Non-radial oscillation modes in hybrid stars
consequences of a mixed phase

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Main Objectives

1. Unvail the Neutron Star (NS) by the non-radial oscillations
2. To study the Neutron Star Matter (NSM) in Relativistic Mean Field (RMF) and Nambu–Jona-Lasinio (NJL) model
3. Getting the equation of state of NSM
4. Effects on the non-radial oscillations by the presence of quark matter

Introduction

NSs are the exciting cosmic laboratories to study the behavior of matter at extreme densities. The properties of NSs like its mass, radius, moment of inertia, tidal deformability and different modes of oscillations etc not only open up many possibilities related to composition, structure and dynamics of cold matter in the observable universe but also throw light on the interaction of matter at the fundamental level. Such compact stars, observed as pulsars, are believed to contain matter of densities few times ρ_0 in its core. To explain and understand the properties of such stars, one needs to connect different branches of physics like low energy nuclear physics, Qunatum chromodynamics (QCD) under extreme conditions, General theory of Relativity (GR) etc.

Mathematical section

Hadronic Phase (HP)

The Lagrangian including baryons (n and p) as the constituents of the nuclear matter and mesons (σ, ω and ρ) as the carriers of the interactions is

$$\mathcal{L} = \sum_b \mathcal{L}_b + \mathcal{L}_l + \mathcal{L}_{\text{int}},$$

$$\begin{aligned}
\mathcal{L}_b &= \bar{\Psi}_b(i\gamma_\mu\partial^\mu - q_b\gamma_\mu A^\mu - m_b + g_\sigma\sigma \\
&\qquad - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu \vec{I}_b\vec{\rho}^\mu)\Psi_b, \\
\mathcal{L}_l &= \bar{\psi}_l(i\gamma_\mu\partial^\mu - q_l\gamma_\mu A^\mu - m_l)\psi_l, \\
\mathcal{L}_{\text{int}} &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{\kappa}{3!}(g_{\sigma N}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma N}\sigma)^4 \\
&\quad - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu, \\
&\quad - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},
\end{aligned}$$

Where $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{R}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are the mesonic and electromagnetic field strength tensors. $\vec{I}_b$ denotes the isospin operator.

Quark Phase (QP)

We use NJL model to describe the quark matter. The Lagrangian of the model with four point interactions is

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}_q(i\gamma^\mu\partial_\mu - m_q)\psi_q \\
&\quad + G_s\left[(\psi_q\bar{\psi}_q)^2 + (\psi_q i\gamma^5\tau\bar{\psi}_q)^2\right] \\
&\quad + G_v\left[(\psi_q\gamma^\mu\bar{\psi}_q)^2 + (\psi_q i\gamma^\mu\gamma^5\tau\bar{\psi}_q)^2\right].
\end{aligned}$$

Here ψ_q is the doublet of u and d quarks.

Mixed Phase (MP)

One can achieve the charge neutrality with a positively charged hadronic matter mixed with a negatively charged quark matter in necessary amounts leading to a global charge neutrality condition.

$$p_{\text{HP}}(\mu_B, \mu_E) = p_{\text{QP}}(\mu_B, \mu_E) = p_{\text{MP}}(\mu_B, \mu_E),$$

The purple (green) surface denotes the pressure in the QP (HP) estimated in NJL (RMF) model.

The two surfaces intersect along the curve AB satisfies the global charge neutrality condition,

$$\chi\,\rho_c^{\text{QP}} + (1-\chi)\,\rho_c^{\text{HP}} = 0,$$

where, ρ_c^{HP} (ρ_c^{QP}) denotes the total charge densities in HP (QP) and χ defines the quark matter fraction in MP.

