MA 125
BUSINESS CALCULUS
FALL 2025

5.4 Bonds & Treasury Bills

Recall that $FV = PV(1 + rt)$, where

- $FV :=$ *Future Value*
- $PV :=$ *Present Value*
- $r :=$ *Simple Interest Rate (Per Year)*
- $t :=$ *Number of Years*

Corporate Bonds

Sometimes, companies borrow money from investors, and in return, they issue bonds. The company promises:

- Repay the **principal** (the original amount borrowed) when the bond *"matures"*.

- Pay **interest** to the bondholders (usually at a fixed rate and regular intervals).

Some key terms.

- **Issuer**: A private or public corporation.

- **Coupon Rate**: The interest rate paid to investors

- **Maturity Date**: When the bond ends and principal is repaid.

- **Risk**: Higher than government bonds because companies can go bankrupt (relatively more common), but they also tend to pay higher interest.

Some of the reasons why investors buy corporate bonds:

- Earn regular income from interest payments.

- Diversify investments.

Example 1. For a corporate bond of $5000 over the period of 20 years at a simple interest rate of 6.05%, find the following. (Round to the nearest cent as needed.)

(i). The semiannual interest on the corporate bond = ...

(ii). The total interest (over the life) of the corporate bond = ...

Present Value for Simple Interest

The *Present Value for Simple Interest* is given by

$$PV = \frac{FV}{1+rt}. \tag{1}$$

Here,

- $FV :=$ *Future Value*
- $PV :=$ *Present Value*
- $r :=$ *Simple Interest Rate (Per Year)*
- $t :=$ *Number of Years*

Example 2. Find the present value of the future value of $\$17,000$ for 15 months at a 6% simple interest rate. (Round to the nearest dollar as needed.)

Simple Discount Loans

In a simple discount loan, the **lender subtracts the interest** (the *discount*) from the loan principal at the start, and the borrower receives the **remaining amount** (called the *proceeds*).

At the end of the loan period (at maturity), the borrower repays the **full face value**. (Some other texts use the term **face value**.)

Remark: The textbook uses **PV (present value)** the same as the face value, but some other texts use **FV (Future Value)**.

Let

$$PV :=\ \text{Face Value or Present Value or simply the amount borrowed}$$
$$r :=\ \text{Discount rate (annual simple discount rate)}$$
$$t :=\ \text{Time (in years)}$$
$$D :=\ \text{Discount.}$$
$$P :=\ \text{Proceeds (the actual amount the borrower gets)}$$

Then,

$$D = PV \cdot r \cdot t. \tag{2}$$

and

$$P = PV - D = PV(1 - rt). \tag{3}$$

Example 3. A 3 month $22,000 treasury bill (called T-bills) with a simple annual interest rate of 0.24% was sold in 2018. Find the following.

(i). The price of the treasury bill (that is, the proceeds) = ...
(Round to the nearest cent as needed.)

(ii). The actual interest rate paid by the Treasury = ...
(Write an integer or decimal rounded to five decimal places as needed.)
(Hint: Use the formula for the simple interest rate: $I = PV \cdot r \cdot t$.)

Example 4. A six month $4800 treasury bill sold for $4579. What was the simple interest discount rate?
(Type an integer or decimal rounded to one decimal place as needed.)
(Hint: Use the formula $D = PV \cdot r \cdot t$).

Zero Coupon Bonds

A **zero-coupon bond** is a type of bond that does not pay periodic interest (coupons). Instead, it is sold at a discount to its face value and pays the **full face value at maturity**.

The difference between the purchase price and the face value represents the investor's earnings.

Remarks.

- **No periodic interest payments**: Unlike regular bonds, there are no coupon payments.

- **Sold at a discount**: For example, you might buy a $1000 bond for $900.

- **Payment at maturity**: At the end of the bond term, you receive the full $1000.

- **Implied interest**: The *interest* you earn is the difference between what you pay now (purchase price\ Present Value (PV)) and what you receive at maturity (face value\ Future Value (FV)).

The formula for the present value (price) of a zero-coupon bond is

$$FV = PV \left(1 + \frac{r}{m}\right)^n \tag{4}$$

where

$$
\begin{aligned}
PV &:= \text{Present value (price you pay now)} \\
FV &:= \text{Face value (amount repaid at maturity)} \\
r &:= \text{Interest rate (per year)} \\
m &:= \text{Number of periods per year} \\
t &:= \text{Total number of periods until maturity.}
\end{aligned}
$$

Remark. According to the textbook, interest is compounded semiannually, even though this may not be explicitly stated in zero-coupon bond problems.

Example 5. Find the face value of the zero-coupon bond.

$$12 \text{ year bond at } 5.5\%; \text{ price } \$11,000.$$

(Round to the nearest dollar as needed.)