# Check your Understanding: Weeks 1 -2

**These slides are intended for review of the material of weeks 1 and 2.**

**Some questions might appear tricky. The intend is to allow students to catch important finer points that could easily be missed at the first reading of the material. The slides include all *Check your Understanding* slides from weeks 1 and 2 lecture slides as well as additional questions.**

**Each slide with a question is followed by a slide with an answer and detailed explanation.**
**Please first try to answer the question on your own before checking an answer.**
**Once checking an answer, please make sure to fully understand the explanation.**

**This review serves as a good preparation for Quiz 1, but only if students understand the reasoning rather than memorize answers.**

**Please make sure to read questions carefully and** ***think before answering*****.** ***Did you get all details right? Remember definitions? Paid attention?***

*Helpful recommendation: read questions aloud!*

# Check of Understanding: Data and Goals of Time Series

**Determine which of the following statements are true:**

**I. Time series analysis refers to the branch of statistics where observations are collected sequentially in time, usually but not necessarily at equally spaces time points.**

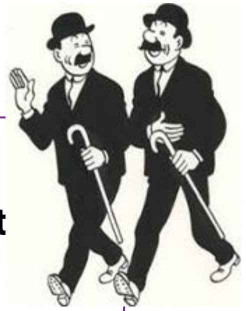
**II. The goal of the time series analysis is to find a model that describes dependence in the data based on a finite sample of data. Once a good model is found and fitted to data, the analyst can use the model to forecast future values, or generate simulations, to guide planning decisions.**

**III. Today, a time series analyst does not need to understand statistical theory because R software is available to perform time series procedures.**

A. **None are true**
B. **I and II only**
C. **I and III only**
D. **II and III only**
E. **The answer is not given by (A), (B), (C) or (D).**

***Check your answer of the next slide***

# Check of Understanding: Data and Goals of Time Series



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**II. The goal of the time series analysis is to find a model that describes dependence in the data based on a finite sample of data. Once a good model is found and fitted to data, the analyst can use the model to forecast future values, or generate simulations, to guide planning decisions.**

**III. Today, a time series analyst does not need to understand statistical theory because R software is available to perform time series procedures.**

**A. None are true**

**B. I and II only**

**C. I and III only**

**D. II and III only**

**E. The answer is not given by (A), (B), (C ) or (D).**

**Short answer: B**
**Long answer:**
**I. and II. are true (see, e.g., summary on slide 23 of week 1.) Although in this class we will consider only equally-spaced observations, this is a usual requirement but is not necessary in general (slide 11 of week 1)**
**III. is false: an analyst must understand a theory of time series analysis in order to be able to correctly choose R procedures and to interpret the outcomes of these procedures.**

# Trend and Seasonality Check of Understanding

**Determine which of the following statements are true:**

**I. Deterministic trends can be modeled using regression.**

**II. Stochastic trends usually have a plausible physical explanation, such as an increase in population.**

**III. Short term extrapolation of deterministic trends can be justified by claiming that underlying trends will usually change slowly in comparison with the forecast lead time.**

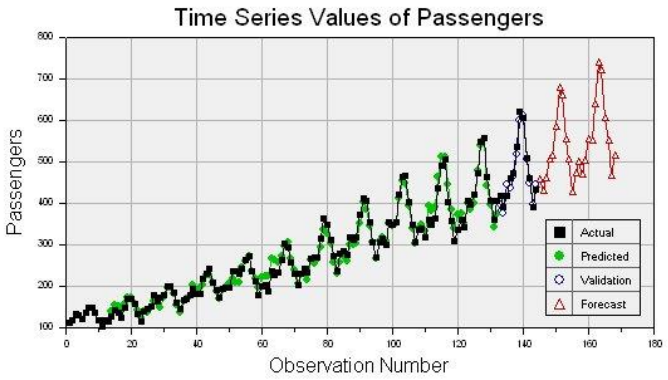
**A. None are true**

**B. I and II only**

**C. I and III only**

**D. II and III only**

**E. The answer is not given by (A), (B), (C ) or (D).**




***Check your answer of the next slide***

# Check your  Understanding: Trend and Seasonality

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**A. None are true**

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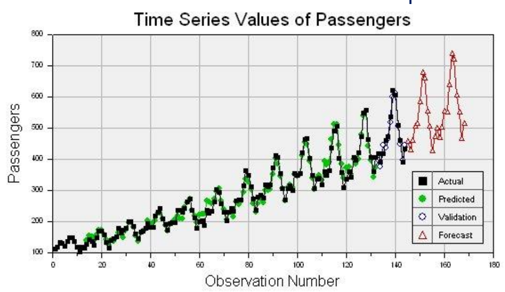



**Short answer: C**

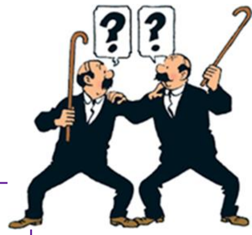
**Long answer:**

**I.   is true: deterministic trend is non-random and can be modeled by regression;**

**III. is true: deterministic trend is a function estimated by regression; it changes slowly and can be extrapolated.**

**II. is incorrect: stochastic trend is caused by a random phenomena. The deterministic trends typically  have plausible physical explanations.**

# Check of Understanding: Stationarity




**Which of the following is false?**

**I. If the second-order characteristics of a stochastic process change over time, then the process is nonstationary.**

**II. Representing a nonstationary time series by a simple algebraic model is often difficult.**

**III. If a time series is stationary, then its mean, variance and, for any lag k, covariance must are constants or depend on lag k only.**

**A. None are true**

**B. I and II only**

**C. I and III only**

**D. II and III only**

**E. The answer is not given by A, B, C or D.**

***Check your answer of the next slide***

# Check of Understanding: Stationarity

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**I. If the second-order characteristics of a stochastic process change over time, then the process is nonstationary.**

**II. Representing a nonstationary time series by a simple algebraic model is often difficult.**

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**A. None are true**

**B. I and II only**

**C. I and III only**

**D. II and III only**

**E. The answer is not given by A, B, C or D.**

**Short answer: E because all three statements are correct.**

**I. is true: if mean, variance or covariance change over time, the process is (weakly) nonstationary (slide 50, week 1).**
**II. is true: mathematical equations for nonstationary times series are complex.**
**III. is true: this statement is a definition of (weakly) stationary process.**

