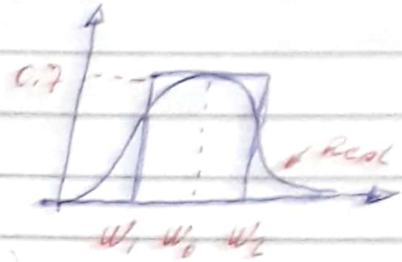
Lista Prof. Renato - Filtro Passa Faixa

1)



a) $V_i = V_L + V_C + V_0$ (EDO)

$V_L = V_i - V_C - I_0$

$L\frac{di}{dt} = V_i - V_C - I_0$

$\frac{di}{dt} = \frac{V_i}{L} - \frac{V_C}{L} - \frac{Ri}{L}$

$I_t = I_L = I_C = I_R$

$I_t = I_C$

$I_t = C\frac{dv}{dt} \Rightarrow \frac{dv}{dT} = \frac{i_T}{C}$

b) E. Estado

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dV}{dt} \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \cdot \begin{bmatrix} i \\ V_C \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} - V_i(t)$$

c) F. Transferência

$\frac{di}{dt} = \frac{V_i}{L} - \frac{V_C}{L} - \frac{Ri}{L}$ $\qquad \frac{dV}{dT} = \frac{i_T}{C}$

$L\frac{di}{dt} = V_i - V_C - Ri$ $\qquad V_S = i_T$

$LSi + Ri = V_i - V_C$ $\qquad V = \frac{i_T}{CS}$

$i(SLR) = V_i - V_0 \Rightarrow i = \frac{V_i - V_C}{RSL}$

d) $\lambda I - A$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} = \begin{bmatrix} -\frac{20}{0.04} & -\frac{1}{0.1} \\ \frac{1}{0.002} & 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -500 & -10 \\ -1000 & 0 \end{bmatrix} = \begin{bmatrix} \lambda + 500 & -10 \\ -500 & \lambda \end{bmatrix}$$

$B \qquad A$

det. $A - B$

$A = (\lambda + 500)\lambda$ $\qquad$ $B = -10 \cdot -1000$

$A = \lambda^2 + 500\lambda$ $\qquad$ $B = -10\,000$

$\rightarrow A - B$

$\lambda^2 + 500\lambda - (-10000) \Rightarrow \lambda^2 + 500\lambda + 10\,000 \Rightarrow \lambda_1 = -20.8 \quad \lambda_2 = -479.12$

$\lambda_1 = -20.8$ $\qquad$ $\lambda_2 = -479.12$

$\lambda_1 = \xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ $\qquad$ $\lambda_2 = \xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

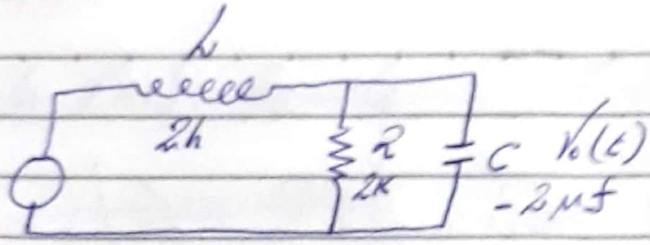
$\xi = \frac{R}{\sqrt{R^2+I^2}} = \frac{-20.8}{\sqrt{20.8^2+0^2}} = -1$ $\qquad$ $\xi = \frac{R}{\sqrt{R^2+I^2}} = \frac{-479.12}{\sqrt{479.12^2+0^2}} = -1$

