Goyal
Roll no. CS20BTECH11014

latex-tikz codes from

enter path

Gate problem no. 42

Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to ...

Solution

Mean = $\mu = 0$

Variance = $\sigma = 1$

Gaussian Probability Distribution function = $p(x)$

$= \frac{1}{\sqrt{2\pi\sigma}} exp(\frac{-(x-\mu)^2}{2\sigma^2})$

$= \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2})$

$$\begin{aligned} E[|X|] &= \int_{-\infty}^{\infty} |x| p(x) \\ &= \int_{-\infty}^{\infty} |x| \frac{1}{\sqrt{2\pi}} \exp(\frac{-x^2}{2}) dx \\ &= 2 \times \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} x \, \exp(\frac{-x^2}{2}) dx \\ &= \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} \exp(-u) du \end{aligned}$$

(Using substitution method)

$$\begin{aligned} &= \sqrt{\frac{2}{\pi}} \times (-1) \times (0 - 1) \\ &= \sqrt{\frac{2}{\pi}} \\ &= 0.799 \end{aligned}$$