# Check your Understanding: Gaussian processes and Stationarity

**True or False:**

I. All random walk processes are nonstationary.

II. All random walk processes are stationary. (Meaning weakly stationary.)

III. All random walk processes are strictly stationary.

IV. For a stationary process its mean does not depend on time, i.e., a constant.

V. For a stationary process variance is a constant but can be infinite.

VI. A strictly stationary process is always weakly stationary.

VII. Any weakly <u>non</u>stationary process is automatically strictly <u>non</u>stationary.

VIII. A strictly stationary Gaussian process is always weakly stationary.

IX. A weakly stationary Gaussian process is always strictly stationary.

X. A seasonal time series may be stationary.

XI. A time series with trend may be stationary.

*Check your answer of the next slide*

# Check your Understanding: Gaussian processes and Stationarity

**True or False:**

I. All random walk processes are nonstationary. True
II. All random walk processes are stationary. (Meaning weakly stationary.) False
III. All random walk processes are strictly stationary. False
IV. For a stationary process its mean does not depend on time, i.e., a constant. True
V. For a stationary process variance is a constant but can be infinite. False
VI. A strictly stationary process is always weakly stationary. False
VII. Any weakly nonstationary process is automatically strictly nonstationary. False
VIII. A strictly stationary Gaussian process is always weakly stationary. True
IX. A weakly stationary Gaussian process is always strictly stationary. True
X. A seasonal time series may be stationary. False
XI. A time series with trend may be stationary. False

Explanation for I-III: We showed in Example 2.1.5 (week 1, slide 55) that the variance of the RW depends on t, that is, not a constant: $\sigma_X^2(t) = \sigma_Z^2 t$. Thus, it is not weakly or strictly stationary.

For IV-VII: (Weakly) stationary processes must have constant and finite mean and variance.
VI is False b/c strictly stationary processes may have $\infty$ variance, e.g., i.i.d. Cauchy r.v.s.
VII is False in cases when time series is strictly stationary but has infinite variance.

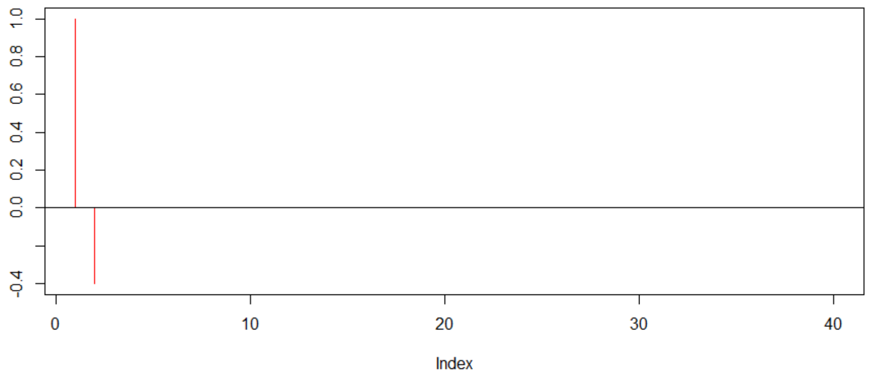
For VIII-IX: The Gaussian distribution is fully determined by its finite mean, variance and correlation (slides 50-51 of week 1).
For X-XI: Time series with trend/seasonality have non constant means, i.e., non-stationary.

# Check your Understanding: Identifying ACF of MA(1)

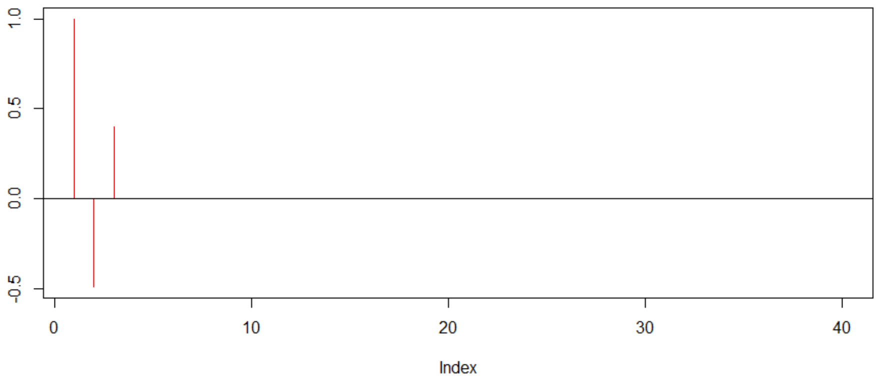

**Figures (a), (b), & (c) below plot acfs for different time series.**

- **Which graphs correspond to MA(1) ?**
- **For graphs corresponding to MA(1) what is the sign of its coefficient $\theta_1$?**

