Autocorrelation Function

TIME SERIES ANALYSIS IN PYTHON

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Autocorrelation Function

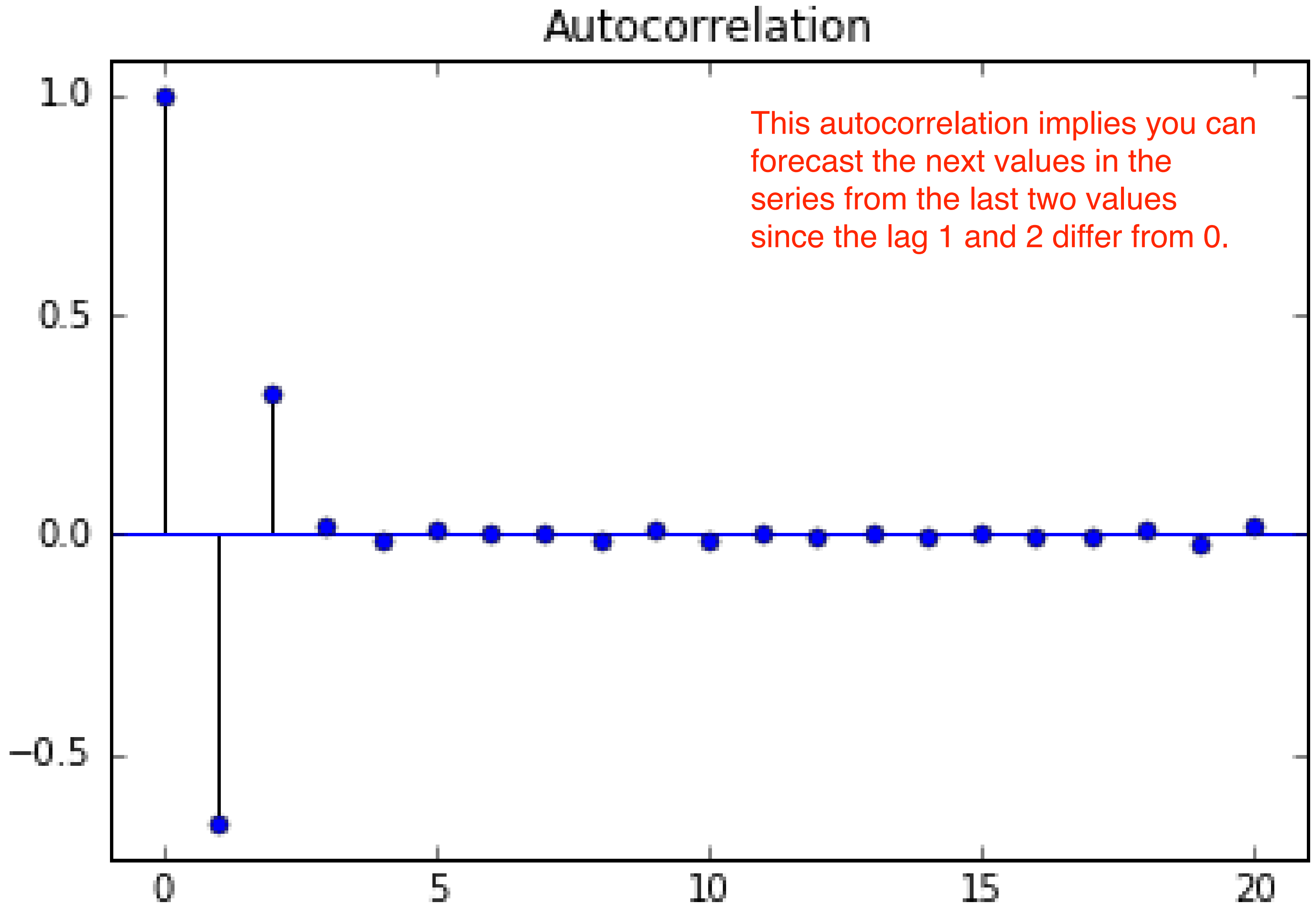
- Autocorrelation Function (ACF): The autocorrelation as a function of the lag

Any significant non-zero autocorrelation implies that the serirs can be forecast from the past.

- Equals one at lag-zero
- Interesting information beyond lag-one

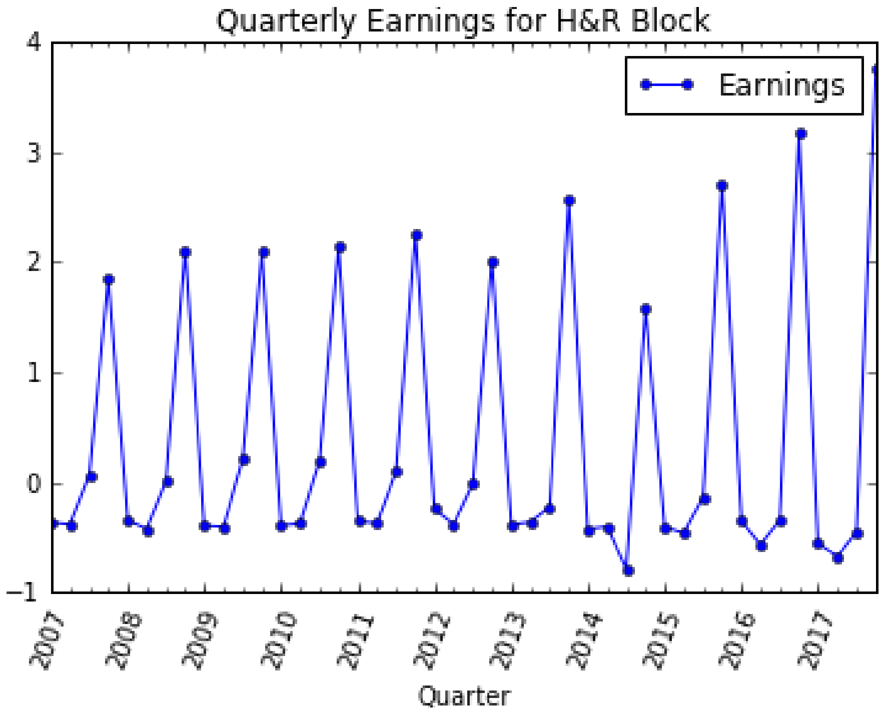
ACF Example 1: Simple Autocorrelation Function

