Bihar Board Class 12 Mathematics Question Paper 2015

INTERMEDIATE EXAMINATION - 2015

(ANNUAL)

MATHEMATICS

Time- $3\frac{1}{4}$ Hours

Full Marks: 100

Instruction for the candidates:

1) Candidates are required to give their answers in their own words as far as practicable.
2) Figures in the right hand margin indicate full marks.
3) 15Minutes of extra time has been allotted for the candidates to read the questions carefully.
4) This question paper is divided into two **section A** and **section B.**
5) In **section-A,** there are **40 objective type questions** which are compulsory, each carry **1 mark.** Darken the circle with blue/black ball pen against the correct option on **OMR** answer Sheet provided to you. **Do not use whitener/Liquid/Blade/Nall etc. on OMR Sheet; otherwise the result will be invalid.**
6) In **Section-B,** there **are 25 short answer** type question (each carrying **2 marks**),out of which **any 15** questions are to be answered. Apart this, there are **8 Long Answer** Type question (Each Carrying **5 Marks**), Out of which **any 4** questions to be answered.
7) Use of any electronic appliances is strictly prohibited.

Section-I : (Objective Type)

For the following Question Nos. 1 to 40 there is only one correct answer against each question. Mark the correct option on the answer sheet. [40*1=40]

1. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2, x_2 \in A$, then the function $f : A \rightarrow R$

(A) One-one **(B) constant**

(C) onto **(D) many one**

Sol.

Correct answer option is (A)

2. The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **is**

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

Sol.

Correct answer option is (D)

3. $\tan^{-1} x =$

(A) $\cos^{-1} x$

(B) $\frac{1}{\cot^{-1} x}$

(C) $\cot^{-1}\frac{1}{x}$

(D) $-\cot^{-1} x$

Sol.

Correct answer option is (C)

4. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} =$

(A) $(x-y)(y+z)(z+x)$

(B) $(x+y)(y-z)(z-x)$

(C) $(x-y)(y+z)(z+x)$

(D) $(x-y)(y-z)(z-x)$

Sol.

Correction answer option is (D)

5. If $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$**, then x=**

(A) 3

(B) 4

(C) 5

(D) 8

Sol.

Correct answer option is (none of these)

6. $\frac{d}{dx}\left(\sin^{-1} x\right)=$

(A) $\frac{1}{\sqrt{1-x^2}}$

(B) $-\frac{1}{\sqrt{1-x^2}}$

(C) $2\left(1-x^2\right)$

(D) $\left(1-x^2\right)$

Sol.

Correct answer option is (A)

7. $\frac{d}{dx}\left(\sin^{-1} x+\cos^{-1} x\right)=$

(A) 0

(B) 1

(C) $\frac{\pi}{2}$

(D) $\frac{1}{\sqrt{1-x^2}}$

Sol.

Correct answer option is (C)

8. If $y=\sin\left(x^3\right)$

(A) $x^3\cos\left(x^3\right)$

(B) $3x^2\sin\left(x^3\right)$

(C) $3x^2\cos\left(x^3\right)$

(D) $\cos\left(x^3\right)$

Sol.

Correct answer option is (D)

9. $y = \tan^2 x$, **then** $\frac{dy}{dx}$

(A) $\sec^2 x$

(B) $\sec^4 x$

(C) $2\tan x \sec x$

(D) $2\tan x \sec^2 x$

Sol.

Correct answer option is (D)

10. $\int 1 \cdot dx$

(A) $x + k$

(B) $1 + k$

(C) $\frac{x^2}{2} + k$

(D) $\log x + k$

Sol.

Correct answer option is (A)

11. $\int \frac{dx}{\sqrt{x}} =$

(A) $\sqrt{x} + k$

(B) $2\sqrt{x} + k$

(C) $x + k$

(D) $\frac{2}{3} x^{3/2+k}$

Sol.

