# Modelagem de sistemas dinâmicos

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**Exercícios**

a) Modele o filtro pelas equações diferenciais ordinárias (tensão no capacitor e corrente no indutor) e implemente em blocos no ambiente Simulink. Simule a resposta da tensão no resistor ($v(0)$). Insira um valor constante de tensão ($v_i$) e discuta os resultados.

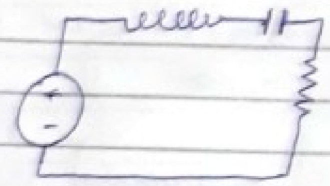
b) Realize o mesmo procedimento solicitado em (a), porém implemente no bloco espaço de estados no ambiente Simulink.

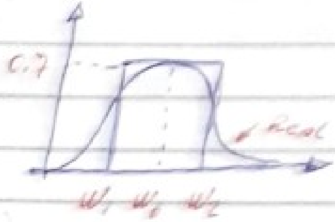
c) Realize o mesmo procedimento solicitado em (a), porém implemente em função de transferência no ambiente Simulink.

d) Realize o cálculo da frequência ressonante do filtro através da análise dos autovalores da matriz A ($\lambda I - A$). Comprove a frequência ressonante através de ensaios no ambiente Simulink.

Lista Prof. Renato - Filtro Passa Faixa

1)






a) $V_t = V_L + V_C + V_0 \quad (C \equiv 0)$

$V_L = V_i - V_C - I_0$

$L \dfrac{di}{dt} = V_i - V_C - I_0$

$\dfrac{di}{dt} = \dfrac{V_i}{L} - \dfrac{V_C}{L} - \dfrac{R_i}{L}$

$I_t = I_L = I_C = I_R$

$I_t = I_C$

$I_t = C \dfrac{dv}{dt} \Rightarrow \dfrac{dv}{dT} = \dfrac{iT}{C}$

b) E. Estado

$$\begin{bmatrix} \dfrac{di}{dt} \\ \\ \dfrac{dV}{dt} \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ \\ 1/C & 0 \end{bmatrix} \cdot \begin{bmatrix} i \\ \\ V_C \end{bmatrix} + \begin{bmatrix} 1/L \\ \\ 0 \end{bmatrix} \cdot V_i(t)$$

c) F. Transferência

$\dfrac{di}{dt} = \dfrac{V_i}{L} - \dfrac{V_C}{L} - \dfrac{R_i}{L}$ $\qquad \dfrac{dV}{dT} = \dfrac{iT}{C}$

$L \cdot \dfrac{di}{dt} = V_i - V_C - R_i$ $\qquad V_S = iT$

$LS_i + R_i = V_i - V_C$ $\qquad V = \dfrac{iT}{CS}$

$i(SLR) = V_i - V_C \Rightarrow i = \dfrac{V_i - V_C}{RSL}$

d) $\lambda I - A$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -500 & -10 \\ 1000 & 0 \end{bmatrix} = \begin{bmatrix} \lambda + 500 & +10 \\ -500 & \lambda \end{bmatrix}$$

det. A - B

$A = (\lambda + 500)\lambda$ $\qquad B = -10 \cdot 1000$

$A = \lambda^2 + 500\lambda$ $\qquad B = -10\,000$

$\rightarrow A - B$

$\lambda^2 + 500\lambda - (-10000) \Rightarrow \lambda^2 + 500\lambda + 10\,000 = 0$ $\lambda_1 = -20.8$ $\lambda_2 = -479.2$

$\lambda_1 = -20.8$ $\qquad \lambda_2 = -479.2$

$\lambda_1 = \xi w_n \pm j w_n \sqrt{1-\xi^2}$ $\qquad \lambda_2 = \xi w_n \pm j w_n \sqrt{1-\xi^2}$

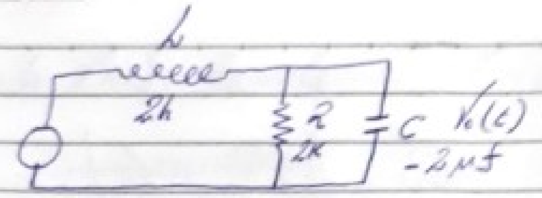
$\xi = \frac{R}{\sqrt{R^2 + I^2}} = \frac{-20.8}{\sqrt{20.8^2 + 0^2}} = -1$ $\qquad \xi = \frac{R}{\sqrt{R^2 + I^2}} = \frac{-479.2}{\sqrt{479.2^2}} = -1$