$\xi\omega_n = -20.8$ $\qquad$ $\xi\omega_n = -479.12$

$-1\,\omega_n = -20.8$ $\qquad$ $-1\,\omega_n = -479.12$

$\omega_n = 0.042$ Rad/s $\qquad$ $\omega_n = 479.12$ Rad/s

2)



a) Paralelo = V_c

$V_{in} = -V_L - V_R = V_C$

$V_{in} = -V_L - V_C$

$+V_L + V_{in} = -V_C$

$V_L = -V_C - V_{in}$

$L\dfrac{di}{dt} = -V_C - V_{in}$

$\dfrac{di}{dt} = \dfrac{-V_C - V_{in}}{L}$

$I_t = I_L = I_R + I_C$

$I_L = I_R + I_C$

$I_L = \dfrac{V_C}{R} + C\dfrac{dV}{dt}$

$C\dfrac{dv}{dt} = I_L - \dfrac{V_C}{R}$

$\dfrac{dV}{dt} = -\dfrac{I_L}{C} - \dfrac{V_C}{RC}$

b) Espaço Estado

$$\begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{dv}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \cdot \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \cdot V_{in}$$

c) F transf.

$\dfrac{di}{dt} = \dfrac{-V_C - V_{in}}{L}$

$L s I = -V_C - V_{iN}$

$I_L = \dfrac{-V_C - V_{in}}{s}$

$\dfrac{dv}{dt} = \dfrac{I_L}{C} - \dfrac{V_C}{RC}$

$s V_C = I_L - \dfrac{V_C}{R}$

$V_C = \dfrac{I_L}{s} - \dfrac{V_C}{Rs}$

d) $\lambda_I - A$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1/L \\ 1/C & -1/RC \end{bmatrix} - \begin{bmatrix} 0 & -1/L \\ 1/[illegible] & -1/[illegible] \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \\ 500000 & -250 \end{bmatrix} = \begin{bmatrix} \lambda & -0.5 \\ -500000 & \lambda + 250 \end{bmatrix}$$

$A = (\lambda + 250) \cdot \lambda \Rightarrow \lambda^2 + 250\lambda$

$B = 0.5 \cdot (-500000) \Rightarrow -250000$

$A - B$

$(\lambda^2 - 250\lambda) - (-250000)$

$\lambda_{1,2} = 125 \pm j\,[illegible] \Rightarrow$ [illegible]

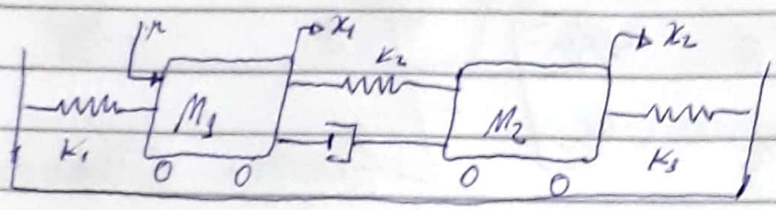
$$\xi = \frac{R}{\sqrt{R^2 + I_m^2}} = \frac{125}{\sqrt{125^2 + [illegible]^2}} = 0.[illegible]$$

$\xi \omega_n = 125$

$0.25\, \omega_n = 125$

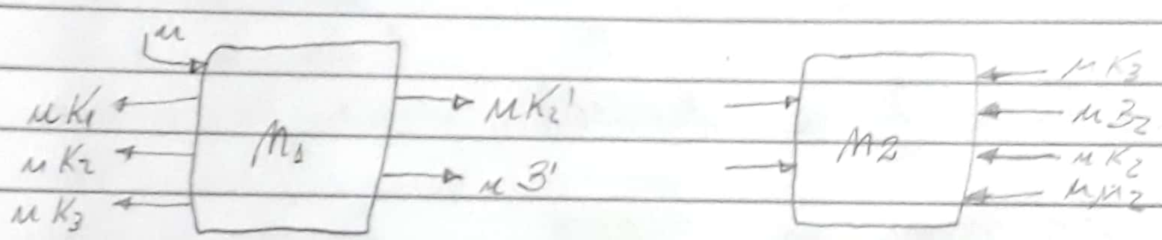
$$\omega_n = \frac{125}{0.25} \Rightarrow 500 \text{ rad/s}$$

