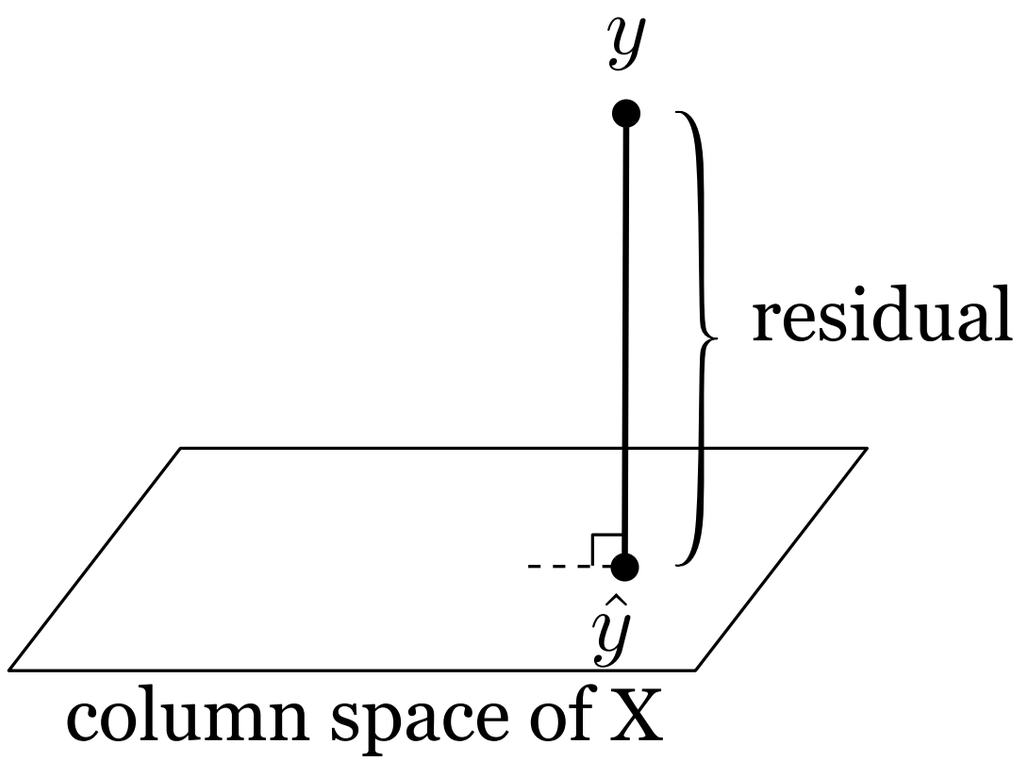
CS109/Stat121/AC209/E-109
Data Science
Bias and Regression

Hanspeter Pfister, Joe Blitzstein, and Verena Kaynig



This Week

- HW1 due tonight at11:59 pm (Eastern Time)
- HW2 posted soon

Census: Everybody's moving into their parents' basements

By Brad Plumer June 20, 2012 Follow @bradplumer

Ever since the financial crisis hit, Americans have found it harder and harder to live on their own. According to a new report (pdf) from the Census Bureau, the number of "shared households" increased by a whopping 2.25 million between 2007 and 2010:

Daniel Sherrett, 28, prepares dinner with his mother as part of his deal to live at home. Parents and children are sharing homes for longer than expected. (Michael Temchine/The Washington Post)

> In spring 2007, there were 19.7 million shared households. By spring 2010, the number of shared households had increased by 11.4 percent, while all households increased by only 1.3 percent.

This number does not include co-habitating or married couples. Rather, it's specifically a measure of the growing fraction of Americans who are either living with roommates or shacking up with relatives. And the bulk of the increase came from kids who are living at home with their parents: "Between 2007 and 2010, the number of adult children who resided in their parents'

Most Read Business

1 Here is everything we know about whether gentrification pushes poor people out 

2 Honey isn't as healthy as we think 

3 These are the hardest places for minimum wage workers to live 

4 Why Americans dress so casually 

5 Is your adviser truly protecting your retirement? 

The Most Popular All Over

The Atlantic
The Rise of Victimhood Culture 

Magazine

1657 COMMENTS

It's Official: The Boomerang Kids Won't Leave

By ADAM DAVIDSON JUNE 20, 2014

Census Data from the Current Population Survey (CPS)

"It is important to note that the CPS counts students living in dormitories as living in their parents' home."

– Census Bureau, http://www.census.gov/prod/2013pubs/p20-570.pdf

If you're not careful, you'd misconstrue the previous example and not see that in actuality there're more college students rather than more kids staying at home.

Some Forms of Bias

If the threshold to publish is .05 p valuethen all the other studies get filed away

Always want to know where the data came from and what data you don't have.

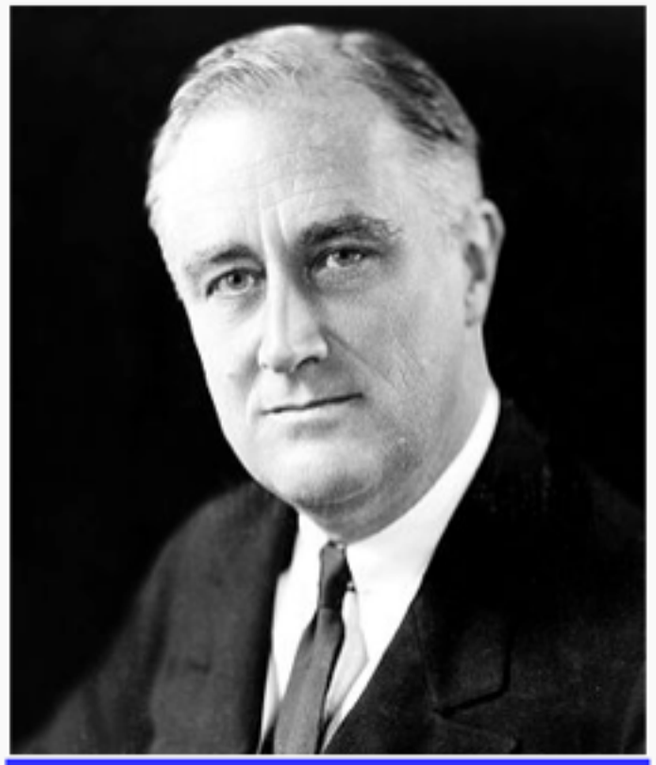
- selection bias
- publication bias (file drawer problem)
- non-response bias
- length bias

