CHAPTER 6

GAIN

No matter what type of gain we're talking about the following two equations are true. First, if we want to calculate gain we use

$$\text{GAIN} = \frac{\text{OUTPUT}}{\text{INPUT}}$$

If we want to use gain we use

$$\text{OUTPUT} = \text{GAIN} \cdot \text{INPUT}$$

I hope that it is clear that these equations are identical, just rearranged a bit. In this chapter we'll consider two categories of gain: 1) DC gain in resistive circuits and 2) Complex gain in circuits with reactive elements.
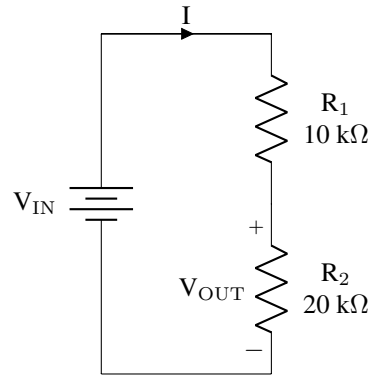
6.1   Real Gain (Resistive, DC circuits)

We'll start with resistive circuits as they are mathematically simple. As we look at these examples keep in mind that we will repeat many of the same steps with reactive elements in the next section.

6.1.1   Voltage Divider

 EXAMPLE 6.1

Find the gain of this circuit.

To begin we write an expression for the output of the circuit, in this case V_{OUT}. In this case the voltage divider is the most direct route.

$$V_{OUT} = V_{IN} \left[\frac{R_2}{R_1 + R_2} \right]$$

This step can become quite lengthy as the circuit analysis becomes more involved. However, for now it is as simple as writing the expression above. Recalling that gain is output over input we can divide both sides of the equation by V_{IN} to find the gain.

$$A = \frac{V_{OUT}}{V_{IN}} = \frac{V_{IN} \left[\frac{R_2}{R_1+R_2} \right]}{V_{IN}}$$

Simplifying this we find the gain to be

$$A = \left[\frac{R_2}{R_1 + R_2} \right] = \frac{2}{3}V/V$$

```
1  clear all
2  close all
3  clc
4
5  format short eng
6
7  R1=10e3;%Ohms
8  R2=20e3;%Ohms
9
10 syms Vi
11 Vo=Vi*(R2/(R1+R2));
12 A=Vo/Vi
```

6.1.1.1   Gain Units   If we look at the units of the gain from the previous example we see that it is unitless. In all cases the numerator of the unit is the unit of the output and the unit of the denominator is the unit of the input. It is possible for a gain to have a unit if the output has a different unit than the input.

6.1.1.2   Useful Alternative Method   The gain of a linear circuit is independent of the input. If you are not comfortable analyzing the circuit for gain symbolically, we can use this to our advantage. Let's find the gain of the previous problem as an example of this method. We will find that the gain is the same regardless of the method used.

Assume a value, any value, for the input. Roll some dice, pick a number out of thin air, or chose a number that makes the analysis easy. The value that makes these problems easy is always 1. If we assume the input is 1, whether we're dealing with voltage or current as input, we'll simplify the last step of this process. In this case we assume the input is $V_{IN} = 1\ V$. Next, we analyze the circuit to find the output. We can use the voltage divider again for this purpose

$$V_{OUT} = V_{IN} \left[\frac{R_2}{R_1 + R_2} \right] = 1\ V \left[\frac{20\ k\Omega}{10\ k\Omega + 20\ k\Omega} \right] = \frac{2}{3}\ V$$

Finally, we recall that gain is always output over input. To find the gain we divide the output by the assumed input
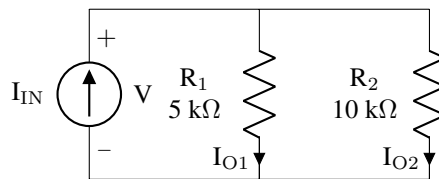
$$A = \frac{V_{OUT}}{V_{IN}} = \frac{2/3\ V}{1\ V} = \frac{2}{3}\ V/V$$

Look! The same result! This approach works for any assumed value of the input but maybe you noticed why the value 1 makes things simple. With this assumption the output shares the value with the gain. The only difference is the unit is adjusted to reflect the input unit.

6.1.2 Current Divider

EXAMPLE 6.2

Find the gain of the circuit

Trick question. A circuit alone isn't enough to determine gain. We must know what to consider as the input and what to consider as the output. This circuit has two separate outputs labeled to illustrate that the result will be different. Let's rephrase the question to include the input/ouput path through the circuit.

Find the gain of the circuit from I_{IN} to I_{O1}.

