# Optimal Trajectory Implementation for VTOL Fixed-Wing Quadcopter

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***Abstract*— This report will outline the implementation process to create an optimal trajectory to minimize the time to rise from hover to climb for a newly designed and manufactured Vertical Takeoff and Landing (VTOL) Fixed-Wing Quadcopter, which will be referenced to as a quadplane throughout this report. The quadplane mission is to deliver medical supplies to rural areas or in congested cities. Consequently, being able to travel from a home location to a target destination as quickly as possible is essential for high-value medical supplies. To resolve this issue, an optimal control problem is created to minimize the time for the quadplane to rise from its hover altitude to its target cruise altitude.**

## I. INTRODUCTION

There is a global need for all people to receive essential medical supplies, and the quadplane unmanned aerial vehicle (UAV) referenced throughout this paper is designed specifically to solve this issue. While all people deserve access to medical supplies, there are often issues that can arise to prevent them from receiving these supplies in a timely manner. Be it congested traffic in large cities, natural disasters blocking roadways to rural areas, or any number of unpredictable events, there is a certain need for alternative modes of transportation to deliver medical supplies. As of now, the most common mode of transportation is through ground vehicles. Introducing a small UAV capable of carrying a small payload will provide an alternative mode of transportation to help deliver smaller lab vials, blood samples, or other lightweight supplies.

Medical supplies are often very time-sensitive and may have a very small window for transportation before the supplies are no longer usable. As such, the quadplane will not be focused on optimizing its energy usage, so much as it will attempt to minimize the travel time to deliver its payload as quickly as possible. To account for this, an optimal control problem can be formulated to minimize the time for the quadplane to rise from its hover altitude to its cruise altitude. By introducing an optimal control problem with a state space model and control input, a technique known as direct collocation can be utilized to convert the optimal control problem into a constrained optimization problem which can be solved to produce the optimal state variables for any given optimal control input. The optimal state variables can then be used to create an optimal trajectory.

The proposed quadplane can be referenced in Fig. 1

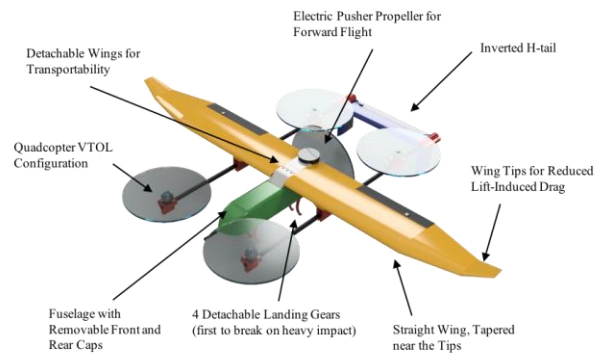


Fig. 1: Quadplane walk-around

The quadplane design was selected for its ease of manufacturing and simplistic design. The other possible designs for convertible VTOL and horizontal flight vehicles include tail-sitters, tilt-rotors, and other designs which require moving parts. The quadplane does not have any moving parts, thus increasing reliability. Also, it should be noted that the quadplane design is entirely 3D-printed, and the only materials used in the design are PLA and carbon fiber rods which act as spars and mounting rods.

## II. QUADPLANE OVERVIEW

### A. *Quadplane Design Specifications*

The quadplane has a set of unique requirements necessary for its specific mission. As it will need to travel through congested cities, it must be capable of takeoff in a very narrow takeoff zone. Because of this, the aircraft has a takeoff zone of a 15m high by 3m diameter cylinder. From here, the vehicle will transition to horizontal flight, which is where it will remain for most of the flight until it reaches its destination, upon which it will transition back to vertical flight to land. Fig. 2 shows the proposed mission profile.

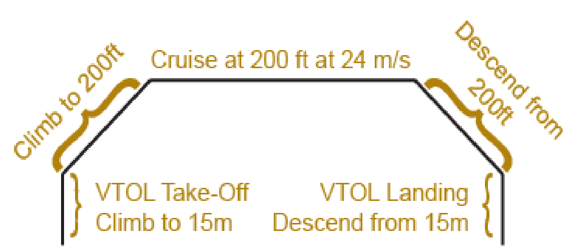


Fig. 2: Mission Profile

As seen from the mission profile, the quadplane has a cruise altitude of 61m (200ft) but is constrained to takeoff in horizontal flight from at least a 15m altitude. Further, the design has a design range of 25 miles (40 km). The full list of design parameters can be seen in Table 1.