1936 Presidential Election, Landon vs. FDR

1932 ← **November 3, 1936** → 1940

531 electoral votes of the Electoral College

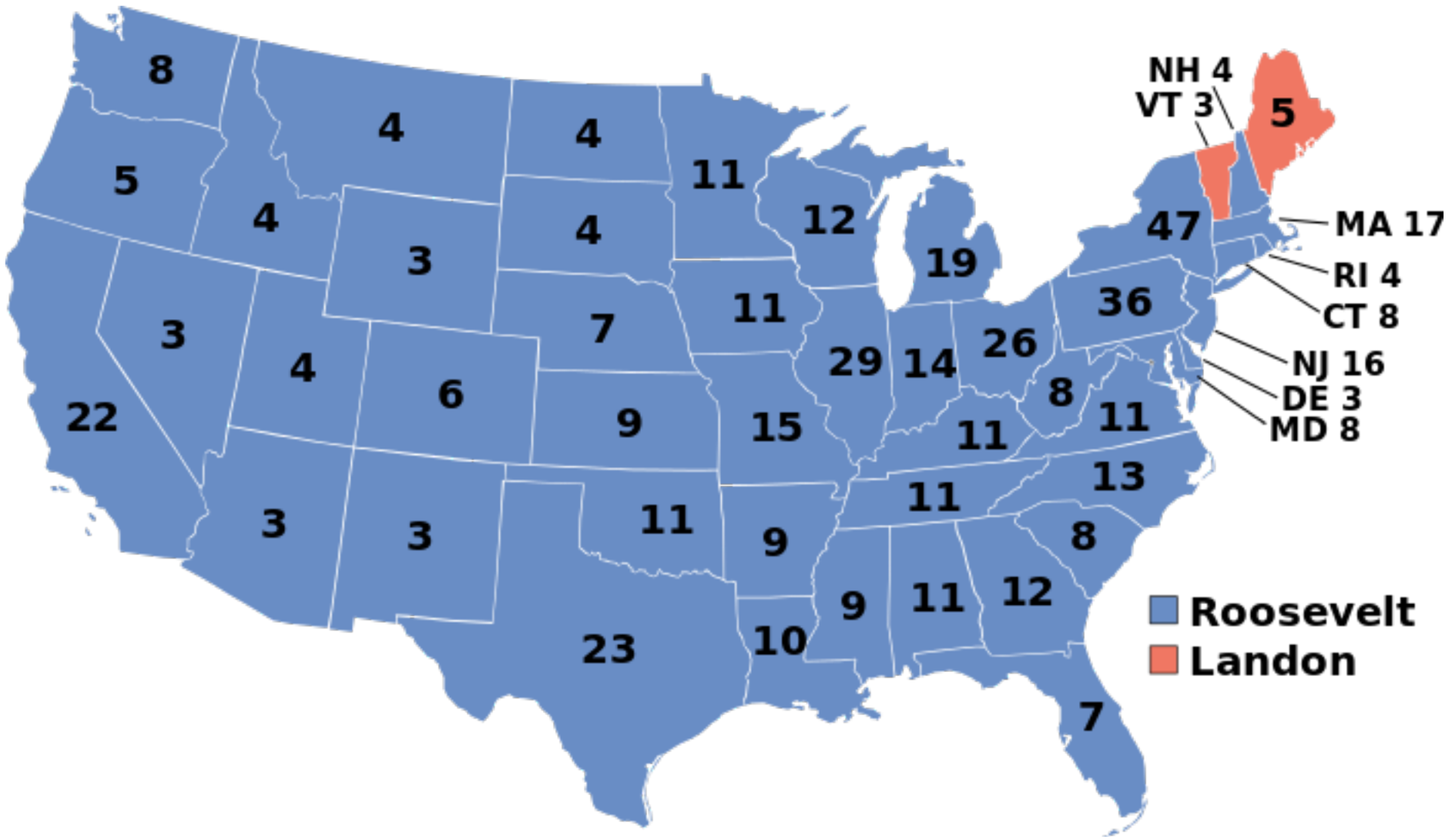
266 electoral votes needed to win



Nominee	**Franklin D. Roosevelt**	Alf Landon
Party	Democratic	Republican
Home state	New York	Kansas
Running mate	**John N. Garner**	Frank Knox

1936 Presidential Election, Landon vs. FDR

Literary Digest predicted Landon
would win with 370 electoral votes,
based on sample size of 2.4 million.



source: https://en.wikipedia.org/wiki/United_States_presidential_election,_1936

1936 Presidential Election, Landon vs. FDR

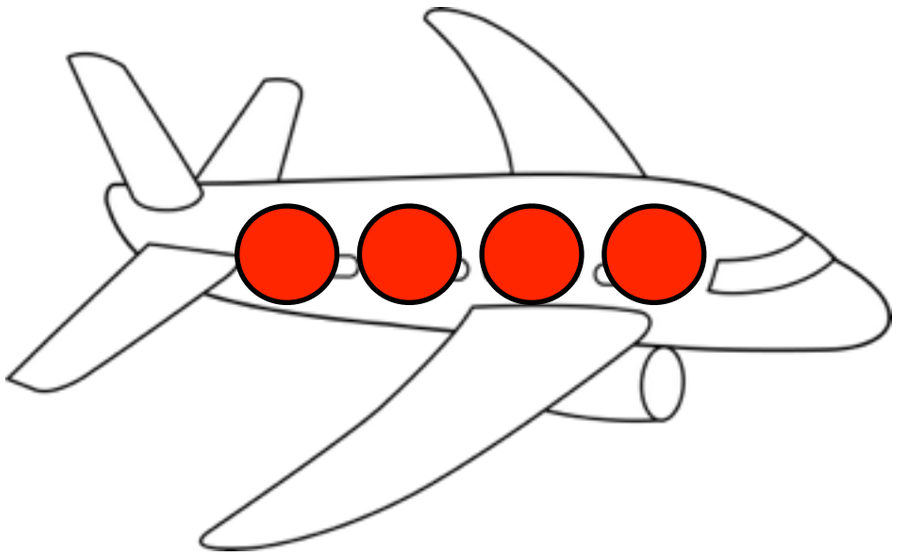
Literary Digest got responses from 2.3 million out of 10 million people surveyed.

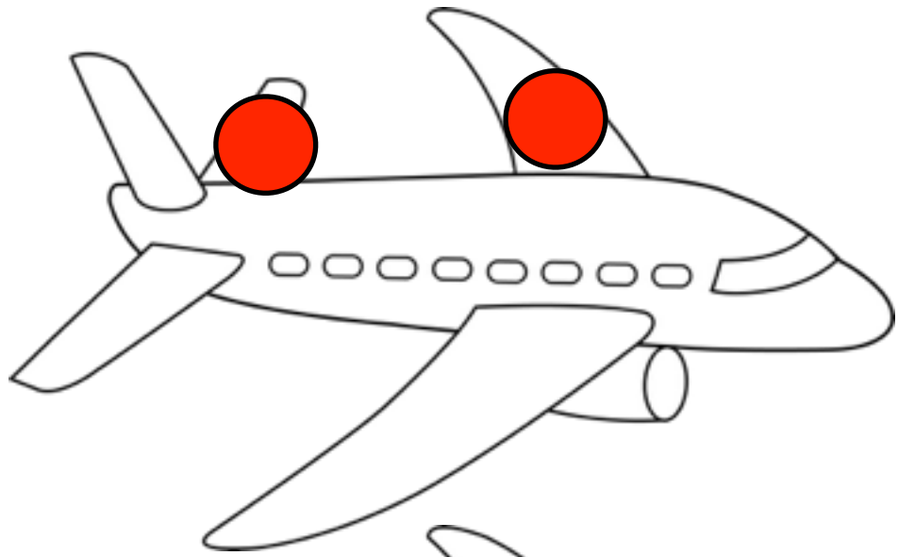
To collect their sample, they used 3 readily available lists:

