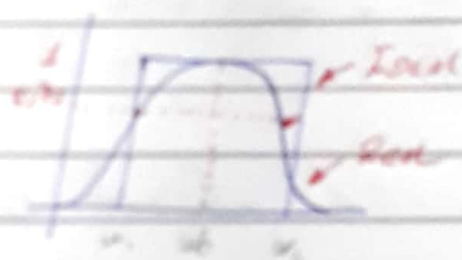
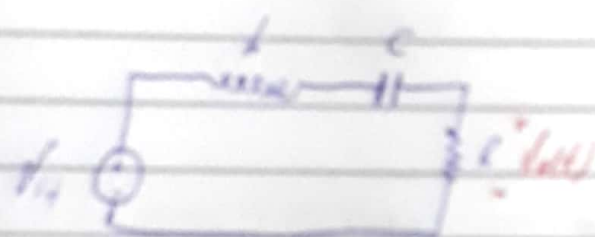


09.10.21

Lista de Exercícios: 01

Modelagem de Sistema Dinâmicos

1) Um circuito RLC em série representado abaixo
um filtro passa-banda



$$R = 50$$

$$L = 100 \times 10^{-3} \text{ H}$$

$$C = 10^{-5}$$

$$\left\{ \begin{array}{l} f_0 = \frac{1}{2\pi R} \\ f_0 = \frac{1}{2\pi L} \end{array} \right\}$$

a)

$$v_s = v_R + v_L + v_C = v_o$$

$$v_C = C \frac{dv_o}{dt}$$

$$I R + L \frac{di}{dt} + \frac{1}{C} = v_s$$

$$[H(s)] \cdot F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$\frac{di}{dt} + \frac{1}{L} i + \frac{R}{L} = \frac{v_s}{L}$$

$$v_{out} = v_o = i \cdot X_C \quad \text{em } v_o = \frac{1}{sC}$$

$$(sL + R) \cdot i = \frac{1}{sC}$$

$$F(s) = \frac{Y(s)}{X(s)}$$

RLC

$$-V + R \cdot I + L \cdot \frac{dI}{dt} + \frac{1}{C} \int I dt = 0$$

$$I = C \cdot \frac{dV}{dt}$$

$$I(R + Ls) = V_0 - \frac{1}{Cs}$$

$$I = \frac{V_0 - \frac{1}{Cs}}{R + Ls}$$

$$\int \frac{di}{dt} = I$$

2) Por E.G. Espaço Estado

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot V(t)$$

3) Por Função de Transferência

$$V_{out} = V_0 \cdot \frac{\kappa}{(\kappa + R) + \kappa} = V_0 \cdot \frac{\frac{1}{sC}}{(sL + R) + \frac{1}{sC}}$$

$$V_{out} = V_0 \cdot \frac{\frac{1}{sC}}{1 + sRC + s^2LC} \quad R + \frac{1}{sC} = \frac{1 + RsC}{sC}$$

$$\frac{V_{out}}{V_0} = \frac{1}{s^2LC + sRC + 1}$$

ES0

$$V_{i(t)} = V_{i(t)} + V_{c(t)} + V_{L(t)} = 0$$

$$V_{i(t)} + L \frac{di}{dt} + V_c = 0$$

$$\frac{V_{i(t)}}{L} + \frac{V_c}{L} + \frac{R}{L} i = \frac{di}{dt}$$

Equation / Extract $\rightarrow x = Ax + Bu$

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{i(t)}$$

$$\frac{di}{dt} = -\frac{Ri}{L} - \frac{V_c}{L} + \frac{V_{i(t)}}{L}, \text{ Eq. 1}$$

