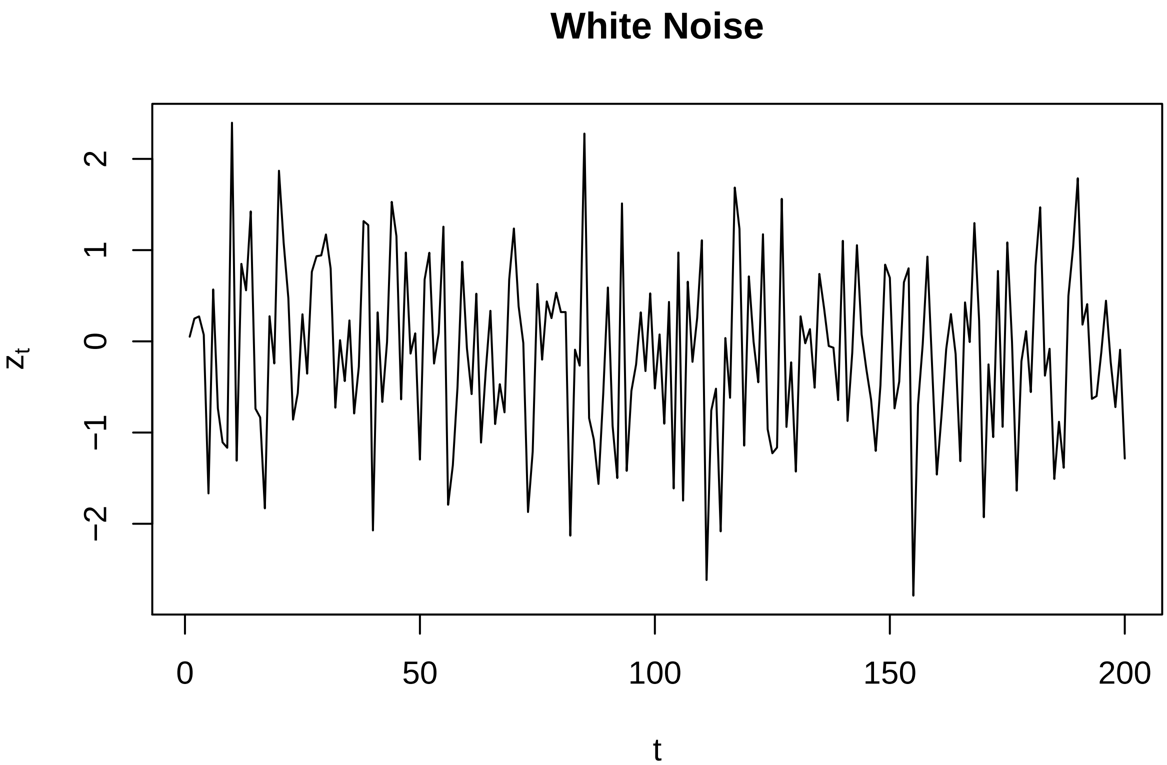
Lab 2

Pstat 174/274

Characteristics of Time Series

(1) **White noise**. Simulate and plot $n = 200$ values of a Gaussian white-noise process with variance $\sigma_Z^2 = 1$, i.e., $X_t = Z_t$, where $Z_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_Z^2)$.

```
z_t <- rnorm(200,0,1)
plot(z_t,xlab = "t",ylab = expression(z[t]),type = "l",main = "White Noise")
```