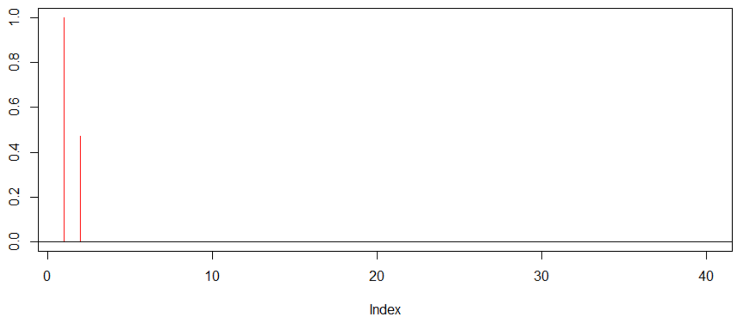
**(a)**




**(b)**




**(c)**

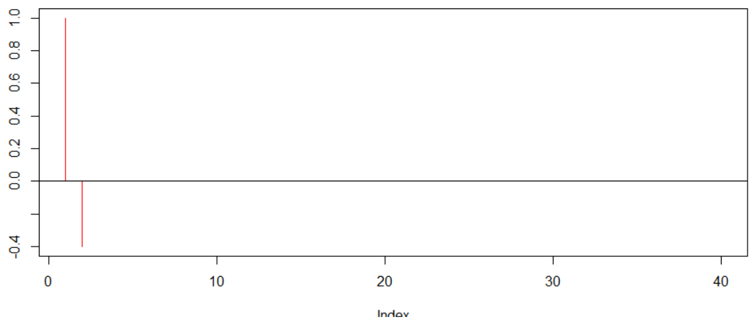


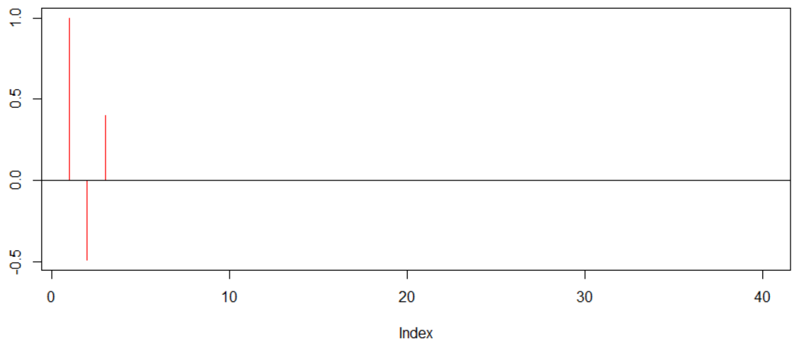
# ACVF and ACF of Moving Average Models: MA(1)

**Figures (a), (b), & (c) below plot acfs for different time series.**

- **Which graphs correspond to MA(1) ?**
- **For graphs corresponding to MA(1) what is the sign of its coefficient $\theta_1$?**

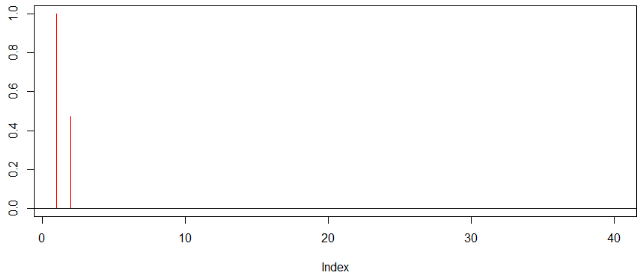
(a)




(b)

(c)



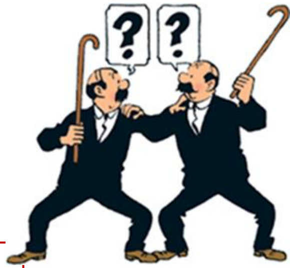

**For MA(1), acf at lags k ≠ 0, 1 are zero:**

**$\rho_X$ (k) = 0 for |k| >1. Sign ($\rho_X(1)$) = Sign ($\theta_1$)**

**Derived: slide 64 week 1; summarized: slide 3 week 2**

- **Graphs (a) & (c) correspond to MA(1) process.**
- **Because (b) has $\rho_X$ (2) ≠ 0, $\rho_X$ (k) = 0, for all lags k >2, it is not MA(1) but MA(2).**
- **Because for MA(1) model, sign ($\rho_X(1)$) = Sign ($\theta_1$), for (a), $\theta_1 < 0$, and for (c), $\theta_1 > 0$ .**

# Check your Understanding: Stationarity




**True or False:**

**I. It is possible to have a nonstationary time series $\{Y_t\}$ with a constant mean and**

**Cov $(Y_t, Y_{t+k})$, $k \geq 1$, free of t.**

**II. It is possible to have a nonstationary time series $\{Y_t\}$ with a constant mean and**

**Cov$(Y_t, Y_{t+k})$, $k \geq 0$, free of t.**

**III. To apply a MA(q) model one must have Gaussian data.**

**IV. AR(p) models are applicable only when one has strictly stationary data.**

**V. Let $X_t$ be a stationary process and $Y_t = 3 + X_t$ . Then, $Y_t$ is no longer stationary.**

**VI. Let $X_t$ be a stationary process and $Y_t = \mu_t + X_t$ where $\mu_t$ is a nonconstant deterministic function. Then, $Y_t$ is no longer stationary.**

**VII. A time series with constant mean and variance is stationary.**

***Check your answer of the next slide***

# Check your Understanding: Stationarity

**True or False:**

**I: It is possible to have a nonstationary time series $\{Y_t\}$ with a constant mean and Cov $(Y_t, Y_{t+k})$, $k \geq 1$, free of t.**

**A: True, it is possible that variance $\sigma_t = \mathrm{Cov}(Y_t, Y_t)$ varies with t while covariance Cov $(Y_t, Y_{t+k})$, $k \geq 1$, does not. For example, take $Y_t$ to be independent normal r.v. with variance t.**

**II: It is possible to have a nonstationary time series $\{Y_t\}$ with a constant mean and Cov$(Y_t, Y_{t+k})$, $k \geq 0$, free of t.**

**A: False. Cov$(Y_t, Y_t) = \sigma^2$ is free of t, the same as mean and covariance. By definition, the process is stationary.**

**III. To apply a MA(q) model one must have Gaussian data.**

**A: False. MA(q) model can be applied to both Gaussian and non-Gaussian data.**

**IV. AR(p) models are applicable only when one has strictly stationary data.**

**A: False. AR models are applicable to weakly stationary data. No requirement of strict stationarity.**

**V. Let $X_t$ be a stationary process and $Y_t = 3 + X_t$. Then, $Y_t$ is no longer stationary.**

**A: False. The only change is that E $Y_t = 3$ while E $X_t = 0$.**

**VI. Let $X_t$ be a stationary process and $Y_t = \mu_t + X_t$ where $\mu_t$ is a nonconstant deterministic function. Then, $Y_t$ is no longer stationary.**

**A: True. Because E $Y_t = \mu_t$ depends on time, the process is nonstationary.**

**VII. A time series with constant mean and variance is stationary.**

**A: False: it is possible that covariance Cov $(Y_t, Y_{t-k})$ depends on t.**

# Check of Understanding: stationarity/invertibility of MA(q), shift operator

**Consider the process $X_t = (1 - 1.6\,B + 0.6\,B^2)\,Z_t$. Determine which of the statements below are true.**

**I. $X_t$ is represented by an AR (2) model.**

**II. $X_t$ can be written in an equivalent form as $X_t = Z_t - 1.6\,Z_{t-1} + 0.6\,Z_{t-2}$.**

**III. $X_t$ can be written in an equivalent form as $X_t - 1.6\,X_{t-1} + 0.6\,X_{t-2} = Z_t$.**

**IV. $X_t$ is invertible and stationary.**

**V. $X_t$ is invertible and nonstationary.**

**VI. $X_t$ is noninvertible but stationary.**


**A. I, II and IV only**

**B. II, III and V only**

**C. II and VI only**

**D. I, II, III and VI only**

**E. The answer is not given by A, B, C or D.**

***Check your answer of the next slide***

# Check of Understanding: stationarity/invertibility of MA(q), shift operator

Consider the process $X_t = (1 - 1.6 B + 0.6 B^2) Z_t$. Determine which of the statements below are true.

