Question 1.1

The Lagrangian and its partial derivatives

The Lagrangian can be formulated as follows:

$$\mathcal{L}(w, \lambda, \mu) = \frac{1}{2}w^T w + \lambda(w \cdot x - 1) + \mu(1 - w \cdot x)$$

which can also be re-written:

$$\mathcal{L}(w, \lambda, \mu) = \frac{1}{2}w^T w + \lambda(w \cdot x - 1) + \mu(1 - w \cdot x)$$

The partial derivatives are:

$$\frac{\partial \mathcal{L}}{\partial w} = w - \lambda x - \mu x$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = w \cdot x - 1$$
$$\frac{\partial \mathcal{L}}{\partial \mu} = 1 - w \cdot x$$

The analytical solution for w^* optimal allocations

From:

$$\frac{\partial \mathcal{L}}{\partial w} = w - \lambda x - \mu x = 0$$

we get:

$$w(1 - \lambda x - \mu x) = 0$$
$$w = 0 \vee \lambda x - \mu x = 1$$
$$w = 0 \vee \frac{1}{x} = \lambda - \mu$$

We know from the constraints that:

$$w \cdot x = 1$$
$$1 \cdot x = 1$$

by replacing w into the equations given:

$$w = 0 \vee \frac{1}{x} = \lambda - \mu \Rightarrow \frac{1}{x} = \lambda - \mu$$
$$1 = \frac{1}{x}\lambda - \mu x \Rightarrow 1 = \lambda - \mu x$$

we have:

$$x = \lambda - \mu$$
$$\mu = \frac{\lambda}{x} - \frac{1}{x^2} \Rightarrow \frac{1}{x^2}$$
$$\lambda = \mu + \frac{1}{x}$$

where λ, μ and $\frac{1}{x}$ are scalars for:

$$w = 0 \vee \frac{1}{x} = \lambda - \mu$$

we get:

$$w = 0 \vee \frac{1}{x} = \mu + \frac{1}{x}$$

we have 2 equations with 2 unknowns and we solve the unknowns:

$$\mu = \frac{\lambda}{x} - \frac{1}{x^2}$$
$$\lambda = \mu + \frac{1}{x}$$

replacing variables into:

$$w = 0 \vee \frac{1}{x} = \lambda - \mu$$

we get:

$$\frac{1}{x} = \frac{\lambda}{x^2} - \frac{1}{x^2} - \frac{1}{x}$$
$$= \lambda - \mu - \frac{1}{x}$$
$$= \lambda - \frac{\lambda}{x} + \frac{1}{x^2} - \frac{1}{x}$$
$$= \frac{\lambda}{x} - \frac{\lambda}{x^2} + \frac{1}{x^2} - \frac{1}{x}$$
$$= \frac{\lambda}{x} - \frac{\lambda}{x^2} + \frac{1}{x^2} - \frac{1}{x}$$

we have replaced quotient into:

$$w = 0 \vee \frac{1}{x} = \lambda - \mu$$

we get:

$$w = 0 \vee \frac{1}{x} = \frac{\lambda}{x^2} - \frac{1}{x^2} - \frac{1}{x}$$

we have replaced quotient into:

$$w = 0 \vee \frac{1}{x} = \lambda - \mu$$

we get:

$$w = 0 \vee \frac{1}{x} = \frac{\lambda}{x^2} - \frac{1}{x^2} - \frac{1}{x}$$
$$= x^2 \cdot w = \lambda x^2 - x^2 - x$$
$$= x^2 \cdot w = \lambda x^2 - x^2 - x$$
$$= x(\lambda x - x) - x$$
$$= \lambda x - x^2 - x$$
$$= \lambda x - \frac{\lambda x}{x} - \frac{x}{x}$$
$$= \lambda x - \frac{\lambda x}{x} - \frac{x}{x}$$

we replacing variable and quotient into:

$$w = 0 \vee \frac{1}{x} = \lambda - \mu$$

gives:

$$w^* = 0 \vee \frac{1}{x} = \frac{\lambda x}{x} - \frac{x}{x}$$
$$= \lambda x - \frac{x}{x}$$
$$= \lambda x - \frac{x}{x}$$
$$= \lambda x

[redacted]

But we can use the expected value of the gaussian variable X which is μ and the variance σ^2 to know that

$$E[X] = \mu$$

$$Var(X) = \sigma^2$$

Now let's plot the original data, fitted to a standard normal distribution and the μ and σ we get

$$\mu = 0.0$$

$$\sigma = 1.0$$

Cool. Now let's repeat the probability plot, but this time with the fitted μ and σ as inputs. So we know we should get a straight line. Yes, we do.