Correct answer option is (B)

12. $\int \frac{dx}{1 + \cos x} =$

(A) $\tan \frac{x}{2} + k$

(B) $\frac{1}{2} \tan \frac{x}{2} + k$

(C) $2\tan \frac{x}{2} + k$

(D) $\tan^2 \frac{x}{2} + k$

Sol.

Correct answer option is (A)

13. $\int_a^b x^5 dx =$

(A) $b^5 - a^5$

(B) $\frac{b^6 - a^6}{6}$

(C) $\frac{a^6 - b^6}{6}$

(D) $a^5 - b^5$

Sol.

Correct answer option is (B)

14. The solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ **is**

(A) $x - y = k$

(B) $x^2 - y^2 = k$

(B) $x^3 - y^3 = k$

(D) $xy = k$

Sol.

Correct answer option is (B)

15. The integrating factor of the linear differential equation $\frac{dy}{dx} = y \tan x \sec^2 x$

(A) $\tan x$

(B) $e^{\tan x}$

(C) $\log \tan x$

(D) $\tan^2$

Sol.

Correct answer option is (B)

16. The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = \frac{d^2 y}{dx^2}$ **is**

(A) 1

(B) 2

(C) 3

(D) 4

Sol.

Correct answer option is (A)

17. **The order of the differential equation** $\frac{d^2y}{dx^2}+x^3\left(\frac{dy}{dx}\right)^3=x^4$ **is**

(A) 1 **(B) 2**

(C) 3 **(D) 4**

Sol.

Correct answer option is (B)

18. **The positive vector of the point (1, 0, 2) is**

(A) $\vec{i}+\vec{j}+2\vec{k}$ **(B)** $\vec{i}+2\vec{j}$

(C) $\vec{i}+3\vec{k}$ **(D)** $\vec{i}+2\vec{k}$

Sol.

Correct answer option is (D)

19. **The modulus of** $7\vec{i}-2\vec{j}+\vec{k}$

(A) $\sqrt{10}$ **(B)** $\sqrt{55}$

(C) $3\sqrt{6}$ **(D)** 6

Sol.

Correct answer option is (C)

20. **If O be the origin and** $\overrightarrow{OP}=2\hat{i}+3\hat{j}-4\hat{k}$ **and** $\overrightarrow{OQ}=5\hat{i}+4\hat{j}-3\hat{k}$ **then** $\overrightarrow{PQ}$ **is equal to**

(A) $7\hat{i}+7\hat{j}-7\hat{k}$ **(B)** $-3\hat{i}-\hat{j}-\hat{k}$

(C) $-7\hat{i}-7\hat{j}+7\hat{k}$ **(D)** $3\hat{i}+\hat{j}+\hat{k}$

Sol.

Correct answer option is (D)

21. **The scalar product of** $5\hat{i}+\hat{j}-3\hat{k}$ **and** $3\hat{i}-4\hat{j}+7\hat{k}$ **is**

(A) 10 **(B) -10**

(C) 15 **(D) -15**

Sol.

Correct answer option is (B)

22. If $\vec{a}\cdot\vec{b}=0$ **then**

(A) $\vec{a}\perp\vec{b}$ **(B)** $\vec{a}\parallel\vec{b}$

(C) $\vec{a}+\vec{b}=\vec{0}$ **(D)** $\vec{a}-\vec{b}=\vec{0}$

Sol.

Correct answer option is (A)

23. $\vec{i}\cdot\vec{j}=$

(A) 0 **(B) 1**

(C) $\vec{k}$ **(D)** $-\vec{k}$

Correct answer option is (A)

24. $\vec{k}\cdot\vec{j}=$

(A) 0 **(B) 1**

(C) $\vec{i}$ **(D)** $-\vec{i}$

Sol.

Correct answer option is (D)

25. $\vec{a}\cdot\vec{a}=$

(A) 0 **(B) 1**

(C) $\left|\vec{a}\right|^2$ **(D)** $\left|\vec{a}\right|$

Sol.