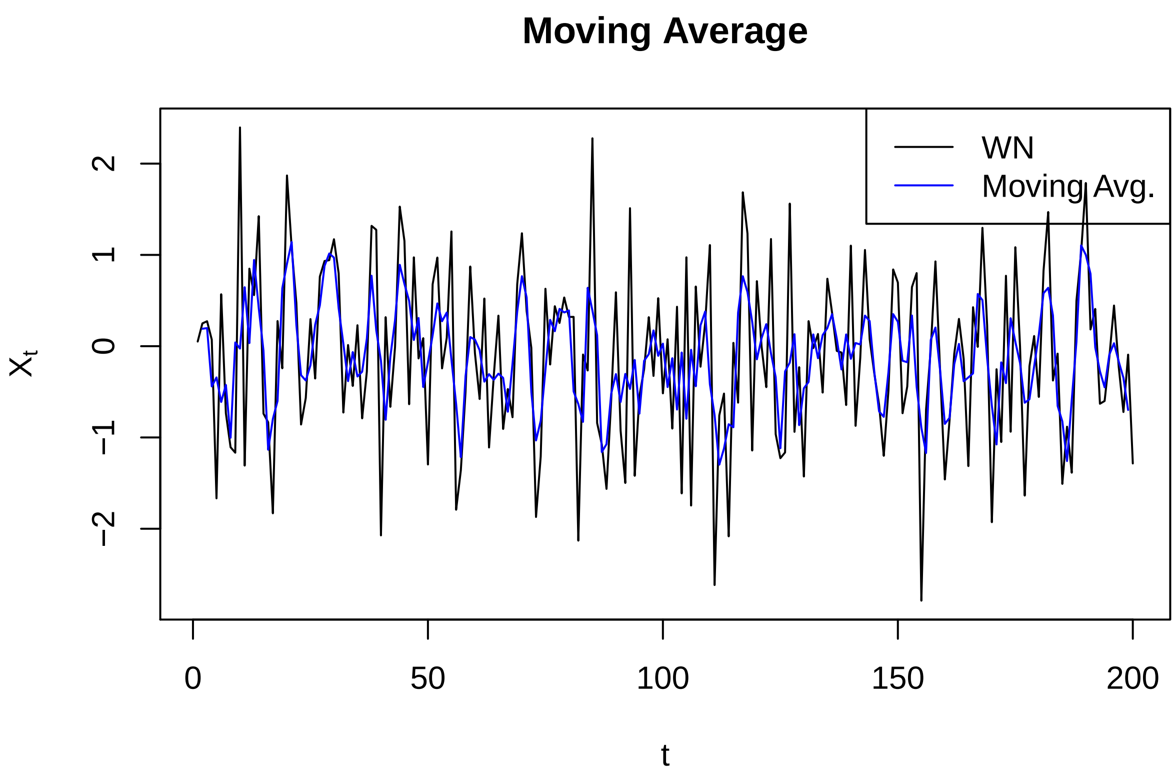


(2) **Moving averages**. Using the above gaussian process x_t, use the `filter` command to construct a moving average process of the form:

$$y_t = (x_{t-1} + x_t + x_{t+1})/3$$

(notice that for this example the current value of y_t is the average of the previous, current and next values of x_t). Then, plot y_t and x_t together with different colors. Do you observe any difference?