Follow the same process as before. Write an expression for the output

$$I_{O1} = I_{IN}\frac{R_2}{R_1 + R_2} = \frac{2}{3}I_{IN}$$

then divide by the input

$$A_1 = \frac{I_{O1}}{I_{IN}} = \frac{2}{3}A/A$$

Now let's consider the gain of the circuit from I_{IN} to I_{O2}. The output is expressed as

$$I_{O2} = I_{IN}\frac{R_1}{R_1 + R_2} = \frac{1}{3}I_{IN}$$

then divide by the input

$$A_2 = \frac{I_{O2}}{I_{IN}} = \frac{1}{3}A/A$$

The result is different. When you analyze a circuit for gain make sure that the input and output are clearly defined and labeled to prevent confusion. This will also be true in the future chapter regarding the related concept of transfer functions.

```
1   clear all
2   close all
3   clc
4
5   format short eng
6
7   R1=5e3;%Ohms
8   R2=10e3;%Ohms
9
10  syms Ii
11  %Find gain from Iin->Io1
12  Io1=Ii*(R2/(R1+R2));
13  A1=Io1/Ii
```

```
14
15  %Find gain from Iin->Io2
16  Io2=Ii*(R1/(R1+R2));
17  A2=Io2/Ii
```

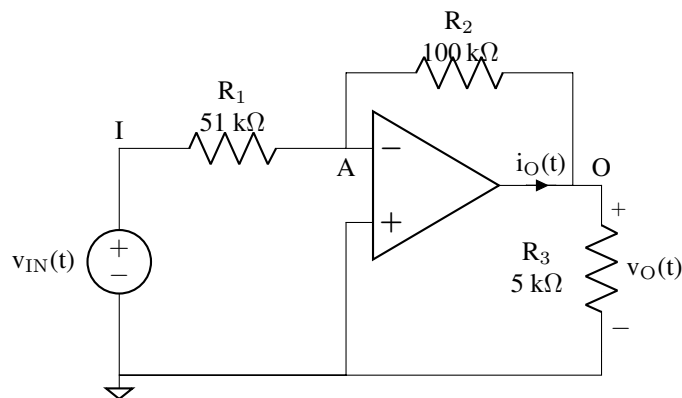### 6.1.3 More Complex Circuits

For more complex circuits we need more methodical analysis. We can use Mesh and Nodal analysis in the approach introduced above.

6.1.3.1 Mesh Analysis

6.1.3.2 Nodal Analysis with Op-Amp   Let's try an example with nodal analysis and an operational amplifier.

EXAMPLE 6.3

Find the gain of the circuit from $v_{IN}(t)$ to $v_O(t)$

The three unknown node voltages for nodes I, A, and O require us to develop a system of three equations. The voltage supply gives us a simple KVL equation

$$\textbf{KVL I} \qquad\qquad V_I = V_{IN}$$
$$\textbf{KVL Op} - \textbf{Amp} \qquad\qquad V_A = 0$$
$$\textbf{KVL A} \qquad\qquad \frac{V_I - V_A}{R_1} - \frac{V_A - V_O}{R_2} = 0$$

```
1   clear all
2   close all
3   clc
4
5   format short eng
6
7   R1=51e3;%Ohms
8   R2=100e3;%Ohms
9   R3=5e3;%Ohms
10
11  syms Vi Va Vo
12  eqn(1)=((Vi-Va)/(R1))-((Va-Vo)/(R2))==0;
13  eqn(2)=Va==0;
14  sol=solve(eqn,Va,Vo)
15
16  eval(sol.Vo/Vi)
```

Reconsider the same circuit. This time find the gain from V_{IN} to I_O. In this case we add another unknown, I_O, and the KCL that includes it

$$\text{\textbf{KCL O}} \qquad V_I = V_{IN}$$

```
1   clear all
2   close all
3   clc
4
5   format short eng
6
7   R1=51e3;%Ohms
8   R2=100e3;%Ohms
9   R3=5e3;%Ohms
10
11  syms Vi Va Vo Io
12  eqn(1)=((Vi-Va)/(R1))-((Va-Vo)/(R2))==0;
13  eqn(2)=Va==0;
14  eqn(3)=Io+((Va-Vo)/(R2))-((Vo)/(R3));
15  sol=solve(eqn,Va,Vo,Io)
16
17  eval(sol.Io/Vi)
```
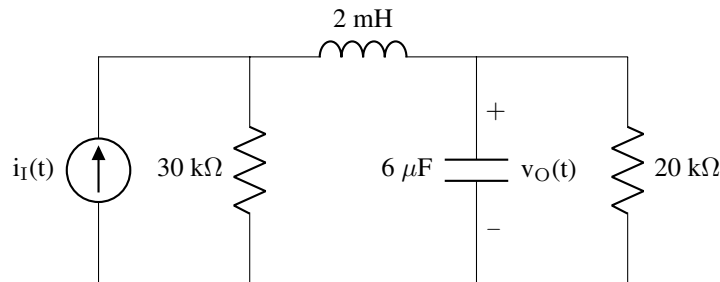
6.2   Complex Gain (Reactive, AC circuits)

6.2.1   Voltage Divider

schematic question analysis solution MATLAB solution Multi frequency table output

