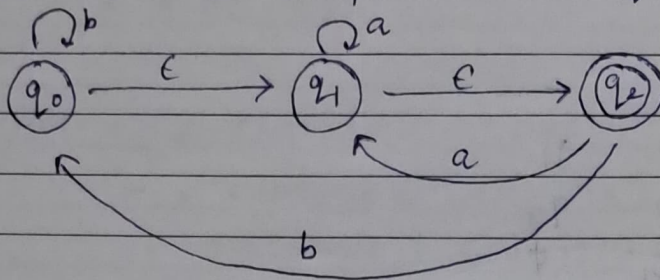


Q1. Convert NFA with Epsilon to equivalent DFA



x $y - \epsilon \text{ closure}(x)$ $\delta(y, a)$ $\delta(y, b)$

| | | | | |
|---|-----------|---------------------|-----------|-----------|
| A | $\{q_0\}$ | $\{q_0, q_1, q_2\}$ | $\{q_1\}$ | $\{q_0\}$ |
| B | $\{q_1\}$ | $\{q_1, q_2\}$ | $\{q_1\}$ | $\{q_0\}$ |

$Q \setminus \epsilon$ a b

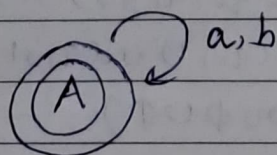
| | | |
|-------|---|---|
| A^* | B | A |
| B^* | B | A |

As the transition of A & B is same and as both of them are final states \therefore A & B can be merged.

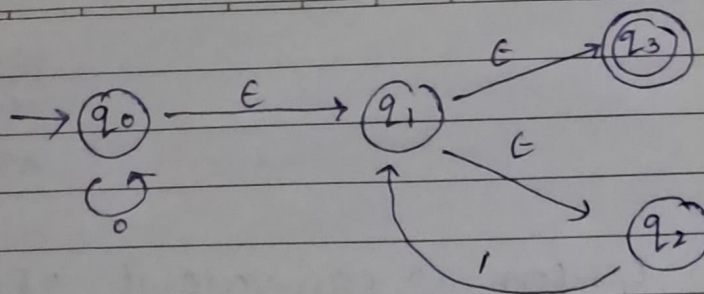
$Q \setminus \epsilon$ a b

| | | |
|-------|---|---|
| A^* | A | A |
|-------|---|---|

\therefore DFA transition diagram



Q2)



| Q \ ε | 0 | 1 | ε |
|----------------|----------------|----------------|----------------|
| q ₀ | q ₀ | ∅ | q ₁ |
| q ₁ | q ₂ | ∅ | q ₃ |
| q ₂ | ∅ | q ₁ | ∅ |
| q ₃ | q ₃ | ∅ | ∅ |

Hence,

$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_3\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_3\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

$$\epsilon\text{-closure}(q_3) = \{q_3\}$$

$$\begin{aligned}
 \delta(q_0, 0) &= \epsilon\text{-closure}(\delta(\delta^*(q_0, \epsilon), 0)) \\
 &= \epsilon\text{-closure}(\delta(q_0, q_1, q_3), 0) \\
 &= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_3, 0)) \\
 &= \epsilon\text{-closure}(q_0 \cup q_2 \cup q_3) \\
 &= \{q_0, q_1, q_2, q_3\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_0, 1) &= \epsilon\text{-closure}(\delta(\delta^*(q_0, \epsilon), 1)) \\
 &= \epsilon\text{-closure}(\delta(q_0, q_1, q_3), 1) \\
 &= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_3, 1)) \\
 &= \epsilon\text{-closure}(\emptyset \cup \emptyset \cup \emptyset) \\
 &= \{\emptyset\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_1, 0) &= \epsilon\text{-closure}(\delta^*(\delta(q_1, \epsilon), 0)) \\
 &= \epsilon\text{-closure}(\delta(q_1, q_3), 0) \\
 &= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_3, 0)) \\
 &= \epsilon\text{-closure}(q_2 \cup q_3) \\
 &= \{q_2, q_3\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_1, 1) &= \epsilon\text{-closure}(\delta^*(\delta(q_1, \epsilon), \phi)) \\
 &= \epsilon\text{-closure}(\delta(q_1, q_3), 1) \\
 &= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_3, 1)) \\
 &= \epsilon\text{-closure}(\phi \cup \phi) \\
 &= \{\phi\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_2, 0) &= \epsilon\text{-closure}(\delta^*(\delta(q_2, \epsilon); \phi)) \\
 &= \epsilon\text{-closure}(\delta(q_2), 0) \\
 &= \epsilon\text{-closure}\{\phi\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_2, 1) &= \epsilon\text{-closure}(\delta^*(\delta(q_2, \epsilon), 1)) \\
 &= \epsilon\text{-closure}(\delta(q_2), 1) \\
 &= \epsilon\text{-closure}(q_1) \\
 &= \{q_1, q_3\}
 \end{aligned}$$