- Can use last two values in series for forecasting

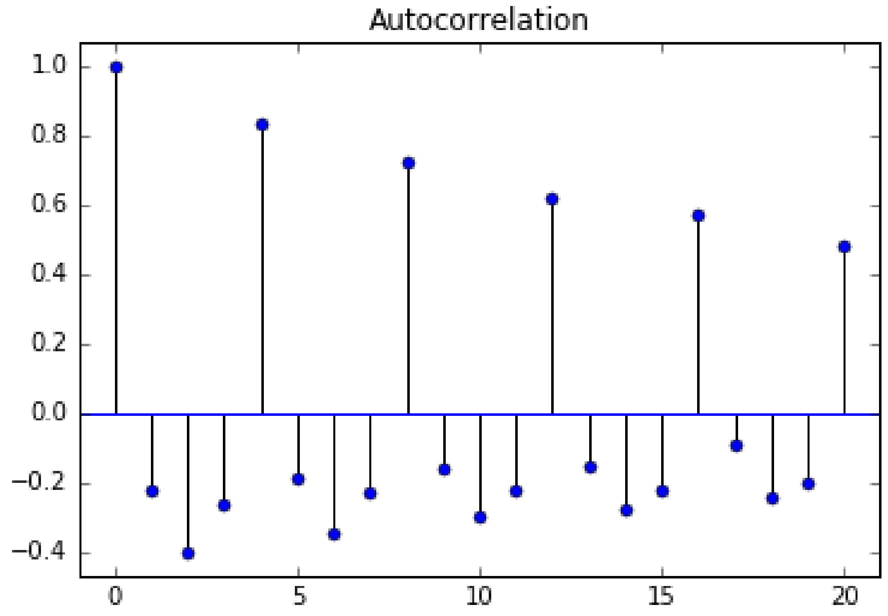


ACF Example 2: Seasonal Earnings

- Earnings for H&R Block

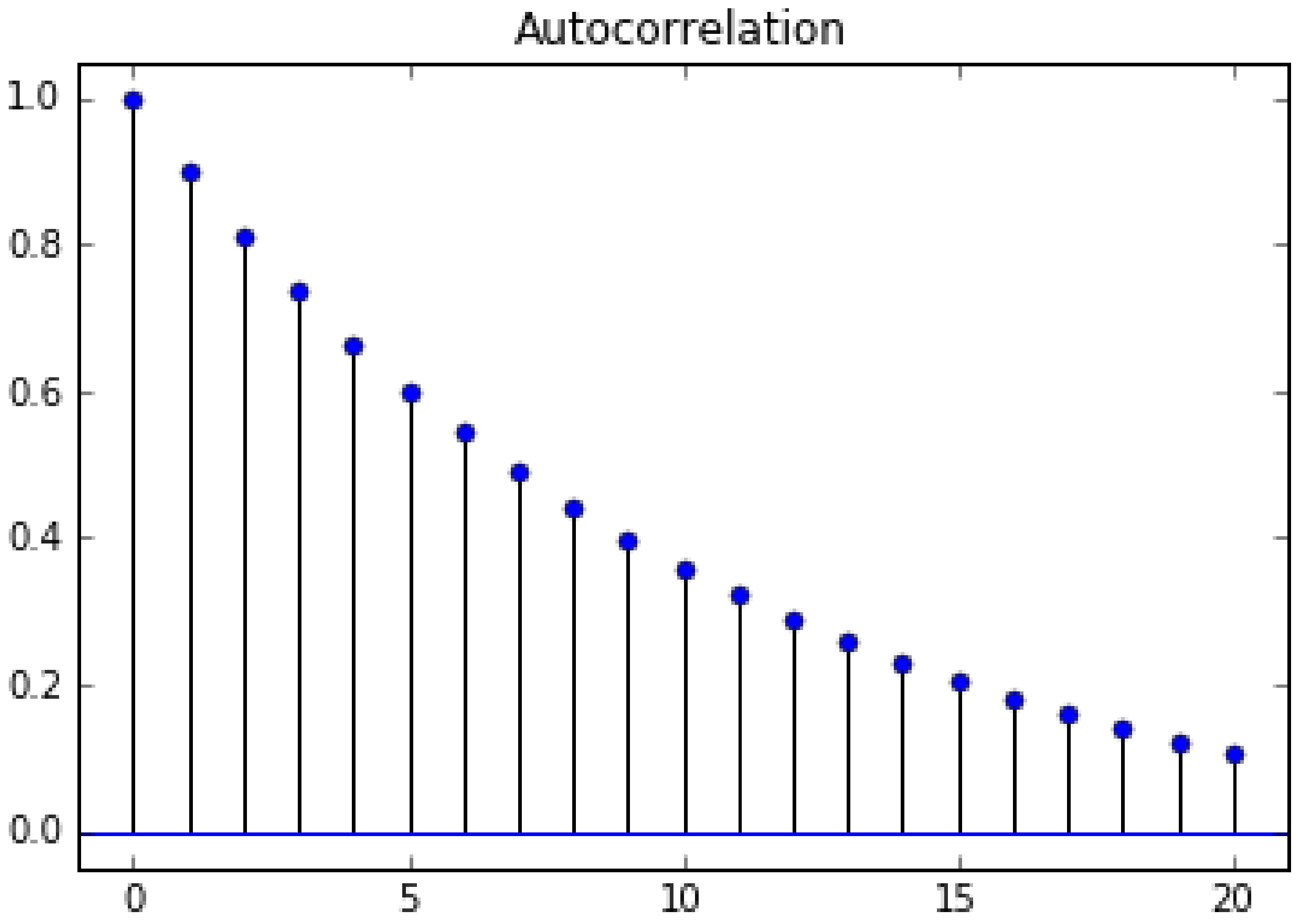


- ACF for H&R Block



This shows strong autocorrelation at lags 4, 8, 12, 16 & 20.

ACF Example 3: Useful for Model Selection



ACF can also be used for a parsimonious model for fitting the data.

- Model selection

Plot ACF in Python

- Import module:

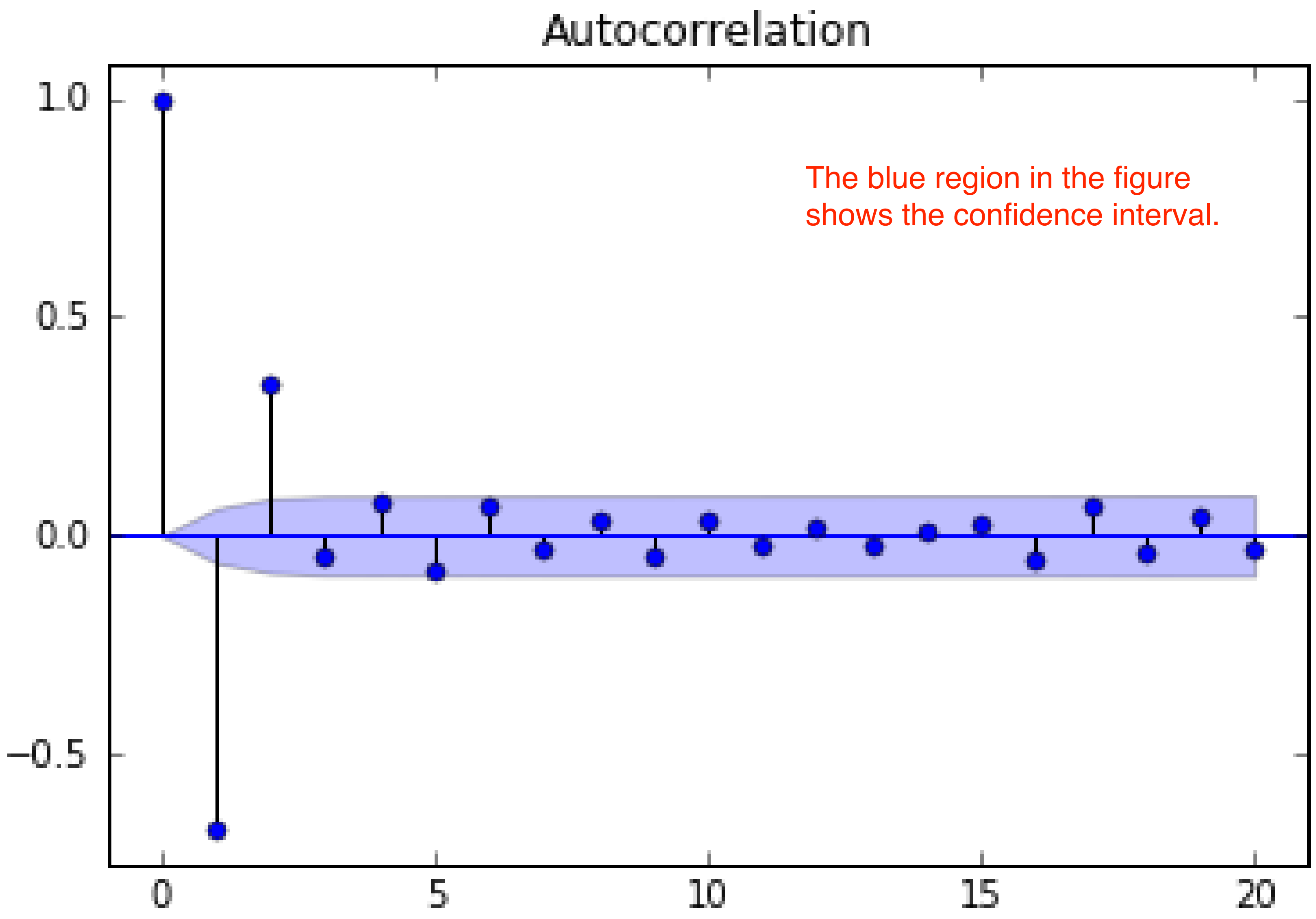
```
from statsmodels.graphics.tsaplots import plot_acf
```