3) Massa mola



$m_1 = 1\,Kg$ $\qquad b = 10\,Ns/m$ $\qquad K_2 = 50\,N/m$

$m_2 = 1{,}5\,Kg$ $\qquad K_1 = 50\,N/m$ $\qquad K_3 = 50\,N/m$



$V = \mu M_1 + \mu K_1 + \mu K_2 + \mu b - \mu K_2' - \mu b'$

$V = M_1\ddot{x}_1 + K_1X_1 + K_2X_1 + b\dot{x}_1 - K_2X_2 - b\dot{x}_2$

$V = M_1\ddot{x}_1 + K_1x_1 + K_2(X_1 - X_2) + b(\dot{x}_1 - \dot{x}_2)$

$0 = \mu M_2 + \mu b + \mu K_1 + \mu K_2 - \mu K_2 - \mu b$

$0 = M_2\ddot{x}_2 + b\dot{x}_2 + K_1X_2 + K_2X_2 - K_2X_1 - bX_1$

$0 = M_2\ddot{x}_2 + K_3X_2 + b(\dot{x}_2 - \dot{x}_1) + K_2(X_2 - X_1)$

Laplace

$V = M_1\ddot{x}_1 + K_1x_1 + K_2(X_2 - X_2) + b(\dot{x}_1 - \dot{x}_2)$

$V_s = M_1 s^2 + 1.S + K_1X_1.S + K_2X_1S - K_2X_2S + b.S\,X_1S - b.S\,X_2S$

$V_s = [M_1s^2 + b.S + K_1 + K_2]\,X_1.S + [-b.S - K_2]\,X_2.S$

$$0 = M_2 \ddot{x}_2 + K_3 x_2 + b(\dot{x}_2 - \dot{x}_1) + K_2 (x_2 - x_1)$$

$$0 = M_2 S^2 X_2 S + K_3 X_2 S + bS X_2 S - bS X_1 S + K_2 X_2 S - K_2 X_1 S$$

$$0 = [-b.S - K_2] X_1 S + [M_2 S^2 + bS + K_3 + K_2] X_2 S$$

$$\begin{bmatrix} M_1 S^2 + b.S + K_1 + K_2 & -bS - K_2 \\ -b.S - K_2 & M_2 S^2 + b.S + K_3 + K_2 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$\det.\Delta = (M_1 S^2 + bS + K_1 + K_2)(M_2 S^2 + bS + K_3 + K_2) - (bS + K_2)^2$$

$M_1 M_2 S^4 + M_1 b S^3 + M_1 K_3 S^2 + M_1 K_2 S^2 + M_2 b S^3 + b^2 S^2 + b K_3 S + \ldots$

$\ldots b K_2 . S + M_2 K_1 S^2 + b K_1 S + K_1 K_3 + K_1 K_2 + M_2 K_2 S^2 + b K_2 S + \ldots$

$\ldots K_2 . K_3 + K_2^2 - [b^2 S^2 + 2 b S K_2 + K_2^2]$

$(M_1 M_2) S^4 + (M_1 b + M_2 b) S^3 + (M_1 K_3 + M_1 K_2 + b^2 + M_2 K_1 + M_2 K_2 - b^2) + \ldots$

$\ldots (b K_3 + b K_2 + b K_1 + b K_2 - 2 b K_2) S + (K_1 K_3 + K_1 K_2 + K_2 K_3 + K_2^2 - K_2^2)$

$1{,}5 S^4 + 25 S^3 + 250 S^2 + 1000 S + 7500 \quad \det \Delta$

$$G_1(S) = \frac{\begin{vmatrix} 1 & -b.S - K_2 \\ 0 & M_2 S^2 + bS + K_3 + K_2 \end{vmatrix}}{\det \Delta} = \frac{1.5 S^2 + [illegible]}{1.5 S^4 + 25 S^3 + 250 S^2 + 1000 S + 7500}$$

$$G_2(S) = \begin{bmatrix} M_1 S^2 + bS + K_1 + K_2 & 1 \\ -bS - K_2 & 0 \end{bmatrix} = \frac{10 S + 50}{1.5 S^4 + 25 S^3 + 250 S^2 + 1000 S + 7500}$$