Modeling Quantitative Judgments of Realistic Stimuli

David Izydorczyk
TeaP 2023
27.03.2023

Part of work with Arndt Bröder and currently under review

"… but what can you do with your research/model ? Can you predict/explain something in the real-world with it ?"

Quantitative Judgments

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Examples:

- “How fast can this bird fly?”
- “How much time do I need to prepare my talk?”
- “How severe are the symptoms of this patient?”

Photo taken by Charles J. Sharp

Quantitative Judgments

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- **A combination of both** (Albrecht et al. 2020; Bröder et al. 2017)

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Computational models require …

- Exemplars
- Cues and cue values of stimuli

Stim.	Cue 1 leave form	Cue 2 petal form	Cue 3 petal color	Crit. price in €	Training
	0	0	0	1	
	1	0	0	3	✓
	0	1	0	4	✓
	1	1	0	6	
	0	0	1	5	✓
	1	0	1	7	
	0	1	1	8	✓
	1	1	1	10	

?

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?

Proposed Solution

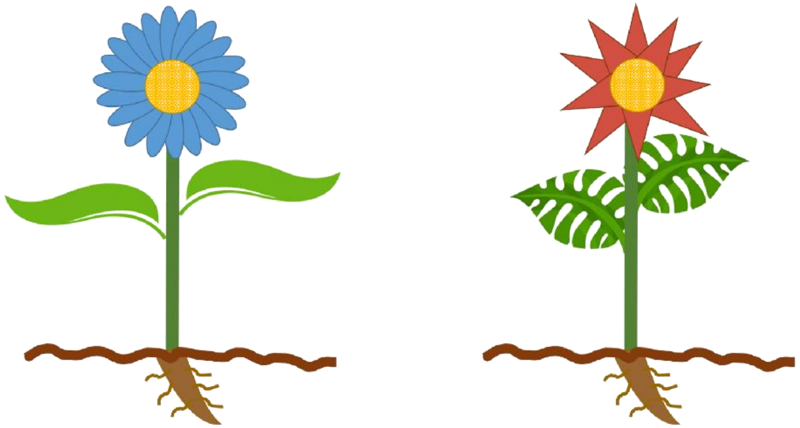
Generating cues with Multidimensional Scaling (MDS), where

- objects are represented as points in a multidimensional (psychological) space
- and more similar objects are located closer together

(Hout et al., 2013; Shepard, 1962; Nosofsky, 1992; Shin et al., 1992; Nosofsky, Sanders, Meagher, et al., 2018)

Two Studies with the same general procedure

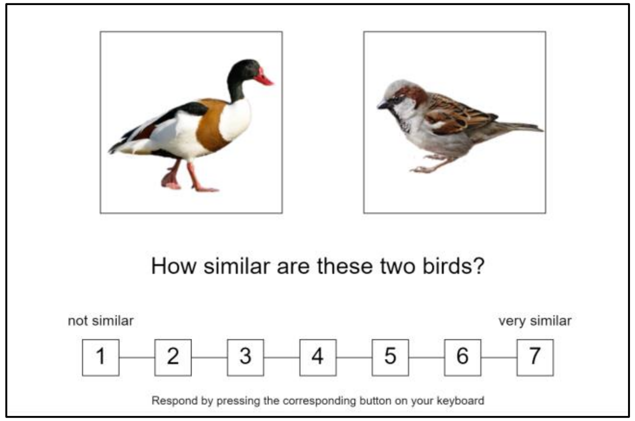
Study 1: Validation
with artificial stimuli &
data from Izydorczyk & Bröder (2022):



Study 2:
with complex stimuli

General Procedure

Pairwise Similarity Ratings of *K* = 32 Birds (*N* = 101)



Every participant rated 124 out of all 496 possible pairs

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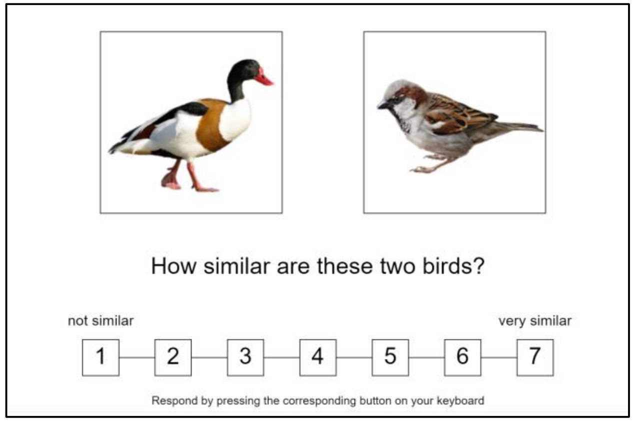

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MDS Analysis

MDS Solution

Bird	Dim. 1	Dim. 2	Dim. 3
1	0.1	-0.2	0.3
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	…		
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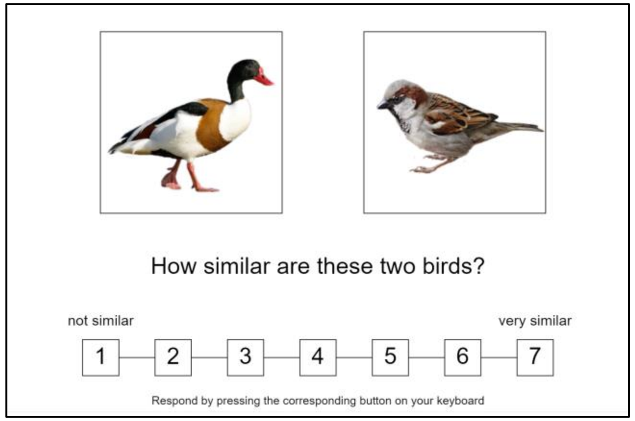

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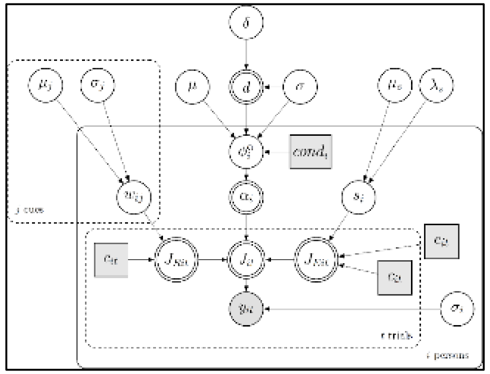

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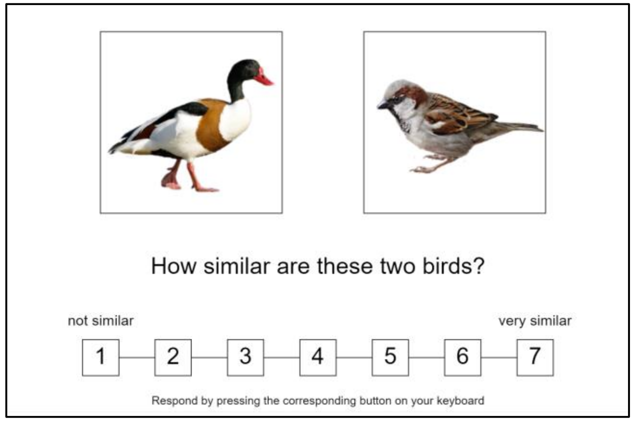