- Plot the ACF:

```
plot_acf(x, lags= 20, alpha=0.05)
```

x is the series or array
lags is how many of the lags of the ACF will be plotted
alpha sets the width of confidence interval.

Confidence Interval of ACF



Confidence Interval of ACF

- Argument `alpha` sets the width of confidence interval
- Example: `alpha=0.05`
  - 5% chance that if true autocorrelation is zero, it will fall outside
  blue band

In other words, can you say that there is less than a 5% chance that we would observe such an autocorrelation value if the true autocorrelation were really zero?

- Confidence bands are wider if:
  - Alpha lower
  - Fewer observations
- Under some simplifying assumptions, 95% confidence bands are
$\pm 2/\sqrt{N}$
- If you want no bands on plot, set `alpha=1`

ACF Values Instead of Plot

```python
from statsmodels.tsa.stattools import acf
print(acf(x))
```

```
[ 1.          -0.6765505   0.34989905 -0.01629415 -0.0250701
 -0.03186545  0.01399904 -0.03518128  0.02063168 -0.0262064
...
  0.07191516 -0.12211912  0.14514481 -0.09644228  0.0521588
```

```
# Import the acf module and the plot_acf module from statsmodels
from statsmodels.tsa.stattools import acf
from statsmodels.graphics.tsaplots import plot_acf

# Compute the acf array of HRB
acf_array = acf(HRB)
print(acf_array)

# Plot the acf function
plot_acf(HRB, alpha = 1)
plt.show()
```

Notice the strong positive autocorrelation at lags 4, 8, 12, 16,20, ...

Let's practice!

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```
# Import the plot_acf module from statsmodels and sqrt from math
from statsmodels.graphics.tsaplots import plot_acf
from math import sqrt

# Compute and print the autocorrelation of MSFT weekly returns
autocorrelation = returns['Adj Close'].autocorr()
print("The autocorrelation of weekly MSFT returns is %4.2f" %(autocorrelation))

# Find the number of observations by taking the length of the returns DataFrame
nobs = len(returns)

# Compute the approximate confidence interval
conf = 1.96/sqrt(nobs)
print("The approximate confidence interval is +/- %4.2f" %(conf))

# Plot the autocorrelation function with 95% confidence intervals and 20 lags using plot_acf
plot_acf(returns, alpha=0.05, lags = 20)
plt.show()
```

Notice that the autocorrelation with lag 1 is significantly negative, but none of the other lags are significantly different from zero

White Noise

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What is White Noise?

- White Noise is a series with:
  - Constant mean
  - Constant variance
  - Zero autocorrelations at all lags
- Special Case: if data has normal distribution, then *Gaussian White Noise*

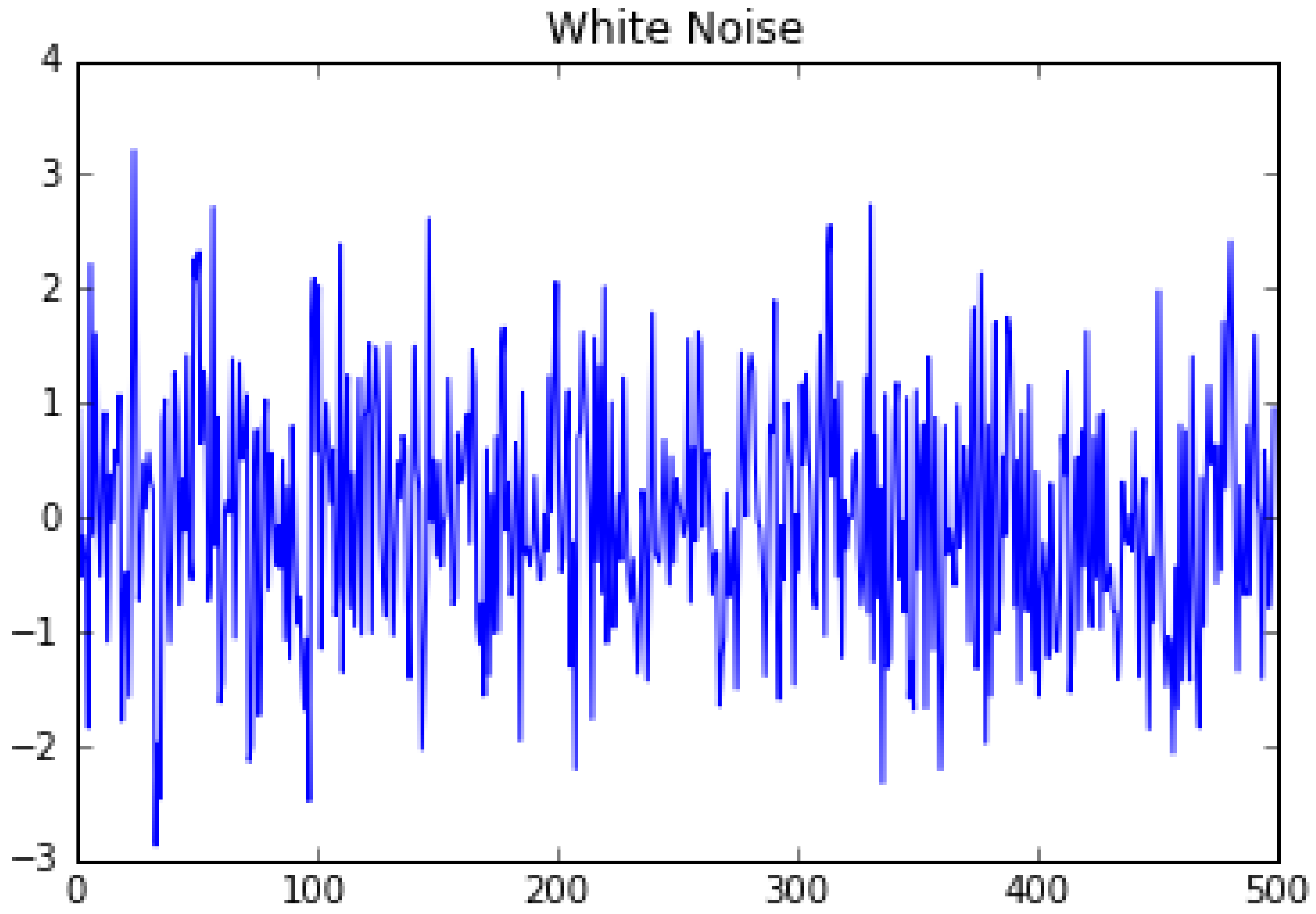
Simulating White Noise

- It's very easy to generate white noise

