PSTAT 174/274 Spring 2023: Homework 7

1. Time series $\{X_t\}$ follows stationary AR(1) model $X_t = \phi_0 + \phi_1 X_{t-1} + Z_t$, $Z_t \sim N(0, \sigma_Z^2)$. Determine which of the following statements about this model is **false.**
A. the parameter ϕ_0 must not equal 1.
B. The absolute value of the parameter ϕ_1 must be less than 1.
C. If the parameter $\phi_1 = 0$, then the model reduces to a white noise process.
D. If the parameter $\phi_1 = 1$, then the model is a random walk.
E. Only the immediate past value X_{t-1}, is used as a predictor for X_t.

2. (Updating forecasts) You use AR(1) model $X_t - 100 = 0.6(X_{t-1} - 100) + Z_t$ to represent a time series of 100 observations. You are given that $X_{100} = 110$. You later observe $X_{101} = 95$. Calculate $P_{101}X_{103} - P_{100}X_{103}$.
(*Hint: review Example 13.1 on slides 9 - 10 of Week 7.*)

3. You are given the AR(3) model for X_t, a company's revenue for year t:
$X_t = 5 + 0.85X_{t-1} - 0.02X_{t-3} + Z_t, Z_t \sim WN(0, \sigma^2)$.

The revenues for the last 4 years are as follows:

Year	Revenue
2019	20
2020	15
2021	22
2022	19

Forecast the expected revenue for 2024. (*Hint: review Example 13.2 of week 7, slide 11.*)

4. A Gaussian AR(1) model was fitted to a time series based on a sample of size n. You are given $\hat{\phi}_1 = 0.8$, $\hat{\mu} = 2$, $\hat{\sigma}_Z^2 = 9 \times 10^{-4}$, $x_n = 2.05$. Write the 95% prediction interval for the observation three periods ahead.
Hint: review Example 13.1 of Week 7; slide 10. Do not forget that the mean is not 0*!*

The following problem is for students enrolled in PSTAT 274 ONLY

G1. A Gaussian AR(1) model was fitted to a time series based on a sample of size $n = 51$. You are given: $\hat{\mu} = 16.75$, $\hat{\phi}_1 = 0.75$. The last observation was $x_{51} = 20.25$, and the sum of the squares of the 51 residuals is 75.7. Determine the upper bound of the shortest 95% probability limit for the forecast of the observation two time periods ahead.