

## PHYS 20323/60323: Fall 2025 - LaTeX Example

1. At time  $t = 0$  a particle is represented by the wave function

$$\Psi(x) = \begin{cases} A \frac{x}{a}, & 0 \leq x \leq a, \\ A \frac{(b-x)}{(b-a)}, & a \leq x \leq b, \\ 0, & \text{otherwise.} \end{cases}$$

where  $A$ ,  $a$ , and  $b$  are constants.

- (a) (3.3 points) Normalize  $\Psi$  (i.e., find  $A$  terms of  $a$  and  $b$ ).
- (b) (3.3 points) Where is the particle likely to be found at  $t = 0$ ?
- (c) (3.4 points) What is the expectation value of  $x$ ?

2. **The following questions refer to stars in the Table below.**

*Note: There may be multiple answers.*

Name	Mass	Luminosity	Lifetime	Temperature	Radius	Variable ?
$\delta$ Scu.	$2.0 M_{\odot}$		$5.0 \times 10^8$ years		$2.0 R_{\odot}$	Y
$\gamma$ Del.	$0.7 M_{\odot}$		$4.5 \times 10^{10}$ years	5000 K		N
$\beta$ Cyg.	$1.3 M_{\odot}$	$3.5 L_{\odot}$				Y
$\eta$ Car.	$60. M_{\odot}$	$10^6 L_{\odot}$	$8.0 \times 10^5$ years			Y
$\epsilon$ Eri.	$6.0 M_{\odot}$	$10^3 L_{\odot}$		20,000 K		N
$\alpha$ Cen.	$1.0 M_{\odot}$			6000 K	$1.0 R_{\odot}$	N

- (a) (4 points) Which of these stars will produce a planetary nebula.
- (b) (4 points) Elements heavier than Carbon will be produced in which stars.

3. An electron is found to be in the spin state (in the  $z$ -basis):  $\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$

- (a) (5 points) Determine the possible values of  $A$  such that the state is normalized.
- (b) (5 points) Find the expectation values of the operators  $S_x$ ,  $S_y$ ,  $S_z$  and  $\vec{S}^2$ .

The matrix representations in the  $z$ -basis for the components of electron spin operators are given by:

$$\mathbf{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \mathbf{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \mathbf{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$