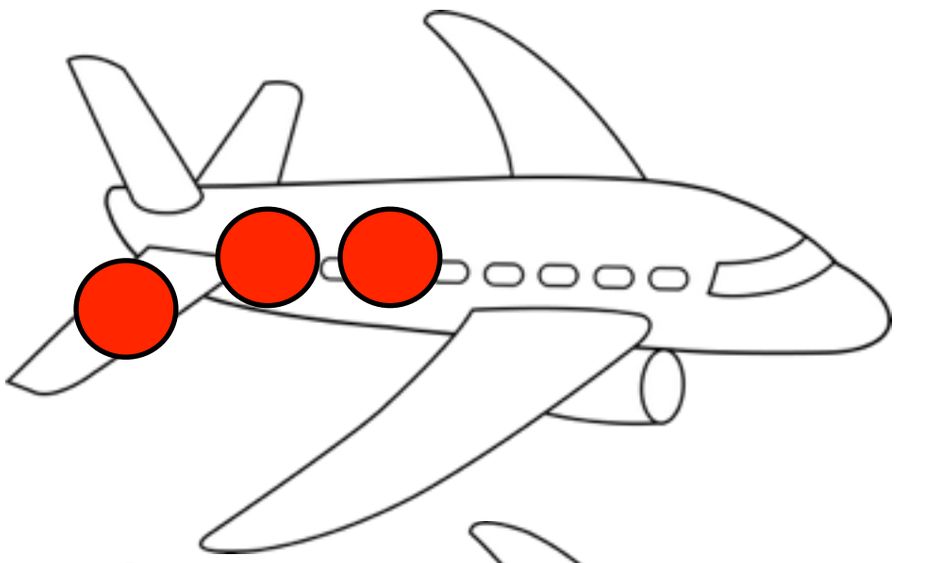
- readers of their magazine
- car registration list
- phone directory

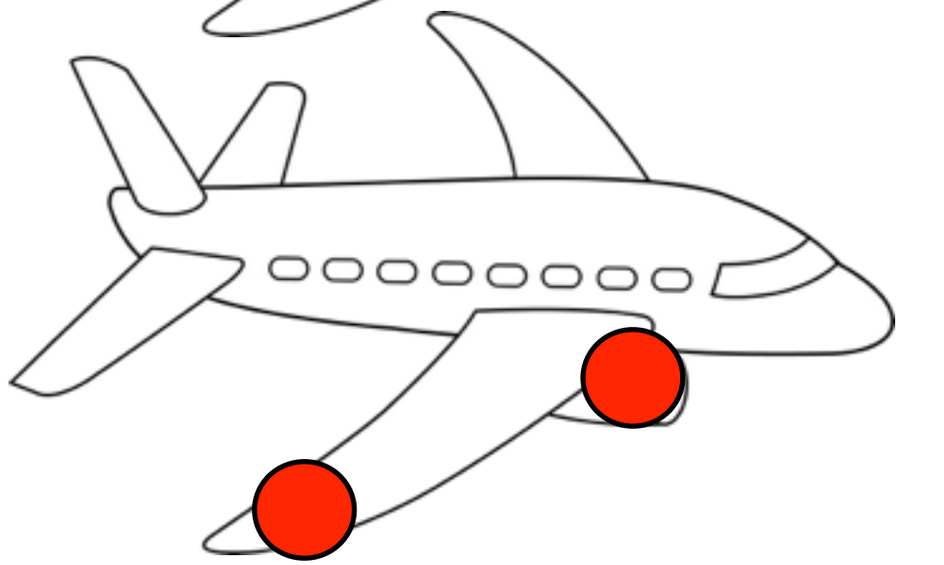
Not everyone subscribes to their magazine and not a lot of people owned cars or had phones in their home.

