Networks Assignment 2 Part 1

Name: Ayodeji Ali

Student ID: 20343733

Q1

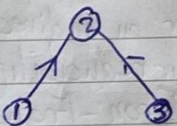
Let $G = (V, E)$ be a connected graph ~~with~~ with $|V| = n$ and $|E| = m$.

It's laplacian is $L = D - A$, where

- A is the $n \times n$ adjacency matrix
- D is the $n \times n$ diagonal matrix of degrees

The oriented incidence matrix B is an $m \times n$ matrix, where each edge $e \in E$ (row of B) has exactly 2 nonzero entries: $+1$ in the column of its "From" vertex and -1 in the entries of its "to" vertex

Graph to illustrate $L = B^T B$.



Adjacency matrix A: $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Degree matrix D:

$\deg(1) = 1, \deg(2) = 2, \deg(3) = 1$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Hence $L = D - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

- Oriented incidence matrix $B (2 \times 3)$:
  - Row for edge $e1 = (1 \to 2)$: $(+1, -1, 0)$
  - Row for edge $e2 = (3 \to 2)$: $(0, -1, +1)$

so

~~Compute B^TB:~~

$$B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \quad B^T = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \end{pmatrix}$$

compute B^TB:

$$B^TB = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} = L$$

Hence $L = B^TB$.

2. Perron - Frobenius Theorem on the adjacency matrix
- A real square matrix $A \geq 0$ is irreducible if it's associated directed graph is strongly connected
- For an undirected, connected graph's adjacency matrix, we get irreducibility.
- Perron - Frobenius then tells us there is a single largest real eaginvalue λ_{max}, and we can choose the corresponding eigenvector to have strictly positive components.

Q3.

Q3. Show the all eigenvalues of L are non-negative

~~To show that all eigenvalues of Laplacian matrix L are non-negative~~

To show that all eigenvalue's of Laplacian matrix L are non-negative

Express L in Terms of the incidence Matrix

~~L = D - A,~~ Recall that for a graph, the Laplacian is defined as.

$$L = D - A,$$

where D is the degree matrix and A is the adjacency matrix.

~~A very useful fact is the it~~

Define oriented incidence matrix B for graph as

$$L = B^T B.$$

We can write ~~$B^T B$~~ L as $B^T B$, for any vector x we write;

$$x^T L x = x^T B^T B x = (Bx)^T (Bx) = \|Bx\|^2$$

Since a squared norm is always non-negative it immediately Follows that.

$$x^T L x \geq 0 \text{ for all } x.$$

Use the Rayleigh Quotient.

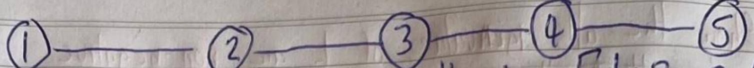
For symmetric matrix L the eigenvalues can be charecterice by Rayleigh Quotient. Then.

$$\lambda = \frac{x^T L x}{x^T x}$$

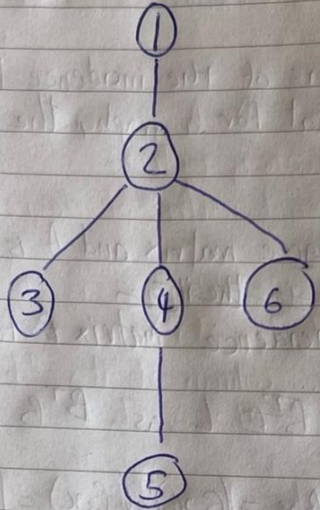
since we have shown the $x^T L x = \|Bx\|^2 \geq 0$ for every x.

$$\lambda = \frac{\|Bx\|^2}{x^T x} \geq 0$$

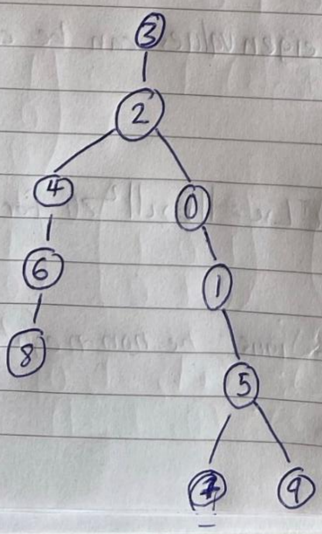
Thus, every eigenvalue λ of L must be non-negative.

4. 

Both DSF and BSF results in [1, 2, 3, 4, 5] as the answer.

5. 

6) [2, 2, 4, 2]

7. 

8. 1. Start at 0
visit order 0

2. From 0 go to neighbour 1 (neighbours {1,2})
visit order : 0,1.
3. At 1, go to neighbour 5 (ignoring 0).
~~3~~ visit order: 0,1, 5
4. ~~At~~ 5, visit smallest neighbour 7 (neighbours: {7, 9})
~~8~~ visit order: 0, 1,5,7
~~3~~ Backtrack to 5 then visit 9
visit order: 0,1,5, 7, 9
5. ~~At~~ Back at 0 next visit neighbour 2
visit order: 0,1,5,7,9,2
6. At 2, visit 3 (neighbours: {3,4}).
Visit order: 0,1,5,7, 9,2,3,
Backtrack to 2, then visit 4
visit order: 0,1,5,7, 9, 2, 3, 4
7. At 4, visit 6 (neighbours: {6}).
visit order : 0,1,5,7,9,2,3,4,6.
8. At 6, visit 8
Final Visit order.
0, 1, 5, 7, 9, 2, 3, 4, 6, 8

$\frac{1}{2}$