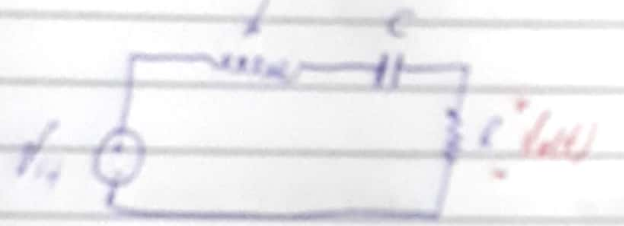
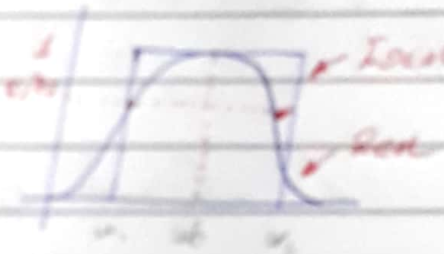
09.10.21

Lista de Exercícios: 01

Modelagem de Sistemas Dinâmicos

1) O circuito RLC em série representa funcionar um filtro passa-faixa





$R = 50$

$L = 100 \times 10^{-3}\ H$

$C = 1 \times 10^{-3}$

$$v_R = IR \qquad v_L = L\frac{di}{dt}$$

$$v_i - v_L - v_C = 0$$

$$i_C = C\frac{dv_C}{dt}$$

a)

$$v_R + v_L + v_C = v_0$$

$$IR + L\frac{di}{dt} + \frac{1}{C} = v_0$$

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}\,dt$$

$$\frac{d_i}{dt} + \frac{1}{L}\frac{di}{dt} + I\frac{R}{L} = \frac{v_0}{L}$$

$$v_{out} = v_0 \cdot \frac{x_c}{(x_L + R) + x_c} \Rightarrow v_i \cdot \frac{\frac{1}{sC}}{(sL + R) + \frac{1}{sC}}$$

$$F(s) = \frac{Y(s)}{X(s)}$$

RLC

$-V + R.I + L.I + V_C = 0$ $\qquad I = C_s V$

$I(R + Ls) = V - V_C$ [illegible] $V_C = \frac{I}{C_s}$

$$\boxed{I = \frac{V - V_C}{R + Ls}}$$

$$\int \frac{di}{dt} = \Delta t$$

2º) Por EE - Espaço Estado

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} i_C \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot V(t)$$

3º) Por Função de Transf.

$$V_{out} = V_0 \cdot \frac{X_C}{(X_L + R) + X_C} \Rightarrow V_0 \cdot \frac{\frac{1}{SC}}{(SL + R) + \frac{1}{SC}}$$

$$V_{out} = V_0 \cdot \frac{\frac{1}{SC}}{\frac{1 + SRC + S^2LC}{SC}} \qquad R + \frac{1}{SC} = \frac{1 + RSC}{SC}$$

$$\frac{V_{out}}{V_0} = \frac{1}{S^2LC + SRC + 1}$$

ESO

$$V_{i(t)} + V_{i(t)} + V_{c,t} + V_{L} = 0$$

$$-V_{i(t)} + L\frac{di}{dt} + V_c + R.I = 0$$

$$\frac{V_{i(t)}}{L} - \frac{V_c}{L} - \frac{R}{L} I = \frac{di}{dt}$$

Espaço / Estado $\rightarrow$ $\dot{x} = Ax + Bu$

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} i_c \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot v(t)$$

$$\frac{di}{dt} = -\frac{R i_c}{L} - \frac{v_c}{L} + \frac{v(t)}{L} \quad , \quad Eq. 1$$

$$\frac{dv}{dt} = \frac{i_c}{C} \qquad Eq. 2$$

Por función de transferencia

$$\frac{di}{dt} = \ldots \qquad \frac{dv}{dt} = \ldots$$

$$-V(t) + R\,i(t) + L\frac{di}{dt} + V_c = 0 \qquad \text{Eq. 1}$$

$$-V + RI + LsI + V_c = 0$$

$$\frac{i_c}{C} = \frac{dV_c}{dt}$$

Despejar I

$$\frac{I}{C} = sV$$

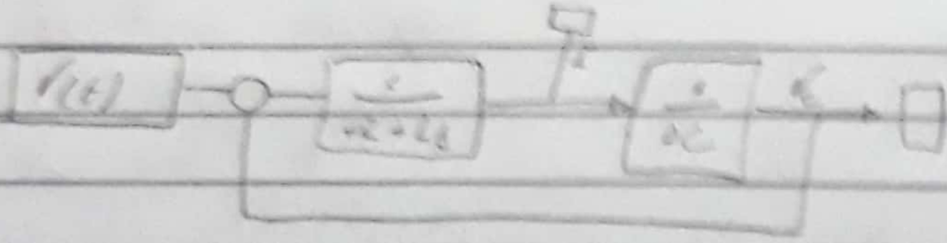
$$I(R + Ls) = (V - V_c)$$

$$I = \frac{(V - V_c)}{(R + Ls)}$$

en función I

$$I = CsV_c$$

$$\boxed{V_c = \frac{I}{Cs}}$$



3) Obtenha as funções de transferência $\frac{x_1}{V_s}$ e $\frac{x_2}{V_s}$ do sistema mecânico

$$V = V_{M_1} + V_{K_1} + V_{K_2} + VB - VK_2' - VB$$

$$V = M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + K_2 x_1 + B\frac{dx_1}{dt} - K_2 x_2 - B\frac{dx_2}{dt}$$

$$V = M_1 \frac{d^2x_1}{dt^2} + B\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + K_2(x_1 - x_2) + K_1 x_1$$