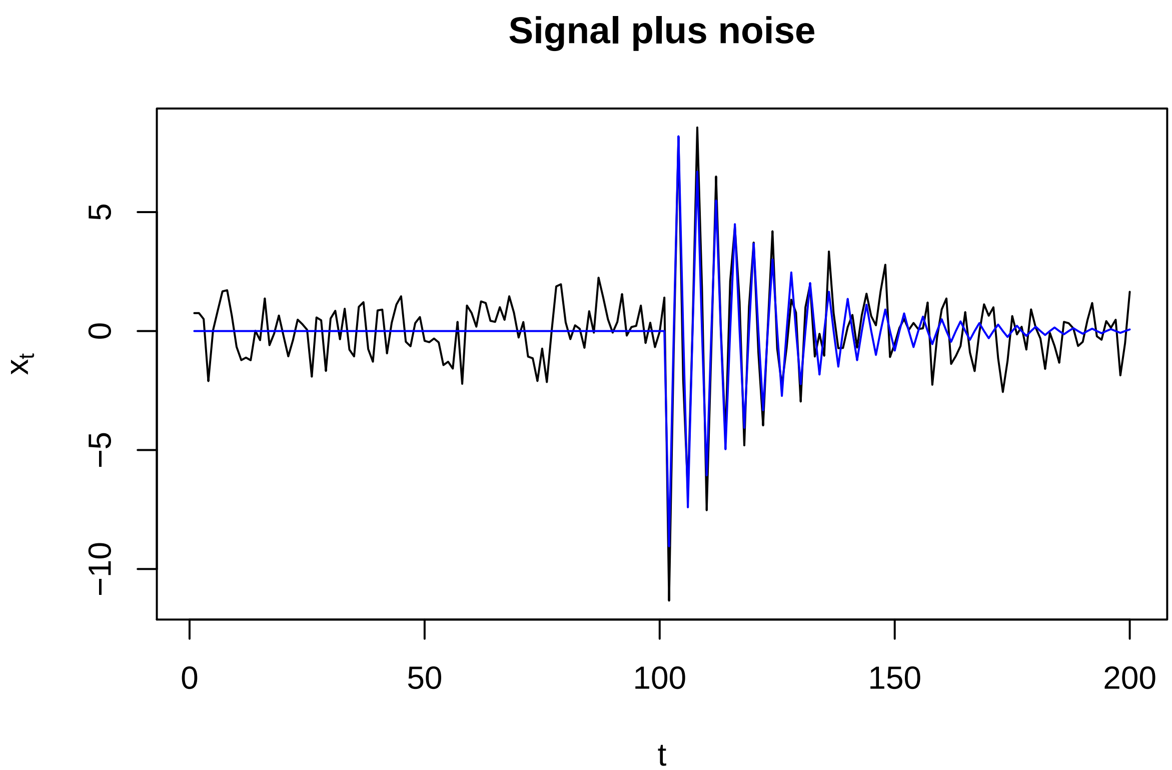
```
# See help for filter
?filter
y_t = filter(z_t, filter = rep(1/3,3), sides = 2, method = "convolution")
# Plot of white-noise
plot(z_t,xlab = "t",ylab = expression(X[t]),type = "l",main = "Moving Average")
# Plot of moving-average
lines(y_t,col = "blue")
# Add legend
legend("topright",c("WN", "Moving Avg."),col = c("black","blue"),lty = 1)
```



(3) **Signal in noise**. Consider a signal-plus-noise model of the general form $x_t = s_t + z_t$; where s_t is regarded as the signal and z_t is Gaussian white noise with $\sigma_Z^2 = 1$. Simulate and plot $n = 200$ observations using

$$s_t = \begin{cases} 0 & \text{if } t = 1, \dots, 100; \\ 10 \exp\{-\frac{(t-100)}{20}\} \cos(2\pi t/4) & \text{if } t = 101, \dots, 200. \end{cases}$$

```
s_t <- c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*101:200/4))
x_t <- ts(s_t + rnorm(200, 0, 1))
# Plot of time series
plot(x_t, xlab  = "t",ylab  = expression(x[t]),type = "l", main = "Signal plus noise")
# Add signal
lines(ts(s_t),col = "blue")
```