Table 1. Design Requirements

| | |
|---|---|
| Design Range | 25 miles |
| Cruise Speed | 24 m/s |
| Payload Weight | 0.500 kg |
| VTOL (Takeoff and Landing) | 15 m above ground level |
| Cruise Altitude | 61 m above ground level |

For the optimization problem solved in this report, the VTOL phases of the project will be ignored, and the optimal control problem will only be solved to generate a trajectory for the horizontal flight. Doing so greatly simplifies the optimization problem, but it can also be reasonably assumed that a minimum-time problem would not offer any substantial impact for the quadplane in the vertical phase. The quadplane is extremely inefficient while in the vertical phase, as it can be modeled as a thin rectangular plate travelling normal to the freestream air. Because it is so inefficient, it can be reasonably assumed that the quickest time to reach the hover altitude of 15m will always be by travelling directly vertically.

### *B. Quadplane design and performance parameters*

To model this optimization problem, it is necessary to know all the design and performance parameters. Beginning with the design parameters, Table 2 shows the design characteristics necessary to solve this problem

Table 2: Design parameters

| | |
|---|---|
| $Mass$ | $4.2\ kg$ |
| $V_{cruise}$ | $24\ m/s$ |
| $V_{max}$ | $27\ m/s$ |
| $Wing\ Area\ (S)$ | $0.275\ m^2$ |
| $Thrust_{MAX}$ | $6.525\ N$ |

Note that the maximum thrust listed is the maximum thrust which can be generated for horizontal flight. Table 3 shows the remaining performance parameters required for this problem.

Table 3: Performance Parameters

| | |
|---|---|
| $\left(\frac{C_L}{C_D}\right)_{\alpha=0}$ | 11.132 |
| $\alpha_{cruise}$ | 3.167° |
| $\alpha_{(C_L/C_D)\ max}$ | 3.96° |
| $C_{L_0}$ | 0.1299 |
| $C_{D_0}$ | 0.013 |
| $C_{L_\alpha}$ | 0.09305 |
| $k$ | 0.0509 |
| $Thrust_{MAX}$ | $6.525\ N$ |

Remembering that the aircraft is made entirely of PLA, the performance parameters were found using the following equations:

$$C_{D_0} = \frac{\sum C_f \cdot FF \cdot FI \cdot S_{wet}}{S_{ref}} \quad (1)$$

$$k = \frac{1}{\pi ARe} \quad (2)$$

$$C_L = C_{L_0} + C_{L_\alpha}\alpha \quad (3)$$

$$C_D = C_{D_0} + kC_L^2 \quad (4)$$

$$D = \frac{1}{2}\rho V^2 S C_D \quad (5)$$

$$L = \frac{1}{2}\rho V^2 S C_L \quad (6)$$

These efficiency factors were all obtained using the open source software XFLR5 to simulate airflow over the wing, elevator, and rudder designs. To further visualize the feasible angle of attacks, observe Fig. 3 and Fig. 4

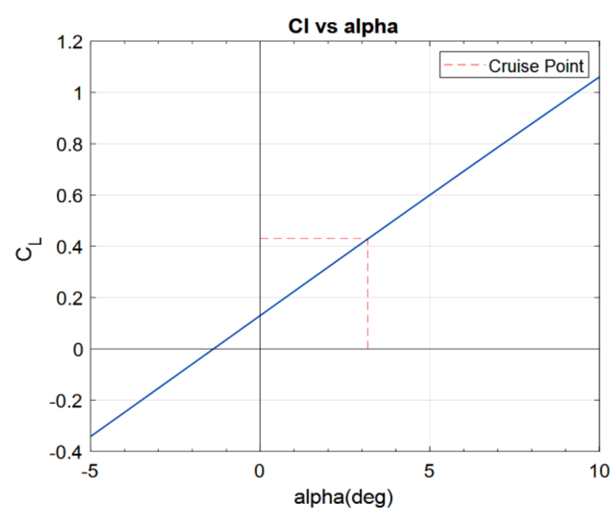


Fig. 3: $C_L$ vs alpha

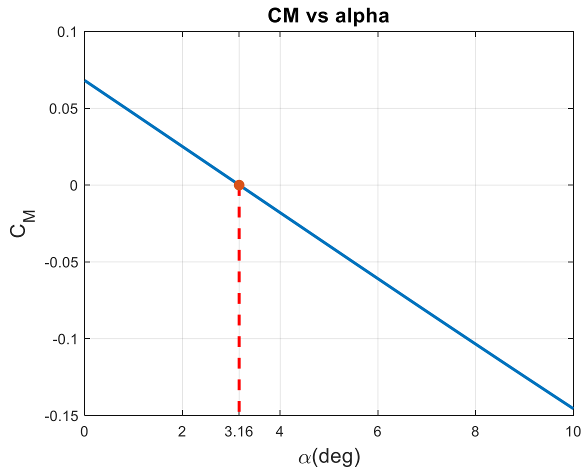

Fig. 4: $C_M$ vs alpha

It is imperative to reference these values while solving the optimal control problem. These values can either be used as constraints to ensure that the aircraft remains stable for the entirety of the trajectory or can be used as a reference to determine if the trajectory generated follows a reasonable path for any given mission.