I. $X_t$ is represented by an AR (2) model.

II. $X_t$ can be written in an equivalent form as $X_t = Z_t - 1.6 Z_{t-1} + 0.6 Z_{t-2}$.

III. $X_t$ can be written in an equivalent form as $X_t - 1.6 X_{t-1} + 0.6 X_{t-2} = Z_t$.

IV. $X_t$ is invertible and stationary.

V. $X_t$ is invertible and nonstationary.

VI. $X_t$ is noninvertible but stationary.

A. I, II and IV only

B. II, III and V only

C. II and VI only

D. I, II, III and VI only

E. The answer is not given by A, B, C or D.

**Short answer: C.**

**Explanation: This is an MA(2) process that can also be written as in II. MA models are always stationary.**

**Because polynomial $\theta(z) = 1 – 1.6z + 0.6z^2$ has a unit root, model is not invertible.**

**Thus, only II and VI are correct, i.e., C.**


# Check of Understanding: ACF for MA and AR processes

**Stationary time series $X_t$ has autocorrelation function with $\rho_X(3) = -0.4$. Which of the following statements displays the model structure corresponding to series X the best:**

**I.** $X_t = Z_t - 0.4\, Z_{t-2}$

**II.** $X_t = Z_t - 0.8\, Z_{t-2} + 0.5\, Z_{t-3}$

**III.** $X_t = Z_t - 0.5\, Z_{t-3}$

**IV.** $X_t - 0.5\, X_{t-1} = Z_t$

**V.** $X_t + 0.4\, X_{t-1} = Z_t$

*Check your answer of the next slide*

# Check of Understanding: ACF for MA and AR processes

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A. $X_t = Z_t - 0.4 Z_{t-2}$

B. $X_t = Z_t - 0.8 Z_{t-2} + 0.5 Z_{t-3}$

C. $X_t = Z_t - 0.5 Z_{t-3}$

D. $X_t - 0.5 X_{t-1} = Z_t$

E. $X_t + 0.4 X_{t-1} = Z_t$

**Short Answer: C**

**Long Answer:**

- Eliminate A because for MA(2) process, $\rho_X(3) = 0$.
- Models D and E are AR(1). Formulas on slide 43 week 2 give: $\rho_X(1) = \phi_1,\ \rho_X(2) = \phi_1^2,\ \rho_X(3) = \phi_1^3$.
  For model D, $\phi_1 = 0.5$ so that its $\rho_X(3) = 0.125$. For model E, $\phi_1 = -0.4$ so that its $\rho_X(3) = -0.064$.
  Conclude: eliminate models D and E.
- Models B and C are MA(3) with $\rho_X(k) = \dfrac{\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \dots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \dots + \theta_q^2}$. (slide 19 of week 2).
  For q=k=3, this formula gives $\rho_X(3) = \dfrac{\theta_3}{1 + \theta_1^2 + \theta_2^2 + \theta_3^2}$. Note that $\rho_X(3)$ has the same sign as $\theta_3$.
  $\rho_X(3) = -0.4 < 0$ implies $\theta_3 < 0$, thus, eliminate B. C is the only remaining option.
- Answer: C is the only appropriate model from this list: for $\theta_1 = \theta_2 = 0$ and $\theta_3 = -0.5$, $\rho_X(3) = \dfrac{-0.5}{1 + 0.5^2} = -0.4$ as given.

# Check of Understanding: AR(1) process

**For AR(1) process $X_t$ with the coefficient $\phi_1 = -0.3$, determine which of the statements below are true.**

**A. $(1 - 0.3B)X_t = Z_t$**

**B. $X_t = -0.3X_{t-1} + Z_t$**

**C. ACF $\rho_X(2) = 0$**

**D. $X_t$ is stationary and invertible**

**E. $X_t$ has a MA ($\infty$) representation and therefore is causal**

**F. ACF $\rho_X(3) < \rho_X(2) < \rho_X(1)$.**

***Check your answer of the next slide***

# Check of Understanding: AR(1) process

For AR(1) process $X_t$ with the coefficient $\phi_1 = -0.3$, determine which of the statements below are true.

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D. $X_t$ is stationary and invertible

E. $X_t$ has a MA ($\infty$) representation and therefore is causal

F. ACF $\rho_X(3) < \rho_X(2) < \rho_X(1)$.

**Short Answer: B, D and E.**

**Long Answer: Check slide 44 of week 2 with a summary of properties of AR(1) model.**

- A is false: with $\phi_1 = -0.3$ the equation is $(1 - (-0.3\,B))\,X_t = Z_t$ that is, $(1 + 0.3\,B)\,X_t = Z_t$ .
- B is correct: $(1 + 0.3\,B)\,X_t = Z_t$ is equivalent to $X_t + 0.3\,X_{t-1} = Z_t$ or $X_t = -0.3\,X_{t-1} + Z_t$
- C is false: ACF is $\rho_X(2) = \phi_1^2 = 0.09$.
- D and E are correct: AR(1) is always invertible. AR(1) with $|\phi_1| < 1$ has a MA ($\infty$) representation and therefore is causal and stationary.
- F is false: $\rho_X(2) = \phi_1^{\,2} = 0.09 > \rho_X(3) = \phi_1^{\,3} = -0.027$, but $\rho_X(2) = \phi_1^{\,2} = 0.09 > \rho_X(1) = \phi_1 = -0.3$.