```
import numpy as np
noise = np.random.normal(loc=0, scale=1, size=500)
```

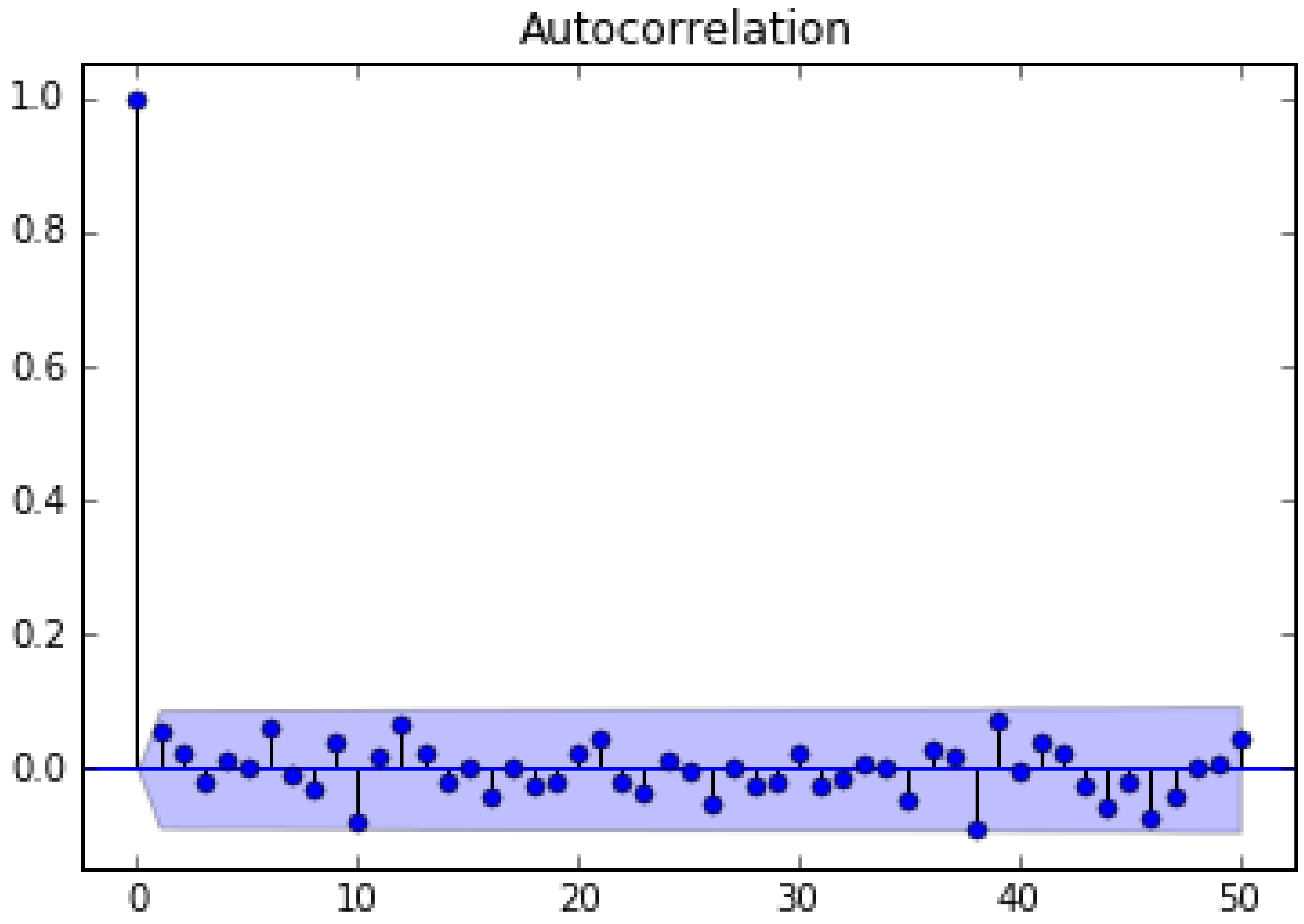
What Does White Noise Look Like?

```
plt.plot(noise)
```



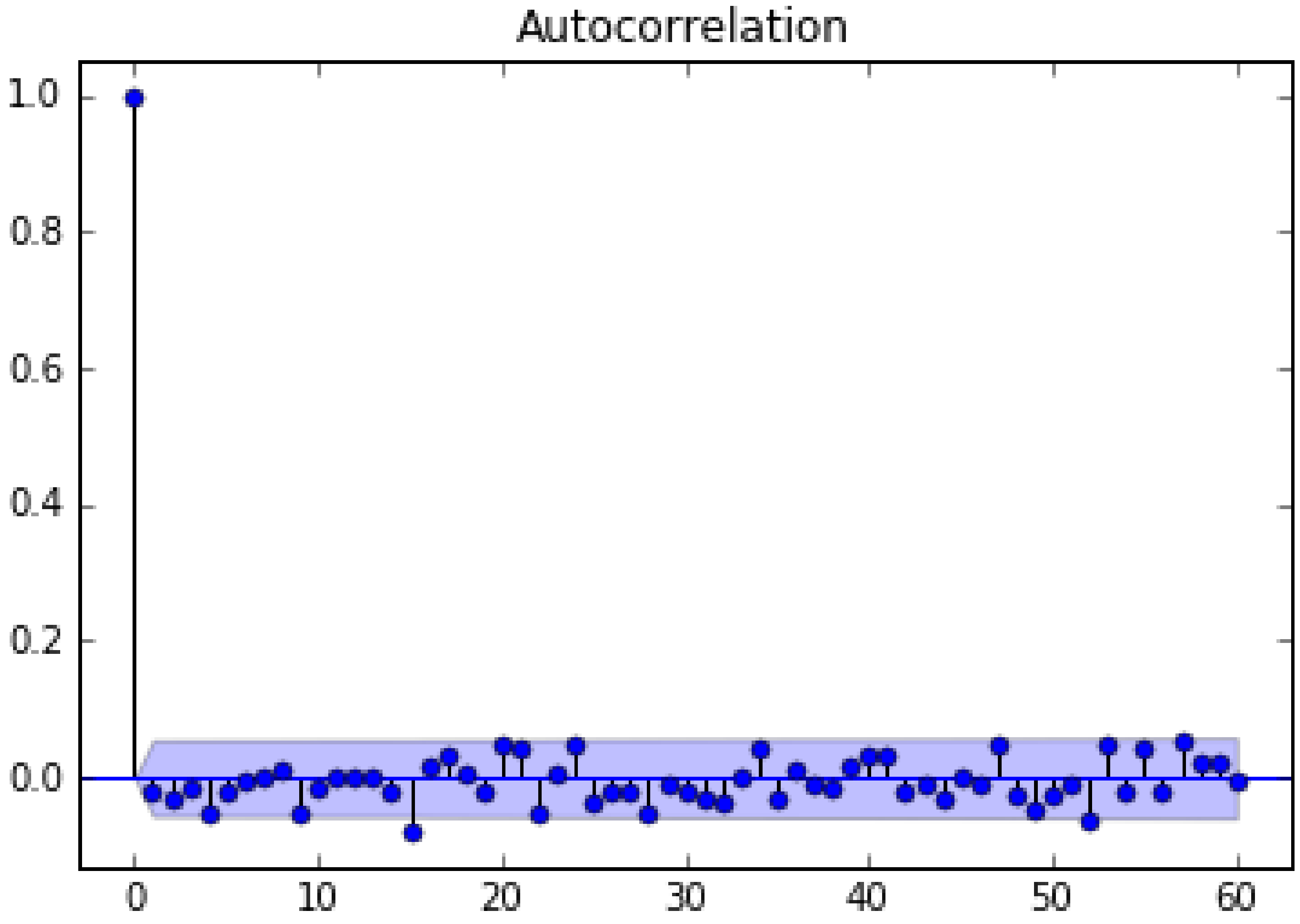
Autocorrelation of White Noise