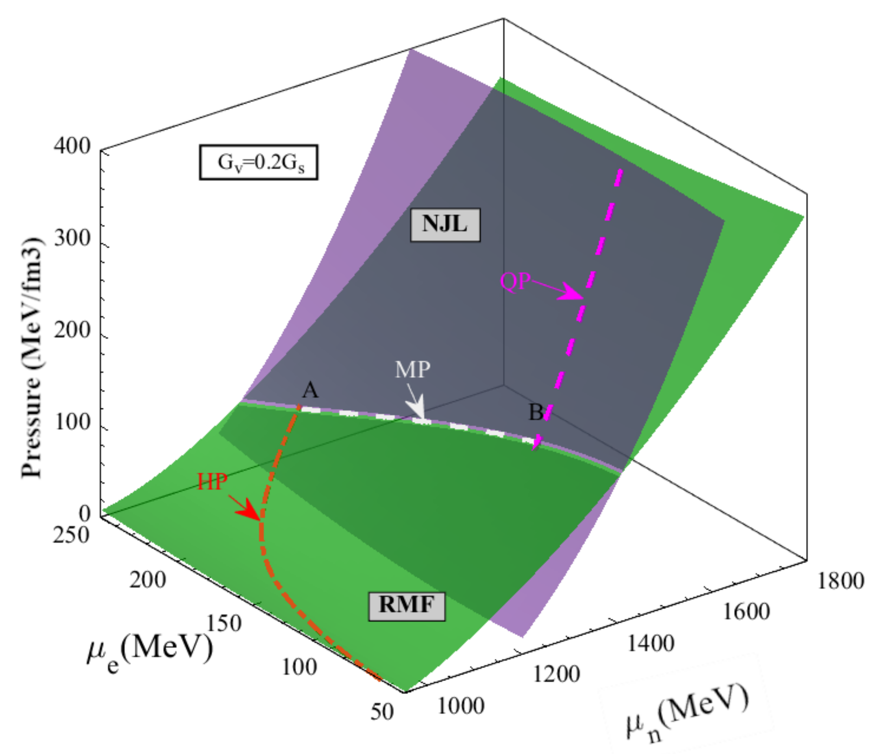

Figure 1: Mixed phase structure in Gibbs construct.

Pulsating equations

The equations governing the oscillation modes of the fluid comprising NSM.

$$Q' = \frac{1}{c_s^2}\left[\omega^2 r^2 e^{\lambda-2\nu} Z + \nu' Q\right] - l(l+1)e^\lambda Z$$

$$Z' = 2\nu' Z - e^\lambda \frac{Q}{r^2} + \frac{\omega_{BV}^2 e^{-2\nu}}{\nu'\left(1 - \frac{2m}{r}\right)}\left(Z + \nu' e^{-\lambda+2\nu}\frac{Q}{\omega^2 r^2}\right)$$

The vanishing of pressure at the surface of the star ($r = R$) leads to the boundary condition