Correct answer option is (C)

26. The direction cosines of the y-axis are

(A) (0, 0, 0) **(B) (1, 0, 0)**

(C) (0, 1, 0) **(D) (0, 0, 1)**

Sol.

Correct answer option is (C)

27. The direction ratios of the line joining the points (x_1, y_1, z_1) **and** (x_2, y_2, z_2) **are**

(A) $x_1 + x_2, y_1 + y_2, z_1 + z_2$ **(B)** $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(C) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$ **(D)** $x_1 - x_2, y_2 - y_1, z_2 - z_1$

Sol.

Correct answer option is (D)

28. The co-ordinates of the midpoint of the line segment joining the points (2,3,4) and (8,-3,8) are

(A) (10, 0, 12) **(B) (5, 6, 0)**

(C) (6, 5, 0,) **(D) (5, 0, 6)**

Sol.

Correct answer option is (D)

29. If the direction cosines of the two straight lines are l_1, m_1, n_1 **and** l_2, m_2, n_2 **then the cosine of the angle** θ **between them or** $\cos\theta$

(A) $(l_1 + m_1 + n_1)(l_2 + m_2 + n_2)$ **(B)** $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(C) $l_1 l_2 + m_1 m_2 + n_1 n_2$ **(D)** $\frac{l_1 + m_1 + n_1}{l_2 + m_2 + n_2}$

Sol.

Correct answer option is (C)

30. The direction ratios of the normal to the plane $7x + 4y - 2z + 5 = 0$ **are**

(A) 7, 4, 5

(B) 7, 4, -2

(C) 7, 4, 2

(D) 0, 0, 0

Sol.

Correct answer option is (B)

31. If the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ **is parallel to the plane** $ax+by+cz+d=0$**, then**

(A) $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$

(B) $al+bm+cz=0$

(C) $al^2+bm^2+cn^2=0$

(D) $a^2l^2+b^2m^2+c^2n^2=0$

Sol.

Correct answer option is (B)

32. If the planes $a_1x+b_1y+c_1z+d_1=0$ **and** $a_2x+b_2y+c_2z+d_2=0$

(A) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(B) $\frac{a_1}{a_2}+\frac{b_1}{b_2}+\frac{c_1}{c_2}=0$

(C) $a_1a_2+b_1b_2+c_1c_2=0$

(D) $a_1^2a_2^2+b_1^2b_2^2+c_1^2c_2^2=0$

Sol.

Correct answer option is (C)

33. The distance of the plane $2x-3y+6z+7=0$ **from the point** $(2,-3,-1)$

(A) 4

(B) 3

(C) 2

(D) $\frac{1}{5}$

Sol.

Correct answer option is (C)

34. If $P(A)=\frac{3}{8}, P(B)=\frac{1}{2}, P(A\cap B)=\frac{1}{4}$ **=**

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{5}$

Sol.

Correct answer option is (D)

35. If A and B are two independent event, then

(A) $P(A \cup B) = 1 - P(A^{'})P(B^{'})$

(B) $P(A \cap B) = 1 - P(A^{'})P(B^{'})$

(C) $P(A \cup B) = 1 + P(A^{'})P(B^{'})$

(D) $P(A \cup B) = \frac{P(A^{'})}{P(B^{'})}$

Sol.

Correct answer option is (A)

36. $\begin{vmatrix} 3 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{vmatrix} =$

(A) 40

(B) 50

(C) 42

(D) 15

Sol.

Correct answer option is (C)

37. The inverse of $A = \begin{bmatrix} 2 & 3 \\ 5 & \mathrm{k} \end{bmatrix}$ **will not be obtained if k has the value**

(A) 2

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{15}{2}$

Sol.

Correct answer option is (D)

38. $\cos^{-1}\frac{1-x^2}{1+x^2} =$

(A) $2\cos^{-1}x$

(B) $2\sin^{-1}x$

(C) $2\tan^{-1}x$

(D) $\cos^{-1}2x$

Sol.