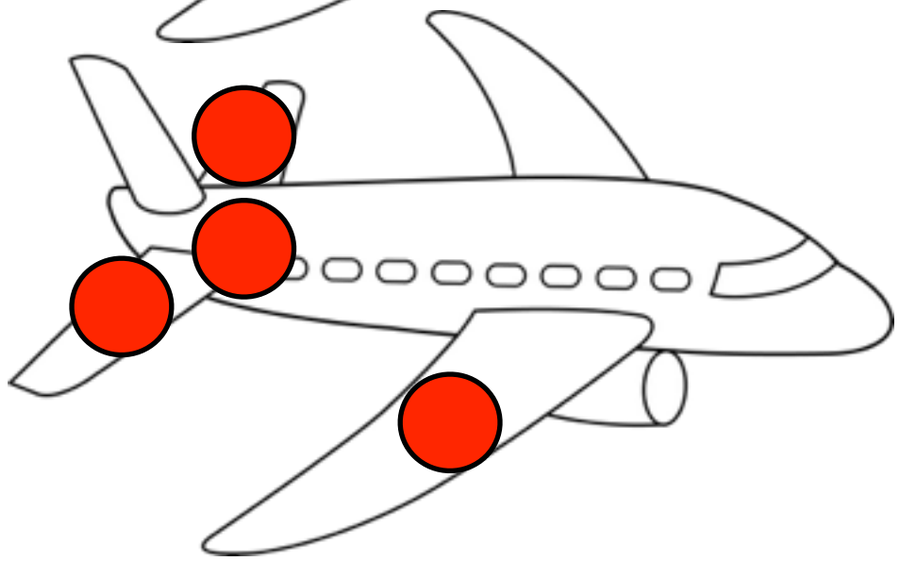
Wald and the Bullet Holes

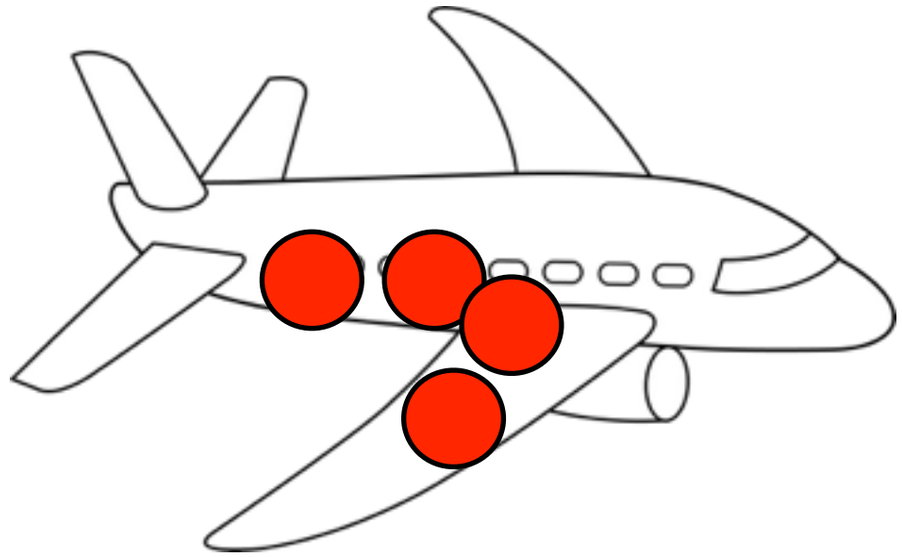








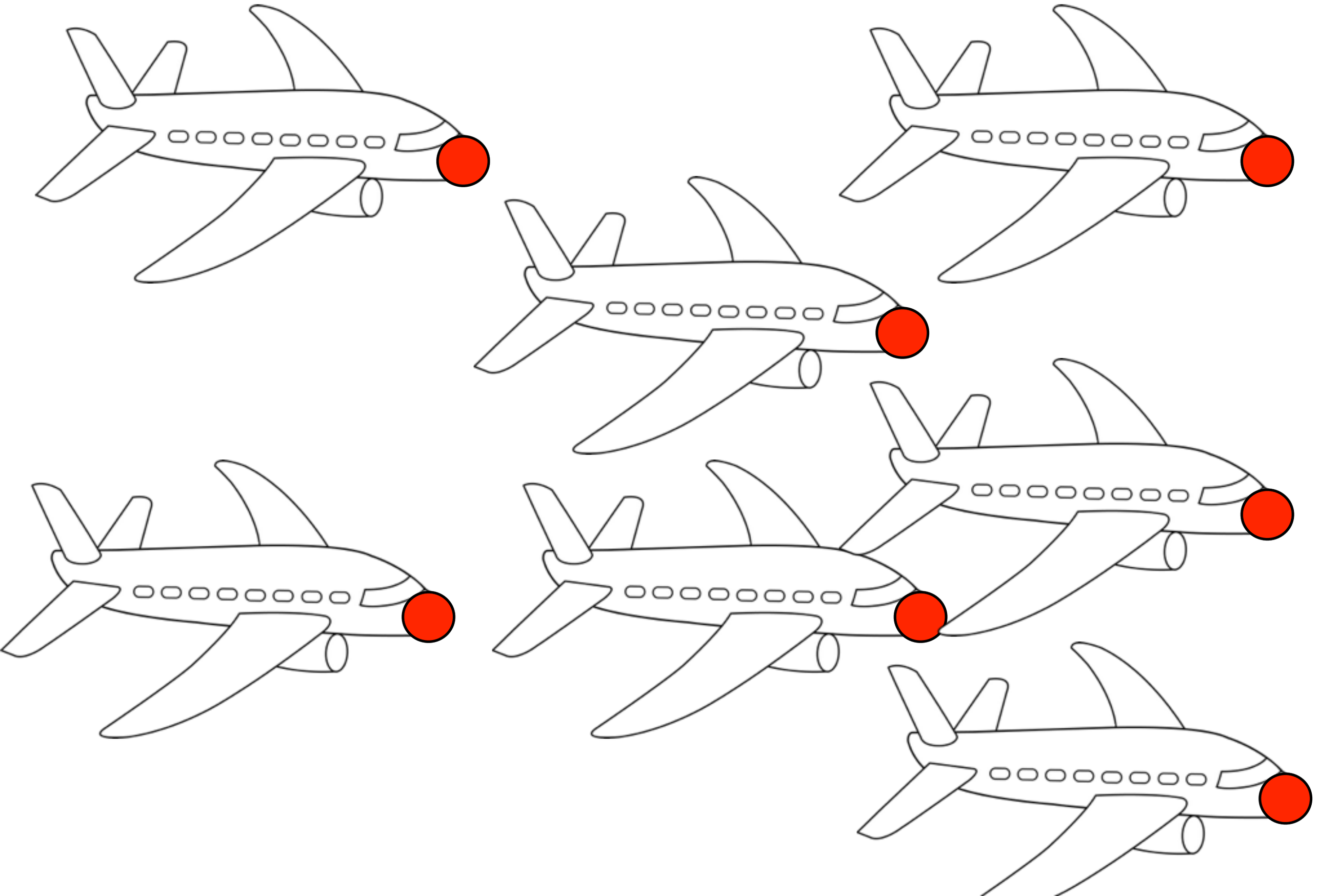




What about the *unobserved* planes? Missing data!

Put the armor where the planes aren’t hit assuming
that that’s wheree planes that didn’t survive were hit.

What about the *unobserved* planes? Missing data!



Longevity Study from Lombard (1835)

Profession	Average Longevity
chocolate maker	73.6
professors	66.6
clocksmiths	55.3
locksmiths	47.2
students	20.2

Mobility across ages for professions wasn't considered.

Sources: Lombard (1835), Wainer (1999), Stigler (2002)

Class Size Paradox

Why do so many schools boast small average class size but then so many students end up in huge classes?

Simple example: each student takes one course; suppose there is one course with 100 students, fifty courses with 2 students.

Dean calculates: (100+50*2)/51 = 3.92

Students calculate: (100*100+100*2)/200 = 51

Depending on which vantage point you're taking, can have huge ramifications for your data.

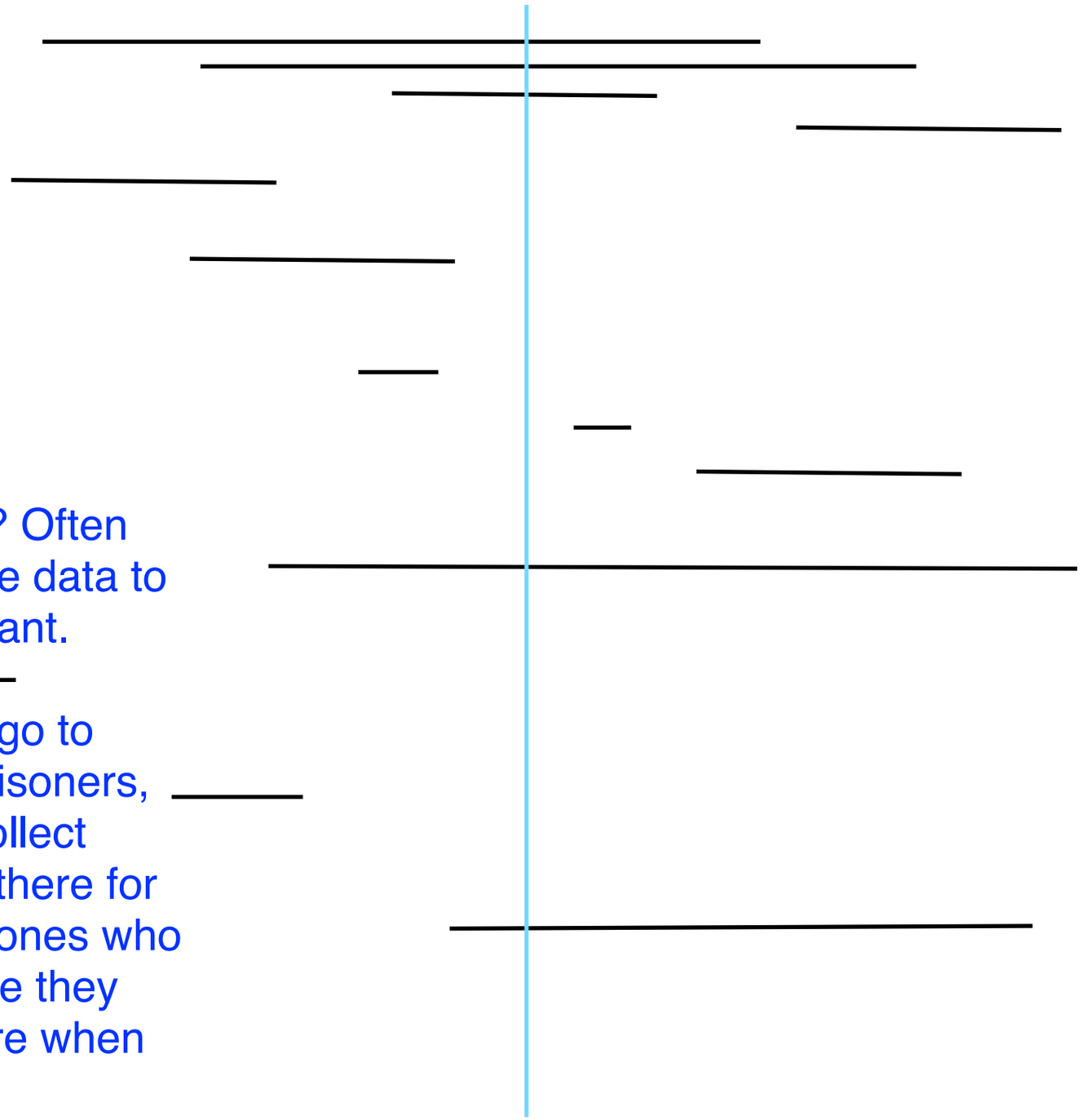
100 students experience a course with 100 students while the other 100 experience it with 2.