$$\frac{dV_c}{dt} = \frac{V_c}{C}, \text{ Eq. 2}$$

San Carlos de Bariloche

di: $\frac{1}{2}$

$$-d\psi = E(\theta) + \frac{1}{2} d\theta^2 + h \cdot c \quad G_1$$

$$-V = K_2 + 147 - 16 = 0$$

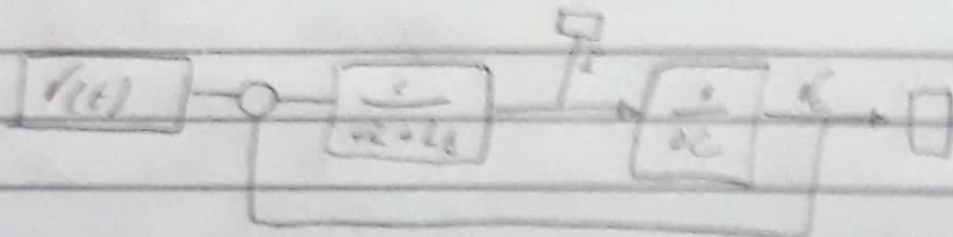
Isolar 5

$$I(R + L_1) = (I - L_2)$$

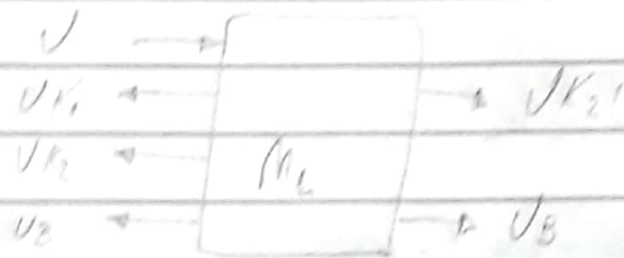
$$T = (V - V_c)$$

$$(2 - 2)$$

| | | |
|---|---|----|
| ✓ | 2 | 5 |
| | | 40 |



3) Obtenha as funções de transferência $\frac{x_1}{U_s}$ e $\frac{x_2}{U_s}$ do sistema mecânico



$$U = U_{m_1} + U_{K_1} + U_{K_2} + U_B - U_{K_2}' - U_R$$

$$U = m_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K_2 x_1 + B \frac{dx_1}{dt} - K_2 x_2 - B \frac{dx_2}{dt}$$

$$U = m_1 \frac{d^2 x_1}{dt^2} + B \left(\frac{dx_1}{dt} - \frac{dx_2}{dt} \right) + K_2 (x_1 - x_2) + K_1 x_1$$

→ Aplicando LAPLACE

$$U(s) = m_1 s^2 x_1(s) + B(s x_1(s) - s x_2(s) + K_2 (x_1(s) - x_2(s))) + K_1 x_1(s)$$

$$U(s) = x_1(s) \cdot (m_1 s^2 + B s + K_2 + K_1) + x_2(s) \cdot (-B s - K_2)$$

Cont. exercícios

Calculando em evidência o $\det A$

$$\det A (M_1 M_2) s^4 + (M_1 B + M_2 B) s^3 + (M_1 K_2 + M_1 K_3 + M_2 K_2 + M_2 K_3 - B^2) s^2$$

$$+ (3K_1 - B K_3 + B K_1 + 3K_1 - B K_1 - B K_1) s + (K_1^2 + K_2 K_3 + K_1 K_1 + K_1 K_3 - K_1^2)$$

$$\det L = (M_1 M_2) s^4 + (M_1 B + M_2 B) s^3 + (M_1 K_2 + M_1 K_3 + M_2 K_2 + M_2 K_3) s^2$$

$$+ (3K_1 + B K_3) s + (K_1 K_2 + K_1 K_3 + K_1 K_3)$$

F. Transf.

$$\begin{bmatrix} 1 & -B s & -K_2 \\ 0 & M_2 s^2 + B s + K_2 + K_3 \end{bmatrix} = M_2 s^2 + B s + K_2 + K_3 - 0$$

$$G_1(s) = \frac{M_2 s^2 + B s + K_2 + K_3}{\det A} \Rightarrow \frac{1,5 s^2 + 10 s + 100}{\det A}$$

$$\begin{bmatrix} M_2 s^2 + B s + K_2 + K_3 & 1 \\ -K_2 - B s & 0 \end{bmatrix} = 0 - (-K_2 - B s) = K_2 + B s$$

$$G_2(s) = \frac{K_2 + B s}{\det A} \Rightarrow \frac{10 s + 50}{\det A}$$

$$\boxed{\det A = 1,5 s^4 + 2,5 s^3 + 2,5 s^2 + 100 s + 7500}$$