TUTORIAL-4

Q1) $T(n) = 3T(n/2) + n^2$

Ans $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2, f(n) = n^2$

$c = \log_b a$

$c = \log_2 3$

$n^{\log_2 3} > n^2$

$T(n) = O(n^2)$ Ans

Q2) $T(n) = 4T(n/2) + n^2$

Ans $a = 4, b = 2, f(n) = n^2$

$c = \log_2 4 \Rightarrow c = \log_2 2^2$

$c = 2\log_2 2 = 2$

$n^2 = n^2$

$T(n) = O(n^2 \log n)$ Ans

Q3) $T(n) = T(n/2) + 2^n$

Ans $a = 1, b = 2, f(n) = 2^n$

$c = \log_2 1 = 0$

$n^0 = 1$

$1 < 2^n$

$T(n) = O(2^n)$ Ans

Q4/ $T(n) = 2^n T(n/2) + n^n$

Ans $a = 2^n$

can't apply masters theorem because 'a' is not constant here.

Q5/ $T(n) = 16T(n/4) + n$

Ans $a = 16, \; b = 4, \; f(n) = n$

$c = \log_4 16$

$c = 2$

$n^2 > n$

$T(n) = O(n^2)$ Ans

Q6/ $T(n) = 2T(n/2) + n \log n$

Ans $a = 2, \; b = 2, \; f(n) = n \log n$

$c = \log_2 2 = 1$

$n < n \log n$

$T(n) = O(n \log n)$ Ans

Q7/ $T(n) = 2T(n/2) + n/\log n$

Ans $a = 2, b = 2, c = \log_2 2 = 1$

$n > \frac{n}{\log n}$

$T(n) = O(n)$ Ans

Q8] $T(n) = 2T(n/4) + n^{0.51}$

Ans $a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_4 2 = 0.5$

$n^{0.5} < n^{0.51}$

$T(n) = O(n^{0.51})$

Ans

Q9] $T(n) = 0.5T(n/2) + \frac{1}{n}$

Ans $a = 0.5$ can't apply master's theorem because it must be greater or equal to 1

Q10] $T(n) = 16T(n/4) + n!$

Ans $a = 16, b = 4, f(n) = n!$

$c = \log_4 16 = 2$

$n^2 < n!$

$T(n) = O(n!)$

Ans

11] $T(n) = 4T(n/2) + \log n$

Ans $a = 4, b = 2, c = \log n$

$c = \log_2 4 = 2$

$n^2 > \log n$

$T(n) = O(n^2)$

Ans

Q12] $T(n) = \sqrt{n}\, T(n/2) + \log n$

Ans $a = \sqrt{n}$ can't apply master's theorem because it must be constant.

Q13] $T(n) = 3T(n/2) + n$

Ans $a = 3,\ b = 2,\ f(n) = n$

$c = \log_2 3$

$n^{\log_2 3} > n$

$T(n) = O(n^{\log_2 3})$ Ans

Q14] $T(n) = 3T(n/3) + \sqrt{n}$

Ans $a = 3,\ b = 3,\ f(n) = \sqrt{n}$

$c = \log_3 2 \approx 1$

$n > \sqrt{n}$

$T = O(n)$ Ans

Q15] $T(n) = 4T(n/2) + cn$

Ans $a = 4,\ b = 2,\ f(n) = cn$

$c = \log_2 4 = 2$

$n^2 > cn$

$T(n) = O(n^2)$ Ans

Q16) $T(n) = 3T(n/4) + n\log n$

Sol $a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_4 3$

$\frac{\log_2 3}{\log_2 4} = 1.5$

$T(n) = O(n\log n)$ Ans

Q17) $T(n) = 3T(n/3) + n/2$

Sol $a = 3,\ b = 3,\ f(n) = n/2$

$c = \log_3 3 = 1$

$T(n) = O(n\log n)$ Ans

Q18) $T(n) = 6T(n/3) + n^2\log n$

Sol $a = 6,\ b = 3,\ f(n) = n^2\log n$

$c = \log_3 6$

$n^2\log n > n^{\log_3 6}$

$T(n) = O(n^2\log n)$ Ans

Q19] $T(n) = 4T(n/2) + \frac{n}{\log n}$

Ans $a = 4, \quad b = 2, \quad f(n) = \frac{n}{\log n}$

$c = \log_2 4$

$c = 2$

$n^2 > \frac{n}{\log n}$

$T(n) = O(n^2)$ Ans

Q20] $T(n) = 64T(n/8) - n^2 \log n$

Ans $a = 64, \quad b = 8, \quad f(n) = -n^2 \log n$

can't apply master's theorem because f(n) must be positive.

Q11] $T(n) = 7T(n/3) + n^2$

Ans $a = 7, \quad b = 3, \quad f(n) = n^2$

$c = \log_3 7$

$c = \frac{\log_8 7}{\log_2 3} \approx 2$

$TC = O(n^2)$ Ans

Q23) $T(n) = T(n/2) + n(2-\cos n)$

Ans $a = 1, \; b = 2, \; f(n) = n(2-\cos n)$

$k = \log_a 1 = 0$

$1 < n(2-\cos n)$

$T(n) = O(n)$