## III. Assumptions

There are several assumptions that had to be made to solve this problem.

1. The quadplane will have fully transitioned from VTOL to horizontal flight
2. The quadplane best rate of climb will be 27 m/s
3. There will not be any objects in the flight path

Assumption 1 is necessary to reiterate that the vertical propellers will not be active during any part of the mission and will never be providing any additional thrust. Assumption 2 is a very optimistic assumption that had to be made due to time limitations to correctly and accurately model the thrust of the aircraft. To model the thrust, several tests needed to be conducted on the motors to gather the correct motor coefficients. Unfortunately, the motors for the aircraft did not arrive until two days before first flight and consequently there was not enough time to model the thrust. Because there is no thrust model, there is no accurate way to use thrust as a control input, and therefore the best rate of climb cannot be measured. Because the maximum velocity of 27 m/s is a conservative parameter for the propulsion system being used in the aircraft, it was determined that a best rate of climb of 27 m/s is a reasonable assumption.

Assumption 3 is a reasonable assumption to be made when the quadplane is travelling in an open area. However, it is certainly not feasible if the quadplane is travelling in a congested city. By making this assumption, the problem can be modelled as a 2D trajectory problem.

## IV. Mathematical Model

This optimization problem is a minimum time problem, which, in its simplest form, can be written as

$$J = \int_{t_0}^{t_f} dt \tag{7}$$

Subject to the following state space model:

$$\begin{bmatrix} \dot{V} = \frac{1}{m}(T\cos(\alpha) - D - mg\sin(\gamma)) \\ \dot{\gamma} = \frac{1}{mv}(T\sin(\alpha) + L - mg\cos(\gamma)) \\ \dot{h} = V\sin(\gamma) \end{bmatrix} \tag{8}$$

control input: $\alpha$

with the following flight constraints:

$$5\ m \leq h \leq 61\ m$$
$$14\frac{m}{s} \leq V \leq 27\frac{m}{s}$$
$$-10° \leq \gamma \leq 10°$$

and the following bounds:

$$h(0) = 15\ m \qquad h(t_f) = 61\ m$$
$$V(0) = \frac{15m}{s} \qquad V(t_f) = 24\frac{m}{s}$$
$$\gamma(0) = 0° \qquad \gamma(t_f) = 0°$$

To solve an optimal control problem of this form, nonlinear programming is used. To reduce this optimal control problem to a nonlinear programming problem, the state variables and input controls are represented as piecewise polynomials. These polynomials for each state can then be represented as cubic polynomials using Hermite interpolation [3]. The values of the states can then be solved using a nonlinear programming solver such as NPSOL. This process is outlined in Fig. 5. NPSOL uses sequential quadratic programming to solve these problems.

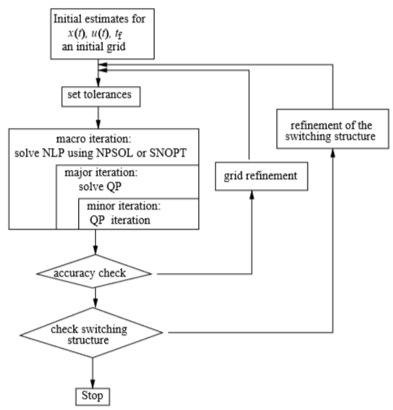


Fig. 5: Direct collocation technique [1]

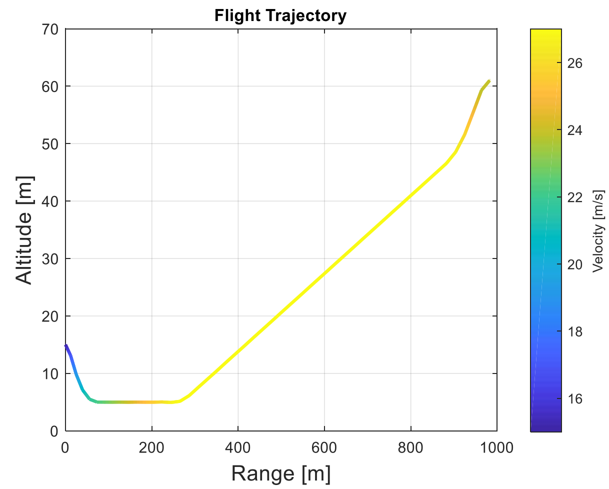


Fig. 6: 2D Optimal Flight Trajectory

Stryk [2] has produced papers outlining direct and indirect collocation methods for trajectory optimization which will go into much greater detail into how the collocation technique works.