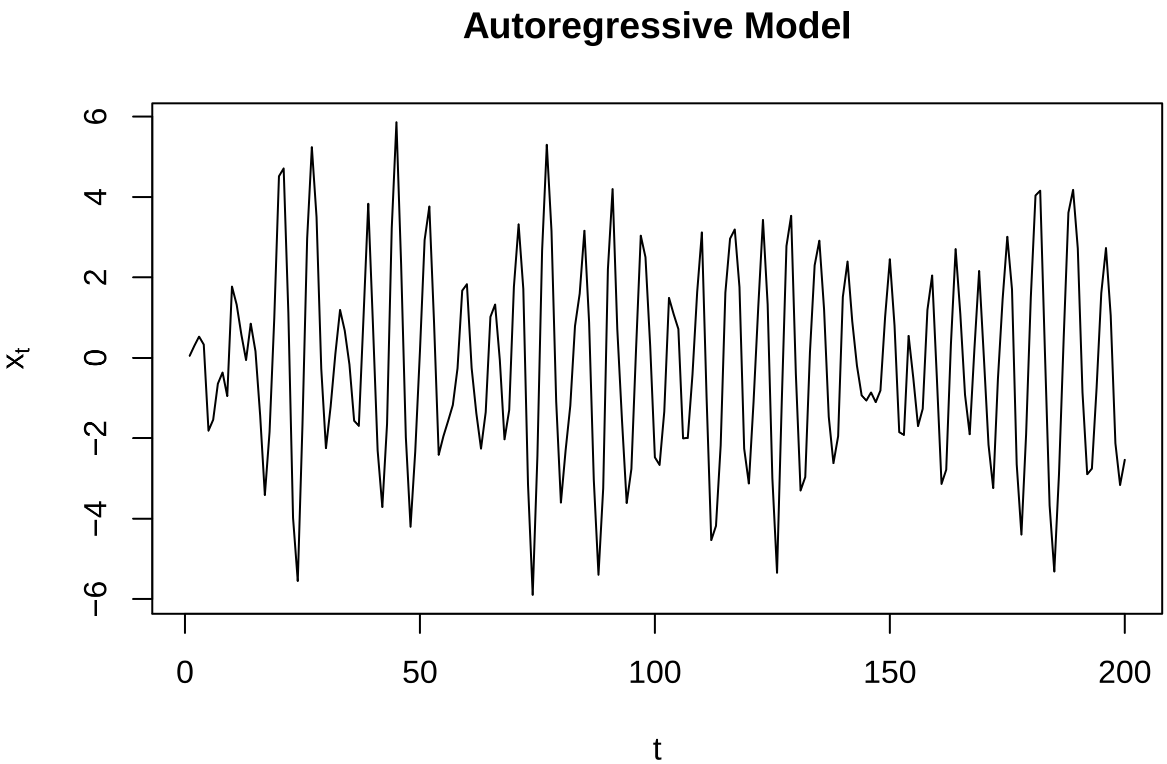


(4) **Autoregressions**. Suppose we consider the white noise series in (1) as input and calculate the output using the equation

$$x_t = x_{t-1} - 0.9x_{t-2} + z_t,\ z_t \overset{iid}{\sim} \mathcal{N}(0,1)$$

successively. The series x_t represents a regression or prediction of the current value x_t of a time series as a function of the past two values of the series, and, hence, the term autoregression is suggested. Simulate and plot the autorregressive process using `filter`.

```
x_t <- filter(z_t,filter = c(1,-0.9),method = "recursive")
plot(x_t,xlab  = "t",ylab  = expression(x[t]),type = "l", main = "Autoregressive Model")
```



(5) **Random Walk Process**.