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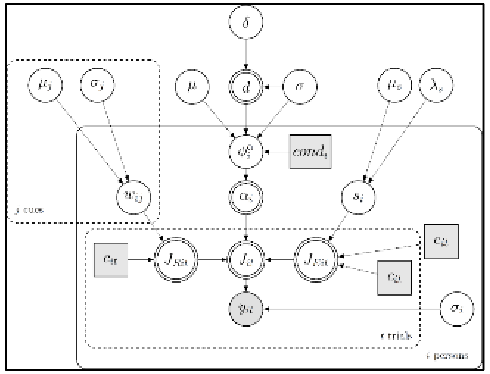
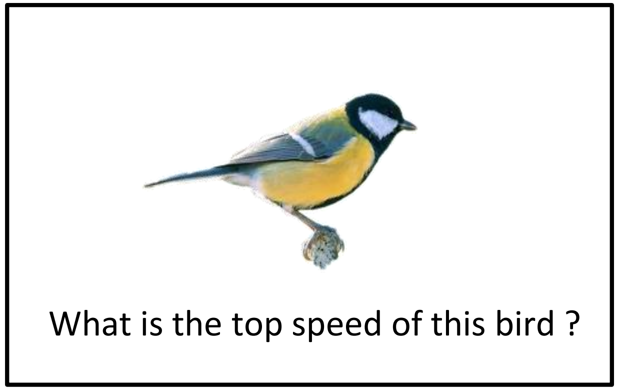


to model/analyze

Judgment Experiment



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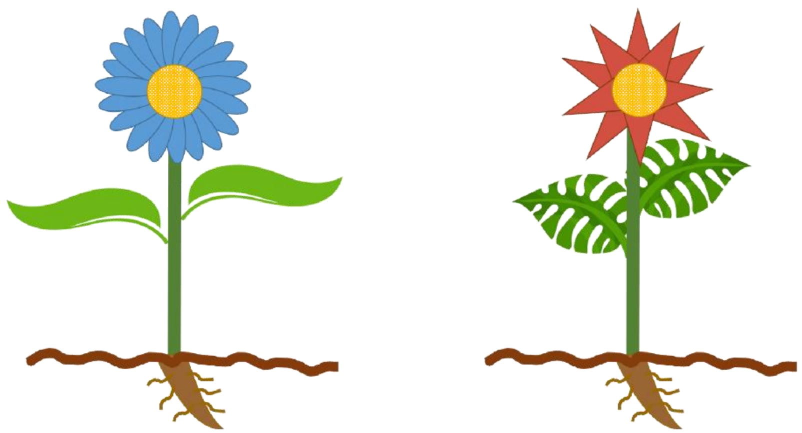

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- Same results when using MDS cues instead of exp. defined cues

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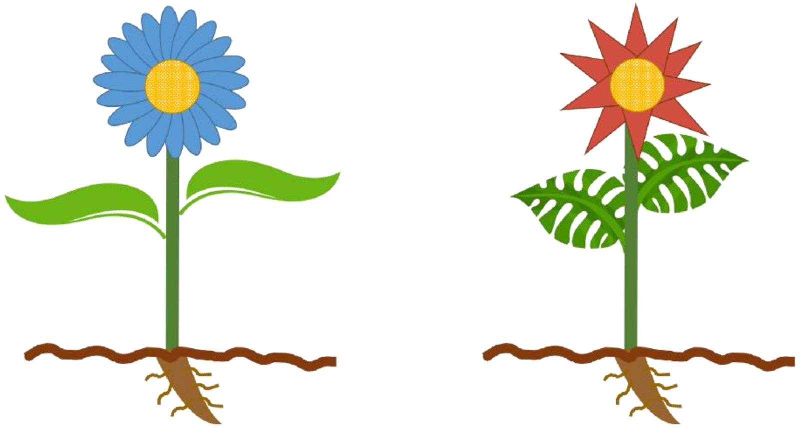

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Study 2:
with complex stimuli

- **Experiment:** Replication of Pachur & Olsson (2012) and Trippas & Pachur (2019)
- **General finding**: More rule-based processing when trained with ***learning by comparison*** than with ***direct criterion learning***

Procedure: Judgment Experiment

Training Phase:
k = 12 exemplar birds

Testing Phase:
K = 32 birds (12 old, 20 new)

learning by comparison (N = 39)

direct criterion learning (N = 39)

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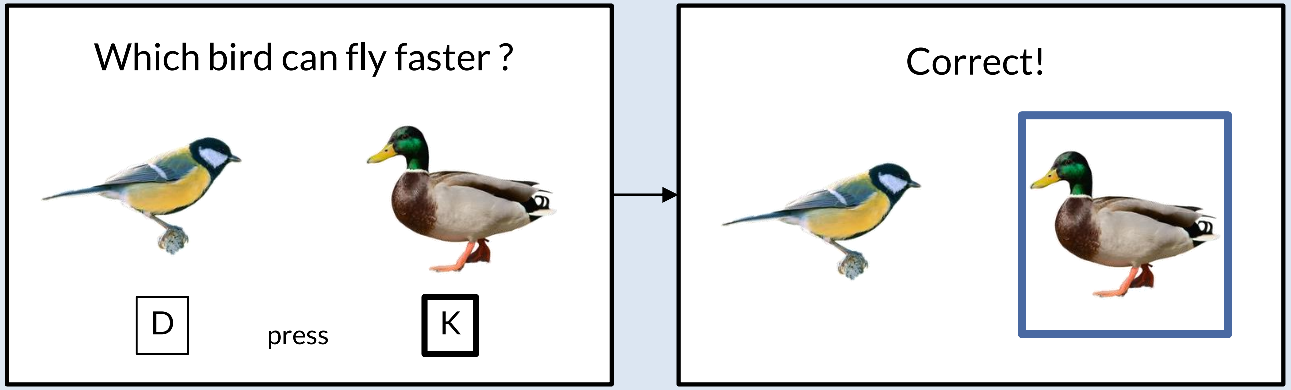

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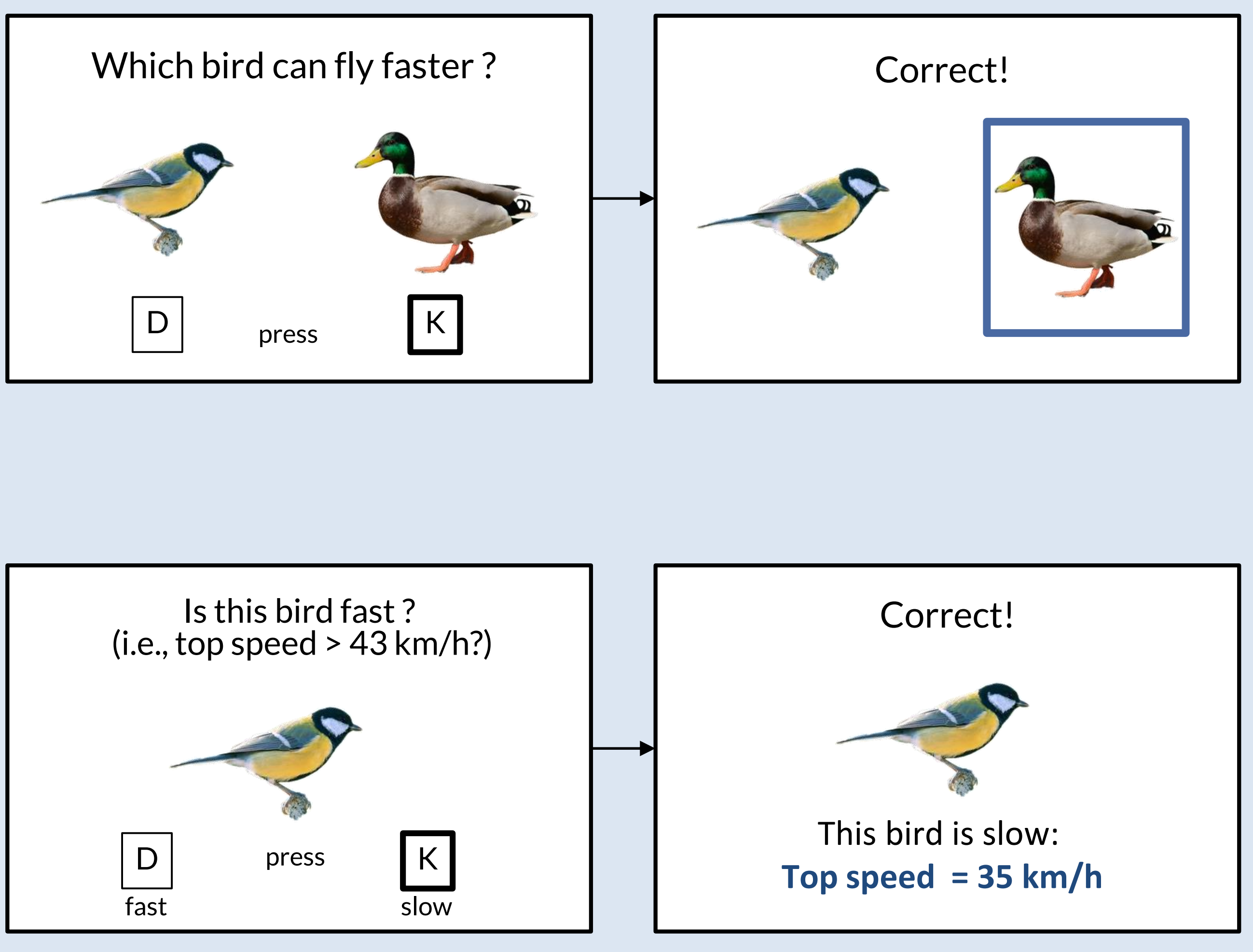