$\xi w_n = -20.8$ $\qquad \xi w_n = -479.2$

$-1 w_n = -20.8$ $\qquad -1 w_n = -479.2$

$w_n = 0.042$ Rad/s $\qquad w_n = 479.2$ Rad/s

2)




a) Paralelo $= V_C$

$V_{in} = -V_L - V_R = V_C$

$V_{in} = -V_L - V_C$

$+V_C + V_{in} = -V_L$

$V_L = -V_C - V_{in}$

$L\frac{di}{dt} = -V_C - V_{in}$

$\frac{di}{dt} = \frac{-V_C - V_{in}}{L}$

$I_t = I_L = I_R + I_C$

$I_L = I_R + I_C$

$I_L = \frac{V_C}{R} + C\frac{dV}{dt}$

$C\frac{dV}{dt} = I_L - \frac{V_C}{R}$

$\frac{dV}{dt} = \frac{-I_L}{C} - \frac{V_C}{RC}$

b) Espaço Estado

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dV}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \cdot \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \cdot V_{in}$$

c) F transf

$\frac{di}{dt} = \frac{-V_C - V_{in}}{L}$

$LSI = -V_C - V_{iN}$

$I_L = \frac{-V_C - V_{in}}{S}$

$\frac{dV}{dt} = \frac{I_L}{C} - \frac{V_C}{RC}$

$SVC = I_L - \frac{V_C}{R}$

$V_C = \frac{I_L}{S} - \frac{V_C}{RS}$

Sistema Fiep | FIEP SESI SENAI IEL | SENAI

d) $\lambda I - A$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & -1/c \\ 1/c & -1/rc \end{bmatrix} = \begin{bmatrix} \text{[illegible]} & -1/c \\ \text{[illegible]} & \text{[illegible]} \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \\ -500000 & -250 \end{bmatrix} = \begin{bmatrix} \lambda & -0.5 \\ -500000 & \lambda + 250 \end{bmatrix}$$

$A = (\lambda + 250) \cdot \lambda = \lambda^2 + 250\lambda$

$B = 0.5 (-500000) = -250000$

$A - B$

$(\lambda^2 - 250\lambda) - (-250000)$

$\lambda_{1,2} = 125 \pm$ [illegible]

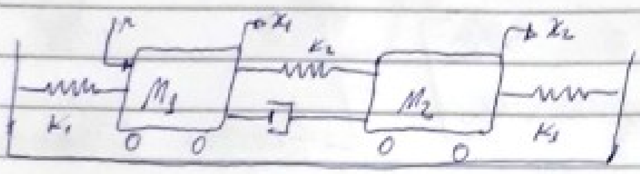
$\xi = \dfrac{\text{[illegible]}}{\sqrt{\text{[illegible]}}} = \dfrac{\text{[illegible]}}{\sqrt{\text{[illegible]}}} = \text{[illegible]}$

$\xi\, \omega_n = 125$

$0.25\, \omega_n = 125$

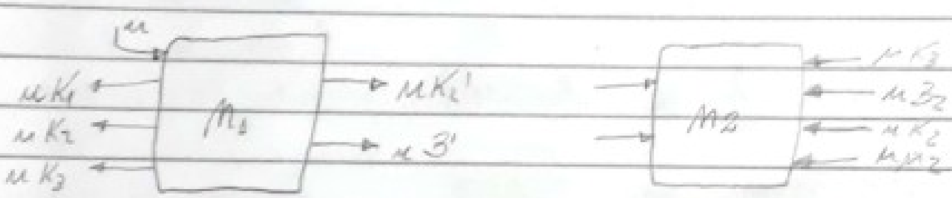
$\omega_n = \dfrac{125}{0.25} = 500$ [illegible]