Correct answer option is (C)

39 For any unit matrix 1

(A) $I^2 = I$

(B) $|I| = 0$

(C) $|I| = 2$

(D) $|I| = 5$

Sol.

Correct answer option is (A)

40. If $x > a, \int \frac{dx}{x^2 - a^2} =$

(A) $\frac{1}{2a}\log\frac{x-a}{x+a} + k$

(B) $\frac{1}{2a}\log\frac{x+a}{x-a} + k$

(C) $\frac{1}{a}\log\left(x^2 - a^2\right) + k$

(D) $\log x + \sqrt{x^2 - a^2} + k$

Sol.

Correct answer option is (A)

Question Nos. 1 to 8 are of short answer type. Each question carries 4 marks.
$[4 \times 8 = 32]$

Short Answer Type Questions

1. Prove that $4\left(\cot^{-1}3 + \cos ec^{-1}\sqrt{5}\right) = \pi$

Sol.

Prove the expression,

$$let\ \ cosec^{-1}\sqrt{5}=\theta \Rightarrow c2os\,ec\theta=\sqrt{5}(squaring, we \text{ get})$$

$$\text{cosec}^2\theta=5\Rightarrow 1+\cot^2\theta=5$$

$$\Rightarrow \cot^2\theta=5-1=4\Rightarrow \cot^2\theta=4\Rightarrow \cot\theta=2$$

$$\Rightarrow \tan\theta=\frac{1}{2}$$

$(now, L.H.S)$

$$4\left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}\right)=4\tan^{-1}\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{6}}$$

$$=4\tan^{-1}\left(\frac{2+3}{6-1}\right)=4\tan^{-1}(1)$$

$$=4\tan^{-1}\left(\tan\frac{\pi}{4}\right)=4\times\frac{\pi}{4}=\pi\left(R.H.S\right)$$

Hence, proved it.

2. Evaluate $A=\begin{vmatrix}1 & \text{bc} & \text{a}(b+c)\\ 1 & \text{ca} & \text{b}(c+a)\\ 1 & \text{ab} & \text{c}(a+b)\end{vmatrix}$

Sol.

Simplify the expression,

$$A=\begin{vmatrix}1 & bc & a(b+c)\\1 & ca & b(c+a)\\1 & ab & c(a+b)\end{vmatrix}$$

Applying $C_3 \rightarrow C_3 + C_2$,

$$A=\begin{vmatrix}1 & bc & ab+ac+bc\\1 & ca & ca+bc+ba\\1 & ab & ab+ca+cb\end{vmatrix}$$

$$A=\begin{vmatrix}1 & bc & ab+bc+ac\\1 & ca & bc+ba+ac\\1 & ab & ab+bc+ac\end{vmatrix}$$

(*Taking* common ab+bc+ac from C_3, *we* get)

$$A=(ab+bc+ac)\begin{vmatrix}1 & bc & 1\\1 & ca & 1\\1 & ab & 1\end{vmatrix}$$

$$\Rightarrow ab+bc+ac\times 0=0$$

Hence, the $C_1 = C_2$

3. Find the value of X and Y :

$$X+Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix} \textbf{ and } X-Y=\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}$$

Sol.

Simplify the expression,

$$X+Y=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}\ldots\ldots\ldots..(i)$$

$$X-Y=\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\ldots\ldots\ldots..(ii)$$

Adding equation (i) and equation (ii), we get

$$X+Y+X-Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\Rightarrow\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}$$

$$2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}\Rightarrow X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

By equation (1), we get

$$X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}\Rightarrow Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$Y=\begin{bmatrix}7-5 & 0-0\\ 2-1 & 5-4\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

$$X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix},Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

4.If $\sin\{\cos[\tan(\cot x)\}$, find $\frac{dy}{dx}$ by first principle.

Sol.