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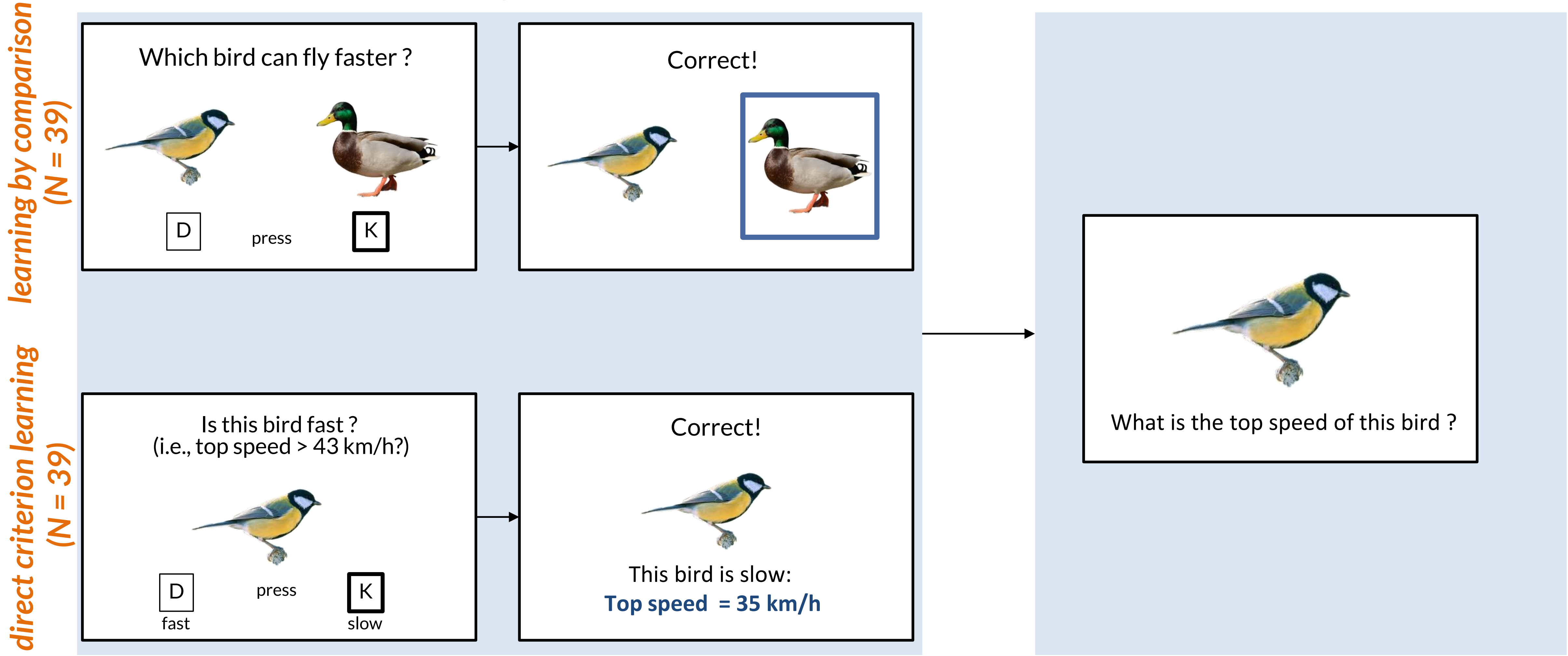
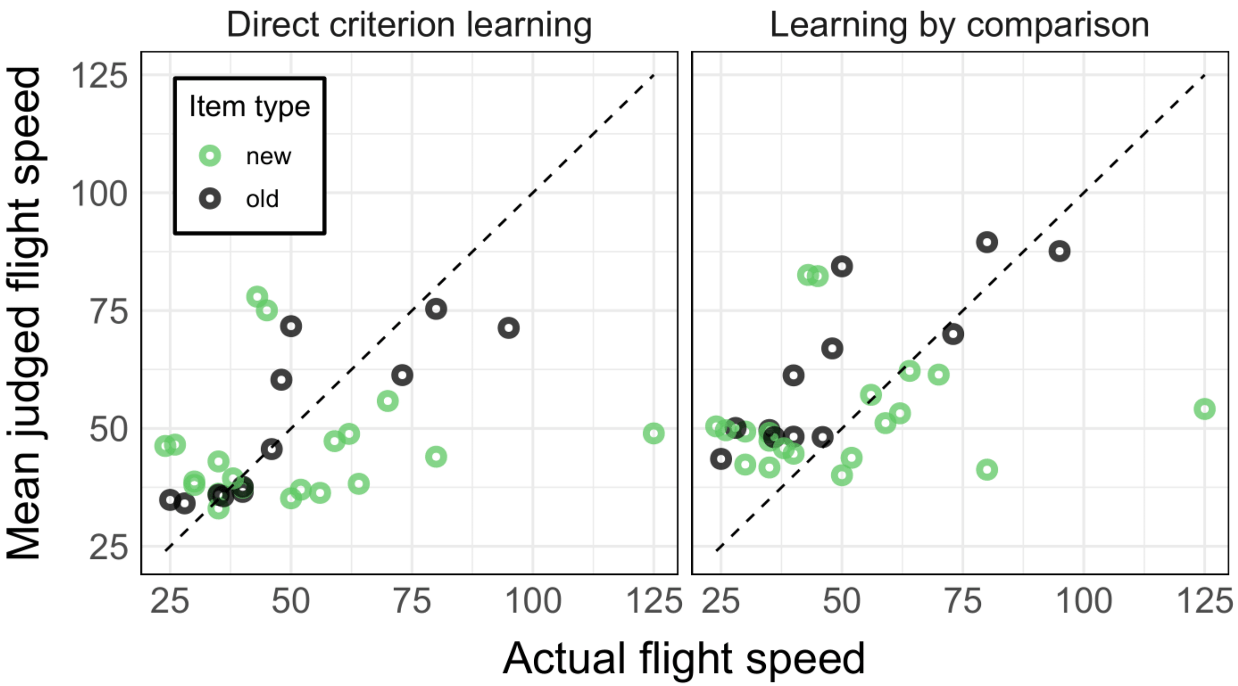


Results – Performance in Testing Phase



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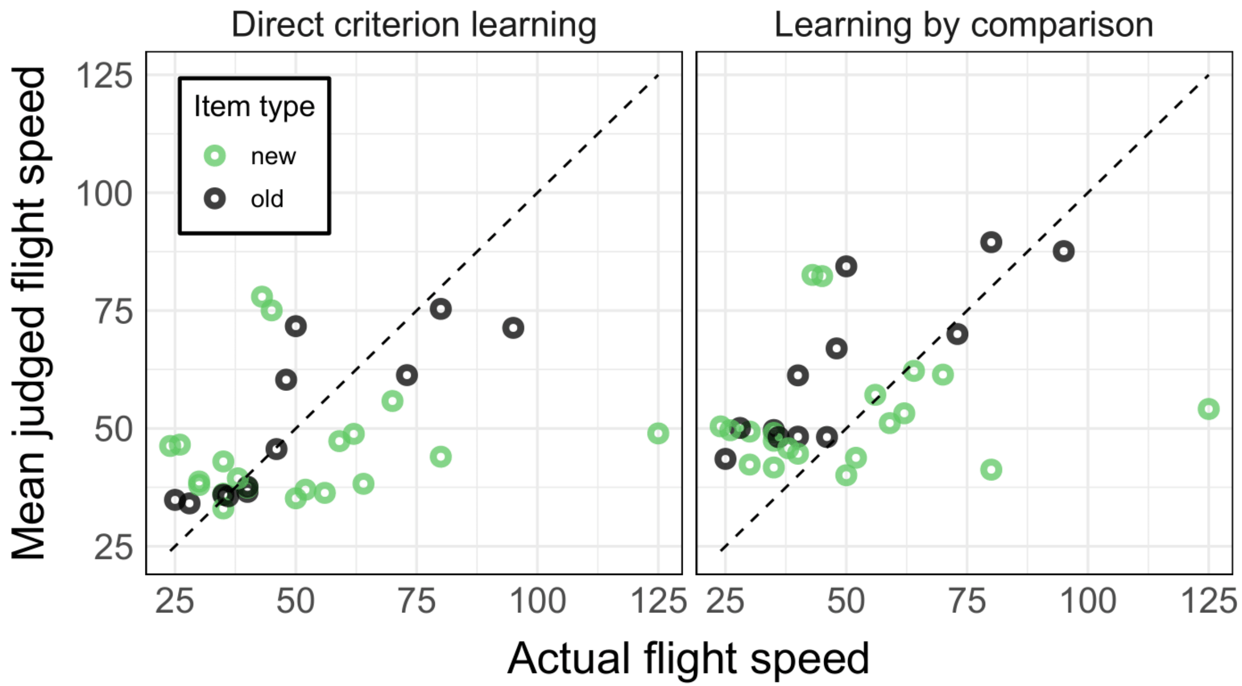