```
plot_acf(noise, lags=50)
```



Stock Market Returns: Close to White Noise

- Autocorrelation Function for the S&P500



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Random Walk

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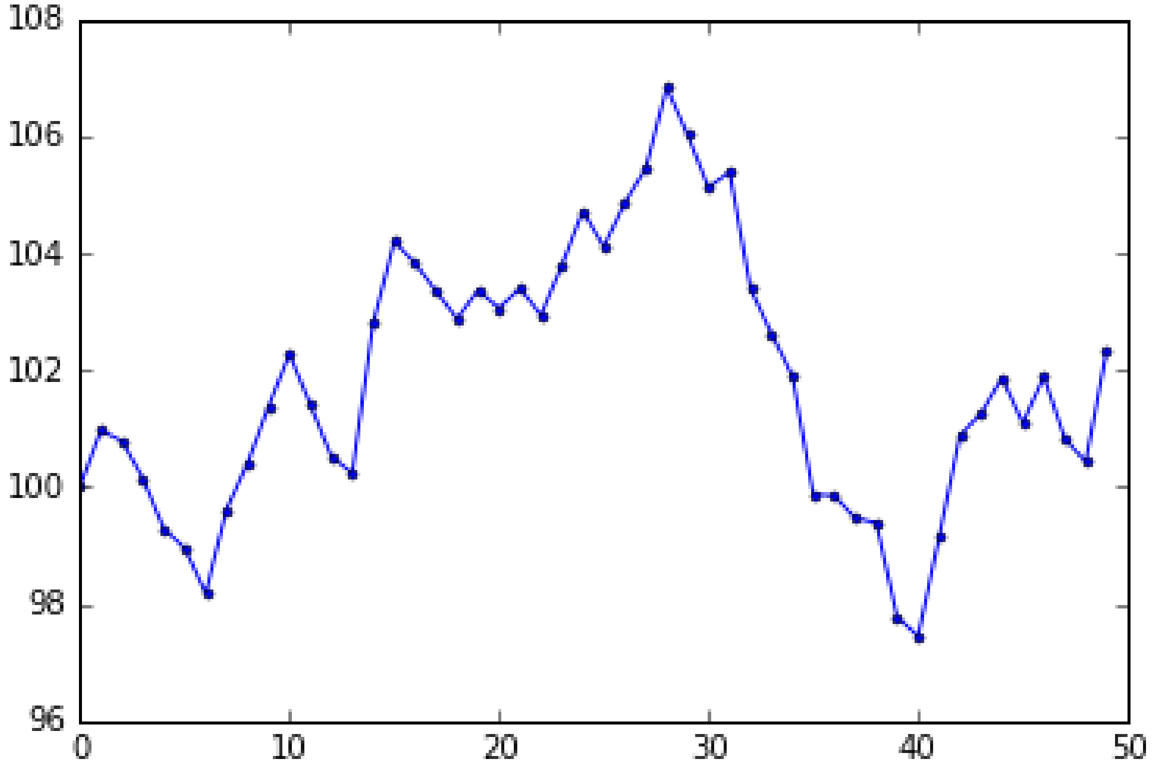
What is a Random Walk?

- Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

- Plot of simulated data



What is a Random Walk?

- Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

- Change in price is white noise

$$P_t - P_{t-1} = \epsilon_t$$

- Can't forecast a random walk
- Best forecast for tomorrow's price is today's price

What is a Random Walk?

- Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

- Random walk with drift:

$$P_t = \mu + P_{t-1} + \epsilon_t$$

- Change in price is white noise with non-zero mean:

$$P_t - P_{t-1} = \mu + \epsilon_t$$

Statistical Test for Random Walk

- Random walk with drift

$$P_t \quad = \quad \mu \quad + \quad P_{t-1} \quad + \quad \epsilon_t$$

- Regression test for random walk

$$P_t \quad = \quad \alpha \quad + \quad \beta \, P_{t-1} \quad + \quad \epsilon_t$$

- Test: $H_0 : \beta = 1$ (random walk) $H_1 : \beta < 1$ (not random walk)

Statistical Test for Random Walk

- Regression test for random walk

$$P_t = \alpha + \beta\, P_{t-1} + \epsilon_t$$

- Equivalent to

$$P_t - P_{t-1} = \alpha + \beta\, P_{t-1} + \epsilon_t$$

- Test: $H_0 : \beta = 0$ (random walk) $H_1 : \beta < 0$ (not random walk)

Statistical Test for Random Walk

- Regression test for random walk

$$P_t - P_{t-1} = \alpha + \beta P_{t-1} + \epsilon_t$$

- Test: $H_0 : \beta = 0$ (random walk) $H_1 : \beta < 0$ (not random walk)
- This test is called the **Dickey-Fuller** test
- If you add more lagged changes on the right hand side, it's the **Augmented Dickey-Fuller** test

ADF Test in Python

- Import module from statsmodels

```
from statsmodels.tsa.stattools import adfuller
```

- Run Augmented Dickey-Test

```
adfuller(x)
```

Example: Is the S&P500 a Random Walk?

```
# Run Augmented Dickey-Fuller Test on SPX data
results = adfuller(df['SPX'])
```