“About 10 percent of the 1.6 million inmates in America’s prisons are serving life sentences; another 11 percent are serving over 20 years.”

source: http://www.nytimes.com/2012/02/26/health/dealing-with-dementia-among-aging-criminals.html?pagewanted=all

Length-Biasing Paradox

How would you measure the average prison sentence?



How to collect all this data? Often you have to find a surrogate data to answer the question you want.

In this example, when you go to collect the data from the prisoners, you're more likely to see/collect data on individuals who're there for longer sentences than the ones who have a short sentence since they most likely wouldn't be there when you arrive.

Bias of an Estimator

A ‘hat’ = ^ means it is your estimator

The *bias* of an estimator is how far off it is on average:

$$\text{bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$$

So why not just subtract off the bias?

You don’t actually know the bias, your theta is unknown.

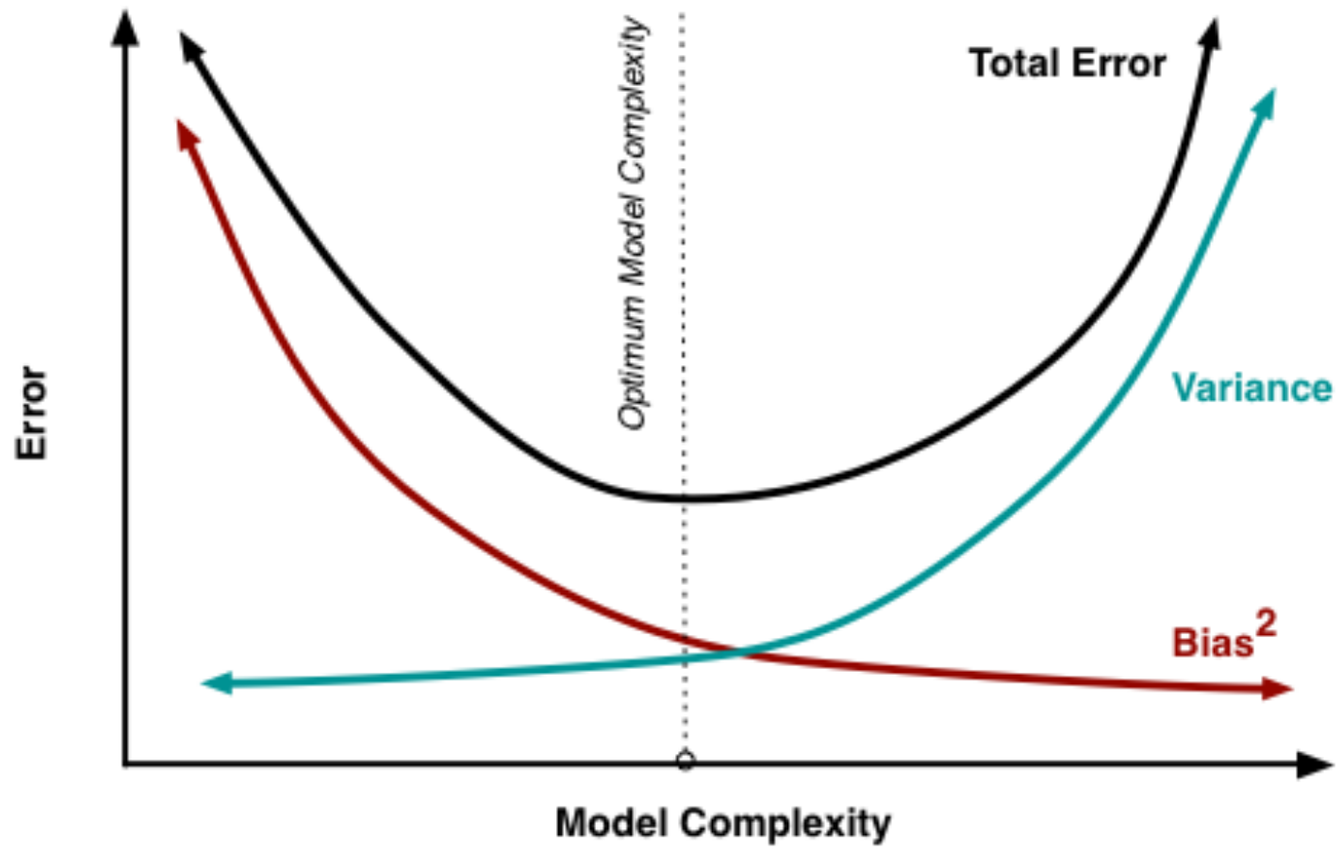
Bias-Variance Tradeoff

one form:

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}(\hat{\theta}) + \mathrm{bias}^2(\hat{\theta})$$

Mean Squared Error = how far off are you from the truth, from the average.

often a little bit of bias can make it possible to have much lower MSE



http://scott.fortmann-roe.com/docs/BiasVariance.html

Unbiased Estimation: Poisson Example

Most widely used distribution for count data

$$X \sim \text{Pois}(\lambda)$$

$$\text{Goal: estimate } e^{-2\lambda}$$

$(-1)^X$ is the best (and only) unbiased estimator of $e^{-2\lambda}$

sensible?

Unbiaseness is not the goal

Fisher Weighting

Often big datasets are composed of small datasets and the trick is to combine those smaller datasets into what you're looking for.

How should we combine independent, *unbiased* estimators for a parameter into one estimator?

$$\hat{\theta} = \sum_{i=1}^{k} w_i \hat{\theta}_i$$

The *weights* should sum to one, but how should they be chosen?

More weights on those values that are more reliable.

The more variance, the less weight it receives.

$$w_i \propto \frac{1}{\mathrm{Var}(\hat{\theta}_i)}$$

(Inversely proportional to variance; why not SD?)

Nate Silver Weighting Method

- Exponential decay based on recency of poll
- Sample size of poll
- Pollster rating

http://fivethirtyeight.com/features/how-the-fivethirtyeight-senate-forecast-model-works/

How do you combine all these data sources? It's a question of a lot of thought and experimentation on how these seemingly disperse data point impact each other.

More recent polls are more useful. The political landscape changes frequently. How much more weight should you put on a poll that's more recent? Silver used an exponential decay, ad hoc number, fine tuned via experimentation.

Bigger sample size, the better.

The bias wasn't really being considered until Nate Silver started looking into the pollster rating. This rates the pollster based on their actual predictive powers for the election. How accurate were they at predicting the outcomes of the elections, for instance.

Multiple Testing, Bonferroni

Fishing expeditions.

How should we handle p-values
when testing multiple hypotheses?