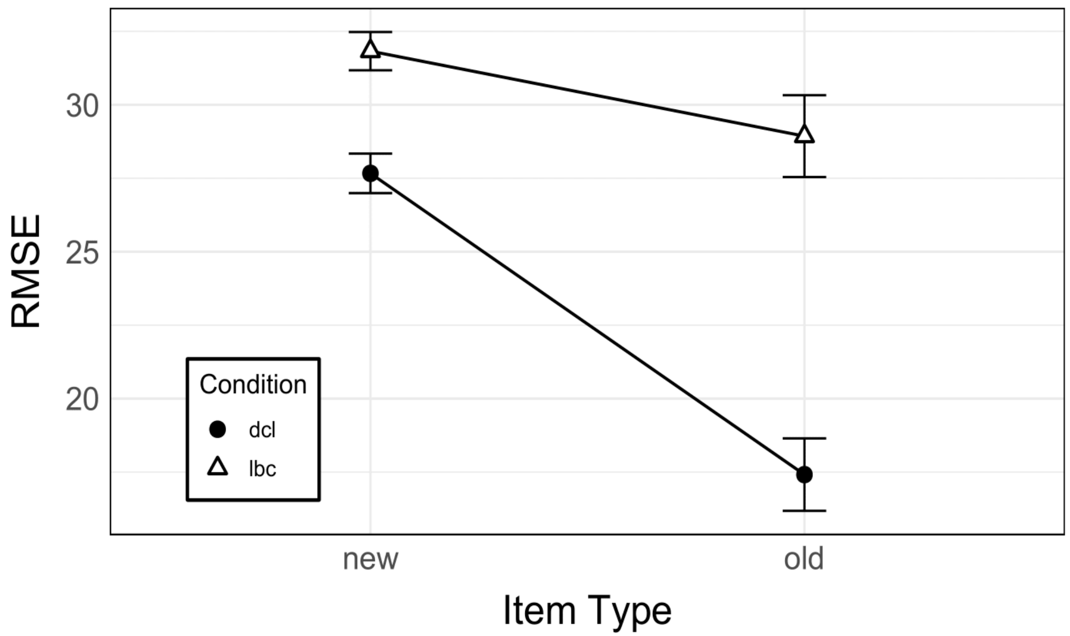

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Main Hypothesis

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Data analyzed with
RulEx-J Model
(Bröder et al., 2017;
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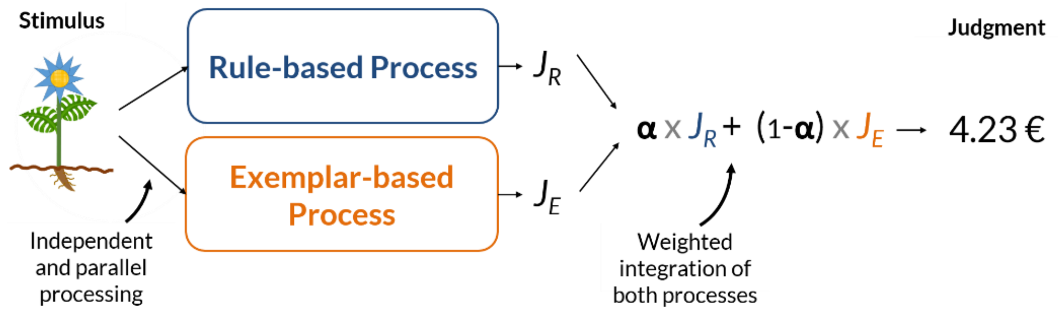

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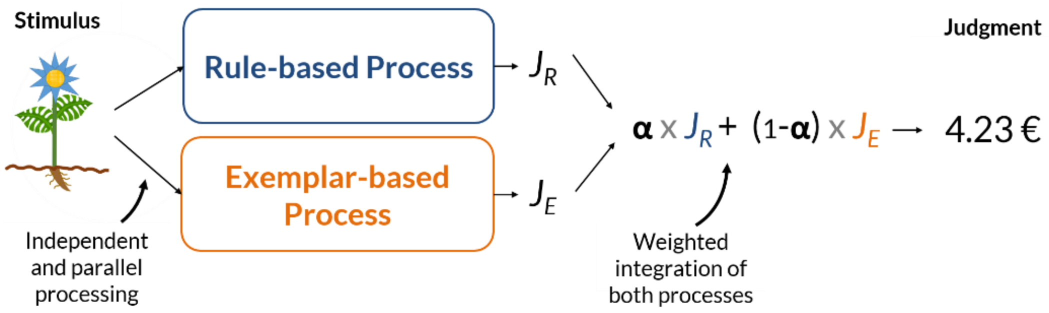
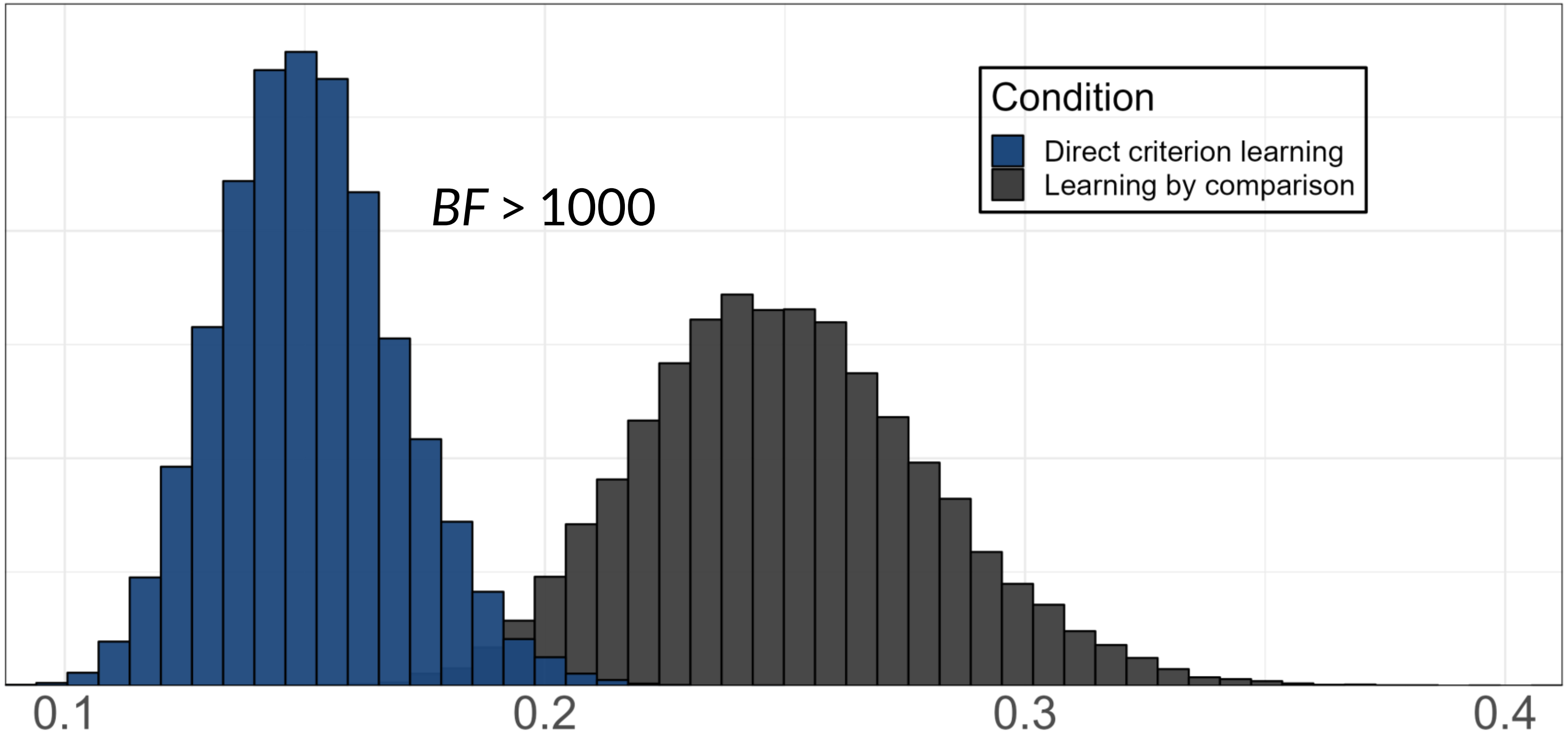


$$\bar{\alpha}_{\textit{learning by comparison}} > \bar{\alpha}_{\textit{direct criterion learning}}$$