Simplify the expression,

$$\Rightarrow\frac{dy}{dx}=\cos\{\cos[\tan(\cot x)\}\times-\sin\left[\tan(\cot x)\right]\times\sec^2\left(\cot x\right)\times-\cos ec^2x$$

$$\Rightarrow\frac{dy}{dx}=\cos ec^2x.\sec^2\left(\cot x\right)\cdot\sin[\tan(\cot x)]\cdot\cos[\cos(\tan(\cot x)]$$

$$\Rightarrow\frac{dy}{dx}=\cos ec^2x.\sec^2\left(\cot x\right).\sin[\tan(\cot x)]\cos[\cos(\tan(\cot x)]$$

5.If $y=\tan^{-1}x,$ **then find** $\frac{dy}{dx}$ **by first principle.**

Sol.

Simplify the expression,

Let $y=\tan^{-1}x\Rightarrow x=\tan y$

If δx be a small change in x and δy be the corresponding change in y,

$x+\delta x=\tan\left(y+\delta y\right)$

Subtracting (1) from (2), we get

$x+\delta x-x=\tan\left(y+\delta y\right)-\tan y$

$$\delta x=\tan\left(y+\delta y\right)-\tan y=\frac{\sin\left(y+\delta y\right)}{\cos\left(y+\delta y\right)}-\frac{\sin y}{\cos y}$$

$$\delta x=\frac{\sin\left(y+\delta y-y\right)}{\cos y\cos\left(y+\delta y\right)}=\frac{\sin\delta y}{\cos y\cos\left(y+\delta y\right)}$$

$$\frac{\delta x}{\delta y}=\frac{\sin\delta y}{\delta y}\cdot\frac{1}{\cos y\cos\left(y+\delta y\right)}$$

$$\frac{\delta y}{\delta x}=\frac{\delta y}{\sin\delta y}\cdot\cos y\cos\left(y+\delta y\right)$$

Taking limit on the both sides, we get

$$\lim_{\delta x\to0}\frac{\delta y}{\delta x}=\lim_{\delta y\to0}\frac{\delta y}{\sin\delta y}\lim_{\delta y\to0}\cos y\cos\left(y+\delta y\right)$$

$$\frac{dy}{dx}=1.\cos y.\cos y=\cos^2 y=\frac{1}{sec^2 y}$$

$$\frac{dy}{dx}=\frac{1}{sec^2 y}=\frac{1}{1+\tan^2 y}=\frac{1}{1+x^2}$$

Hence, the value is $y=\tan^{-1}x$ then $\frac{dy}{dx}=\frac{1}{1+x^2}$.

6. Integrate $\int e^x\cos xdx$

Sol.

Simplify the expression,

Take e^x as the first function and cosx as second function. Then integrating by part, we have

$$I=\int e^x\cos dx=e^x\left(\sin x\right)+\int e^x\sin xdx$$
$$=e^x\left(\sin x\right)+I_1......(i)$$

Taking e^x and sinx as the first and second functions, respectively, in I_1, we get

$$I_1=e^x\cos x-\int e^x\cos xdx$$

Substituting the value of I_1 in (i), we get

$I_1 = e^x \sin x + e^x \cos x - I$ or $e^x(\sin x + \cos x)$

$I = \int e^x \cos x dx = \frac{e^x}{2}(\sin x + \cos x) + C$

$I = \frac{e^x}{2}(\sin x + \cos x) + C$

Hence, the value is $I = \frac{e^x}{2}(\sin x + \cos x) + C$

If Take e^x as the first function and $\cos x$ as second function. Then integrating by part, we have

$I = \int e^x \cos dx = e^x(\sin x) + \int e^x \sin x dx$

$= e^x(\sin x) + I_1 ......(i)$

Taking e^x and $\sin x$ as the first and second functions, respectively, in I_1, we get