```
# Print p-value
print(results[1])
```

```
0.782253808587
```

```
# Print full results
print(results)
```

```
(-0.91720490331127869,
 0.78225380858668414,
 0,
 1257,
 {'1%': -3.4355629707955395,
  '10%': -2.567995644141416,
  '5%': -2.8638420633876671},
```

Let's practice!

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Stationarity

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What is Stationarity?

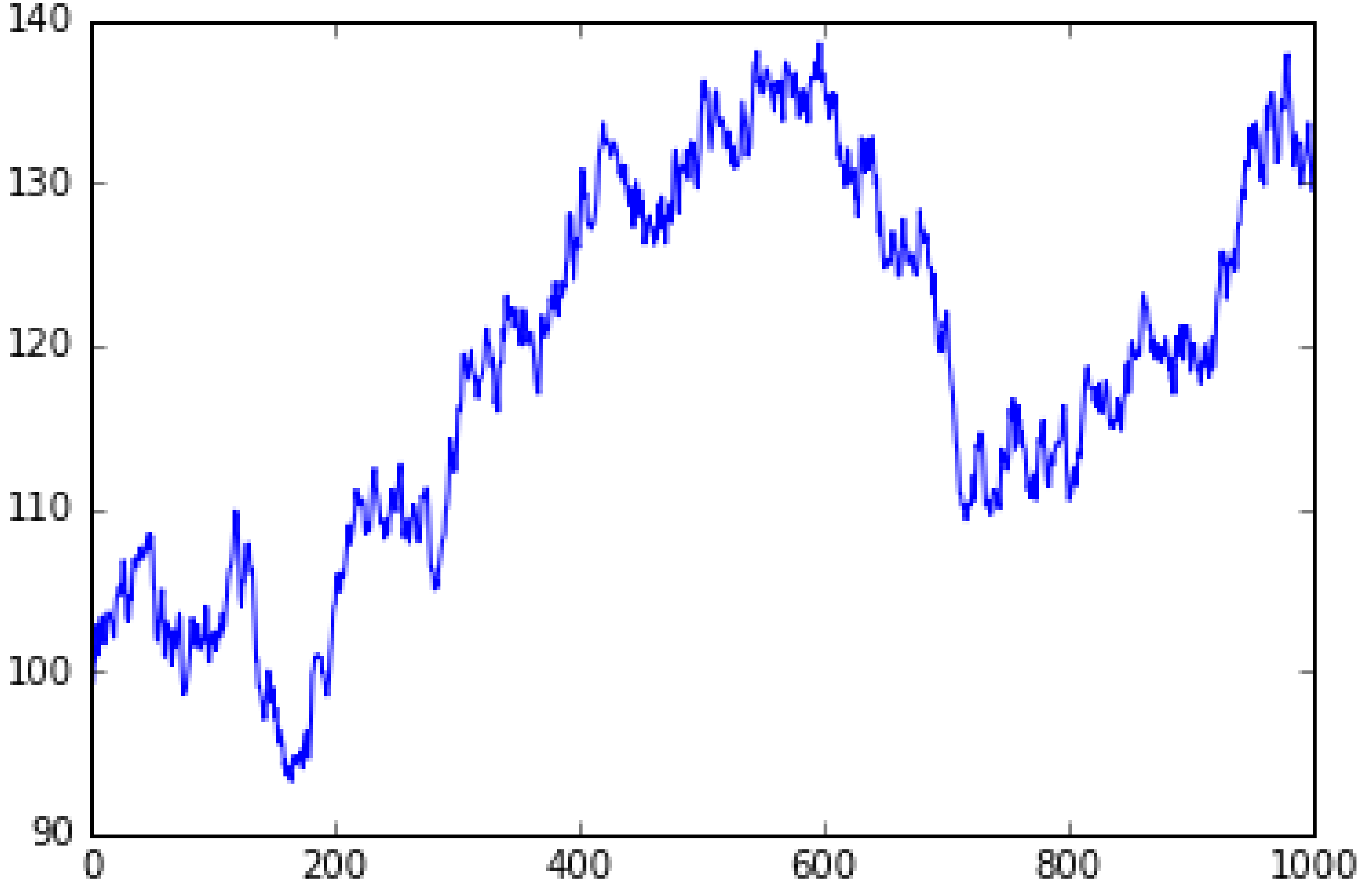
- **Strong stationarity**: entire distribution of data is time-invariant
- **Weak stationarity**: mean, variance and autocorrelation are time-invariant (i.e., for autocorrelation, corr$(X_t, X_{t-\tau})$ is only a function of τ)

Why Do We Care?

- If parameters vary with time, too many parameters to estimate
- Can only estimate a parsimonious model with a few parameters

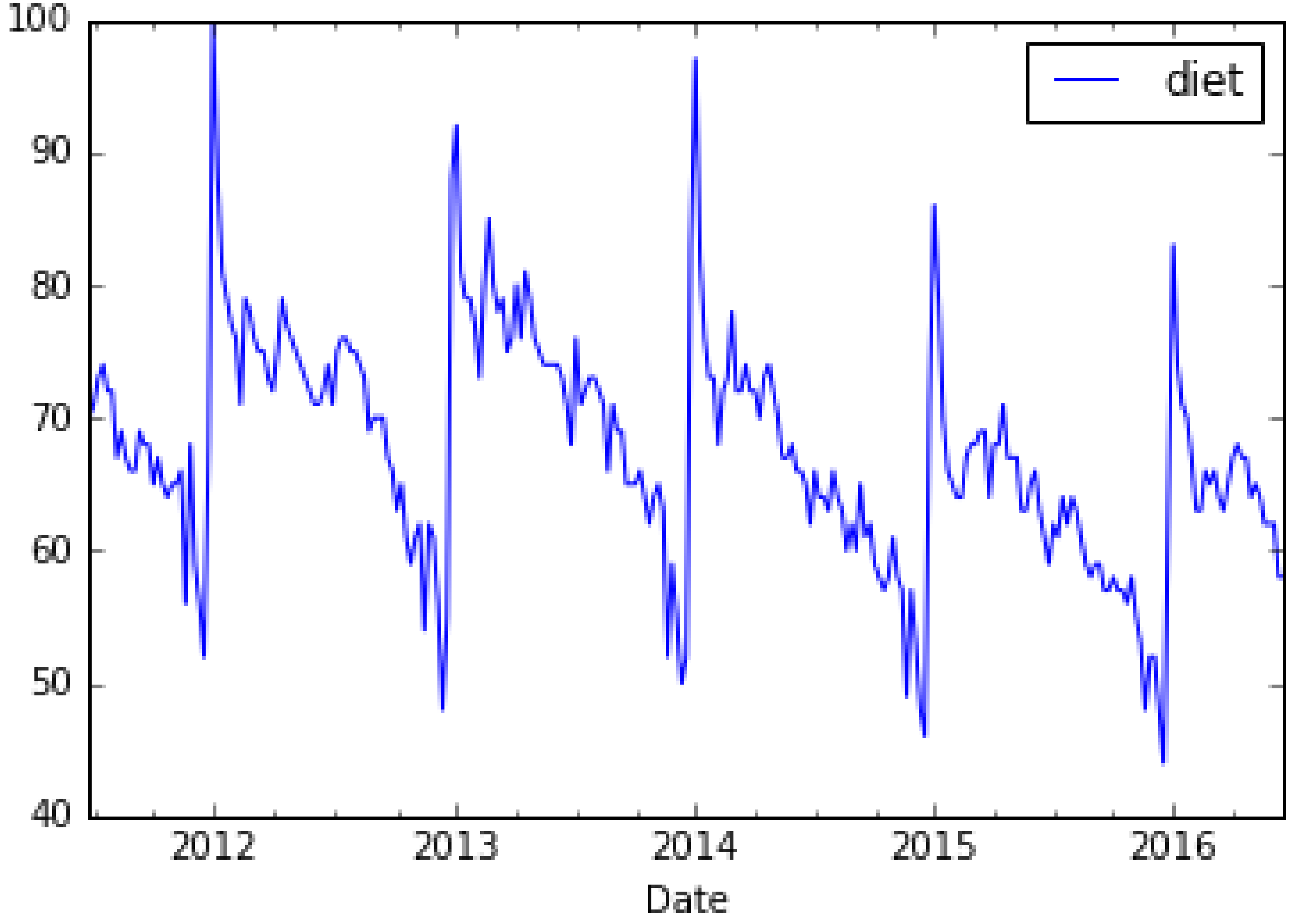
Examples of Nonstationary Series

- Random Walk



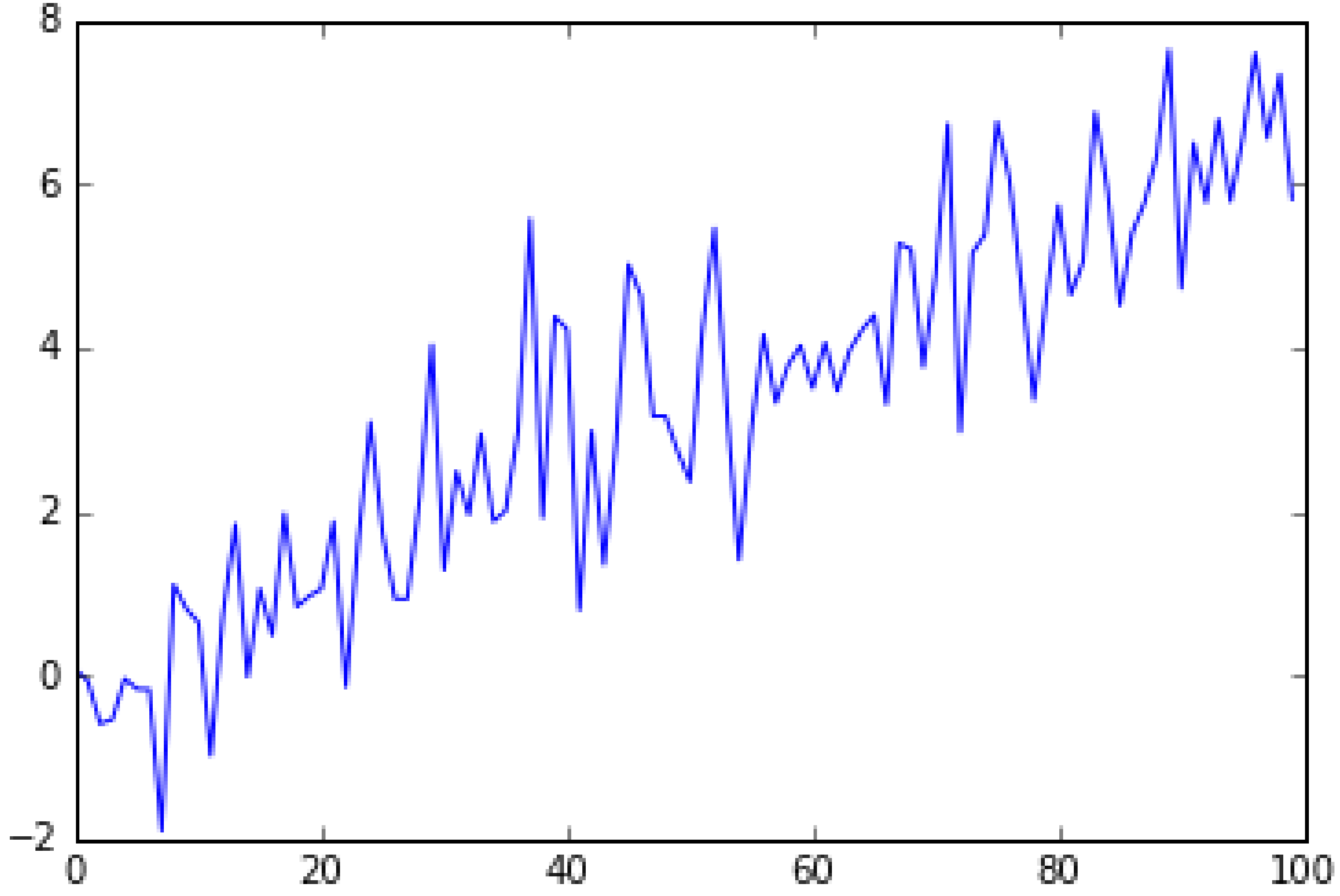
Examples of Nonstationary Series

- Seasonality in series



Examples of Nonstationary Series

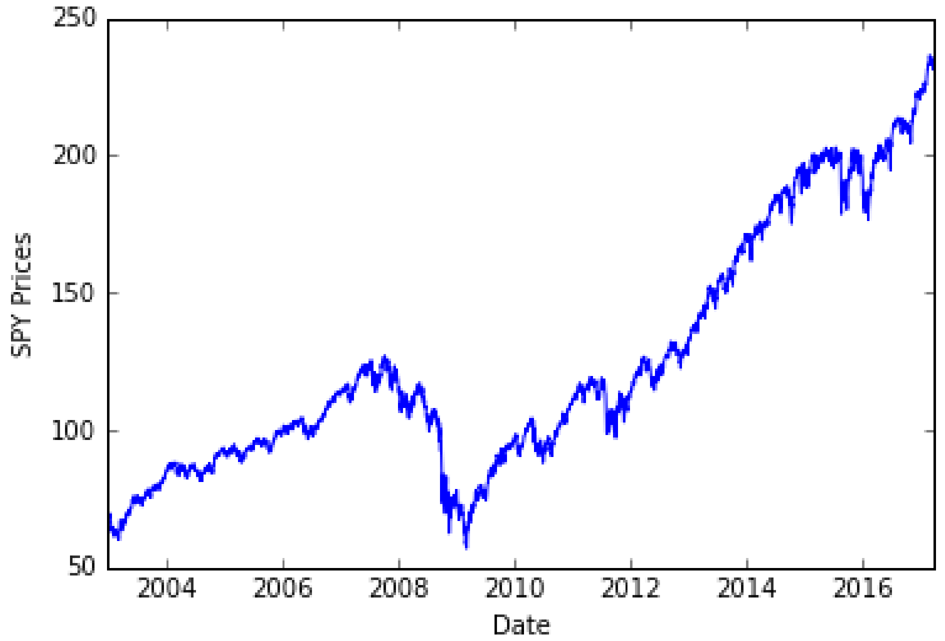
- Change in Mean or Standard Deviation over time



Transforming Nonstationary Series Into Stationary Series

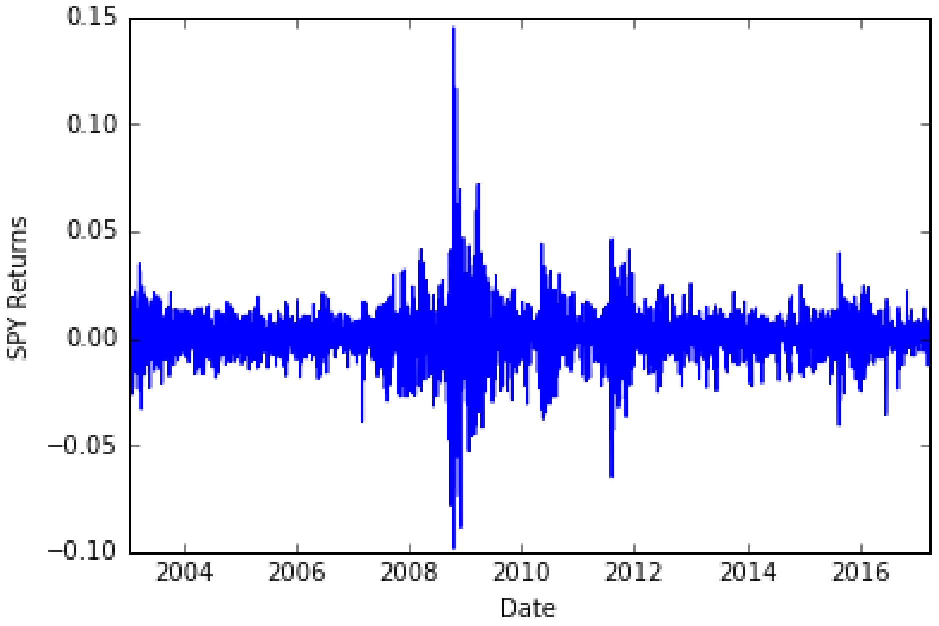
- Random Walk