$$\omega^2 r^2 e^{\lambda-2\nu} Z + \nu' Q\Big|_{r=R} = 0.$$

Results

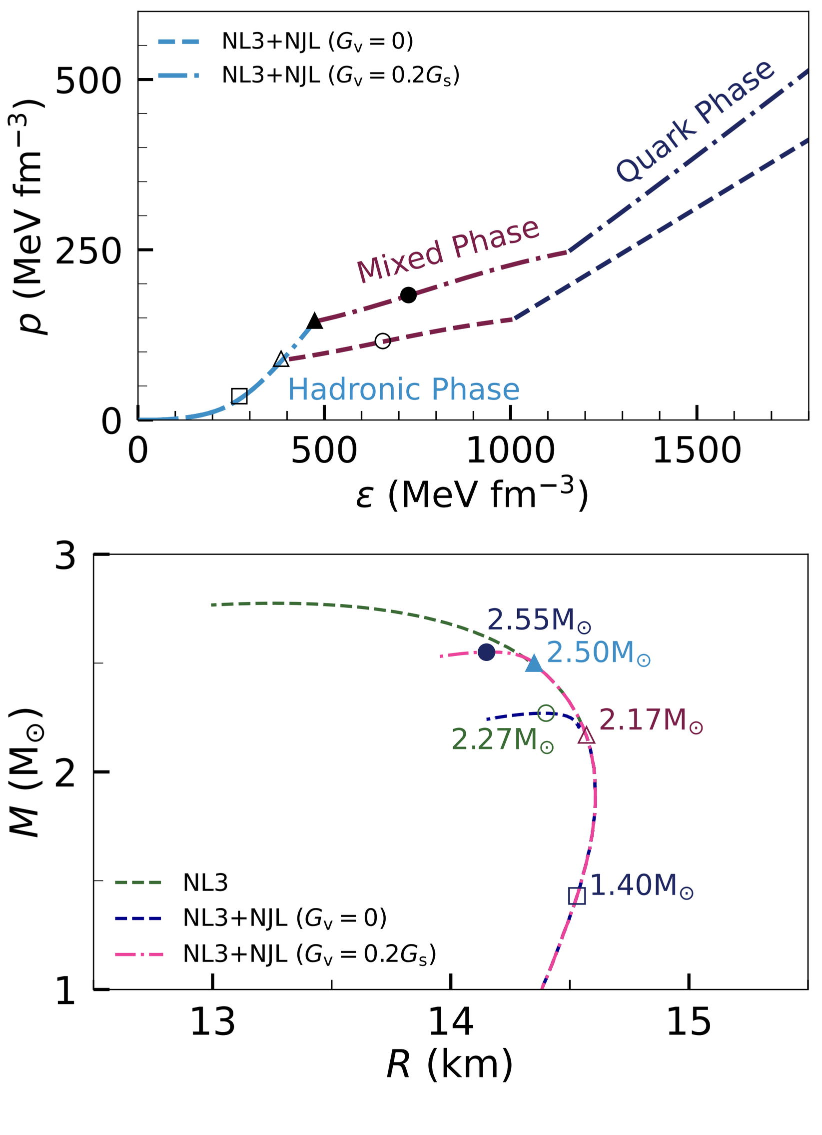

Figure 2: Equations of states and mass-radius curve

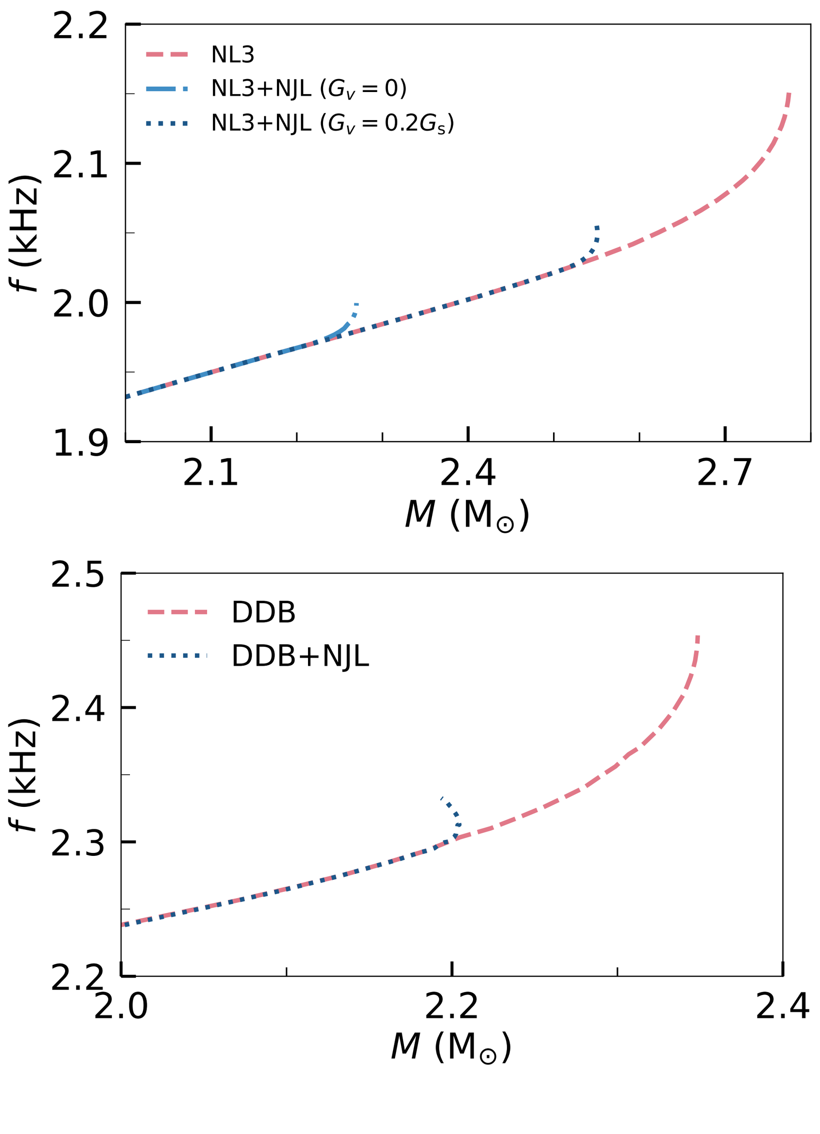

Figure 3: The oscillation frequencies of f mode

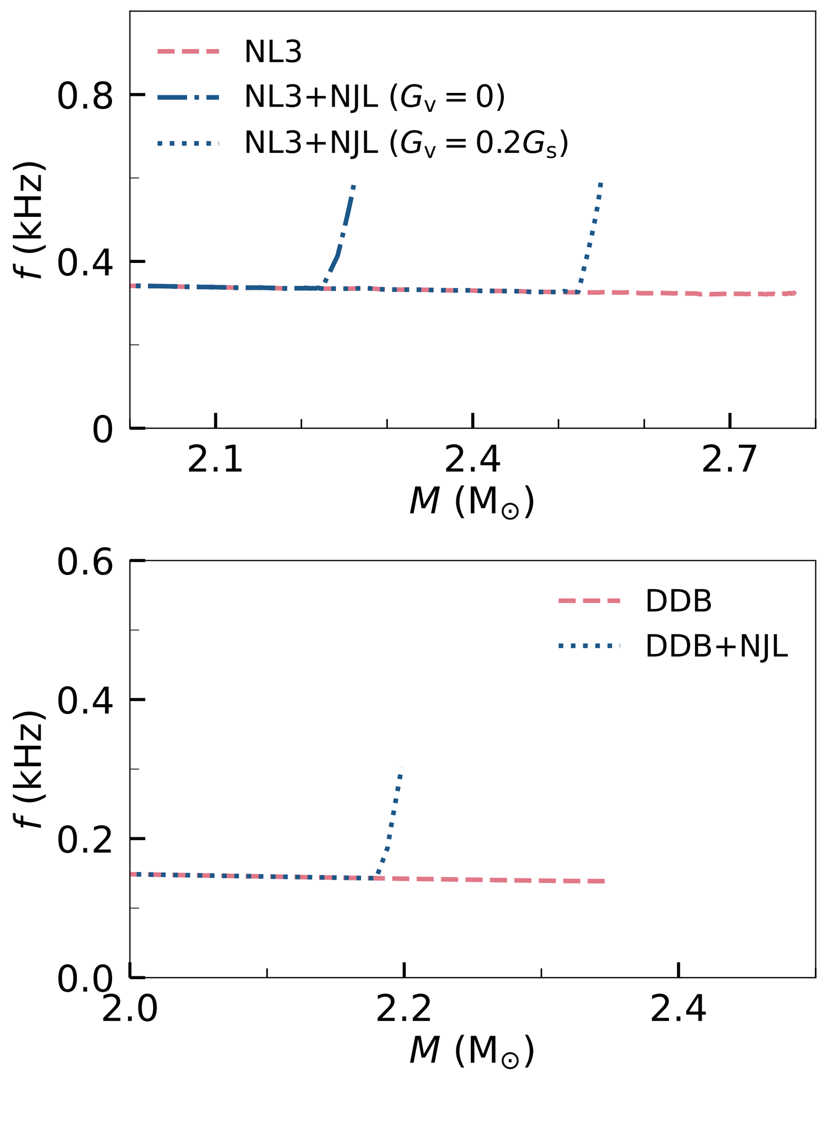

Figure 4: The oscillation frequencies of g mode

Conclusions

We find the quark matter in the core of maximum mass stars in the MP which enhances the non-radial oscillation mode frequencies significantly.

Forthcoming research

I am working on the quark-hadron phase transition in the context of NSs. I am trying to fit the NJL model parameters from the astrophysical observations using Bayesian approach.