6.2.2   Mesh Nodal

1.5 kHz $i_I(t)=5\cos(wt)$ mA w_c=9128.7 rad/s

```
1   clear all
2   close all
3   clc
4
5   format short eng
6
7   R1=30e3;%Ohms
8   R2=20e3;%Ohms
9   L=2e-3;%H
10  C=6e-6;%F
11  f=1.5e3;%Hz
12  w=2*pi*f;%rad/s
13  Zl=j*w*L;%Ohms
```
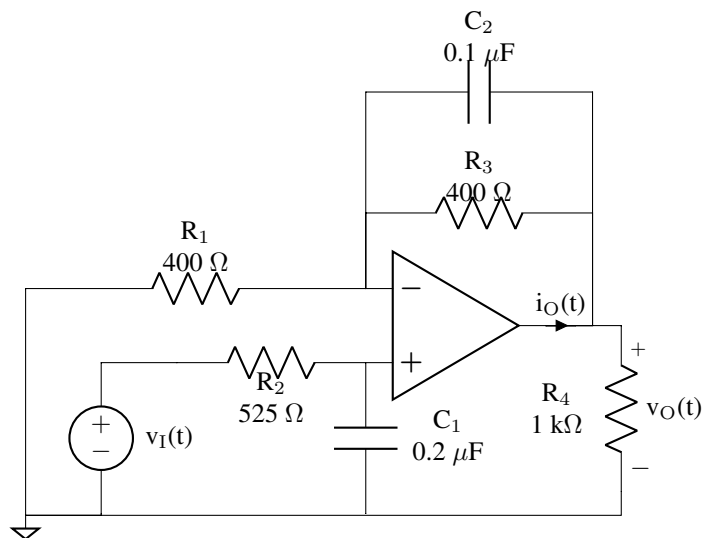
```
14  Zc=inv(j*w*C);%Ohms
15
16  syms I1 I2 I3 Ii
17  eqn(1)=I1==Ii;
18  eqn(2)=(-R1)*I1+(-R1-Zl-Zc)*I2+(-Zc)*I3==0;
19  eqn(3)=(-Zc)*I2+(Zc+R2)*I3==0;
20  sol=solve(eqn,I1,I2,I3);
21
22  Vo=(sol.I2-sol.I3)*Zc;
23  A=Vo/Ii;
24  fprintf('%f at an angle of %f degrees\n',abs(A),angle(A)*(180/pi))
```

6.2.3 Nodal with Op-Amp

EXAMPLE 6.4

What is the gain of the circuit at 1830 Hz?

```
1   clear all
2   close all
3   clc
4
5   format short eng
6
7   R1=400;%Ohms
8   R2=525;%Ohms
9   R3=400;%Ohms
10  R4=1e3;%Ohms
11  C1=0.2e-6;%F
12  C2=0.1e-6;%F
13  f=1.83e3;%Hz
14  w=2*pi*f;%rad/s
15  Zc1=inv(j*w*C1);%Ohms
16  Zc2=inv(j*w*C2);%Ohms
17
18
19  syms Va Vb Vc Vo Vi
```

```
20 eqn(1)=Va==Vi;
21 eqn(2)=Vb==Vc;
22 eqn(3)=((Va-Vc)/(R2))-((Vc)/(Zc1))==0;
23 eqn(4)=((0-Vb)/(R1))-((Vb-Vo)/(R3))-((Vb-Vo)/(Zc2))==0;
24 sol=solve(eqn,Va,Vb,Vc,Vo);
25
26 A=eval(sol.Vo/Vi);
27 fprintf('%f at an angle of %f degrees\n',abs(A),angle(A)*(180/pi))
```

```
1  clear all
2  close all
3  clc
4
5  format short eng
6
7  R1=400;%Ohms
8  R2=525;%Ohms
9  R3=400;%Ohms
10 R4=1e3;%Ohms
11 C1=0.2e-6;%F
12 C2=0.1e-6;%F
13 f=1.83e3;%Hz
14 w=2*pi*f;%rad/s
15 Zc1=inv(j*w*C1);%Ohms
16 Zc2=inv(j*w*C2);%Ohms
17
18
19 syms Va Vb Vc Vo Vi Io
20 eqn(1)=Va==Vi;
21 eqn(2)=Vb==Vc;
22 eqn(3)=((Va-Vc)/(R2))-((Vc)/(Zc1))==0;
23 eqn(4)=((0-Vb)/(R1))-((Vb-Vo)/(R3))-((Vb-Vo)/(Zc2))==0;
24 eqn(5)=Io-((Vb-Vo)/(Zc2))-((Vb-Vo)/(R3))-((Vo)/(R4))==0;
25 sol=solve(eqn,Va,Vb,Vc,Vo,Io);
26
27 A=sol.Io/Vi;
28 fprintf('%f at an angle of %f degrees\n',abs(A),angle(A)*(180/pi))
```