$I_1 = e^x \cos x - \int e^x \cos x dx$

Substituting the value of I_1 in (i), we get

$I_1 = e^x \sin x + e^x \cos x - I$ or $e^x(\sin x + \cos x)$

$I = \int e^x \cos x dx = \frac{e^x}{2}(\sin x + \cos x) + C$

$I = \frac{e^x}{2}(\sin x + \cos x) + C$

Hence, the value is $I = \frac{e^x}{2}(\sin x + \cos x) + C$

7. If $\vec{a} = 2\vec{i} - 3\vec{j} - 5\vec{k}$ and $\vec{b} = -7\vec{i} + 6\vec{j} + 8\vec{k}$, find $\vec{a} \times \vec{b}$

Sol.

Simplify the expression,

$\vec{a} = 2\vec{i} - 3\vec{j} - 5\vec{k}$ and $\vec{b} = -7\vec{i} + 6\vec{j} + 8\vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -5 \\ -7 & 6 & 8 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = (-24+30)\hat{i} - (16-35)\hat{j} + (12-21)\hat{k}$$

$$\vec{a} \times \vec{b} = +\hat{i} + 19\hat{j} - 9\hat{k}$$

Hence, the value $+\hat{i} + 19\hat{j} - 9\hat{k}$.

8. A speak the truth in 75% cases and dB B in 80% of the cases. In what percentage of the cases are they likely to contradict each other in stating the same fact?

Sol.

Simplify the expression,

Contradict in statement if. One speak truth and other speak lie

Then $n(E_1) = 75$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{75}{100} = \frac{3}{4}$$

Again $E_2 = B$ Speak lie then $n(E_2) = 100 - 80 = 20$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{20}{100} = \frac{1}{5}$$

Thus the probability of a speak truth and B speak lie

$$P(E_1 \cap E_2) = P(E_1)P(E_2) = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20} \text{........(1)}$$

Again the probability of B speak truth and B speak truth and A speak lie is

$$P\left(E'_1 \cap E'_2\right) = P\left(E'_1\right) - P\left(E'_2\right)$$

$$P\left(E'_1\right) = 1 - P\left(E_1\right) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P\left(E'_2\right) = 1 - P\left(E_2\right) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\Rightarrow P\left(E'_1 \cap E'_2\right) = P\left(E'_1\right) P\left(E'_2\right) = \frac{1}{4} \times \frac{4}{5} = \frac{1}{5} ....(2)$$

Then from (1) and (2)

The probability of contradict

$$= P\left(E_1 \cap E_2\right) \cup P\left(E'_1 \cap E'_2\right)$$

$$= P\left(E_1 \cap E_2\right) + P\left(E'_1 \cap E'_2\right)$$

$$= \frac{3}{20} + \frac{1}{5} = \frac{3+4}{20} = \frac{7}{20} = \frac{35}{100} = 35\%$$

Hence, percentage of contradicts is 35%

Long Answer Type Questions $[4 \times 7 = 28]$

9. Evaluate $\int_0^{\pi/4} \sin^2 dx$

Sol.

Let $I = \int_0^{\pi/4} \sin^2 dx$

$$I = \int_0^{\pi/4} \frac{(1-\cos 2x)}{2} dx = \frac{1}{2}\int_0^{\pi/4} dx = \frac{1}{2}\int_0^{\pi/4} \cos 2x dx =$$

$$I = \frac{1}{2}[x]_0^{\pi/4} - \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$I = \frac{1}{2}\left[\frac{\pi}{4} - 0\right] - \frac{1}{4}\left[\sin^2 \cdot \frac{\pi}{4} - \sin 0\right]$$

$$\Rightarrow \frac{\pi}{8} - \frac{1}{4}\left[\sin^2 \cdot \frac{\pi}{4} - \sin 0\right] = \frac{\pi}{8} - \frac{1}{4}$$

$$I = \frac{\pi}{8} - \frac{1}{4}$$

Hence the value of $I = \frac{\pi}{8} - \frac{1}{4}$.

