CSCI2100 Data Structures

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Abstract

This is a note for **CSCI2100 Data Structures** for self-revision purpose ONLY. Some contents are taken from lecture notes and reference book.
Mistakes might be found. So please feel free to point out any mistakes.
Contents are adapted from the lecture notes of CENG3420, prepared by Irwin King, as well as some online resources.

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Chapter 1

Introduction

1.1 Overview

A **data structure** is a way to organize and store data in a computer program, allowing for efficient access and manipulation.

An **algorithm** is different from a **program**. An algorithm is a process or set of rules used for calculation or problem-solving. It is a step-by-step outline or flowchart showing how to solve a problem. A program, on the other hand, is a series of coded instructions that control the operation of a computer or other machines. It is the implemented code of an algorithm.

For example, to solve the greatest common divisor (GCD) problem, we can use the following algorithm.

Algorithm 1.1: Euclid's Algorithm

```
Data: m, n ∈ ℤ⁺
Result: GCD(m, n)
while m > 0 do
    if n > m then
        swap m and n
    subtract n from m
return n
```

By using a mathematical method to prove this algorithm, we can show that it is correct, provided that it terminates.

Having proved the correctness, we also need to use different test cases to check if there is anything wrong with the coding or the proof. We should consider special cases, including large values, swapped values, etc.

We are also interested in the time and space (computer memory) it uses, which we call **time complexity** and **space complexity**. Typically, complexity is a function of the values of the inputs, and we would like to know which function. We can also consider the best case, average case, and worst-case scenarios.

For example, in the above algorithm, the best case would be $m = n$, with just one iteration. If $n = 1$, there are m iterations, which is the worst case. However, for the average case, it is difficult to analyze.

Also, for space complexity, it is constant since we only use space for the three integers: m, n, and t.

To improve the above algorithm, we can use `mod`, so we don't need to keep doing subtraction.

1.2 Algorithm

An algorithm is a finite set of instructions which, if followed, accomplishes a particular task. Every algorithm must satisfy the following criteria:

- Input: There are zero or more quantities that are externally supplied.

- Output: At least one quantity is produced.

- Definiteness: Each instruction must be clear and unambiguous.

- Finiteness: If we trace out the instructions of an algorithm, then for all cases, the algorithm will terminate after a finite number of steps.

- Effectiveness: Every instruction must be sufficiently basic that it can, in principle, be carried out by a person using only pencil and paper.

We also define an algorithm as any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output. It is thus a sequence of computational steps that transform the input into output.

It can also be viewed as a tool for solving a well-specified computational problem. The problem statement specifies, in general terms, the desired input or output relationship, and the algorithm describes a specific computational procedure for achieving that input or output relationship.

An algorithm is said to be correct if, for every input instance, it halts with the correct output. It can solve the given computational problem. In contrast, an incorrect algorithm might not halt at all on some input instances, and sometimes it can even produce useful results.

1.3 Study of Data

A data structure is a particular way of storing and organizing data in a computer so that it can be used efficiently.

A data structure is a set of domains D, a designated domain $d \in D$, a set of functions F, and a set of axioms A.

An implementation of a data structure d is a mapping from d to a set of other data structures e.

Chapter 2

Analysis

2.1 Complexity

Before, we talked about the definition of an algorithm. In this part, we would like to know how we can estimate the time required for a program, how to reduce the running time of a program, what the storage complexity is, and how to deal with trade-offs.

We can analyze the runtime by comparing functions. For example, given two functions $f(N)$ and $g(N)$, we can compare their relative rates of growth. There are three types of comparisons that we can make: $f(n) = \Theta(g(n))$ represents the exact bound, $f(n) = O(g(n))$ represents the upper bound, and $f(n) = \Omega(g(n))$ represents the lower bound.

By using bounds, we can establish a relative order among functions. Here, we often use $O(n)$ to analyze time complexity.

