Correction TD 2

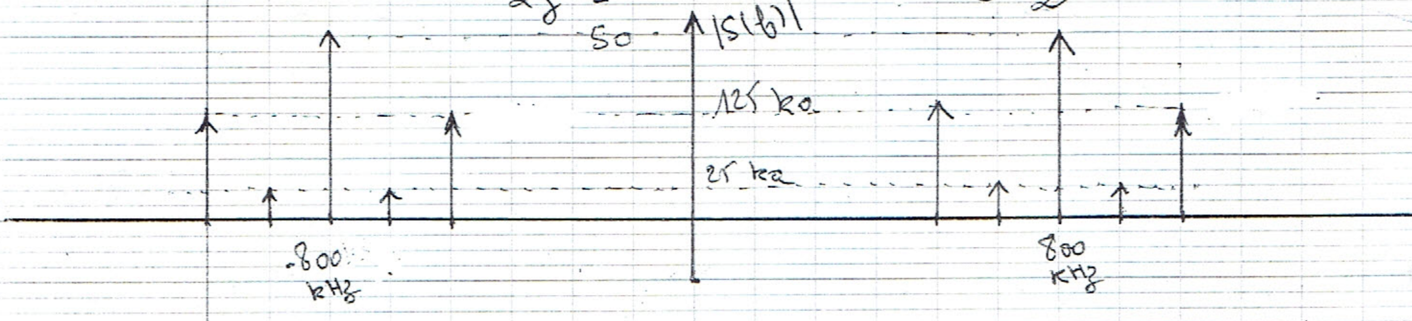
Exercice : 1 :

$$S_{DBAP}(t) = 100[1+k_a m(t)]\cos(2\pi f_0 t)$$
$$m(t) = \sin(2000\pi t) + 5\cos(4000\pi t)$$

1)

$$S_{DBAP}(f) = TF[100\cos(2\pi f_0 t) + 100 k_a m(t)\cos(2\pi f_0 t)]$$
$$= 50\,\delta(f-f_0) + 50\,\delta(f+f_0) + 50 k_a M(f) * [\delta(f-f_0)+\delta(f+f_0)]$$
$$= 50\,\delta(f-f_0) + 50\,\delta(f+f_0) + 50 k_a M(f-f_0) + 50 k_a M(f+f_0)$$

avec $M(f) = \frac{1}{2j}[\delta(f-1000) - \delta(f+1000)] + \frac{5}{2}[\delta(f-2000) + \delta(f+2000)]$



2) Pour faire une démodulation par détection d'enveloppe il faut que $|k_a m(t)| < 1$

$-6 < m(t) < 6 \Rightarrow -k_a 6 < k_a m(t) < +k_a 6$

$\Rightarrow |k_a m(t)| < k_a 6 < 1 \Rightarrow \underline{k_a < 1/6}$

Exercice n°2 :

1) $y(t) = a x(t) + b x^2(t)$

où $x(t) = m(t) + p(t)$

$\Rightarrow y(t) = a m(t) + a p(t) + b m^2(t) + b p^2(t) + 2b\, m(t)\, p(t)$

$= a m(t) + b m^2(t) + b p^2(t) + a\left[1 + \frac{2b}{a} m(t)\right] p(t)$

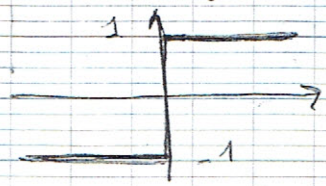
2) filtrage pass-bande autour de la freq porteuse