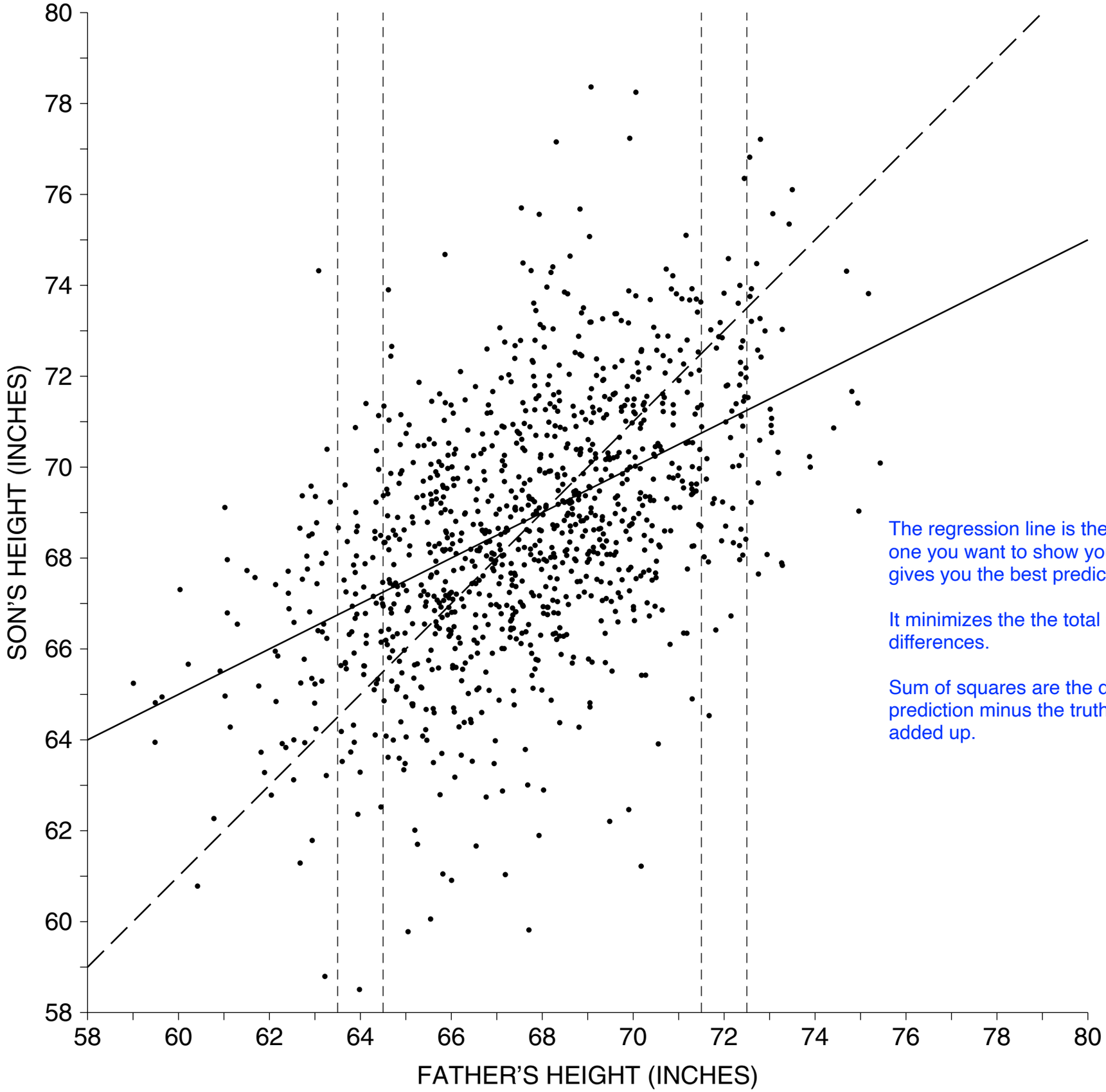
For example, what if we are looking
at diet (with 10 kinds of food) and
disease (with 10 diseases)?

A simple, conservative approach is Bonferroni: divide significance level by number of hypotheses being tested.

$$FWER = Pr\left\{\bigcup_{I_o}\left(p_i \leq \frac{\alpha}{m}\right)\right\} \leq \sum_{I_o}\left\{Pr\left(p_i \leq \frac{\alpha}{m}\right)\right\} \leq m_0\frac{\alpha}{m} \leq m\frac{\alpha}{m} = \alpha$$

If you end up testing 100 hypothesis, then divide your significance level, say 0.05 by 100 to determine which hypothesis is worth looking at.

https://en.wikipedia.org/wiki/Bonferroni_correction



The regression line is the solid one, it's the one you want to show you the best fit. It gives you the best prediction of y from X

It minimizes the the total sum of squares differences.

Sum of squares are the difference from the prediction minus the truth, squared and then added up.

plot from Freedman, data from Pearson-Lee

Regression Toward the Mean (RTTM)

The term, regression, comes from regression towards the mean.

Examples are everywhere...

Test scores

Sports

Inherited characteristics, e.g., heights

Traffic accidents at various sites

Pre-test takers who did well are happy and their scores go down when they take the real SAT, the people who did poorly, their scores go up… and it's how 'the universe' functions in this case.

Some sites will have more collisions some times and things fluctuate.

Daniel Kahneman Quote on RTTM

I had the most satisfying Eureka experience of my career while attempting to teach flight instructors that praise is more effective than punishment for promoting skill-learning....

[A flight instructor objected:] "On many occasions I have praised flight cadets for clean execution of some aerobatic maneuver, and in general when they try it again, they do worse. On the other hand, I have often screamed at cadets for bad execution, and in general they do better the next time. So please don't tell us that reinforcement works and punishment does not..."

This was a joyous moment, in which I understood an important truth about the world: because we tend to reward others when they do well and punish them when they do badly, and because there is regression to the mean, it is part of the human condition that we are statistically punished for rewarding others and rewarded for punishing them.

Regression Paradox

y: child's height (standardized)
x: parent's height (standardized)

Standardized means they have a standard deviation of 1 and a mean of 0

Regression line: predict y = rx;
think of this as a weighted average of
the parent's height and the mean

Now, what about predicting the parent's height from the child's height? Use x = y/r?

Mathematically, the regression paradox stays the same even when the variables are flipped. Forward in time and backwards in time.

Regression line is x = ry, the r stays the same!

Linear Model

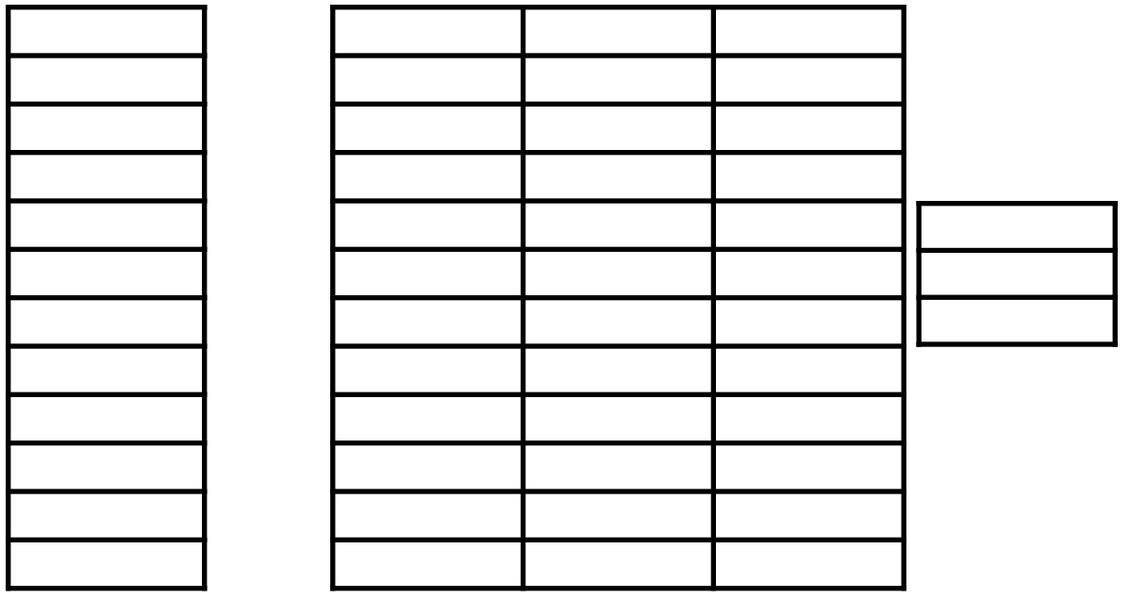
often called "OLS" (ordinary least squares), but that puts the focus on the procedure rather than the model.

