Probability Functions

PMF/PDF, CDF, Expected Value, Variance

General

(*independent assumed)

$$\mathbb{E}[X] = \mu = \sum_x x p_X(x) = \int_{-\infty}^{\infty} x f_X(x) dx$$

$$\mathbb{E}[\alpha X + \beta Y] = \alpha \mathbb{E}[X] + \beta \mathbb{E}[Y]$$

$$*\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$$

$$\mathrm{Var}[X] = \mathbb{E}\left[(X-\mu)^2\right] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$*\,\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$$

$$\sigma_X \coloneqq \sqrt{\mathrm{Var}[X]} = \sqrt{\mathbb{E}\left[(X-\mu)^2\right]}$$

$X \sim$ **Bernoulli**(p)

$$p_X(x) = \begin{cases} 1-p & \text{if } x = 0 \\ p & \text{if } x = 1 \end{cases}$$

$$\mathbb{E}[X] = p \qquad \mathrm{Var}[X] = p(1-p)$$

$X \sim$ **Binomial**(n, p)

$$p_X(k) = \mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mathbb{E}[X] = np \qquad \mathrm{Var}[X] = np(1-p)$$

$X \sim$ **Geometric**(p)

$$p_X(k) = \mathbb{P}(X = k) = p(1-p)^{k-1}$$

$$\mathbb{E}[X] = \frac{1}{p} \qquad \mathrm{Var}[X] = \frac{1-p}{p^2}$$

$X \sim$ **Poisson**(λ)

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, 2, 3$$

$$\mathbb{E}[X] = \lambda \qquad \mathrm{Var}[X] = \lambda$$

$X \sim$ **Uniform**(a, b)

$$F_X(x) = \begin{cases} 0, & \text{if } x < a \\ \dfrac{x-a}{b-a}, & \text{if } a \le x \le b \\ 1, & \text{if } b < x \end{cases}$$

$$f_X(x) = \begin{cases} 0, & \text{if } x < a \\ \dfrac{1}{b-a}, & \text{if } a \le x \le b \\ 0, & \text{if } b < x \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2} \qquad \mathrm{Var}[X] = \frac{(b-a)^2}{12}$$

$X \sim$ **Exponential**(λ)

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \ge 0 \\ 0, & \text{if } x < 0 \end{cases}$$

$$F_X(x) = 1 - e^{-\lambda x}$$

$$\mathbb{E}[X] = \frac{1}{\lambda} \qquad \mathrm{Var}[X] = \frac{1}{\lambda^2}$$

Joint PMF

$$p(x, y) = \mathbb{P}(X = x, Y = y)$$

Normal Distribution

$X \sim \mathcal{N}(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathbb{E}[X] = \mu \qquad \mathrm{Var}[X] = \sigma^2$$

Standard Normal Distribution

$\mathcal{N}(0, 1)$

$$f_X(x) \coloneqq \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\mathbb{E}[Z] = 0 \qquad \mathrm{Var}[Z] = 1.$$

Convert:

$$Z = \frac{X-\mu}{\sigma}$$

Central Limit Theorem

$$Z_n = \frac{S_n - n\mu}{\sigma\sqrt{n}}, \; S_n = X_1 + \cdots + X_n$$

$$\lim_{n\to\infty} \mathbb{P}(Z_n \le z) = \Phi(z)$$

Bayesian Statistic