→ Aplicando Laplace

$$V(s) = M_1 s^2 . x_1(s) + B(s x_1(s) - s x_2(s)) + K_2 (x_1(s) - x_2(s)) + K_1 x_1 (s)$$

$$V(s) = x_1(s).(M_1 s^2 + Bs + K_2 + K_1) + x_2(s).(-Bs - K_2)$$

Continuação exemplo 3

$0 = M_2 - K_2 - K_2 - K_3 - K_2 - K_3$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B\frac{dx_2}{dt} + K_2 x_2 + K_3 x_2 - K_2 x_1 - B\frac{dx_1}{dt}$$

→ Aplicando Laplace temos:

$$0 = M_2 s^2 x_2(s) + Bs\,x_2(s) + K_2 x_2(s) + K_3 x_2(s) - K_2 x_1(s) - Bs\,x_1(s)$$

$$0 = x_2(s)\left(M_2 s^2 + Bs + K_2 + K_3\right) + x_1(s)\left(-K_2 - Bs\right)$$

→ Equação E.E

$$\begin{bmatrix} M_1 s^2 + Bs + K_1 + K_2 & \underline{-Bs - K_2} \\ -K_2 - Bs & M_2 s^2 + Bs + K_2 + K_3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$\det \Delta = \left[(M_1 s^2 + Bs + K_1 + K_2)(M_2 s^2 + Bs + K_2 + K_3)\right] - \left[(-Bs - K_2)(-K_2 - Bs)\right]$

$\det \Delta = \left[(M_1 M_2 s^4 + M_1 Bs^3 + M_1 K_2 s^2 + M_1 K_3 s^2 + M_2 Bs^3 + B^2 s^2 + BK_2 s + BK_3 s +\right.$

$\left. M_2 K_2 s^2 + BK_2 s + K_2^2 + K_2 K_3 + M_2 K_1 s^2 + BK_1 s + K_1 K_2 + K_1 K_3\right) -$

$- \left(+BK_2 s + B^2 s^2 + K_2^2 + BK_2 s\right)$

Cont. exercício 3

Calculando em evidência o det Δ

$$\det \Delta\, (M_1 M_2)s^4 + (M_1 B + M_2 B)s^3 + (M_1 K_2 + M_1 K_3 + \cancel{B^2} + M_2 K_2 + M_2 K_1 - \cancel{B^2})s^2$$

$$+ (\cancel{BK_2} + BK_3 + \cancel{BK_2} + BK_1 - \cancel{BK_2} - \cancel{BK_2})s + (\cancel{K_2^2} + K_2 K_3 + K_1 K_2 + K_1 K_3 - \cancel{K_2^2})$$

$$\det \Delta = (M_1 M_2)s^4 + (M_1 B + M_2 B)s^3 + (M_1 K_2 + M_1 K_3 + M_2 K_2 + M_2 K_1)s^2$$

$$+ (BK_1 + BK_3)s + (K_1 K_2 + K_2 K_3 + K_1 K_3)$$

F. transf.

$$\begin{bmatrix} 1 & -Bs - K_2 \\ 0 & M_2 s^2 + Bs + K_2 + K_3 \end{bmatrix} = M_2 s^2 + Bs + K_2 + K_3 - 0$$

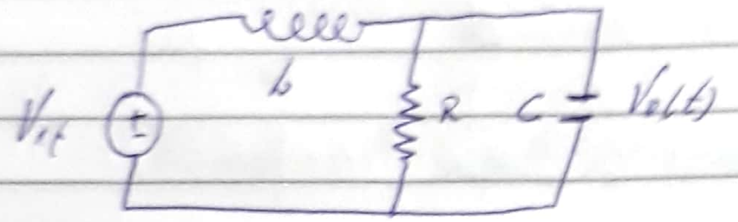
$$G_1(s) = \frac{M_2 s^2 + Bs + K_2 + K_3}{\det \Delta} \Rightarrow \frac{1{,}5s^2 + 10s + 100}{\det \Delta}$$

$$\begin{bmatrix} M_1 s^2 + Bs + K_2 + K_1 & 1 \\ -K_2 - Bs & 0 \end{bmatrix} = 0 - (-K_2 - Bs) = K_2 + Bs$$

$$G_2(s) = \frac{K_2 + Bs}{\det \Delta} \Rightarrow \frac{10s + 50}{\det \Delta}$$

$$\boxed{\det \Delta = 1{,}5s^4 + 25s^3 + 255s^2 + 1000s + 7500}$$

Q2) Determine que tipo de filtro é mostrado



$$G(s) = \frac{V_o}{V_i}$$

$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \qquad R//C = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} \qquad R//C = \frac{\frac{R}{sC}}{\frac{RsC+1}{sC}}$$

$$R//C = \frac{\frac{R}{sC}}{\frac{RsC+1}{sC}} \qquad \boxed{R//C = \frac{R}{sRC+1}}$$

$$V_o = V_i \cdot \frac{Z_1}{Z_1 + Z_2}$$

$$V_o = V_i \cdot \frac{\frac{R}{sRC+1}}{sL + \frac{R}{sRC+1}}$$

$$\frac{V_o}{V_i} = \frac{\frac{R}{sRC+1}}{\frac{sL(sRC+1)+R}{sRC+1}} \Rightarrow \frac{V_o}{V_i} = \frac{R}{s^2LRC + sL + R}$$

$$\boxed{Ft(s) = \frac{R}{LRCs^2 + Ls + R}}$$

A[illegible] análise no Matlab, observamos
[illegible] passa-baixa.
[illegible] a amplitude e a fase [illegible]
[illegible] se comporta como a variação [illegible]
[illegible] de entrada.
A configuração do circuito mostra o [illegible]
[illegible] com a entrada, se comporta como
filtro passa-baixa.