Results – Cognitive Modeling



Posterior of μ_α for both conditions

α = 1: Only rule-based processing
α = 0: Only exemplar-based processing

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Discussion & Open Questions

Goldstein and Hogarth (1997, p.37): *"To what extent can we generalize from laboratory studies with abstract tasks [and artificial stimuli] to behavior in the real-world domains?"*

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Main Results:

- More rule-based processing when trained with *learning by comparison* than with *direct criterion learning* → Replication of previous experiments with complex naturalistic stimuli
- More exemplar-based processing in general → in line with literature (e.g., Hoffmann et al., 2016; Juslin et al., 2008; Platzer & Bröder, 2013; von Helversen et al., 2013)

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Open Questions:

- Quality of extracted cues
- Differences between methods to extract features or collect similarity ratings
- Improvable model fit (for some participants)

Thank you for your attention !
Any Questions ?

STATISTICAL MODELING in PSYCHOLOGY
FREIBURG HEIDELBERG LANDAU MANNHEIM TÜBINGEN

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Thanks to my Co-Author:
Arndt Bröder

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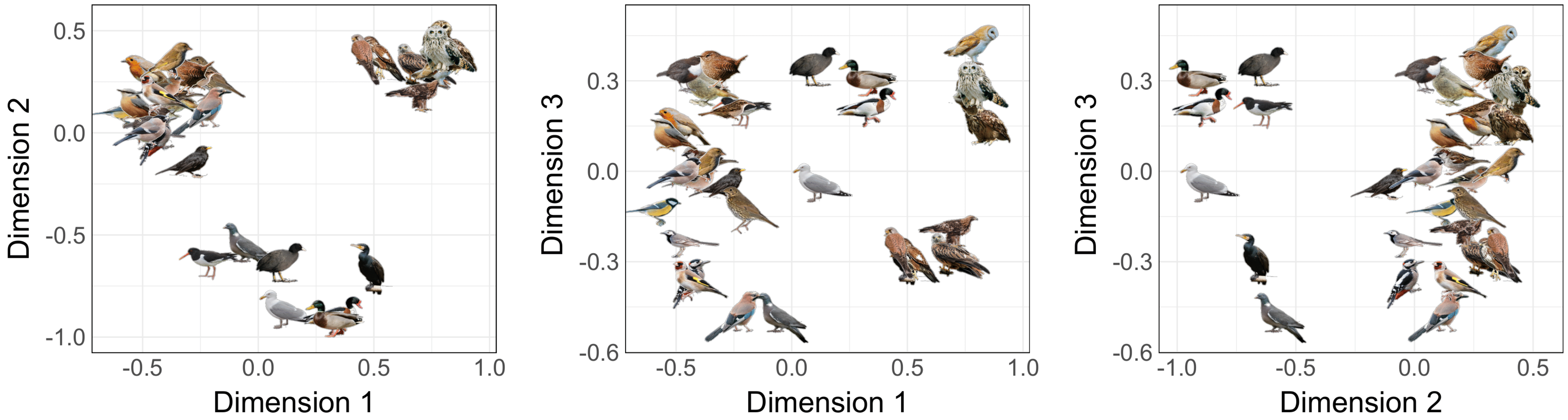
Appendix



0.5
0.0
-0.5
-1.0
Dimension 2
-0.5
0.0
0.5
1.0
Dimension 1
0.3
0.0
-0.3
-0.6
Dimension 3
-0.5
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Dimension 3
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0.0
0.5
Dimension 2

Appendix

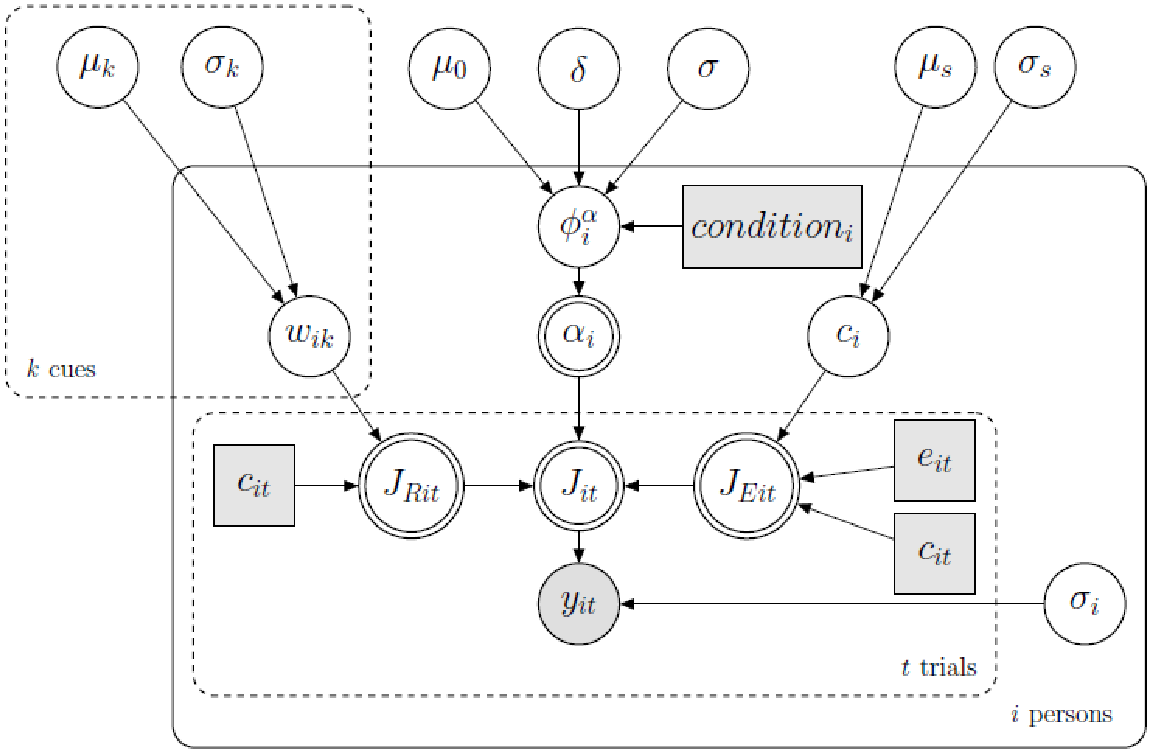
Table 1

Model comparison using log(BF).

Condition	$\mathcal{M}_1$: RulEx-J	$\mathcal{M}_1$: Exemplar	vs. $\mathcal{M}_0$
DCL	118.96		Exemplar
	314.72	195.76	Rule
LBC	289.82		Exemplar
	307.10	17.29	Rule

Note. Positive values of log(Bayes Factor) indicating evidence in favor of $\mathcal{M}_1$ and negative values indicating evidence in favor of $\mathcal{M}_0$. DCL = direct criterion learning, LBC = learning by comparison.