## V. Software Implementation

Implementing this method in MATLAB was done using an optimization platform known as TOMLAB [6]. TOMLAB uses the nonlinear solver SNOPT, which behaves similarly to NPSOL. After providing the bounds and constraints, the program was set to create a trajectory using 50 points. First supply an initial guess as required by SNOPT and then supply the state space model and control input. Finally, identify the minimization variable as the final time, and solve.

TOMLAB will output the control input history along with the corresponding state variables. The corresponding outputs for the quadplane optimization problem can be seen in Fig. 7.

Remembering the targeted cruise angle of attack is 3.16°, the optimal trajectory will force the control surfaces to be nearly fully deployed to maintain stability throughout the entire climb, but the corresponding states for the control inputs are reasonable for the given mission.

With the output produced, the trajectory will first always attempt to reach the lowest allowable altitude. Once it reaches this altitude, it will then speed up at this altitude until it reaches it maximum velocity. Once it reaches its maximum velocity, it will then climb at its highest permissible velocity until it reaches the cruise altitude. Upon reaching the cruise altitude, it will expectedly slightly decrease in speed to account for the shifting angle of attack to reach the targeted cruise angle of attack. This process can be visualized as seen in Fig. 6.

Solving the optimization problem yields a minimum time to go from hover to cruise of 38.73 seconds. This number is reasonable and greatly out-performs the previous estimate of 60 seconds to reach the cruise altitude.

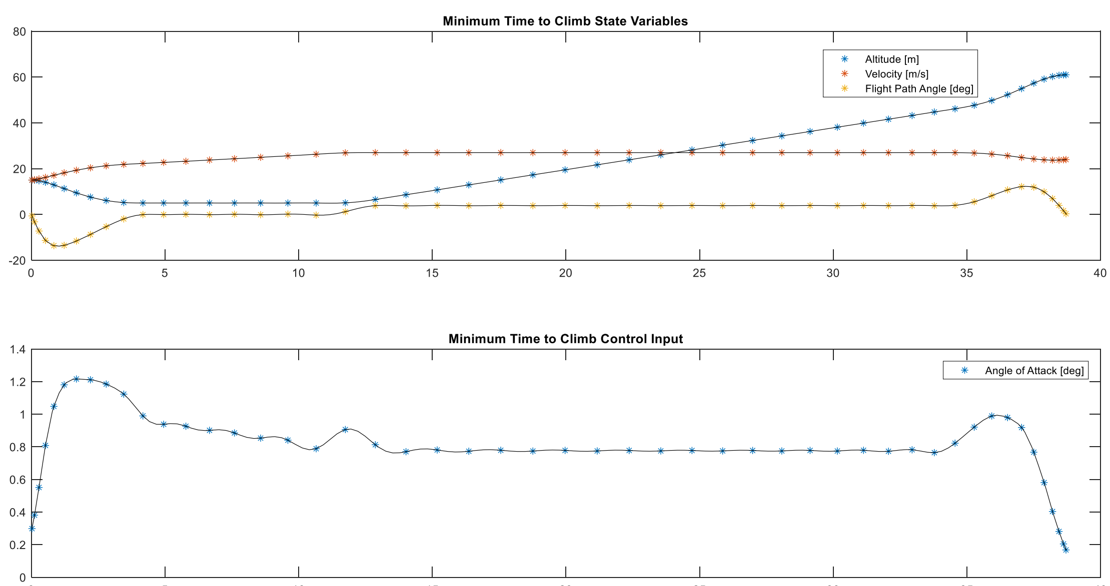


Fig. 7: TOMLAB Output for Quadplane Minimum-Time Problem

Unfortunately, when the quadplane made its maiden flight, it was not capable of reaching the required 15 m altitude and consequently experimental data could not be collected to analyze the trajectory produced. The propulsion system lacked the thrust necessary to reach the full 15 m height, and thus the flight was not successful. If the quadplane would have had a successful first flight, the maximum rate of climb could have been more accurately identified, and the optimization problem could have been re-analyzed to obtain a more accurate trajectory.

