PSTAT 174/274, Spring 2023: Homework # 3.

Note: $\{Z_t\} \sim WN(0, \sigma_Z^2)$ denotes white noise.

1. You are given the following time series model: $X_t = \frac{2}{3}X_{t-1} + \frac{1}{2}X_{t-2} + Z_t$.
Determine whether this time series is stationary and/or invertible.

2. You are given the following statements about a time series modeled as an AR(3) process:
I. Partial Autocorrelation for lag 3 is always equal to zero.
II. Partial Autocorrelation for lag 4 is always equal to zero.
III. Partial Autocorrelation for lag 4 is always greater than zero.
Determine which of the above statements are true.

3. For a stationary ARMA(1,1) model, you are given the following information: $\rho_X(1) = 0.7$, $\rho_X(2) = 0.3$. Calculate $\phi_1$.
*Hint: Formulas for ACF of ARMA (1,1) model $X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$ are given in §5.1 of lecture notes and slide 24 of week 3. Can you determine a recursive relation between $\rho_X(k)$ and $\rho_X(k-1)$? Use it!*

4. You are given PACF for a stationary process: $\phi_{11} = -0.60$, $\phi_{22} = 0.36$, $\phi_{kk} = 0$ for $k \geq 3$. What time series model could have this PACF? Identify model's coefficients and write model equation.
*Hint: • §6 of Lecture Notes and slides 32-33 of Week 3 provide relationship between PACF and ACF, allowing to calculate ACFs of the model from given PACFs.*
*• Yule-Walker equations, in §4.3 of Lecture Notes and on slide 14, Week 3, provide relationship between ACF and model coefficients.*

5. You are given the following time-series model: $X_t = 0.8X_{t-1} + 2 + Z_t - 0.5Z_{t-1}$. Which of the following statements about this model is false?

A. $\rho_X(1) = 0.4$. B. $\rho_X(k) < \rho_X(1), k \geq 2$. C. The model is ARMA(1,1).
D. The model is stationary. E. The mean, $\mu_X$, is 2.

6. The Notion of *parameter redundancy* pertains to the situation when AR and MA characteristic polynomials $\phi(z)$ and share $\theta(z)$ share a common factor, in which case model may be simplified. Determine which of the following models are parameter redundant:
I. $X_t = \frac{1}{2}X_{t-1} + Z_t - \frac{1}{2}Z_{t-1}$;
II. $X_t = \frac{1}{2}X_{t-1} + Z_t - \frac{1}{9}Z_{t-2}$;
III. $X_t = -\frac{5}{6}X_{t-1} - \frac{1}{6}X_{t-2} + Z_t + \frac{8}{12}Z_{t-1} + \frac{1}{12}Z_{t-2}$.

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**The following problems are for students enrolled in PSTAT 274 ONLY**

**GE 1.** Show that the two MA(1) processes

$X_t = Z_t + \theta Z_{t-1}$, $\{Z_t\} \sim WN(0, \sigma^2)$, and $Y_t = \tilde{Z}_t + \frac{1}{\theta}\tilde{Z}_{t-1}$, $\{\tilde{Z}_t\} \sim WN(0, \sigma^2\theta^2)$,

where $0 < |\theta| < 1$, have the same autocovariance functions.

**GE 2.** You are given the following time series model: $X_t = \phi X_{t-1} + Z_t$. Let $Y_t = X_{2t}$, that is, the original process observed only at even times. Show that $Y_t$ is also an AR(1) model $Y_t = \phi^* Y_{t-1} + Z_t^*$ and determine $\phi^*$ and $Z_t^*$, that is, express $\phi^*$ and $Z_t^*$ via $\phi$ and $Z_t$.