RulEx-J Model



Rule Module

$$w_k \sim Normal(\mu_k, \sigma_w)$$

$$\mu_k \sim Normal(0, 5)$$

$$\sigma_w \sim Exp(0.5)$$

Exemplar Module

$$d_{pe} = \sqrt{\sum_{k=1}^{n_k} (p_k - e_k)^2}$$

$$s_{pe} = e^{-cd_{pe}}$$

$$c \sim Normal_{[0,]}(mu_c, \sigma_c)$$

$$\mu_c \sim Normal(0, 2)$$

$$\sigma_c \sim Exp(1)$$

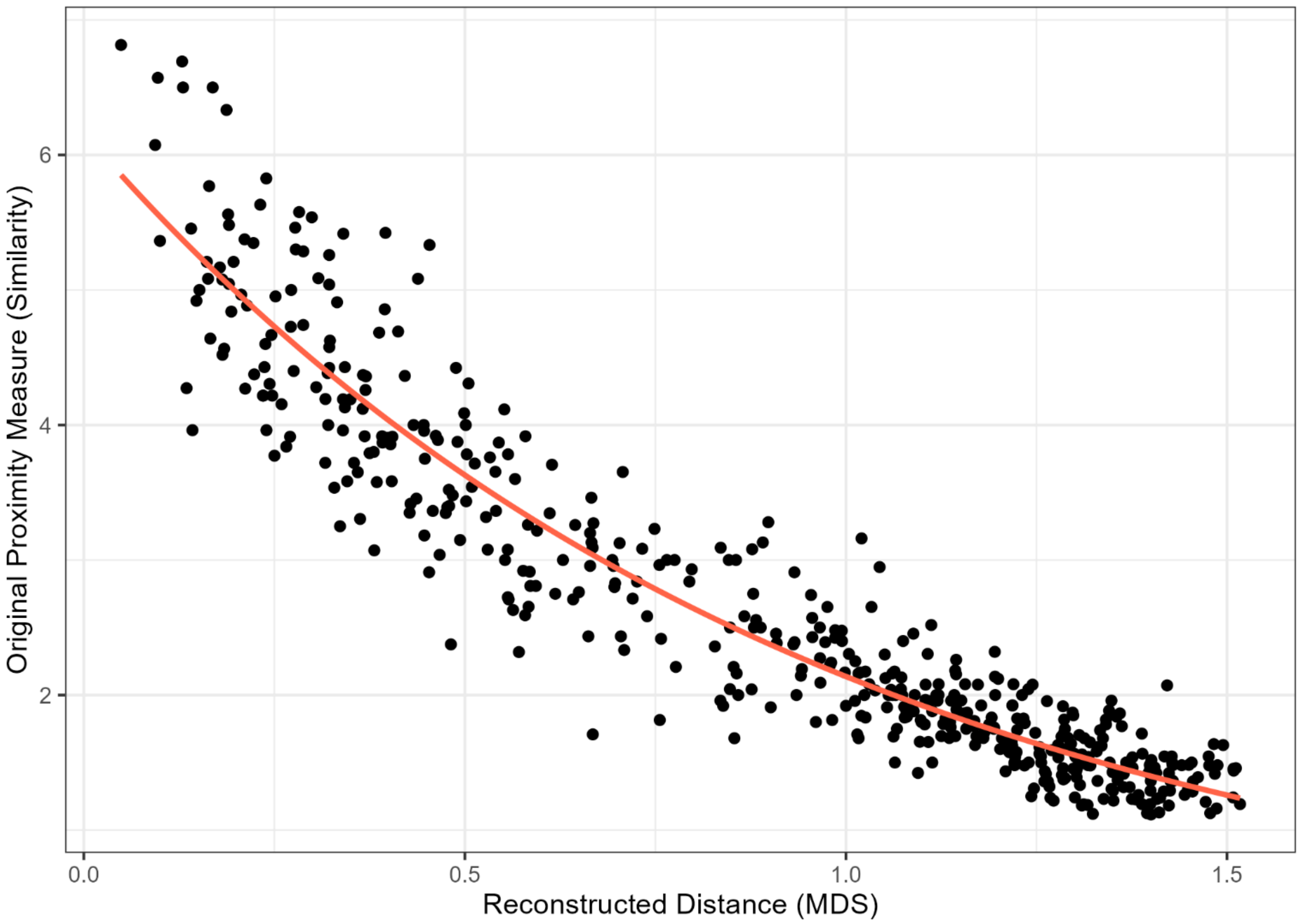
MDS Birds

Dimension	CV	BIC	Stress	R^2
1	.69	1695	.36	.54
2	.86	673	.16	.79
3	.88	740	.11	.86
4	.87	887	.09	.89
5	.86	1053	.07	.92

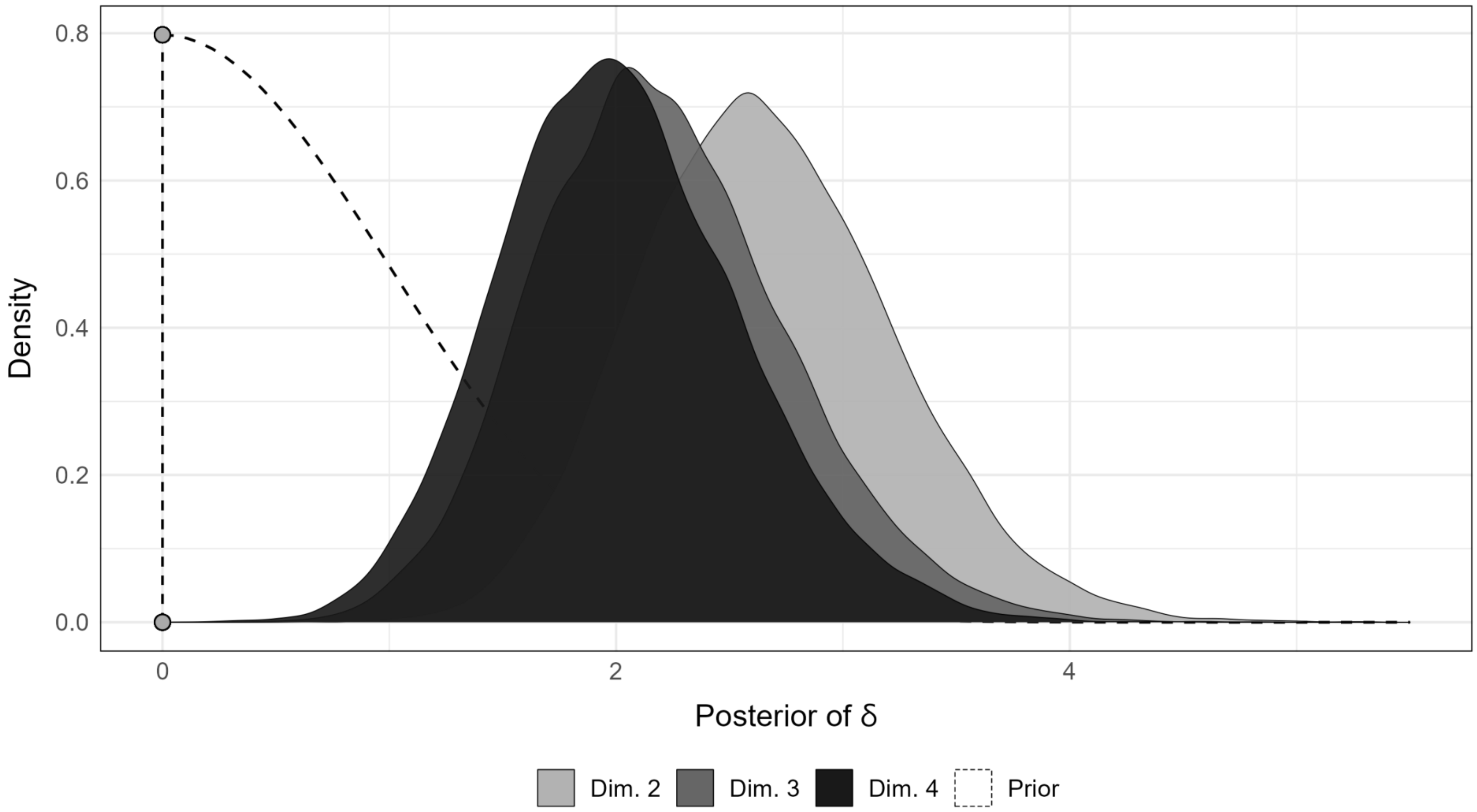
Estimates Birds

Parameter	M	SD	$95\% HDI$	$\hat{R}$	n_{eff}
δ	2.20	0.55	2.17 [1.2, 3.37]	1	13655
$\delta \times \sigma_\alpha$	0.34	0.08	0.34 [0.19, 0.5]	1	8868
$\mu_{\alpha_{overall}}$	-0.86	0.08	-0.86 [-1, -0.7]	1	2584
$\mu_{\alpha_{rule}}$	-0.69	0.09	-0.69 [-0.86, -0.5]	1	2849
$\mu_{\alpha_{exemplar}}$	-1.03	0.08	-1.03 [-1.18, -0.87]	1	3478
σ_α	0.16	0.04	0.16 [0.09, 0.24]	1	7832
μ_c	4.65	0.28	4.64 [4.12, 5.22]	1	16233
λ_c	0.53	0.20	0.5 [0.25, 1.01]	1	17031
μ_{w_0}	9.00	4.69	9.04 [-0.22, 18.07]	1	10227
μ_{w_1}	12.38	4.57	12.43 [3.41, 21.27]	1	16813
μ_{w_2}	14.41	4.48	14.41 [5.57, 23.22]	1	19382
μ_{w_3}	-15.03	4.67	-15.08 [-24.18, -5.83]	1	17119
σ_w	71.73	8.30	71.28 [56.83, 89.47]	1	2916