```
plot.plot(SPY)
```



- First difference

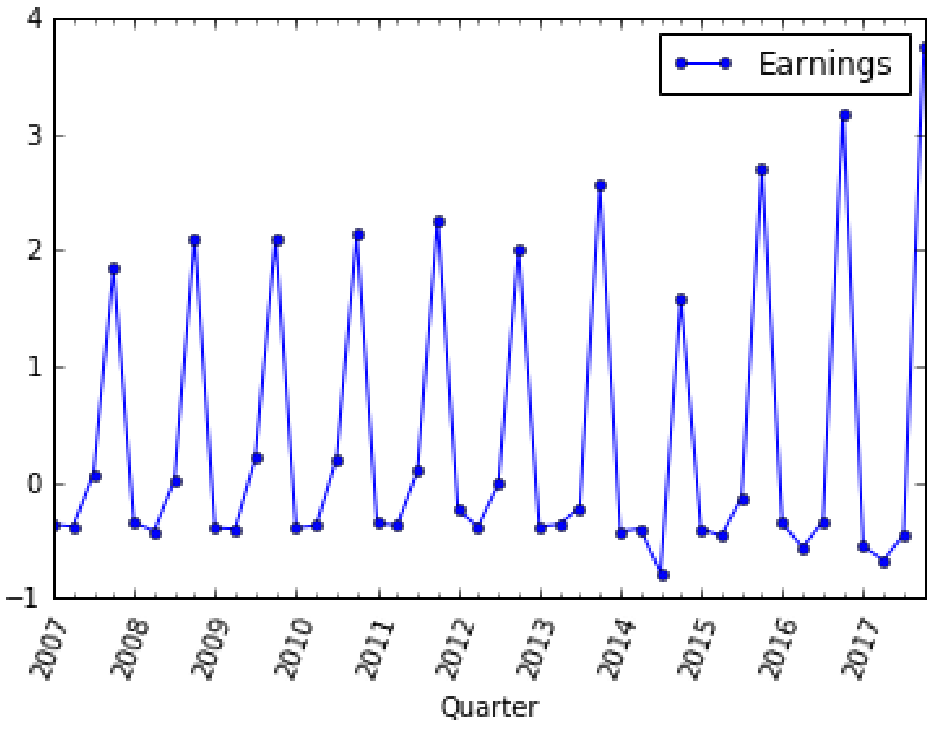
```
plot.plot(SPY.diff())
```



Transforming Nonstationary Series Into Stationary Series

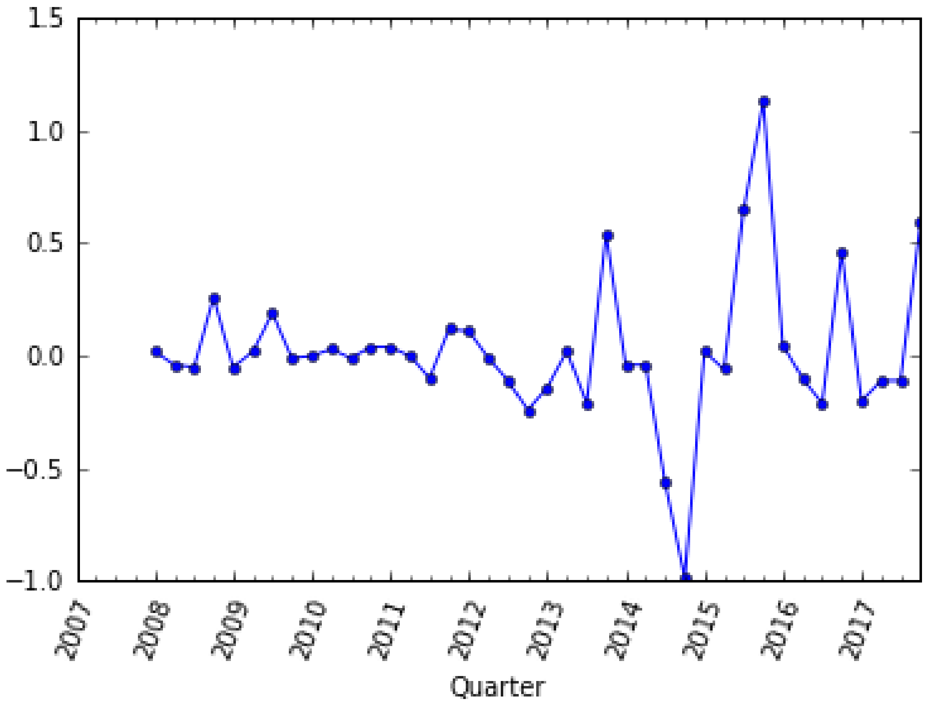
- Seasonality