For the definition of the upper bound, it says that there is some point n_0 past which $cf(N)$ is always at least as large as $T(N)$. Then we say that $T(N) = O(f(N))$, where $f(N)$ is the upper bound on $T(N)$.

Definition 2.1.1. We say that $f(n) = O(g(n))$ iff there exists a constant $c > 0$ and an $n_0 \geq 0$ such that

$$f(n) \leq cg(n) \quad \text{for all } n \geq n_0$$

Or we can use the following notation

$$\exists c > 0, n_0 \geq 0 \text{ such that } f(n) \leq cg(n) \forall n \geq n_0$$

There are some rules to follow:

- Transitivity

$$\text{If } f(n) = O(g(n)) \text{ and } g(n) = O(h(n)), \text{ then } f(n) = O(h(n))$$

- Rule of sums

$$f(n) + g(n) = O(max\{f(n), g(n)\})$$

- Rule of products

$$\text{If } f_1(n) = O(g_1(n)), f_2(n) = O(g_2(n)), \text{ then } f_1(n)f_2(n) = O(g_1(n)g_2(n))$$

We do not include constants or lower-order terms inside Big-O notation.

If $f(n)$ is a polynomial in n with degree r, then $f(n) = O(n^r)$, but for $s < r$, $f(n) \neq O(n^s)$.

Also, any logarithm of n grows more slowly than any positive power of n as it increases. Hence, $\log n$ is $O(n^k)$ for any $k > 0$, but n^k is never $O(\log n)$ for any $k > 0$.

Order	Time
$O(1)$	constant time
$O(n)$	linear time
$O(n^2)$	quadratic time
$O(n^3)$	cubic time
$O(2^n)$	exponential time
$O(\log n)$	logarithmic time
$O(\log^2 n)$	log-squared time

On a list of length n, sequential search has a running time of $O(n)$.

On an ordered list of length n, binary search has a running time of $O(\log n)$.

The sum of the sums of integer indices of a loop from 1 to n is $O(n^2)$.

In summary, Big-O notation provides an upper bound of the complexity in the **worst-case**, helping to quantify performance as the input size becomes arbitrarily large. However, it doesn't measure the actual time, but represents the number of operations an algorithm will execute.

2.2 Recurrence Relations

Recurrence relations are useful in certain counting problems, for example, recursive algorithms. They relate the n-th element of a sequence to its predecessors.

By definition, a recurrence relation for the sequence $a_0, a_1, \cdots$ is an equation that relates a_n to certain of its predecessors $a_0, a_1, \cdots, a_{n-1}$. Initial conditions for the sequence are explicitly given values for a finite number of the terms of the sequence.

To solve a recurrence relation, we can use iteration. We use the recurrence relation to write the n-th term a_n in terms of certain of its predecessors. We then successively use the recurrence relation to replace each of $a_{n-1}, \cdots$ by certain of their predecessors. We continue until an explicit formula is obtained.

For example, the Fibonacci sequence is also defined by the recurrence relation.

Example (Tower of Hanoi)**.** Find an explicit formula for a_n, the minimum number of moves in which the n-disk Tower of Hanoi puzzle can be solved.

Given $a_n = 2a_{n-1} + 1$, $a_1 = 1$, by applying the iterative method, we obtain:

$$\begin{aligned}
a_n &= 2a_{n-1} + 1 \\
&= 2(2a_{n-2} + 1) + 1 \\
&= 2^2 a_{n-2} + 2 + 1 \\
&= 2^2(2a_{n-3} + 1) + 2 + 1 \\
&= 2^3 a_{n-3} + 2^2 + 2 + 1 \\
&= \cdots \\
&= 2^{n-1} a_1 + 2^{n-2} + 2^{n-3} + \cdots + 2 + 1 \\
&= 2^{n-1} + 2^{n-2} + 2^{n-3} + \cdots + 2 + 1 \\
&= 2^n - 1
\end{aligned}$$