Shepards Law

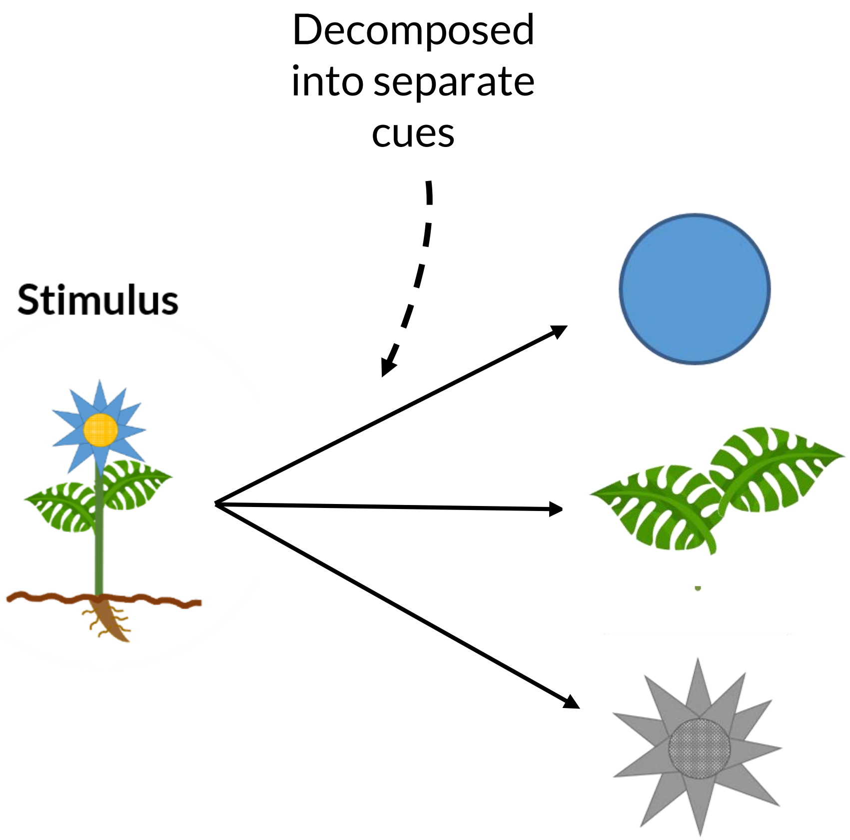


Robustness Dimension - δ



Rule-based Processes

(e.g., Juslin et al., 2003)



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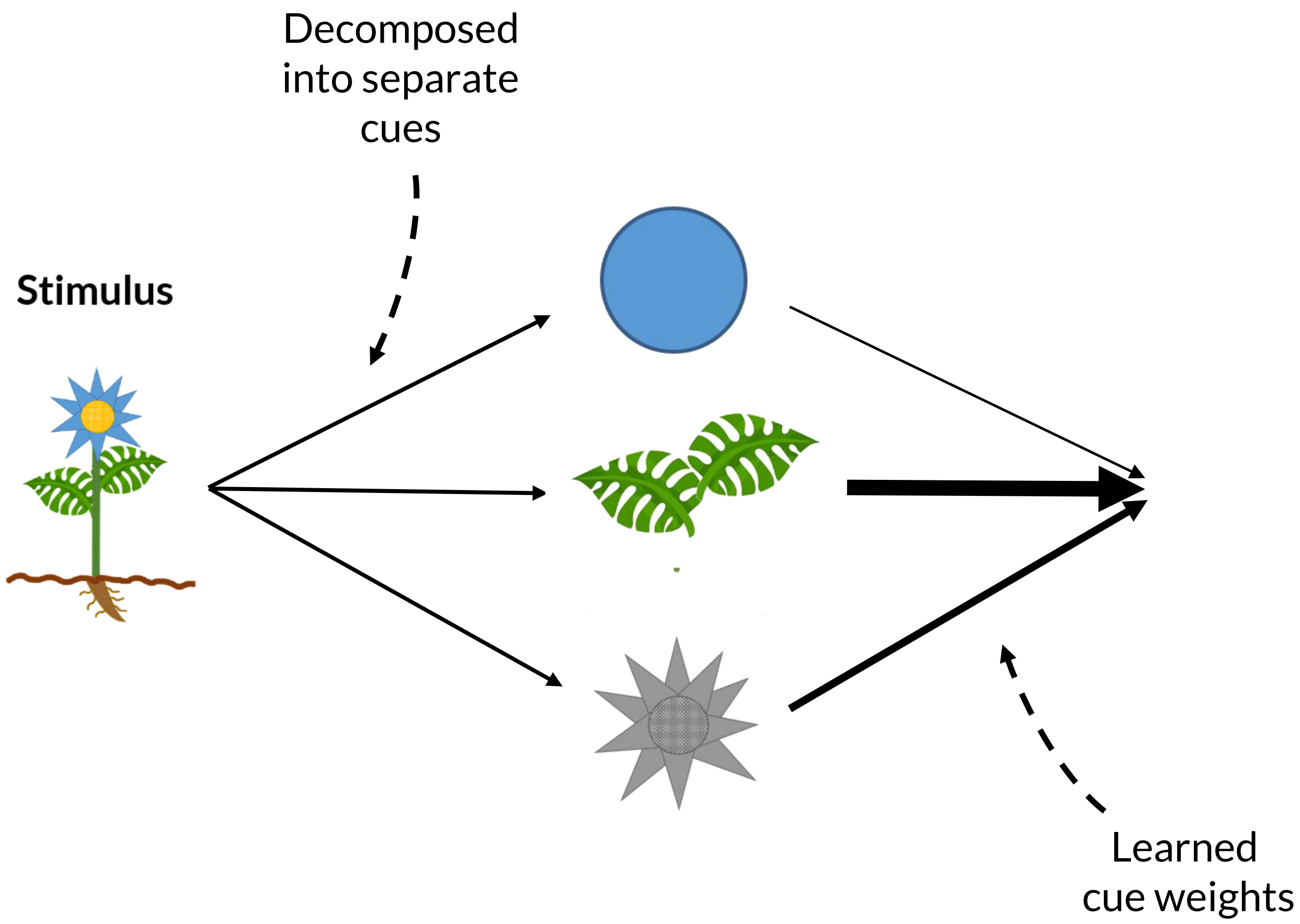

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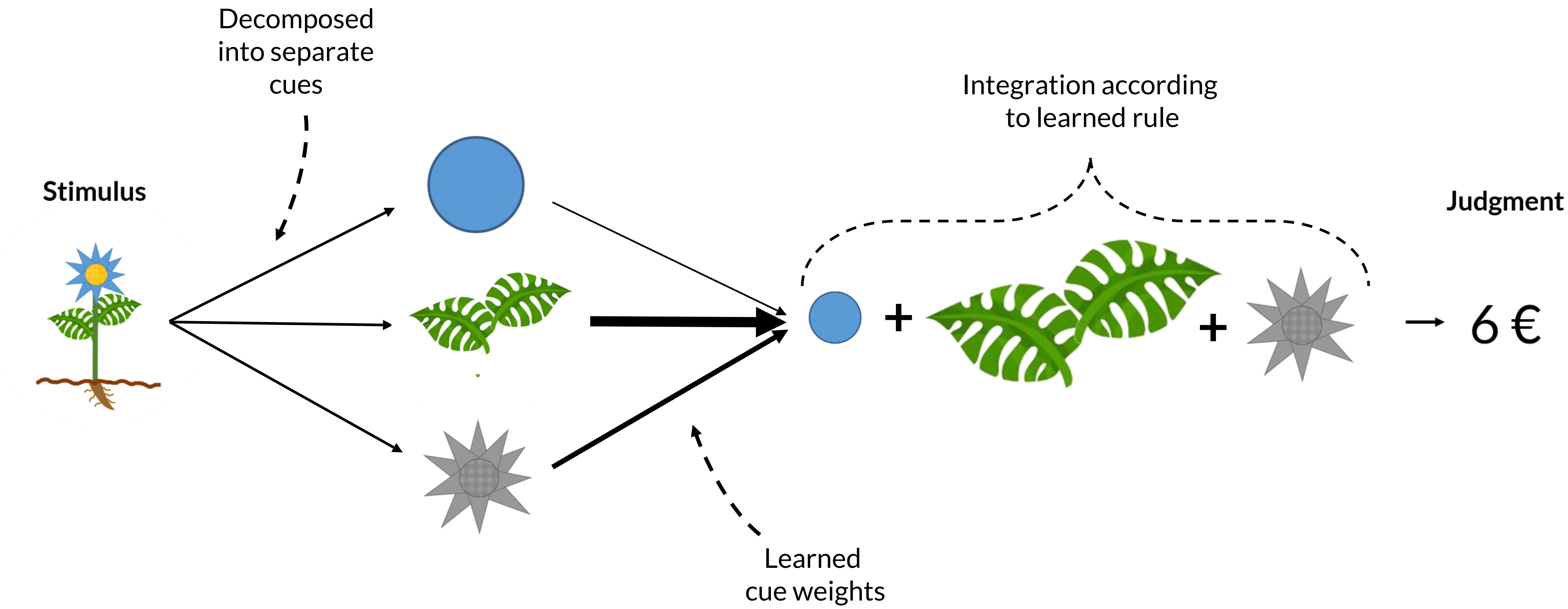