**Conclude: B, D and E are correct.**

# Check your understanding : ACF for MA(q)

**The MA(6) model $X_t = Z_t + \theta_1 Z_{t-1} + \theta_3 Z_{t-3} + \theta_6 Z_{t-6}$ is being analyzed, with $\theta_1$, $\theta_3$ & $\theta_6$ all non-zero.**
**How many of the following autocorrelations must be 0:**
**I. $\rho_X(2)$ II. $\rho_X(3)$ III. $\rho_X(4)$ IV. $\rho_X(5)$**


*Check your answer of the next slide*

*$E(Z_t Z_s) = 0$, $t \neq s$.*
*$E(Z_t^2) = \sigma_Z^2$ for all* t

*One way to think about this problem:*

If an MA model does not have any pair of Z terms that are k time lags apart,

then $\rho_X(k)$ must be 0:

$$E(X_t X_{t+k}) = E\{(Z_t + \theta_1 Z_{t-1} + \theta_3 Z_{t-3} + \theta_6 Z_{t-6})(Z_{t+k} + \theta_1 Z_{t+k-1} + \theta_3 Z_{t+k-3} + \theta_6 Z_{t+k-6})\} =$$

$$= E(Z_t Z_{t+k}) + \theta_1 E(Z_t Z_{t+k-1}) + \theta_3 E(Z_t Z_{t+k-3}) + \theta_6 E(Z_t Z_{t+k-6})$$

$$+ \theta_1 \{E(Z_{t-1} Z_{t+k}) + \theta_1 E(Z_{t-1} Z_{t+k-1}) + \theta_3 E(Z_{t-1} Z_{t+k-3}) + \theta_6 E(Z_{t-1} Z_{t+k-6})\}$$

$$+ \theta_3 \{E(Z_{t-3} Z_{t+k}) + \theta_1 E(Z_{t-3} Z_{t+k-1}) + \theta_3 E(Z_{t-3} Z_{t+k-3}) + \theta_6 E(Z_{t-3} Z_{t+k-6})\}$$

$$+ \theta_6 \{E(Z_{t-6} Z_{t+k}) + \theta_1 E(Z_{t-6} Z_{t+k-1}) + \theta_3 E(Z_{t-6} Z_{t+k-3}) + \theta_6 E(Z_{t-6} Z_{t+k-6})\}$$

Pairs of Z terms that are k time lags apart: 0, 1 ($Z_t$ & $Z_{t-1}$), 2 ($Z_{t-1}$ & $Z_{t-3}$), 3 ($Z_{t-3}$ & $Z_{t-6}$),
5 ($Z_{t-1}$ & $Z_{t-6}$), 6 ($Z_t$ & $Z_{t-6}$).

Indices are the same

(sign does not matter) if

k=0, 1, 3, 6

k= -1 (or 1), 0, 2, 5

k = -3 (or 3), -2(or 2), 0, 3

k= -6, -5, -3, 0 (or 6, 5, 3)

Index k = 4 is missing

# Check your understanding : ACF for MA(q)

The MA(6) model $X_t = Z_t + \theta_1 Z_{t-1} + \theta_3 Z_{t-3} + \theta_6 Z_{t-6}$ is being analyzed, with $\theta_1$, $\theta_3$ & $\theta_6$ all non-zero. How many of the following autocorrelations must be 0:
I. $\rho_X(2)$ II. $\rho_X(3)$ III. $\rho_X(4)$ IV. $\rho_X(5)$

This MA (6) does not have any pair of Z terms that are 4 time lags apart, so that $\rho_X(4)$ must be 0.

$Z_{t-1}$ & $Z_{t-3}$ are 2 lags apart and for k=2 term $E(Z_{t-1} Z_{t+k-3}) = E(Z_{t-1} Z_{t+2-3}) = \sigma_Z^2$, so that $\rho_X(2) \neq 0$.

$Z_t$ & $Z_{t-3}$ are 3 lags apart and for k=3 term $E(Z_t Z_{t+k-3}) = E(Z_t Z_{t+3-3}) = \sigma_Z^2$, so that $\rho_X(3) \neq 0$.

$Z_{t-1}$ & $Z_{t-6}$ are 5 lags apart and for k=5 term $E(Z_{t-1} Z_{t+k-6}) = E(Z_{t-1} Z_{t+5-6}) = \sigma_Z^2$, so that $\rho_X(5) \neq 0$.

*$E(Z_t Z_s) = 0$, $t \neq s$.*
*$E(Z_t^2) = \sigma_Z^2$ for all t*

Answer: Only one acf $\rho_X(4)$ must be zero.

$$
\begin{aligned}
E(X_t X_{t+k}) &= E\{(Z_t + \theta_1 Z_{t-1} + \theta_3 Z_{t-3} + \theta_6 Z_{t-6})(Z_{t+k} + \theta_1 Z_{t+k-1} + \theta_3 Z_{t+k-3} + \theta_6 Z_{t+k-6})\} = \\
&= E(Z_t Z_{t+k}) + \theta_1 E(Z_t Z_{t+k-1}) + \theta_3 E(Z_t Z_{t+k-3}) + \theta_6 E(Z_t Z_{t+k-6}) \\
&\quad + \theta_1 \{E(Z_{t-1} Z_{t+k}) + \theta_1 E(Z_{t-1} Z_{t+k-1}) + \theta_3 E(Z_{t-1} Z_{t+k-3}) + \theta_6 E(Z_{t-1} Z_{t+k-6})\} \\
&\quad + \theta_3 \{E(Z_{t-3} Z_{t+k}) + \theta_1 E(Z_{t-3} Z_{t+k-1}) + \theta_3 E(Z_{t-3} Z_{t+k-3}) + \theta_6 E(Z_{t-3} Z_{t+k-6})\} \\
&\quad + \theta_6 \{E(Z_{t-6} Z_{t+k}) + \theta_1 E(Z_{t-6} Z_{t+k-1}) + \theta_3 E(Z_{t-6} Z_{t+k-3}) + \theta_6 E(Z_{t-6} Z_{t+k-6})\}
\end{aligned}
$$