```
plot.plot(HRB)
```



- Seasonal difference

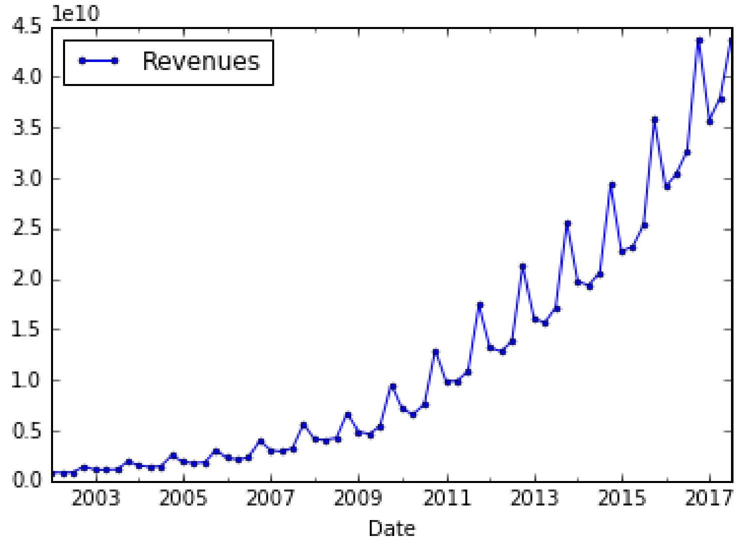
```
plot.plot(HRB.diff(4))
```



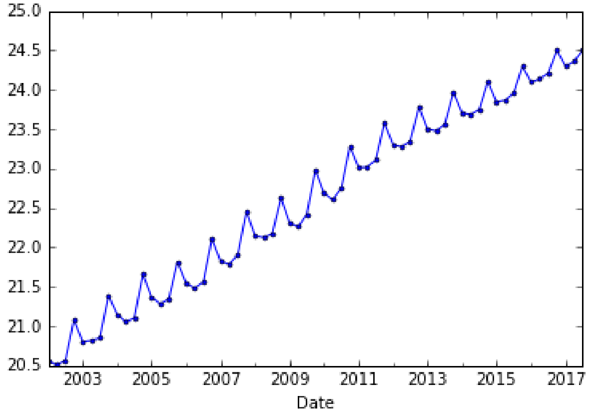
Transforming Nonstationary Series Into Stationary Series

- AMZN Quarterly Revenues

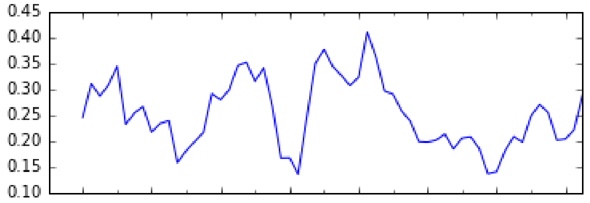
```
plt.plot(AMZN)
```




```
# Log of AMZN Revenues
plt.plot(np.log(AMZN))
```




```
# Log, then seasonal difference
plt.plot(np.log(AMZN).diff(4))
```



Let's practice!

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