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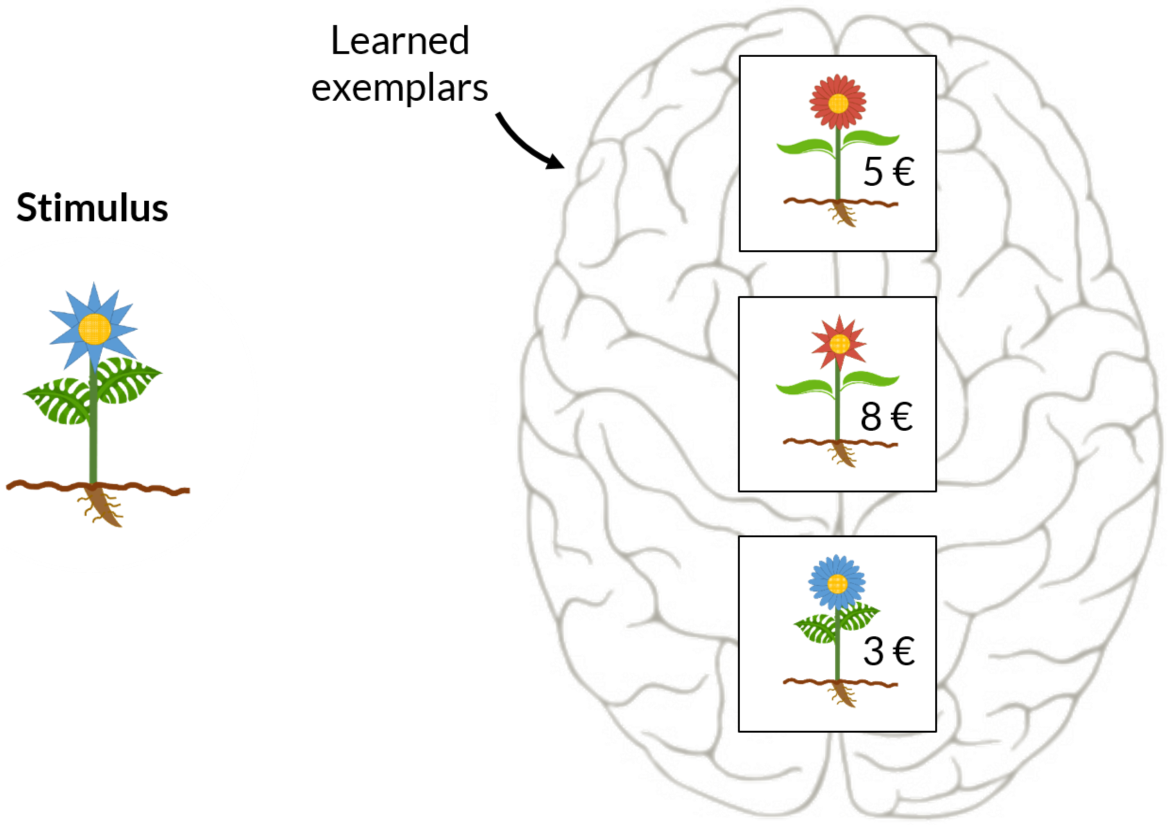

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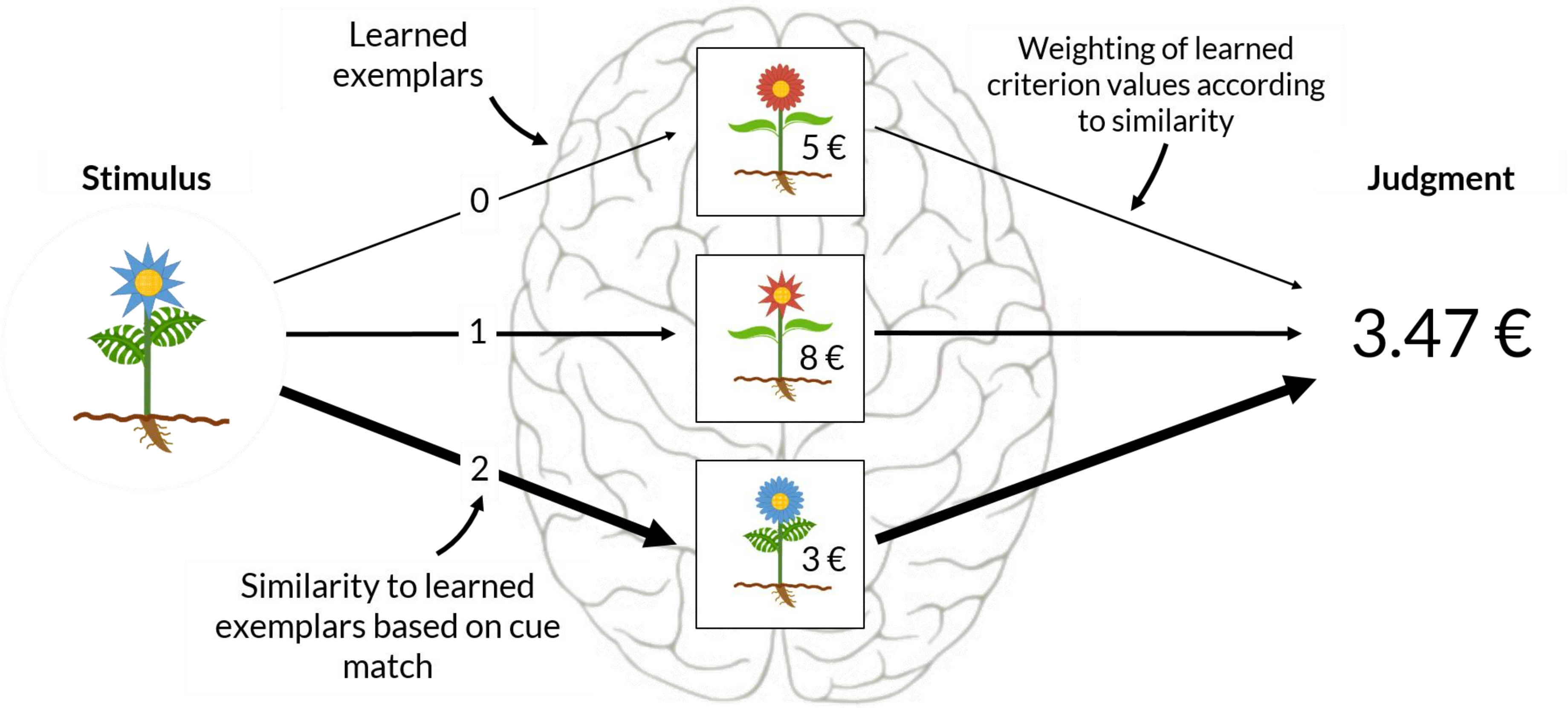

A combination of both

(Albrecht et al. 2020; Bröder et al. 2017)

E.g.: RulEx-J Model

(Bröder et al., 2017)

Stimulus

Independent and parallel processing

Rule-based Process → J_R

Exemplar-based Process → J_E

$\alpha \times J_R + (1-\alpha) \times J_E \rightarrow$ 4.23 €

Judgment

Weighted integration of both processes

α = 1: Only rule-based processing

α = 0: Only exemplar-based processing