Epsilon is the error.

There're specific criterion for OLS. The process is to minimize the sum of square differences and then there're algorithms for doing that.

Matrix notation

$$\underbrace{y}_{n\times 1} = \underbrace{X}_{n\times k}\underbrace{\beta}_{k\times 1} + \underbrace{\epsilon}_{n\times 1}$$



What's linear about it?

You're presenting it in linear algebra notation

$$\underbrace{y}_{n\times 1} = \underbrace{X}_{n\times k}\underbrace{\beta}_{k\times 1} + \underbrace{\epsilon}_{n\times 1}$$

Linear refers to the fact that we're taking linear combinations of the predictors. Still linear if, e.g., use both x and its square and its cube as predictors.

In traditional stats you're assuming you have more N (observations) than k (variables.

In machine learning that tends to get flipped where you have say, 100 observations but a ton of variables for those observations.

Sample Quantities vs. Population Quantities

sample version (think of x and y as data vectors)

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

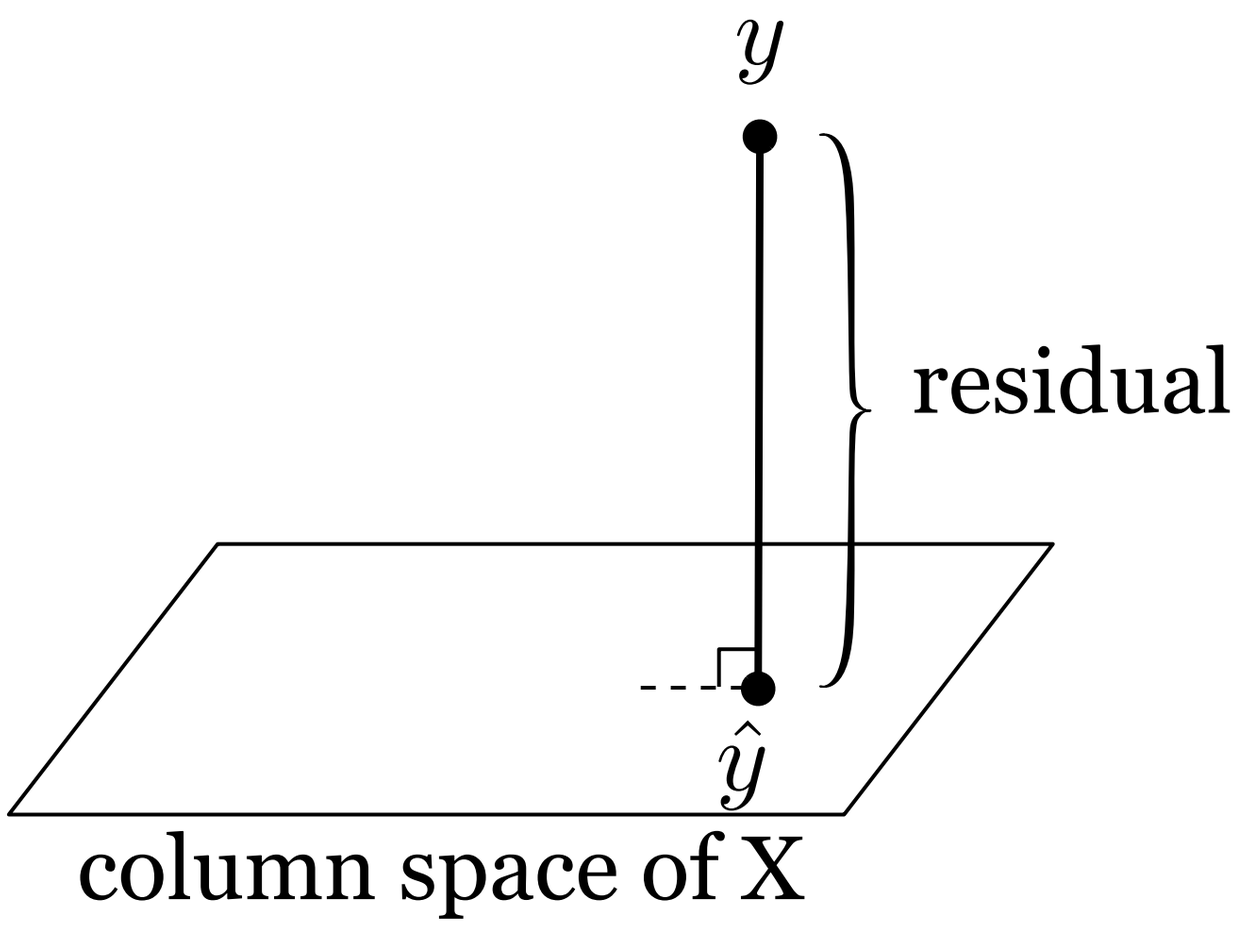
population version (think of x and y as r.v.s)

$$y = \beta_0 + \beta_1 x + \epsilon$$

$$E(y) = \beta_0 + \beta_1 E(x)$$

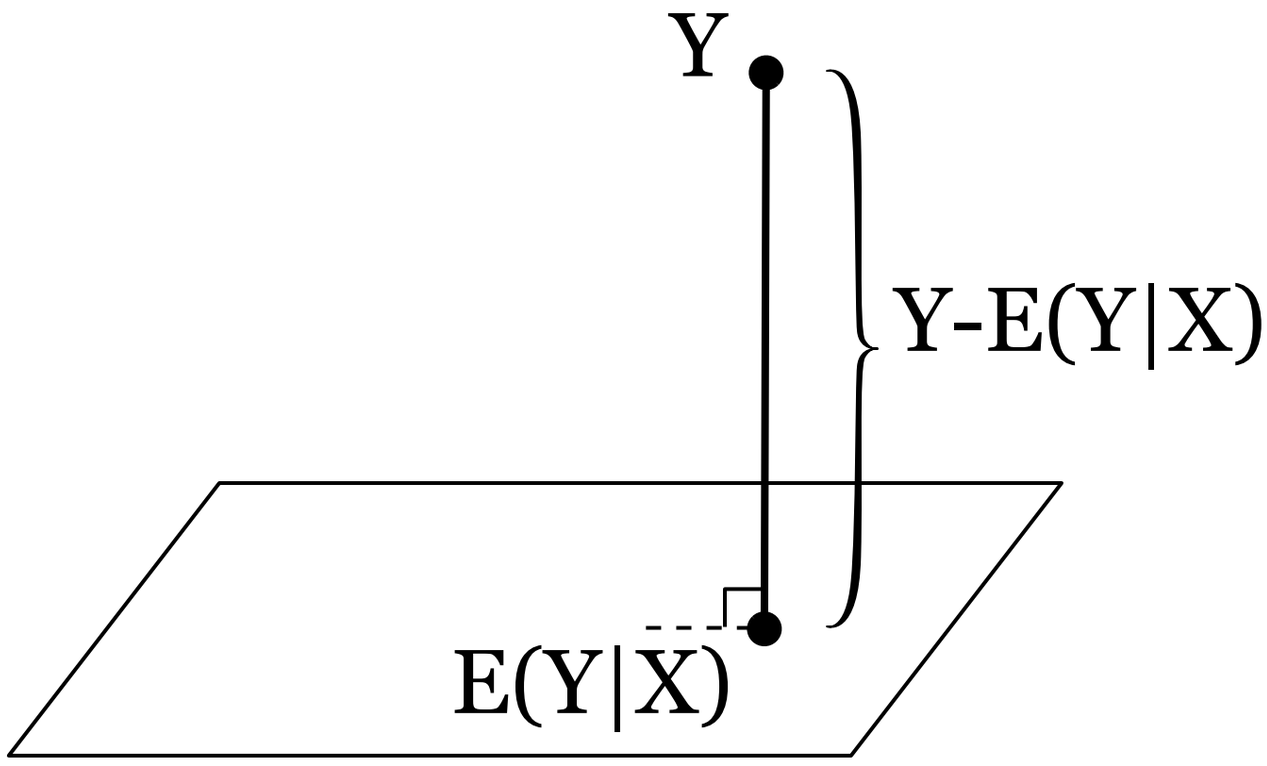
$$\text{cov}(y, x) = \beta_1 \text{cov}(x, x)$$