a. Show that the random walk

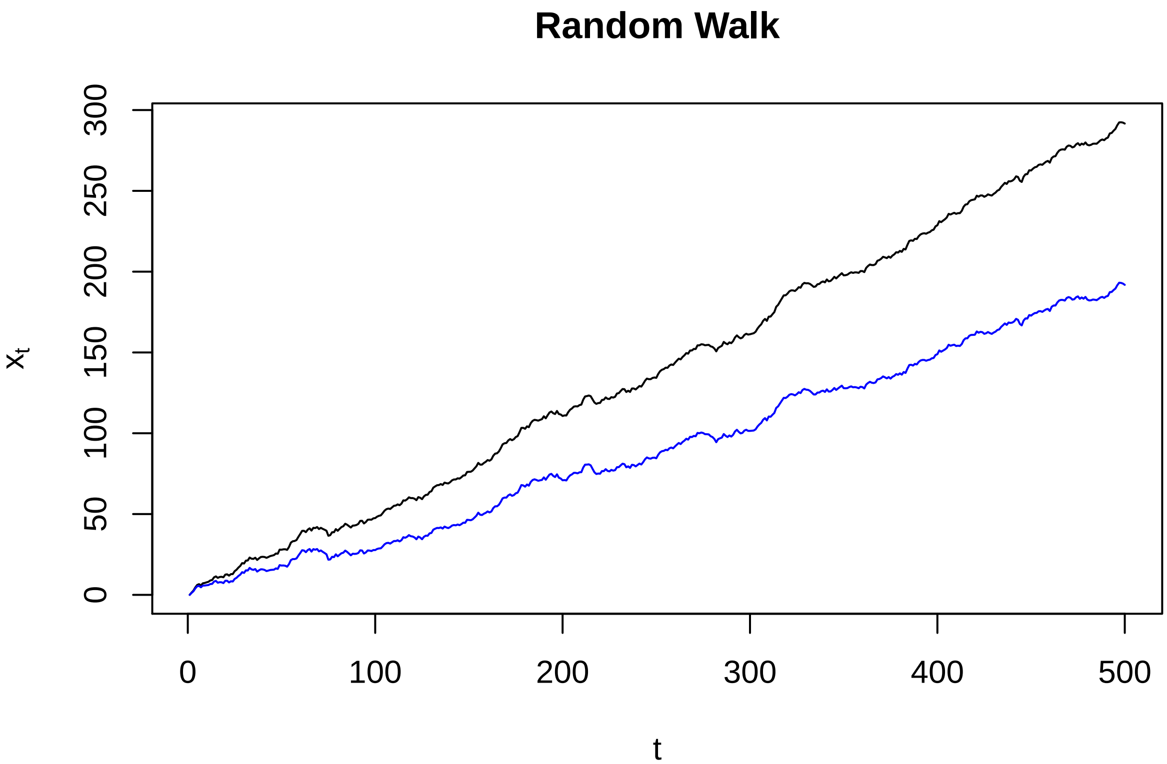
$$X_t = \delta + X_{t-1} + Z_t,\ Z_t \overset{iid}{\sim} WN(0, \sigma_Z^2)$$

can be re-written as the cumulative sum of white noise variates: $X_t = \delta t + X_0 + \sum_{j=1}^{t} Z_j$.

b. Simulate $n = 200$ observations of a random-walk with *drift* $\delta = 0.6, 0.4$; initial condition $x_0 = 0$ and $\sigma_Z^2 = 1$. Then, plot both realizations using different colors.

c. Is the random-walk with drift δ a (weakly) stationary process?

```
z_t <- rnorm(499,0,1)
x_t <- c(0,cumsum(z_t))
rw1 <- 0:499*0.6 + x_t
rw2 <- 0:499*0.4 + x_t
plot(rw1,type = "l",xlab = "t",ylab = expression(x[t]),
          main = "Random Walk")
lines(rw2,col = "blue",type = "l")
```



Measures of Dependence of a Time Series

(6) Consider an MA(2) process, given by

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}, \text{ where } Z_t \overset{iid}{\sim} \mathcal{N}(0,1).$$

Using `R`, simulate a time series of length 100 from an MA(2) process with the following values of the coefficients:

a. $\theta_1 = 0.45, \theta_2 = 0.55$

b. $\theta_1 = -0.45, \theta_2 = 0.55$

For each simulated time series, plot the sample ACF using `acf` and the theoretical ACF. What do you notice?