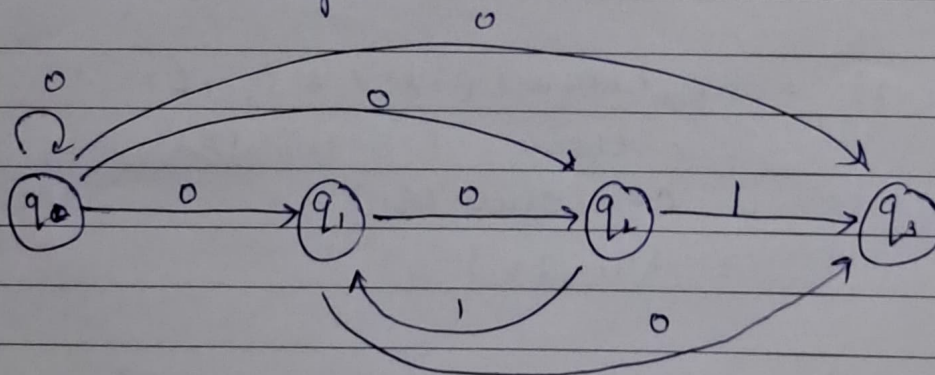
$$\begin{aligned}
 \delta(q_3, 0) &= \epsilon\text{-closure}(\delta^*(\delta(q_3, \epsilon), 0)) \\
 &= \epsilon\text{-closure}(\delta(q_3), 0) \\
 &= \epsilon\text{-closure}(\delta(q_3, 0)) \\
 &= \epsilon\text{-closure}(q_3) \\
 &= \{q_3\}
 \end{aligned}$$

$$\begin{aligned}
 \delta(q_3, 1) &= \epsilon\text{-closure}(\delta(q_3, \epsilon), 1) \\
 &= \epsilon\text{-closure}(q_3, 1) \\
 &= \epsilon\text{-closure}(\phi) \\
 &= \{\phi\}
 \end{aligned}$$

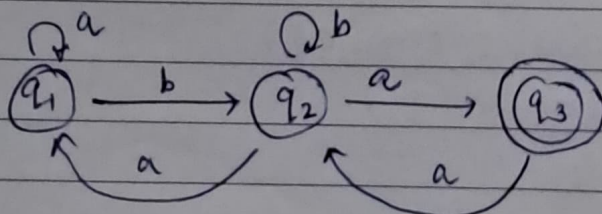
state transition diagram

| Q \ E | 0 | 1 |
|-------|--------------------------|----------------|
| q_0 | $\{q_0, q_1, q_2, q_3\}$ | ϕ |
| q_1 | q_2, q_3 | ϕ |
| q_2 | ϕ | $\{q_1, q_3\}$ |
| q_3 | q_3 | ϕ |

transition diagram



Q3)



state transition equation is given as

$$q_1 = \epsilon + q_1 \cdot a + q_2 \cdot a \quad \dots \textcircled{1}$$

$$q_2 = q_1 \cdot b + q_2 \cdot b + q_3 \cdot a \quad \dots \textcircled{2}$$

$$q_3 = q_2 \cdot a \quad \dots \textcircled{3}$$

substituting $\textcircled{3}$ in $\textcircled{2}$

$$q_2 = q_1 \cdot b + q_2 \cdot b + q_2 \cdot a \cdot a$$

$$R = Q + RP \quad \{ \text{Arden's theorem} \}$$

$$\therefore q_2 = q_1 \cdot b (b + aa)^* \quad \dots \textcircled{4}$$

sub. eq $\textcircled{4}$ in $\textcircled{1}$

$$q_1 = \epsilon + q_1 \cdot a + q_1 \cdot b (b + aa)^* \cdot a$$

$$q_1 = \epsilon + (a + b(b + aa)^* \cdot a) q_1$$

$$R = Q + RP$$

$$\therefore q_1 = (a + b(b + aa)^* \cdot a)^* \textcircled{5} \quad \{ \text{Arden's theorem} \}$$