Indices are the same (sign does not matter) if

k= 0, 1, 3, 6

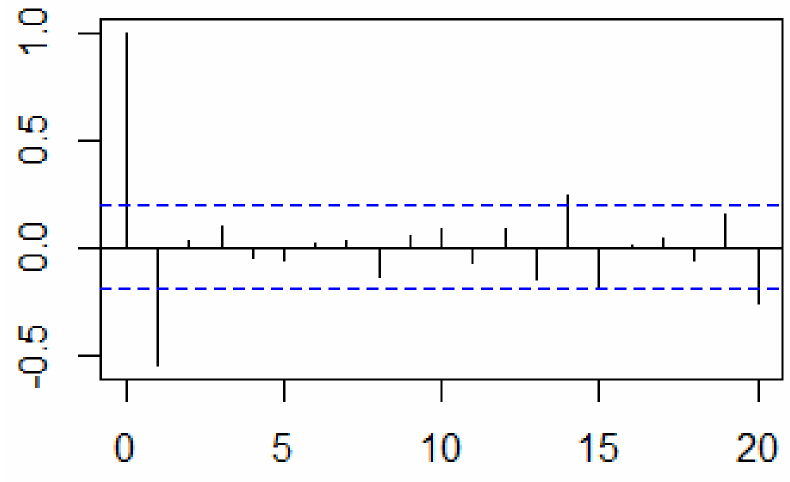
k= -1 (or 1), 0, 2, 5

k = -3 (or 3), -2(or 2), 0, 3

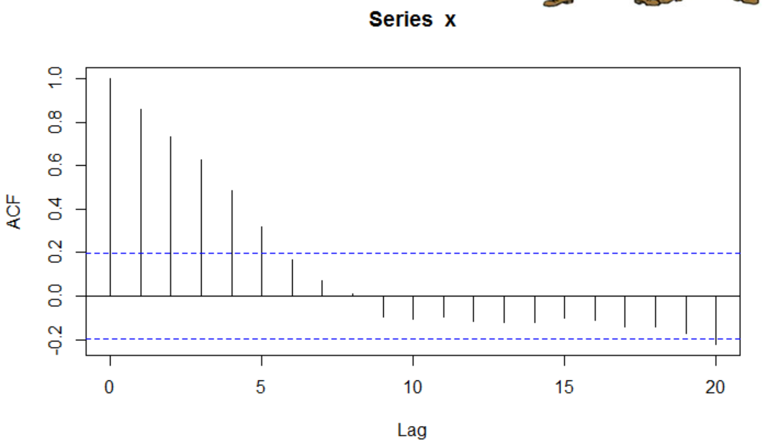
k= -6, -5, -3, 0 (or 6, 5, 3)

# Check your Understanding:
# MA(q): Model Identification from Sample ACF

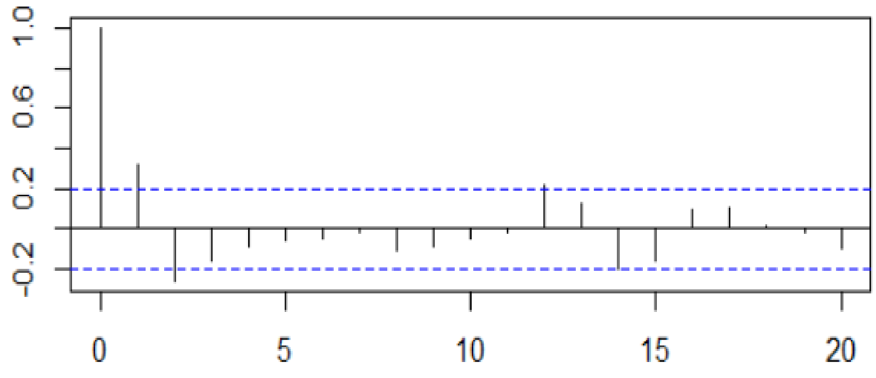
A



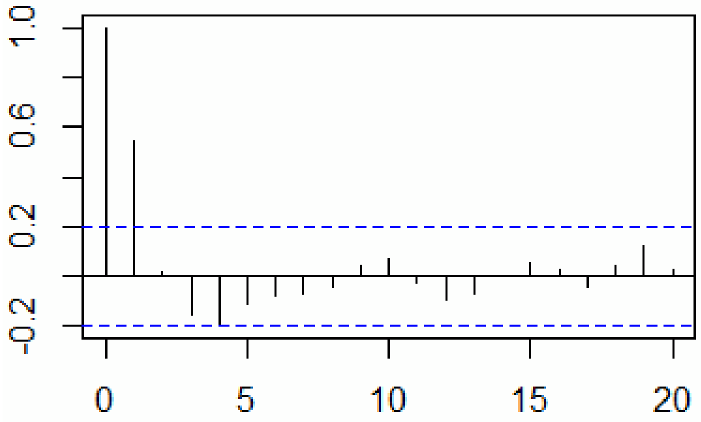

C




B




D

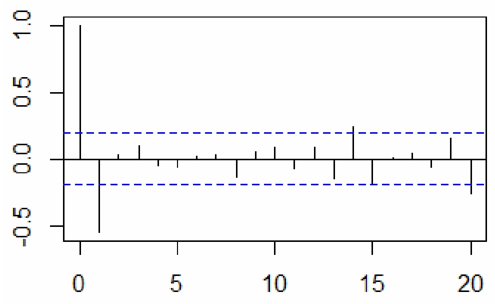



**Identify graphs of sample ACFs corresponding to MA(q) processes. Determine q.**

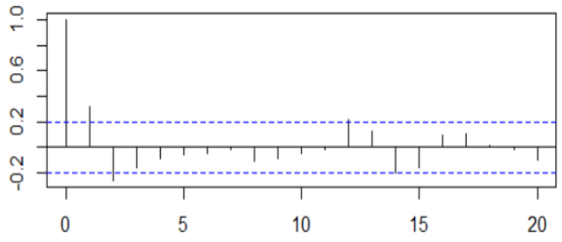
**… check your answers on the next slide**

# MA(q): Model Identification from Sample ACF

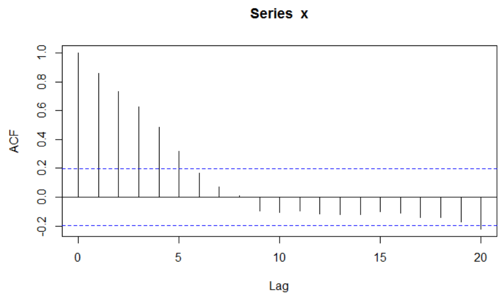
A



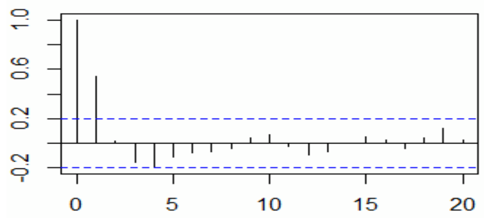

B




C




D




**Identify graphs of sample ACFs corresponding to MA(q) processes. Determine q.**

**Discussion:**

**(A) Sample ACFs at lag 1 is outside the confidence interval, but is inside the intervals for lags k >1.**
**Thus, the model is MA(1). ($\rho_X(14)$ and $\rho_X(20)$ are almost in the interval.)**