Note: The (theoretical) ACF of an MA(2), $\rho_X(k) := Corr(X_t, X_{t+k})$, is given by

$$\rho_X(k) = \begin{cases} 1 & \text{if } k = 0 \\ \frac{\theta_1 + \theta_1\theta_2}{1+\theta_1^2+\theta_2^2} & \text{if } |k| = 1 \\ \frac{\theta_2}{1+\theta_1^2+\theta_2^2} & \text{if } |k| = 2 \\ 0 & \text{if } |k| > 2 \end{cases}$$

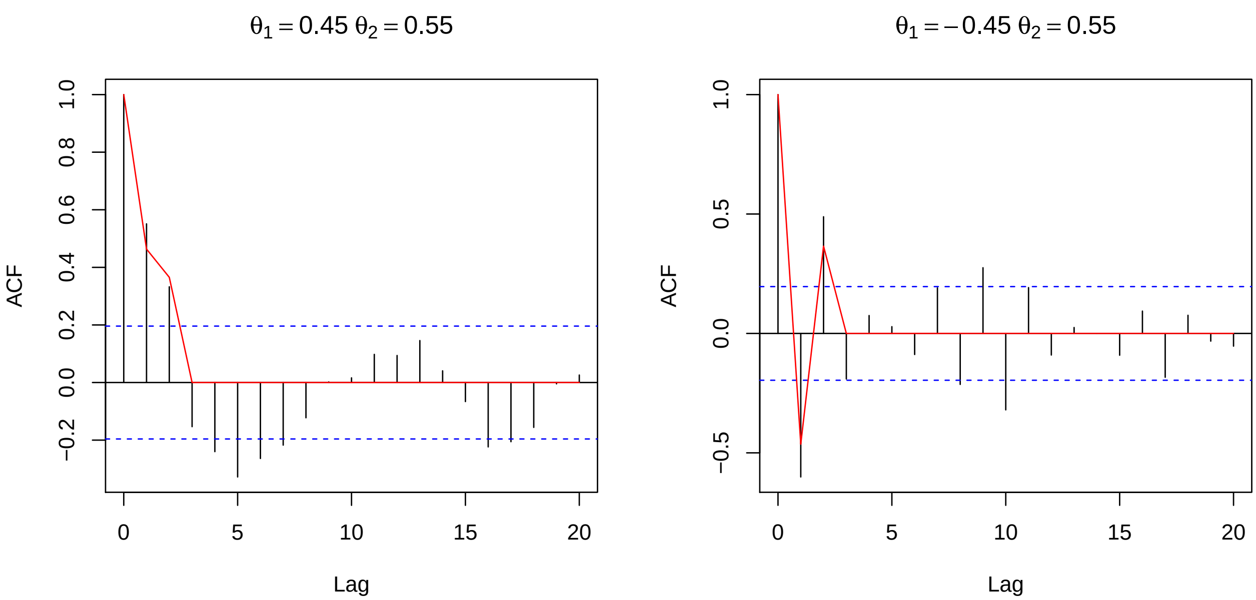
```r
# a
theta_1 <- 0.45
theta_2 <- 0.55
var_ma <- 1+theta_1^2+theta_2^2

# Simulate MA
x1 <- arima.sim(n = 100,model = list(ma=c(theta_1,theta_2)))
# Theoretical ACF
theo_acf1 <- c(var_ma,(theta_1 + theta_1*theta_2),theta_2,rep(0,18))/var_ma
```

```r
# b
theta_1 <- -0.45
theta_2 <- 0.55
var_ma <- 1+theta_1^2+theta_2^2
x2 <- arima.sim(n = 100,model = list(ma=c(theta_1,theta_2)))
theo_acf2 <- c(var_ma,(theta_1 + theta_1*theta_2),theta_2,rep(0,18))/var_ma

# Plot both ACFs
op <- par(mfrow = c(1,2))
acf(x1,main = expression(theta[1] == 0.45~theta[2] == 0.55)) # Sample auto-correlation
lines(x = 0:20,y = theo_acf1,col = "red") # Add theoretical ACF

acf(x2,main = expression(theta[1] == -0.45~theta[2] == 0.55))
lines(x = 0:20,y = theo_acf2,col = "red")
```




```r
par(op)
```