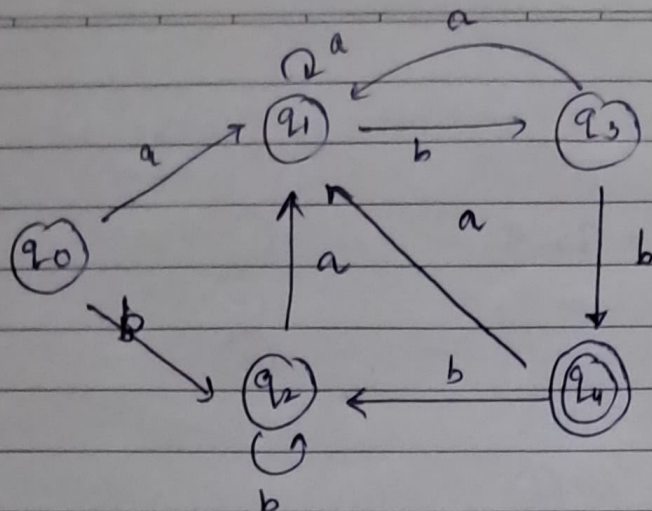
sub $\textcircled{5}$ in $\textcircled{4}$

$$q_2 = (a + b(b + aa)^* \cdot a)^* \cdot b (b + aa)^* \quad \dots \textcircled{6}$$

sub $\textcircled{6}$ in $\textcircled{3}$

$$q_3 = (a + b(b + aa)^* \cdot a)^* \cdot b (b + aa)^* \cdot a$$

Q4)



| $q \backslash e$ | a | b |
|------------------|-------|-------|
| q_0 | q_1 | q_2 |
| q_1 | q_1 | q_3 |
| q_2 | q_1 | q_2 |
| q_3 | q_1 | q_4 |
| q_4^* | q_1 | q_2 |

| | | | | |
|-------|-------|-------|-------|-------|
| q_1 | 3 | | | |
| q_2 | X | 3 | | |
| q_3 | 2 | 2 | 2 | |
| q_4 | 1 | 1 | 1 | 1 |
| | q_0 | q_1 | q_2 | q_3 |

for (q_0, q_1)

check

$$\left. \begin{array}{l} \delta(q_0, a) = q_1 \\ \delta(q_1, a) = q_1 \end{array} \right\} \times \quad \begin{array}{l} \delta(q_1, b) = q_3 \\ \delta(q_1, a) = q_2 \end{array}$$

$\therefore (q_2, q_3)$ is not marked, $(q_2, q_3) \Rightarrow (q_0, q_1)$

for (q_0, q_2)

$$\left. \begin{array}{l} \delta(q_0, a) = q_1 \\ \delta(q_2, a) = q_1 \end{array} \right\} \times \quad \begin{array}{l} \delta(q_0, b) = q_2 \\ \delta(q_2, b) = q_1 \end{array} \right\} \times$$

\therefore cannot be marked.

→ for (a_1, a_2)

$$\left. \begin{array}{l} s(a_1, a) = a_1 \\ s(a_2, a) = a_1 \end{array} \right\} \times \quad \left. \begin{array}{l} s(a_1, b) = a_3 \\ s(a_2, b) = a_2 \end{array} \right\}$$

$\therefore (a_1, a_3)$ isn't marked $(a_2, a_3) \Rightarrow (a_1, a_2)$

→ for (a_0, a_3)

$$\left. \begin{array}{l} s(a_0, a) = a_1 \\ s(a_3, a) = a_1 \end{array} \right\} \times \quad \left. \begin{array}{l} s(a_0, b) = a_2 \\ s(a_3, b) = a_4 \end{array} \right\}$$

Now (a_2, a_4) is marked as 1

$\therefore (a_0, a_3) \Rightarrow 2$

→ for (a_1, a_3)

$$\left. \begin{array}{l} s(a_1, a) = a_1 \\ s(a_3, a) = a_1 \end{array} \right\} \times \quad \left. \begin{array}{l} s(a_1, b) = a_2 \\ s(a_3, b) = a_4 \end{array} \right\}$$

$\therefore (a_2, a_4)$ is marked.