**(B) Sample ACFs at lag 2 is outside the confidence interval, but is inside the intervals for lags k >2.**
**Thus, the model is MA(2).**

**(C) Sample ACFs are large and do not follow pattern of MA(q).**

**(D) Sample ACFs at lag 1 is outside the confidence interval, but is inside the intervals for lags k >1.**
**Thus, the model is MA(1).**

# Check of Understanding: MA(q) and Stationarity:

**You are given the following statements about stationarity:**

**I. Linear models for time series are stationary when they include functions of time.**

**II. All moving averages processes are stationary.**

**III. All random walk processes are nonstationary.**

**Determine which of the above statements are true:**

**A. None are true**

**B. I and II only**

**C. I and III only**

**D. II and III only**

**E. The answer is not given by (A), (B), (C ) or (D).**

**… check your answers on the next slide**

# MA(q) and Stationarity : Check of Understanding -- Discussion

**You are given the following statements about stationarity:**

**I. Linear models for time series are stationary when they include functions of time.**

**II. All moving averages processes are stationary.**

**III. All random walk processes are nonstationary.**

**Determine which of the above statements are true:**

**A. None are true**

**B. I and II only**

**C. I and III only**

**D. II and III only**

**E. The answer is not given by (A), (B), (C ) or (D).**

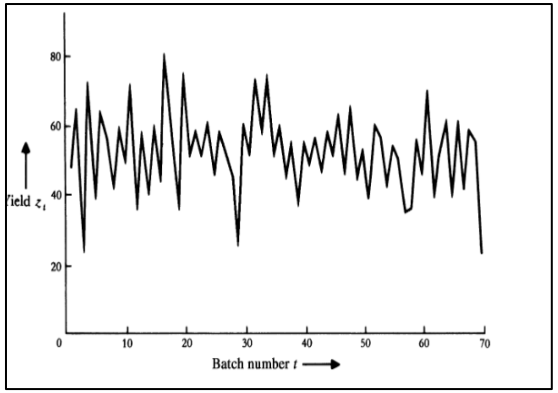
**Quick answer: D**

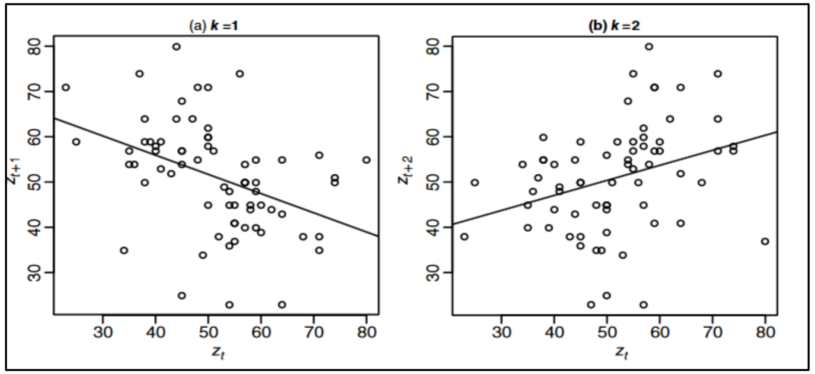
**I is false: Random walk (discussed in § 2.1.5, slide 55 of week 1) is linear but not stationary.**

**II is correct: All MA(q) models are stationary, (see slide 17 of week 2)**

**III is correct: Random walk is not stationary, its variance depends on time t (see slide 55 of week 1)**

# Check of Understanding: Autocorrelation




Scatter diagrams at lags (a) $k$ = 1 and (b) $k$ = 2 for the data $Z_t$ presented at the graph on the left.

**Based on the above graphs, choose a model from the list below that is appropriate or this data sample.**

**Notations: Observed process is denoted as $Z_t$. $W_t$ is a mean zero unit variance white noise.**

**A.** $Z_t = W_t - 0.2\, W_{t-1}$.

**B.** $Z_t = W_t - 0.2\, W_{t-2.}$

**C.** $Z_t = -\, 0.2\, Z_{t-1} + W_t$

**D.** $Z_t = 0.2\, Z_{t-1} + W_t$

**E.** $Z_t = Z_{t-1} + W_t\, ;\, Z_1 = W_1$ .

***Check your answer of the next slide***

# Check of Understanding: Autocorrelation

**Based on the graphs below, choose a model from the list below that is appropriate or this data sample.**

**Notations: Observed process is denoted as $Z_t$. $W_t$ is a mean zero unit variance white noise.**

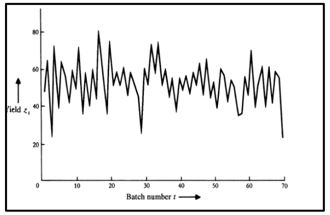
**A. $Z_t = W_t - 0.2\ W_{t-1}$.**

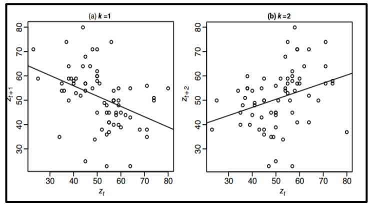
**B. $Z_t = W_t - 0.2\ W_{t-2.}$**

**C. $Z_t = -\ 0.2\ Z_{t-1} + W_t$**

**D. $Z_t = 0.2\ Z_{t-1} + W_t$**

**E. $Z_t = Z_{t-1} + W_t\ ;\ Z_1 = W_1$ .**





**Scatter diagrams at lags (a) $k$ = 1 and (b) $k$ = 2 for the data $Z_t$ presented at the graph on the left.**

**Analysis:**

**Step 1: This is a stationary data, thus it is appropriate to apply MA and AR models.**

- **Conclude: Eliminate random walk model (E), it is non-stationary. (See slide 55, week 1)**

