PSTAT 174/274 Spring 2023 Homework 6

1. The three models AR(2), MA(1), and ARMA(2,1) are fitted to the following time series:

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2010	112	104	100	96
2011	101	101	105	94
2012	106	106	108	110

The results using R are as follows:

AR(2):

	ar1	ar2	intercept
	0.2618	0.0868	104.227
st. error	0.3289	0.3326	2.372
$\sigma^2 = 25.02$	log likelihood = -36.4		

MA(1):

	ma1	intercept
	0.1899	103.798
st. error	0.2574	1.7373
$\sigma^2 = 25.66$		log likelihood = -36.51

ARMA(2,1):

	ar1	ar2	ma1	intercept
	-0.5547	0.4437	0.9779	104.4542
st. error	0.3329	0.33267	0.235	2.4584
$\sigma^2 = 21.31$		log likelihood = -36.02		

The models are ranked using Akaike Information Criterion, Corrected for bias (AICC) (see §11.3 of Lecture Notes or slide 32 of week 5).

AICC $= -2\times$ log-likelihood$+2\times\frac{rn}{n-r-1}$ where n is the sample size and r is the number of free parameters, most often $r = p + q + 1$.

Determine the order from the best to worst model. Give full explanation on how you arrived to your answer. Show calculations.

A. AR(2), MA(1), ARMA (2,1)
B. AR(2), ARMA (2,1), MA(1)
C. MA(1), AR(2), ARMA (2,1)
D. MA(1), ARMA (2,1), AR(2)
E. ARMA (2,1), AR(2), MA(1).

2. In modeling the weekly sales of a certain commodity over the past six months, the time series model $X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$ was thought to be appropriate. Suppose the model was fitted and the autocorrelations of the residuals were:

k	1	2	3	4	5	6	7	8
$\hat\rho_{\hat W}(k)$	-.04	-.50	.03	-.01	.01	.02	.03	-.01
st. dev $\hat\rho_{\hat W}(k)$	.08	.10	.11	.11	.11	.11	.11	.11

Is the assumed model really appropriate? If not, how would you modify the model? Explain.

Hint: Check slides 8 - 9 of Lecture 11 for the 95% confidence intervals for autocorrelation function of the fitted residuals. You might also find slide 26 of Week 2 and slide 15 of Lecture 11 useful.

3. Suppose that in a sample of size 100 from an AR(1) process with mean μ, $\phi = 0.6$, and $\sigma^2 = 2$, we obtain $\bar{x}_{100} = 0.271$. Construct an approximate 95% confidence interval for μ. Are the data compatible with the hypothesis that $\mu = 0$?

Help: (i) Here $\sigma^2 \equiv \sigma_Z^2$ is the variance of noise Z_t.
(ii) Large sample distribution of $\bar{X}_n$ is given as follows (See, for example, slide 48 or §10.1 of Week 4): For n large, distribution of the sample mean $\bar{X}_n$ is approximately normal with mean $\mu \equiv EX_t$ and variance $n^{-1}v$, where $v \approx \gamma_X(0) + 2\sum_{h=1}^{\infty}\gamma_X(h)$.
(iii) For ACVF for AR(1) process see slide 43 of Week 2.

The following problem is for students enrolled in PSTAT 274 ONLY

G1. Two hundred observations of a time series, $X_1, \ldots, X_{200}$, gave the following sample statistics: sample mean: $\bar{x}_{200} = 3.82$; sample variance: $\hat{\gamma}(0) = 1.15$; sample ACF: $\hat{\rho}(1) = .427$, $\hat{\rho}(2) = .475$, $\hat{\rho}(3) = .169$.

(a) Based on these sample statistics, is it reasonable to suppose that $\{X_t - \mu\}$ is a white noise?
(b) Assuming that $\{X_t - \mu\}$ can be modeled as the AR(2) process,

$$X_t - \mu - \phi_1(X_{t-1} - \mu) - \phi_2(X_{t-2} - \mu) = Z_t,$$

where $Z_t \sim \mathrm{IID}(0, \sigma_Z^2)$, find estimates of μ, ϕ_1, ϕ_2, and σ_Z^2.
(c) Assuming that the data was generated from an AR(2) model, derive estimates of the PACF for all lags $h \geq 1$.