TOMLAB is only one software package which focuses on solving optimization problems but is certainly not the only way to solve optimal control problems of this form. For more in-depth discussion for computational optimal control, see the text provided by Subchan and Zbikowski [1].

## VI. Future Work

If this project were to continue in the future, there are several ways to proceed going forward. First and foremost, the quadplane design needs to be finalized, tested, and proven capable of completing its proposed mission. With a finalized and proven aircraft, experimental data can be collected to analyze the correctness and accuracy of the solution to the optimization problem. Next, an accurate thrust model will greatly increase the precision of the minimum-time optimization problem. To do so, motor parameters will need to be obtained for the propulsion system being used in the quadplane. Once the thrust can be accurately modeled, it can also be used as a control input to further increase the fidelity of the optimization problem solution.

Further, to solve the optimal trajectory problem for the quadplane mission in its entirety, the VTOL propulsion system can also be modeled to obtain a separate thrust model for when the aircraft is in its vertical phase. There are many resources to model the thrust of a quadcopter, and the quadcopter force model has been well documented, as shown in Fig. 8 [4].

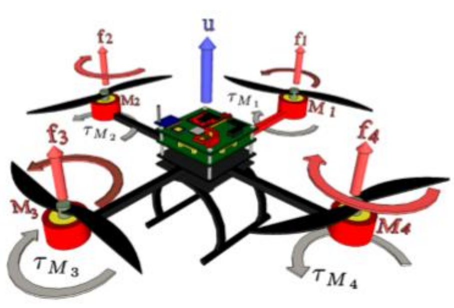


Fig. 8: Quadcopter Force Model

Another potential phase of the project going forward would be to introduce object detection and avoidance. To do so would add a much higher degree of complexity to this problem and would require several additional techniques to find the optimal path in a 3D coordinate space with obstacles. Further, the quadplane would require a more powerful autopilot system, and a high-quality camera would need to be introduced. Following this path will offer several major alterations to the current problem and is not recommended.

Another route to take for the project going forward is to solve the minimum-energy problem for the same mission of the quadplane. Direct collocation techniques are mostly used for minimum-time and minimum-fuel problems and will easily be able to solve the minimum-energy problem for the horizontal phase of the mission profile. However, here the VTOL phases of the mission have a massive impact on energy usage and cannot be ignored. For reference, the $C_D$ of the quadplace in VTOL is 1.21, which is extremely inefficient. To solve this problem, the force model in Fig. 8 will need to be used to create an objective function which models the energy usage of the quadplane in vertical flight. Once the objective function is created, an optimization solver can then be used find the motor thrust inputs to minimize the energy-usage of the UAV in VTOL.

If the minimum-energy problem is solved, then a final step could be taken in finding the optimal trajectory to identify the best path to follow for minimizing a combination of energy usage and time-taken. To do so, weights would need to be introduced for each aspect of the problem respectively, and then an objective function could be formulated and then solved to produce another trajectory.

Stengel [5] has research specifically targeted towards minimum-time and minimum-fuel problems and produced an in-depth textbook on optimal control and estimation which can be used as an added reference to add a theoretical foundation to any future work with optimal control problems.

## VII. Conclusion

This paper outlines a very high-level implementation of how an optimal control problem can be solved by transforming the problem into a constrained nonlinear optimization problem. Though experimental data could not be collected to prove the validity of the optimization solution, there is clearly much room for improvement in this particular project. As outlined in the previous section there are several paths forward for this project which could result in massive value if introduced to a fleet of UAVs.

If the quadplane had a successful maiden flight, then the quadplane would have followed a pre-planned fully autonomous mission using its autopilot system. With the data collected from this, the performance and precision of the trajectory produced could be further analyzed and potentially improved based on the data gathered. Further, it may have been discovered that the optimization problem was producing state outputs which are infeasible and will result in instability and a resulting crash.

To address this, it is not recommended to attempt to solve an optimal control problem for an aircraft which has not yet proved that it is capable of flying and has no proven track record of completed missions. As seen with the quadplane in this paper, it is impossible to evaluate the correctness of the produced trajectory without any previous data with which to compare the theoretical results.

The optimization problem can have a massive impact if successfully implemented with the quadplane mission. Though it may not seem as if the optimization solution adds much value to the single quadplane used throughout the analysis of this report, that is only because the optimization problem is being used to analyze a single UAV. If the quadplane were to be mass-produced and included as a fleet of hundreds of UAVs being used to deliver medical samples, then saving even a few seconds of time to deliver a payload will add up exponentially and save the sponsoring company money, as well as prevent labs from losing valuable lab samples or potentially life-saving medical supplies for those who have been struck by disaster.

## REFERENCES


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