**Step 2: From scatter diagrams determine that correlation at lag 1 negative and at lag 2 positive: ρ(1) < 0, ρ(2) > 0.**

- **Conclude: Eliminate A because MA(1) processes have ρ(2) = 0.**

**Step 3: Recall that for MA(2) model (slide 19 of week 2):** $\rho_X(1) = \dfrac{\theta_1+\theta_1\theta_2}{1+\theta_1{}^2+\theta_2{}^2}$ $\quad \rho_X(2) = \dfrac{\theta_2}{1+\theta_1{}^2+\theta_2{}^2}$

- **Conclude: Eliminate B because for MA(2) models with $\theta_2 < 0$ holds: ρ(2) < 0.**

**Step 4: Recall that for AR(1) model (slide 43 week 2):** $\rho_X(1) = \phi_1,\ \rho_X(2) = \phi_1^2.$

- **Conclude: Eliminate D, because it has $\phi_1 = 0.2 > 0$ so that $\rho_X(1) = \phi_1 > 0$.**
- **Conclude: AR(1) model with $\phi_1 < 0$ has ρ(1) < 0, ρ(2) > 0, so it is an appropriate model.**

**Final answer: Choose C ($\phi_1 = -0.2 < 0$ )**

# Check of Understanding: More on ACF, MA, AR, etc., etc., etc. ...


**Determine which of the following statements are true:**

**A. Values of the sample autocovariance function $\widehat{\gamma}_X(k)$, k = 1, 2, …, lie between -1 and +1.**

**B. Values of the theoretical autocorrelation function $\rho_X(k)$ , k = 1, 2, …, lie between -1 and +1, but the sample autocorrelation function $\widehat{\rho}_X(k)$ , k = 1, 2, …, can take values outside the unit interval.**

**C. Sample autocorrelation function can be calculated for any time series, including non-stationary.**

**D. Autocorrelation function $\rho_X(k)$ , k = 1, 2, …, can be calculated for any time series.**

**E. An invertible moving average model of order one can be written as $Y_t = C + Z_t + \theta_1 Z_{t-1}$.**
**(i) The parameter $\theta_1$ must take values in the interval [ -1, 1].**
**(ii) Mean of Y is C.**
**(iii) To assure stationarity of the model, the absolute value of the parameter $\theta_1$ must be less than 1.**

**F. A stationary autoregressive model of order one can be written as $X_t= C+ \phi_1 X_{t-1}+Z_t$ .**
**(i) The absolute value of the parameter $\phi_1$ must be less than 1 for the model to be invertible.**
**(ii) If the parameter $\phi_1 = 0$, then the model reduces to a white noise process.**
**(iii) If the parameters C=0 and $\phi_1 = 1$, then the model reduces to a random walk.**
**(iv) Mean of X is C.**

**… check your answers on the next two slides**

# Check of Understanding: More on ACF, MA, AR, etc., etc., etc. ...

**Determine which of the following statements are true:**

**A. Values of the sample autocovariance function $\widehat{\gamma}_X(k)$, k = 1, 2, …, lie between -1 and +1. (False)**

**B. Values of the theoretical autocorrelation function $\rho_X(k)$ , k = 1, 2, …, lie between -1 and +1, but the sample autocorrelation function $\widehat{\rho}_X(k)$ , k = 1, 2, …, can take values outside the unit interval. (False)**

**C. Sample autocorrelation function can be calculated for any time series, including non-stationary. (True)**

**D. Autocorrelation function $\rho_X(k)$ , k = 1, 2, …, can be calculated for any time series. (False)**

**Explanations:**

**A is false: values of both theoretical and sample acvf can range from - ∞ to +∞ (slides 36, 38, 41 and 52 of week 1)**

**B is false: both theoretical and sample acf are normalized to have values between -1 and +1. (slides 36, 38, 41 and 52 of week 1)**

**C is true: one may always substitute values from the sample into the formula for sample acf on slide 38 of week 1.**

**D is false: for nonstationary times series, autocorrelation function $\rho_X(t,s)$ may depend on times t and s. In this case it is impossible to represent is as a function of a lag k (slide 52 of week 1).**

# Check of Understanding: More on ACF, MA, AR, etc., etc., etc. ...

**Determine which of the following statements are true:**

**E. An invertible moving average model of order one can be written as $Y_t = C + Z_t + \theta_1 Z_{t-1}$.**

**(i) The parameter $\theta_1$ must take values in the interval [ -1, 1]. (False)**

**(ii) Mean of Y is C. (True)**

**(iii) To assure stationarity of the model, the absolute value of the parameter $\theta_1$ must be less than 1. (False)**

**Long Answers (review properties of MA(1) model on slides 3-4 of week 2)**

**(i) Is false: for invertibility, parameter $\theta_1$ must take values in the interval (-1, 1), i.e., $\theta_1$ cannot take values -1 or 1.**

**(ii) Is true: $E\,Y_t = C + E\,Z_t + \theta_1 E\,Z_{t-1} = C + 0 = C$.**

**(iii) Is false: MA is always stationary as a linear combination of values of WN $Z_t$, which is stationary.**

**F. A stationary autoregressive model of order one can be written as $X_t = C + \phi_1 X_{t-1} + Z_t$.**

**(i) The absolute value of the parameter $\phi_1$ must be less than 1 for the model to be invertible. (False)**

**(ii) If the parameter $\phi_1 = 0$, then the model reduces to a white noise process. (True)**

**(iii) If the parameters C=0 and $\phi_1 = 1$, then the model reduces to a random walk. (True)**

**(iv) Mean of X is C. (False)**

**Long Answers (review properties of AR(1) on slide 51 of week 2)**

**(i) Is false: AR models are always invertible: $Z_t = - C + X_t - \phi_1 X_{t-1}$. $|\phi_1| < 1$ is required to assure stationarity.**

**(ii) Is true: $X_t = C + Z_t$ is a WN with added constant C to make mean C.**

**(iii) Is true: $X_t = X_{t-1} + Z_t$ is a definition of the random walk (slide 55 of week 1)**

**(iv) Is false: $\mu := E\,X_t = C + \phi_1 E\,X_{t-1} + E\,Z_t = C + \phi_1 \mu + 0$, that is, $\mu = C + \phi_1 \mu$ so that $\mu = C/(1 - \phi_1)$.**