10. Solve the differential equation: $(x-y)dy - (x+y)dx = 0$

Sol.

Simplify the expression,

Which is homogenous differential equation then putting $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{x+vx}{x-vx} = \frac{x(1+v)}{x(1-v)} = \frac{v+1}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{1-v} \Rightarrow \frac{1-v}{1+v^2}dv = \frac{dx}{x} \Rightarrow -\int \frac{v-1}{v^2+1} = \int \frac{dx}{x}$$

$$= -\frac{1}{2}\int \frac{2v}{v^2+1} + \frac{dv}{v^2+1} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{v}{x} = -\frac{1}{2x^2} + C$$

Where $v = \frac{1}{y}$

Hence, the required solution of differential equation.

11. Find the equation of the straight line perpendicular to the two lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1};\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ and passing through their point of intersection.

Sol.

Given lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}=\lambda_1$$

$$\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}=\lambda_2$$

Let, l, m, n be the direction rations of the line through the intersection the lies (1) and (2). Then

$$-3l+2m+n=0$$
$$l-3m+n=0$$
$$\frac{l}{4+3}=\frac{m}{1+6}=\frac{n}{9-2}\Rightarrow\frac{l}{7}=\frac{m}{7}=\frac{n}{7}$$

Therefore direction ratios of required line is (7, 7, 7)

Now, the co-ordinates of any point on line (1) are

$$\left(-3\lambda_1-1,2\lambda+3,\lambda_1-2\right)$$

Therefore point of intersection of (1) and (2)

$$-3\lambda_1-1=\lambda_2$$
$$2\lambda+3=-3\lambda_2+7$$
$$\lambda_1-2=2\lambda_2-7$$

Putting the value of λ_2 from (3) in (4), we get

$$2\lambda_1+3=-\left(-3\lambda_1-1\right)+7=9\lambda_1+3+7$$
$$\Rightarrow 2\lambda_1-9\lambda_1=10-3\Rightarrow-7\lambda_1=7\Rightarrow\lambda_1=-1$$

On putting the value of λ_1 and λ_2 in $\left(-3\lambda_1-1,2\lambda_2+3,\lambda_1-2\right)$ and $\left(\lambda_2,-3\lambda_2+7,2\lambda_2-7\right)$, we get the point of intersection of the lines (1) and (2) is (2, 1, -3)

Then the equation line through the intersection of the line (1) and (2) and perpendicular to each of them is