3) $\left|\frac{2b}{a} m(t)\right| < 1$

Ex3: $m(t) = \cos(2000\pi t) + 2\sin(2000\pi t)$

1) $\tilde{m}(t) = \frac{1}{\pi t} * m(t) \Rightarrow \tilde{M}(f) = -j\,\mathrm{sig}(f) \cdot M(f)$

ou sign(f):



$$M(f) = \frac{1}{2}\delta(f-1000) + \frac{1}{2}\delta(f+1000) + \frac{1}{j}\delta(f-1000) - \frac{1}{j}\delta(f+1000)$$

$$\tilde{M}(f) = -\frac{j}{2}\delta(f-1000) + \frac{j}{2}\delta(f+1000) - \delta(f-1000) - \delta(f+1000)$$

$$TF^{-1}(\tilde{M}(f)) = \tilde{m}(t) = -\frac{j}{2}e^{j2000\pi t} + \frac{j}{2}e^{-j2000\pi t} - e^{+j2000\pi t} - e^{-j2000\pi t}$$

$$= -\frac{j}{2}\left[2j\sin(2000\pi t)\right] - 2\cos(2000\pi t)$$

$$\boxed{\tilde{m}(t) = \sin(2000\pi t) - 2\cos(2000\pi t)}$$

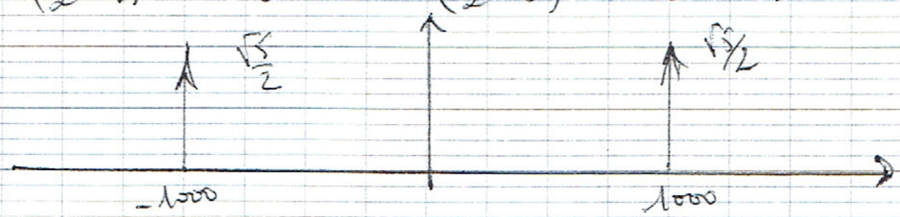
2) $s_{BLU,sup}(t) = \frac{A_0}{2} m(t)\cos(2\pi f_0 t) - \frac{A_0}{2}\hat{m}(t)\sin(2\pi f_0 t)$

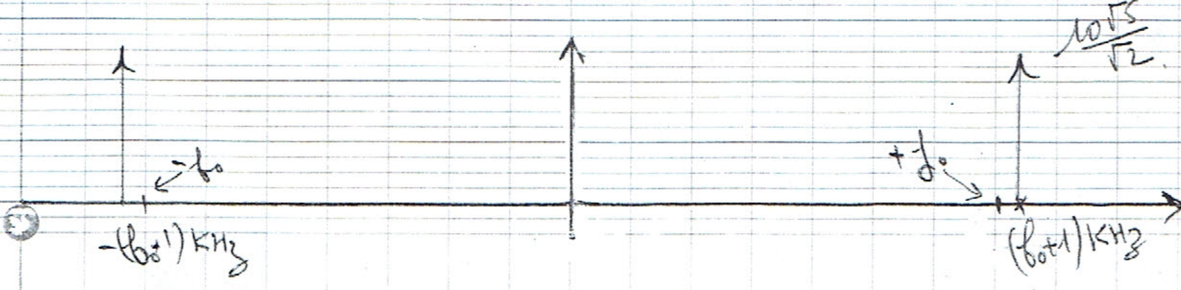
$$= \frac{100}{2}\left(\cos(2000\pi t) + 2\sin(2000\pi t)\right)\cos(2\pi f_0 t) - \frac{100}{2}\left(\sin(2000\pi t) - 2\cos(2000\pi t)\right)\sin(2\pi f_0 t)$$

$$= \frac{100}{2}\cos(2000\pi t)\cos(2\pi f_0 t) + \frac{200}{2}\sin(2000\pi t)\cos(2\pi f_0 t) - \frac{100}{2}\sin(2000\pi t)\sin(2\pi f_0 t) + \frac{200}{2}\cos(2000\pi t)\sin(2\pi f_0 t)$$

$$= \frac{100}{2}\cos\left(2\pi(f_0 + 1KHz)t\right) + \frac{200}{2}\sin\left(2\pi(f_0 + 1KHz)t\right)$$