\therefore marking (a_3, a_4) as 2

Now again checking for

→ (a_0, a_1)

$$\left. \begin{array}{l} s(a_0, a) = a_1 \\ s(a_1, a) = a_1 \end{array} \right\} \times \quad \left. \begin{array}{l} s(a_0, b) = a_2 \\ s(a_1, b) = a_3 \end{array} \right\}$$

\therefore marked. we mark (a_0, a_1) as 3

→ for (a_0, a_2)

$s(a_0, a_2)$

$$\left. \begin{array}{l} s(a_0, a) = a_1 \\ s(a_2, a) = a_1 \end{array} \right\} \times \quad \left. \begin{array}{l} s(a_0, b) = a_2 \\ s(a_2, b) = a_1 \end{array} \right\} \times$$

cannot be marked -

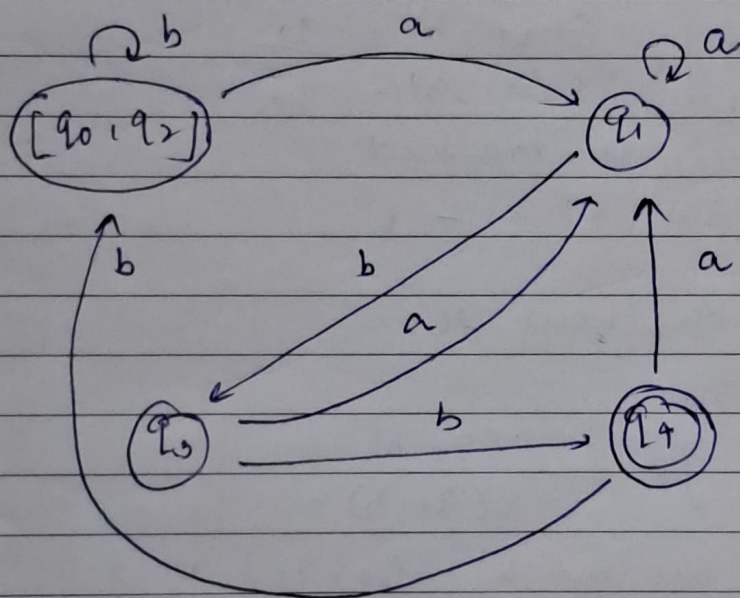
for (a_1, q_2)

$$\left. \begin{array}{l} \delta(q_1, a) = q_1 \\ \delta(q_2, a) = q_1 \end{array} \right\} \times \quad \left. \begin{array}{l} \delta(a_1, b) = q_3 \\ \delta(a_2, b) = q_2 \end{array} \right\}$$

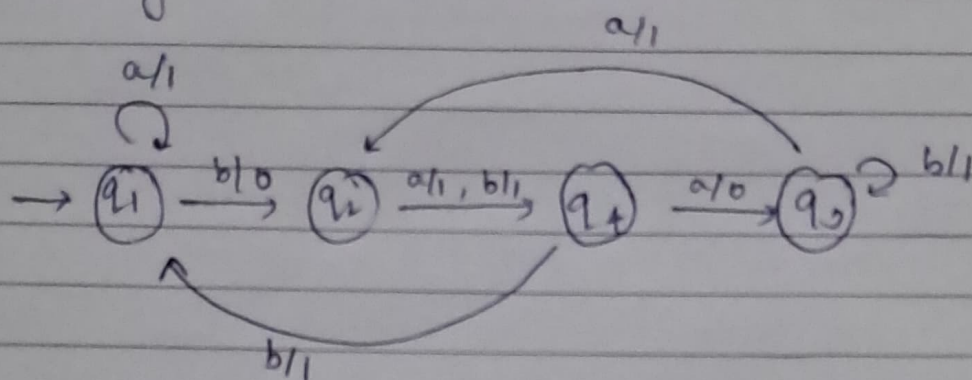
$\therefore (q_2, q_3)$ is marked
marking (a_1, q_2) as 3

merging (q_0, q_2)

\therefore minimized DFA



Q5) mealy to moore machine.



moore machine.