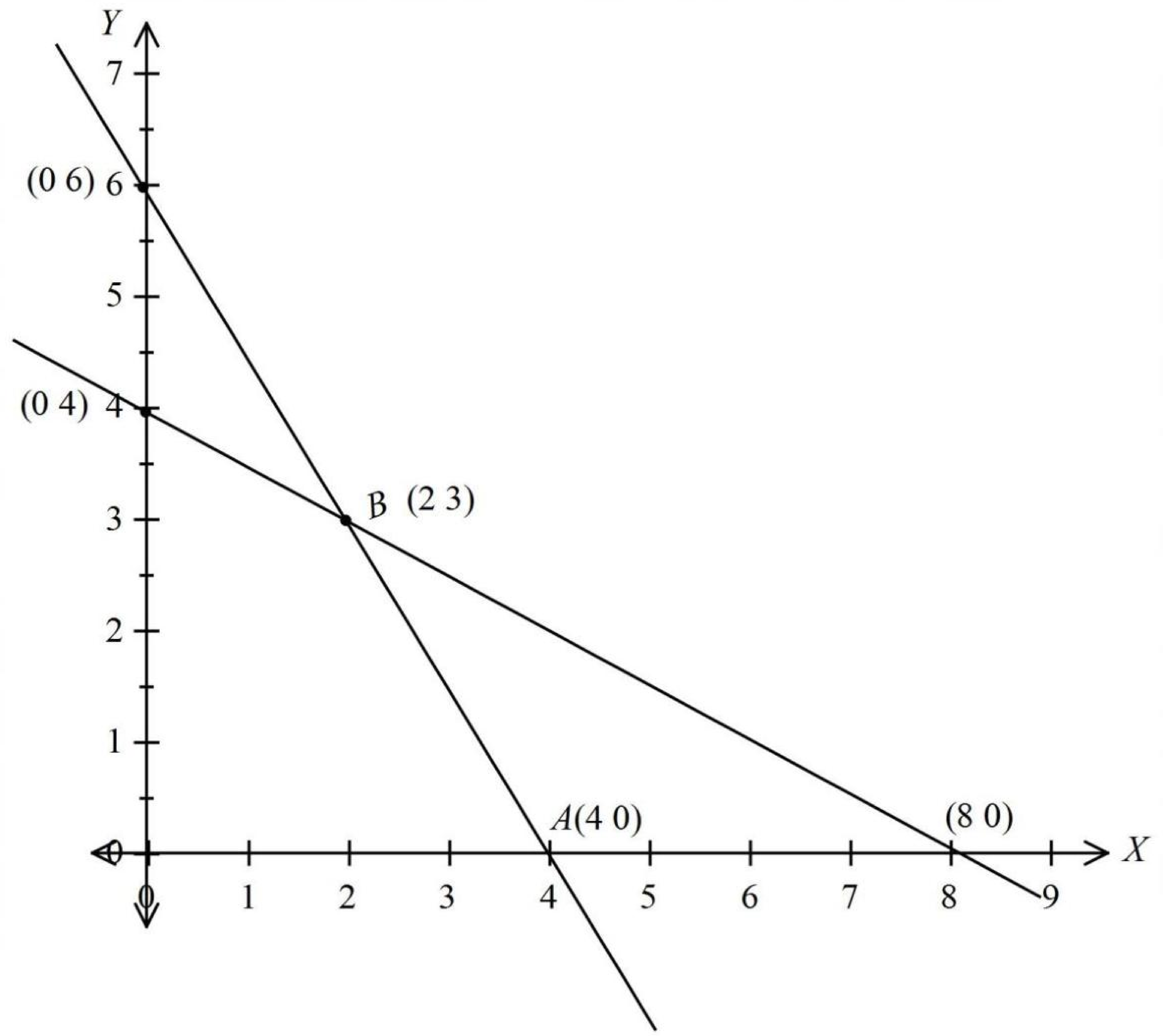
$$\frac{x-2}{7}=\frac{y-1}{7}=\frac{z+3}{7}.$$

Hence, the straight line of equation in perpendicular of two lines is $\frac{x-2}{7}=\frac{y-1}{7}=\frac{z+3}{7}$.

12. Minimize $Z=-3x+3y$

$$\textbf{Subject to}\quad \begin{aligned} x+2y&\le 8\\ 3x+2y&\le 8\\ x\ge 0,\ y&\ge 0. \end{aligned}$$

Sol.



Simplify the equation,

$x+2y \le 8 \Rightarrow x+2y=8$.......(1)
$3x+2y \le 12 \Rightarrow 3x+2y=12$........(2)
$x \ge 0, y \ge 0 \Rightarrow x=0, y=0$...........(3)
$z=-3x+3y$

First of all draw graph of equation corresponding to inequation (1) to (2) as thrown in figure.

It is clear from figure (1) and (2) intersect at the point (2,3). So, we get feasible region OABC which is bounded. The co-ordinates of corner points are O(0, 0) A (4, 0) and C(0, 4) Lastly applying corner point method to find the value of objective function Z are as follows.

$(Corner\ \text{point})$	Z$=-3x+3y$
$O(0,0)$	z=0
$A(4,0)$	z$=-12$
$B(2,3)$	z$=-3.2+3.3=3$
$C(0,4)$	z=0+12=12

In is clear from above table that the minimum value of Z is -12 at the Point (-4, 0).

Hence, the minimum value of objective function Z is -12.