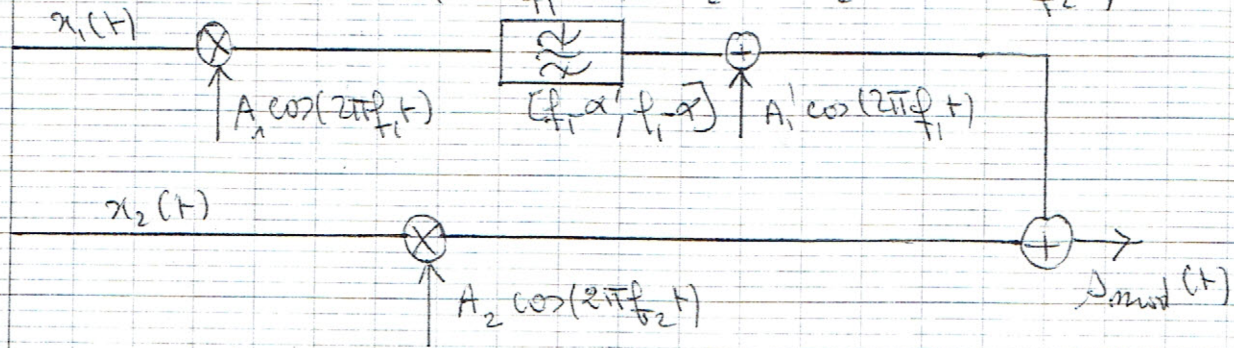
3) $M(f) = \left(\frac{1}{2} - j\right)\delta(f-1000) + \left(\frac{1}{2} + j\right)\delta(f+1000)$





Exercice n° 5 :

1) $s_{mod}(t) = A_1 x_1(t)\cos(2\pi f_1 t) + A_1 \tilde{x}_1(t)\sin(2\pi f_1 t) + A_1'\cos(2\pi f_1 t) + x_2(t) A_2 \cos(2\pi f_2 t)$



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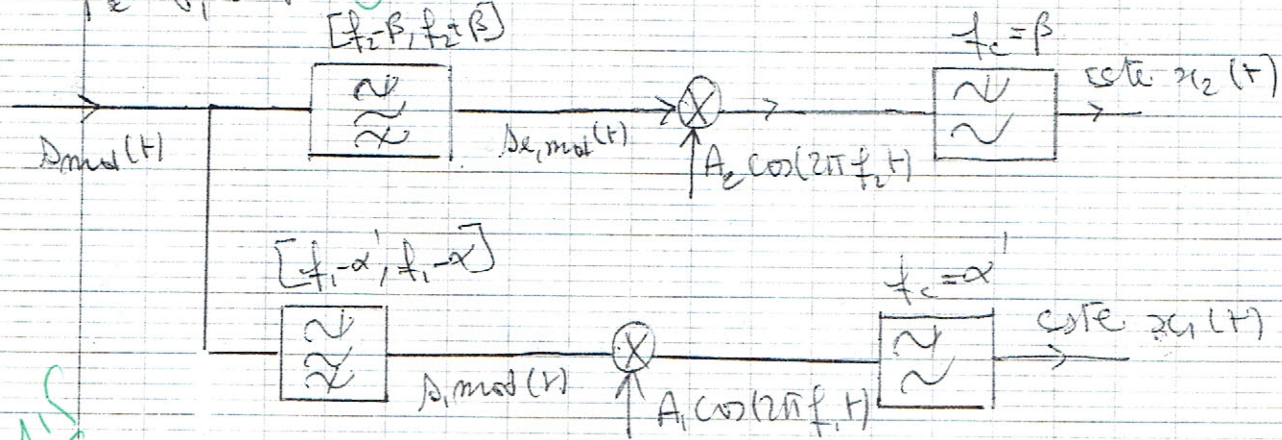
2) $f_2 - f_1 > \beta$ 1

3)



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Ex6: $S_+(f) = 2\,u(f)\,S(f)$

