PSTAT 174/274, Spring 2023: Homework # 4.

Note: In all problems, $\{Z_t\} \sim WN(0, \sigma_Z^2)$ denotes white noise, and B denotes the backshift operator $BX_t = X_{t-1}$.

1. For each of the models (1)-(3), write corresponding characteristic polynomials $\theta(z)$ and $\phi(z)$. In other words, write models (1)-(3) as $\phi(B)X_t = \theta(B)Z_t$ and specify polynomials $\theta(z)$ and $\phi(z)$. For each, identify the model as a certain (S)ARIMA model and determine whether the model in invertible and stationary:
(1) $X_t = -0.2X_{t-1}+Z_t-2Z_{t-1}$; (2) $X_t = X_{t-1}+Z_t-0.2Z_{t-1}-0.7Z_{t-2}$; (3) $X_t = X_{t-3}+Z_t+0.4Z_{t-1}-0.45Z_{t-2}$.

2. Identify the model equation corresponding to SARIMA $(0, 0, 2)(0, 0, 1)_{12}$ model.
A. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_1 2 Z_{t-2}$.
B. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \theta_1\theta_2 Z_{t-3}$.
C. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \theta_1\theta_2 Z_{t-12}$.
D. $X_t = Z_t + \theta_1 Z_{t-2} + \theta_2 Z_{t-12} + \theta_1\theta_2 Z_{t-14}$.
E. $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-11} + \theta_1\theta_2 Z_{t-12}$.

3. (a) For the following SARIMA$(p, d, q)\times(P, D, Q)_s$ models, specify parameters p, d, q, P, D, Q and s, and write corresponding equations. It is OK to leave equations in the form $\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D X_t = \theta(B)\Theta(B^s)Z_t$, $Z_t \sim WN(0, \sigma_Z^2)$.

(i) SARIMA $(1, 1, 0) \times (1, 1, 2)_6$, (ii) SARIMA$(0, 1, 1) \times (0, 0, 3)_{12}$, (iii) SARIMA$(2, 1, 2) \times (2, 0, 1)_4$.

(b) For the following processes $\{X_t\}$, identify SARIMA $(p, d, q) \times (P, D, Q)_s$ model:

(i) $(1 - B^{12})^2 X_t = (1 - 0.3B)Z_t$; (ii) $X_t = 0.5X_{t-6} + Z_t$; (iii) $(1 - 0.8B)(1 + 0.5B^4)X_t = (1 - 1.5B)Z_t$.

(c) You are given a time series model where PACF is zero except for lags 12 and 24. Which model will have this pattern?

4. For the processes $X_t = 0.4X_{t-1} + Z_t - 0.7Z_{t-1}$,
(i) Simulate and plot 100 values of the processes;
(ii) Compute and graph their theoretical ACF and PACF using R.
(iii) Compute and graph their sample ACF and PACF using R. How do sample functions compare to their theoretical counterparts?
(iv) Analyze smoothness of the simulated processes using their ACF's.

Please include the code with clear comments explaining the meaning of the code. Make sure to label the graphs.

The following problems are for students enrolled in PSTAT 274 ONLY

GE 1. Let $\{Y_t\}$ be the AR(1) plus noise time series defined by $Y_t = X_t + W_t$, where $\{W_t\} \sim WN(0, \sigma_W^2)$, $\{X_t\}$ is the AR(1) process $X_t - \phi X_{t-1} = Z_t$, $\{Z_t\} \sim WN(0, \sigma_Z^2)$, $|\phi| < 1$, and $E(W_s Z_t) = 0$ for all t, s.
(a) Show that $\{Y_t\}$ is stationary and find its autocovariance function.
(b) Show that the time series $U_t := Y_t - \phi Y_{t-1}$ is 1-correlated (i.e., $\gamma_U(h) = 0$ for $h > 1$) and hence is MA(1) process.
(c) Conclude from (b) that $\{Y_t\}$ is an ARMA(1,1) process and express the three parameters of this model in terms of ϕ, σ_W^2 and σ_Z^2.

GE 2. Find the ACVF, ACF and PACF for $\{X_t\}$ when $X_t = \Phi X_{t-4} + Z_t$, $|\Phi| < 1$.