3) Massa mola




$m_1 = 1\,Kg$ $\quad b = 10\,Ns/m$ $\quad K_2 = 50\,N/m$

$m_2 = 1{,}5\,Kg$ $\quad K_1 = 50\,N/m$ $\quad K_3 = 50\,N/m$




$U = \mu M_1 + \mu K_1 + \mu K_2 + \mu b - \mu K_2' - \mu b'$

$U = M_1\ddot{x}_1 + K_1 x_1 + K_2 x_1 + b\dot{x}_1 - K_2 x_2 - b\dot{x}_2$

$U = M_1\ddot{x}_1 + K_1 x_1 + K_2(x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2)$

$0 = \mu M_2 + \mu b + \mu K_1 + \mu K_2 - \mu K_2 - \mu b$

$0 = M_2\ddot{x}_2 + b\dot{x}_2 + K_1 x_2 + K_2 x_2 - K_2 x_1 - b x_1$

$0 = M_2\ddot{x}_2 + K_3 x_2 + b(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1)$

Laplace

$U = M_1\ddot{x}_1 + K_1 x_1 + K_2(x_2 - x_1) + b(\dot{x}_1 - \dot{x}_2)$

$U_S = M_1 S^2 + 1.S + K_1 X_1.S + K_2 X_2 S - K_2 X_2 S + b.S\,X_1 S - b S X_2 S$

$U_S = [M_1 S^2 + b.S + K_1 + K_2] X_1.S + [-b.S - K_2] X_2.S$

$$0 = M_2 \ddot{x}_2 + K_3 x_2 + b(\dot{x}_2 - \dot{x}_1) + K_2 (x_2 - x_1)$$

$$0 = M_2 S^2 X_2 S + K_3 X_2 S + bS X_2 S - bS X_1 S + K_2 X_2 S - K_2 X_1 S$$

$$0 = [-bS - K_2] X_1 S + [M_2 S^2 + bS + K_3 + K_2] X_2 S$$

$$\begin{bmatrix} M_1 S^2 + bS + K_1 + K_2 & -bS - K_2 \\ -bS - K_2 & M_2 S^2 + bS + K_3 + K_2 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$\det\Delta = (M_1 S^2 + bS + K_1 + K_2)(M_2 S^2 + bS + K_3 + K_2) - (bS + K_2)^2$$

$$M_1 M_2 S^4 + M_1 b S^3 + M_1 K_3 S^2 + M_1 K_2 S^2 + M_2 b S^3 + b^2 S^2 + bK_3 S + bK_2 S + \ldots$$

$$\ldots bK_2 S + M_2 K_1 S^2 + bK_1 S + K_1 K_3 + K_1 K_2 + M_2 K_2 S^2 + bK_2 S + \ldots$$

$$\ldots K_2 K_3 + K_2^2 - [b^2 S^2 + 2bS K_2 + K_2^2]$$

$$(M_1 M_2) S^4 + (M_1 b + M_2 b) S^3 + (M_1 K_3 + M_1 K_2 + b^2 + M_2 K_1 + M_2 K_2 - b^2) S^2 + \ldots$$

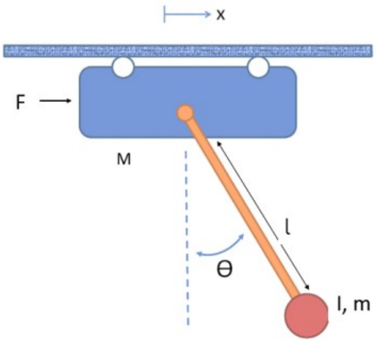
$$\ldots (bK_3 + bK_2 + bK_1 + bK_2 - 2bK_2) S + (K_1 K_3 + K_1 K_2 + K_2 K_3 + K_2^2 - K_2^2)$$

$$1{,}5 S^4 + 25 S^3 + 250 S^2 + 1000 S + 7500 \quad \det\Delta$$

$$G_1(S) = \frac{\begin{vmatrix} 1 & -bS - K_2 \\ 0 & M_2 S^2 + bS + K_3 + K_2 \end{vmatrix}}{\det} = \frac{1{,}5 S^2 + 10S + 50 + 50}{1{,}5 S^4 + 25 S^3 + 250 S^2 + 1000 S + 7500}$$

$$G_2(S) = \begin{vmatrix} M_1 S^2 + bS + K_1 + K_2 & 1 \\ -bS - K_2 & 0 \end{vmatrix} = \frac{10S + 50}{1{,}5 S^4 + 25 S^3 + 250 S^2 + 1000 S + 7500}$$