visualize regression as a *projection*



Regressino is a projection, the idea of regression, you're finding the closest point of column space where all vectors you can get from taking linear combinations of values of x. YOu're finding the closest point, which is perpendicular.

or as a *conditional expectation*



The function of X is the best predictor of Y. Conditional what function of X does the best job predicting Y. Normal distributions, epsilon is normal, the best predictor of X is linear. Best in the sense of minimizing mean squared error.

Gauss-Markov Theorem

Consider a linear model

$$y = X\beta + \epsilon$$

where y is n by 1, $\mathbf{X}$ is an n by k matrix of covariates, β is a k by 1 vector of parameters, and the errors ϵ_j are uncorrelated with equal variance, $\epsilon_j \sim [0, \sigma^2]$. The errors do not need to be assumed to be Normally distributed.

Then it follows that...

$$\hat{\beta} \equiv (X'X)^{-1}X'y$$

is **BLUE** (the **B**est **L**inear **U**nbiased **E**stimator).

For Normal errors, this is also the MLE.

Residuals

You're never going to know the true values of epsilon or Beta.

$$y = X\hat{\beta} + e$$

mirrors

$$y = X\beta + \epsilon$$

The residual vector e is *orthogonal* to all the columns of X.

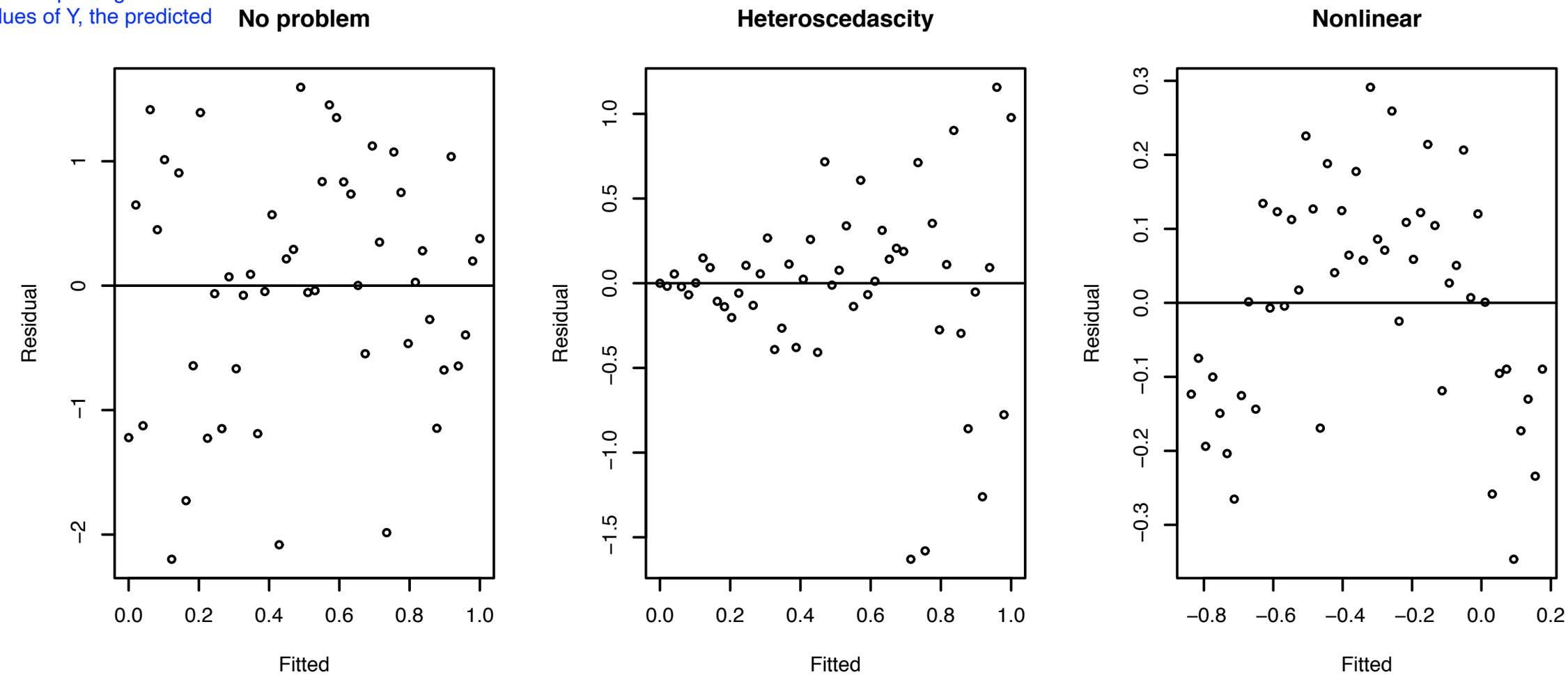
You then use estimators, e.g. the '^'

Residual Plots

Always plot the residuals! (Plot residuals vs. fitted values, and residuals vs. each predictor variable)

Typically you're plotting the residuals on the vertical axis and the horizontal plotting are the fitted values of Y, the predicted values.

'Heteroscedasticity' refers to a model such that the variability of a variable is equal across the range of values of a second variable predicting it



You're looking for no pattern, so the plot on the far left is the what's best. The middle plot shows your values starting getting more erroneous/variable as you move to the right. The plot on the far right indicates that a non-linear mdodel should be chosen.

Faraway, http://cran.r-project.org/doc/contrib/Faraway-PRA.pdf

"Explained" Variance

R2 = the ratio of how much of the variance was captured by the model vs the total variance of the y variable.

Not good at telling you how you will do at future data, just how good you are with current data.

$$\mathrm{var}(y) = \mathrm{var}(X\hat{\beta}) + \mathrm{var}(e)$$

$$R^2 = \frac{\mathrm{var}(X\hat{\beta})}{\mathrm{var}(y)} = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

R^2 measures goodness of fit, but
it does *not* validate the model.
Adding more predictors can only increase R^2.