1) $s_+(t) = 2\,TF^{-1}(u(f)) * s(t)$

or on a : $TF(u(t)) = \frac{1}{2}\delta(f) + \frac{1}{2\pi j f}$

par dualité on a :

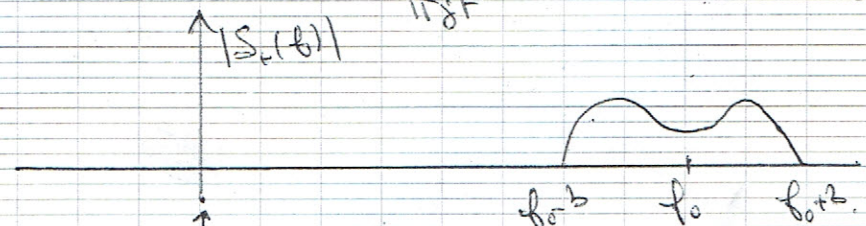
$$TF(X(-t)) = x(f)$$

$$\Rightarrow TF\left(\frac{1}{2}\delta(t) - \frac{1}{2\pi j t}\right) = u(-f)$$

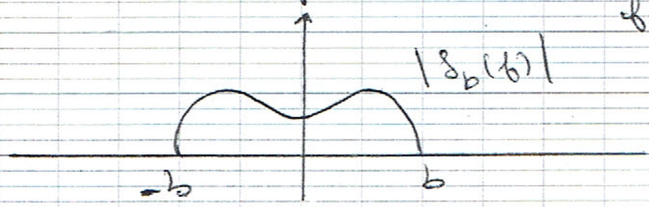
$$\Rightarrow TF^{-1}(u(f)) = \frac{1}{2}\delta(t) - \frac{1}{2\pi j t}$$

Donc : $s_+(t) = \left(\delta(t) - \frac{1}{\pi j t}\right) * s(t)$

$$= s(t) - \frac{1}{\pi j t} * s(t) = s(t) + j\hat{s}(t)$$



2)



$$S_b(f) = S_+(f+f_0) = S_+(f) * \delta(f+f_0)$$

$$\Rightarrow s_b(t) = s_+(t)\,e^{-2\pi j f_0 t}$$
$$= (s(t) + j\hat{s}(t))\,e^{-2\pi j f_0 t}$$

3)

$$s(t) + j\hat{s}(t) = s_b(t)\,e^{2\pi j f_0 t}$$

$$\Rightarrow s(t) = Re\left[s_b(t)\,e^{2\pi j f_0 t}\right]$$
$$= \frac{1}{2}\left[s_b(t)\,e^{2\pi j f_0 t} + s_b^*(t)\,e^{-2\pi j f_0 t}\right]$$

↳ $S(f) = \frac{1}{2}\left[S_b(f-f_0) + S_b^*(-f-f_0)\right]$

4) $\xrightarrow{s(t)} \boxed{h(t)} \xrightarrow{r(t)}$

$$R(f) = H(f)\,S(f)$$

$$= \frac{1}{2}\left(H_b(f-f_0) + H_b^*(-f-f_0)\right) \cdot \frac{1}{2}\left[S_b(f-f_0) + S_b^*(-f-f_0)\right]$$

$$= \frac{1}{4} H_b(f-f_0)\,S_b(f-f_0) + \frac{1}{4} H_b^*(-f-f_0)\,S^*(-f-f_0)$$

$$+ \frac{1}{4} \underbrace{H_b(f-f_0)\,S_b^*(-f-f_0)}_{0} + \frac{1}{4} \underbrace{H_b^*(-f-f_0)\,S_b(-f-f_0)}_{0}$$

$$= \frac{1}{2}\left(R_b(f-f_0) + R_b^*(-f-f_0)\right)$$

avec $R_b(f) = \frac{1}{2} H_b(f)\,S_b(f)$

$$\Downarrow$$

$$r_b(t) = \frac{1}{2} h_b(t) * s_b(t)$$