4) Para o sistema da Figura 4, a entrada de controle é a força F que move o carrinho horizontalmente e as saídas são a posição angular do pêndulo $\theta$ e a posição horizontal do carrinho $x$.




$$\ddot{\theta}(I+ml^2)+mgl\theta=-ml\ddot{x} \qquad \text{Eq pêndulo}$$

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}=F \qquad \text{Eq carrinho}$$

Figura 4: Circuito para análise.

### Índice


Model - Pendulo_001
System - Pendulo_001
Appendix


### Lista de Tabelas


1. Constant Block Properties
2. Gain Block Properties
3. Integrator Block Properties
4. Trigonometry Block Properties
5. Block Type Count
6. Model Functions


# Model - Pendulo_001

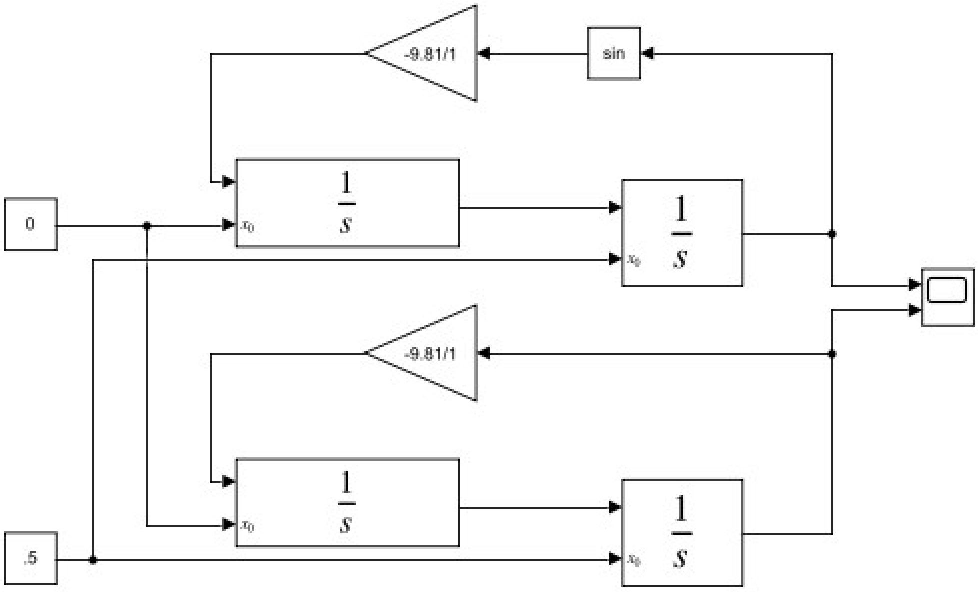
### Full Model Hierarchy

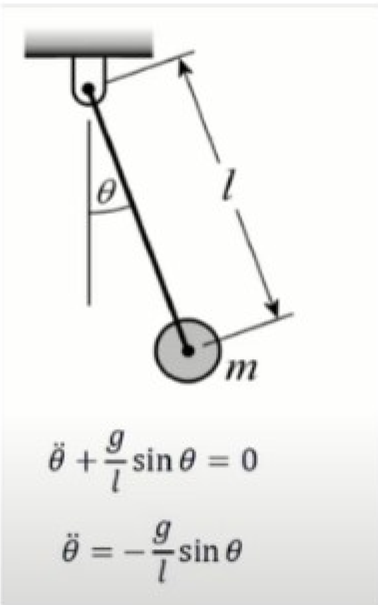
1. Pendulo_001

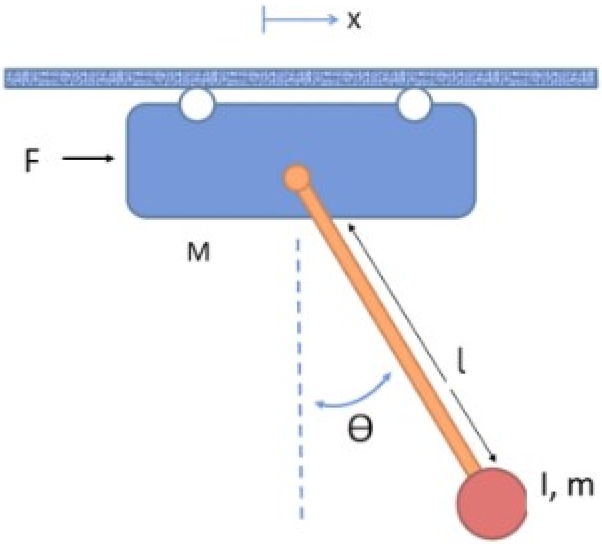
| Simulation Parameter | Value |
|---|---|
| Solver | VariableStepAuto |
| RelTol | 1e-3 |
| Refine | 1 |
| MaxOrder | 5 |
| ZeroCross | on |

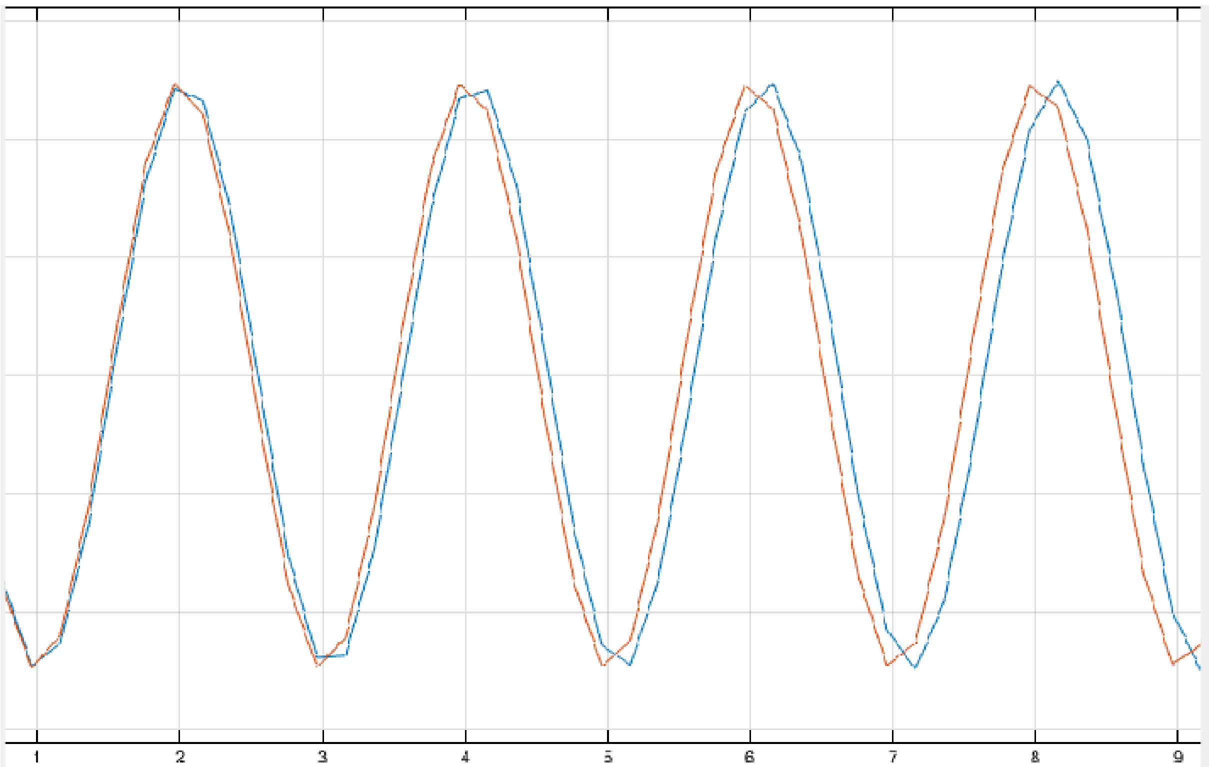
# Pendulo_001

## Details for Pendulo_001





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