Circuits Lab 1

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1 Experiment 1: Resistance Measurement

For this experiment, we used a 499Ω, .25W, through-hole resistor. We first determined the resistance as indicated by the manufacturer's color coding. Its color code was yellow, white, white, black, and brown, indicating the resistor had a resistance of $499\times10^0\Omega$ with a tolerance rating of 1%. Our goal for this experiment was to compare this indicated resistance to the resistance we measured with two tools: the Keithley 2400 SourceMeter and the SMU.

1.1 Keithley 2400 SourceMeter

Using the Keithley 2400 SourceMeter, we recorded a value of 0.498kΩ: an error of .21%. This number was generally consistent during multiple tests with the trailing digits varying slightly.

1.2 SMU

To test the resistance with our SMU, we set up the circuit shown in Figure 1.

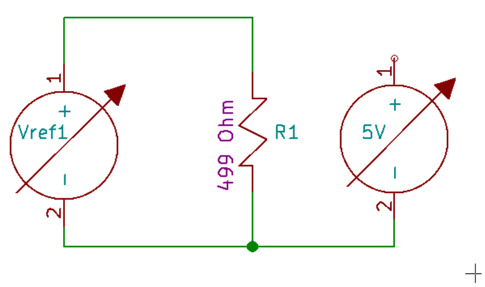

Figure 1: Circuit 1. Channel 1 on the SMU served as Vref1. Channel 1 is both providing the voltage across the resistor and measuring the current through the resistor.

The SMU, which we used as a voltage source in this experiment, allowed us to measure the current through the resistor at various voltages. Using Ohm's law ($V = I * R$), we can find the measured resistance by dividing each voltage with the current measured when we applied that voltage. To get a more accurate reading, we took multiple measurements, shown in Figure 2, and fit a line to our current-voltage pairs from 0V to 5V using Matlab's polyfit function. This best-fit line agrees with our data over the entire range. Because the applied voltage was our X-axis and the resultant current our Y-Axis, the slope of the line of best fit actually gives us conductivity $(1/R)$.

As you can see below, with a slope of $.002 Mhos$, our measured resistance was the reciprocal, or 499.18Ω: an error of .04%.

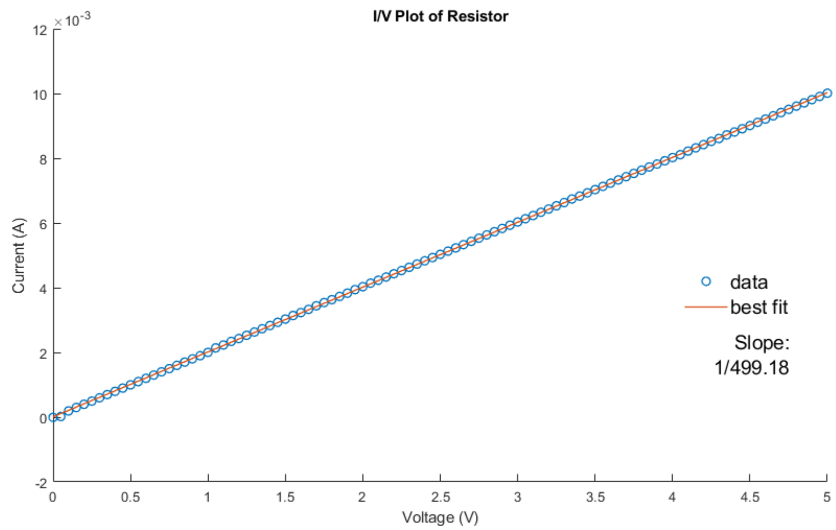

Figure 2: Plot showing the recorded (V, I) data points along with a line of best fit. The slope of the line is 1/499.18. The reciprocal of this slope, 499.8, is the measured resistance in Ohms.

1.3 Observations

The specific resistor we selected had a tolerance rating of 1%. For our resistor, that meant the range of expected resistance values would be between 494Ω and 504Ω. Both the Keithley SourceMeter and the SMU recorded resistive values in that range as expected. The difference in measured resistance values could be related to the style of measurement, the setup of each test, or the tolerances of each tool. According to the user manual for the SourceMeter, the meter outputs current and measures voltage across the resistor. The default test current for a resistance between 200Ω and $2k$ is $10mA$.The setup for this measurement was also relatively simple with two leads connected across the resistor. With our SMU, we measured the current at specific voltages. The setup for this lab also required the use of a breadboard and pin-connectors to which we clipped the probes; all of these could have increased the resistance between the measurement points. It is likely most accurate to utilize the Keithley SourceMeter because the probes are directly connected to the two ends of the resistor.

2 Experiment 2: Resistive Voltage Division

Our goal of this experiment was to test the voltage-divider ratio by measuring the ratio between the output and input voltages of a simple voltage divider circuit. This experiment also required the use of a Bourns resistor array chip: an integrated circuit consisting of 8 resistors of similar values.

2.1 Circuit

We first measured the resistances of each of the resistors within our two Bourns chips. Within each chip, the resistances stay well within their 2% tolerance ratings, as they vary by only 24Ω on the first chip and only 242Ω on the second chip. From the first chip to the second, the values remain close, as the total variation of the resistances across both chips is 246Ω. To test the voltage divider ratio, we picked the three resistors from the Bourns chip with the closest values and set two of them in parallel with the third in series. Measuring across the third resistor would give us a measure output voltage that we could compare to a theoretical output voltage that we calculated using the voltage divider ratio.

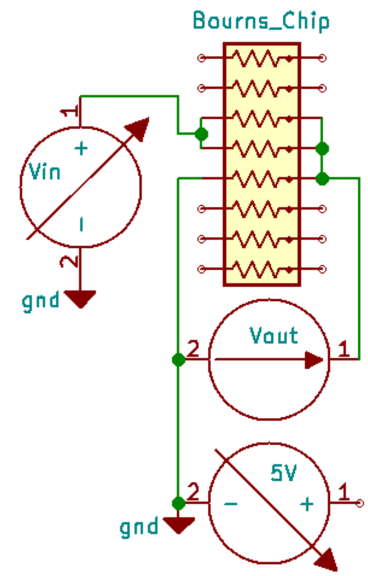

Figure 3: Circuit diagram for the voltage-divider. The two resistors in parallel had values of 9931Ω and 9932Ω, respectively. The third resistor in series has a value of 9932Ω.

	Chip 1	**Chip 2**
Resistor 1	9.947kΩ	10.169kΩ
Resistor 2	9.930kΩ	9.929kΩ
Resistor 3	9.932kΩ	9.927kΩ
Resistor 4	9.931kΩ	9.933kΩ
Resistor 5	9.932kΩ	9.946kΩ
Resistor 6	9.923kΩ	9.929kΩ
Resistor 7	9.929kΩ	9.930kΩ
Resistor 8	9.930kΩ	9.932kΩ

Table 1: The resistance values of each resistor on the two Bourns resistor array chips we used, measured using the Keithley 2400 SourceMeter. The resistances are all within 12Ω of 9.935kΩ, with the exception of the first resistor on Chip 2.

In our setup, shown in Figure 3, we have three nearly identical resistors, two in parallel and one in series with those two. Ideally, the voltage across the last resistor should be equal to two-thirds the input resistance by Ohm's law. Though our resistors were not exactly identical, their measured values were close enough that our theoretical V_{out} was actually equal to two-thirds V_{in} to several significant digits. Figure 5 shows the voltage across the third resistor, in series with the two in parallel, measured at various V_{in}s.

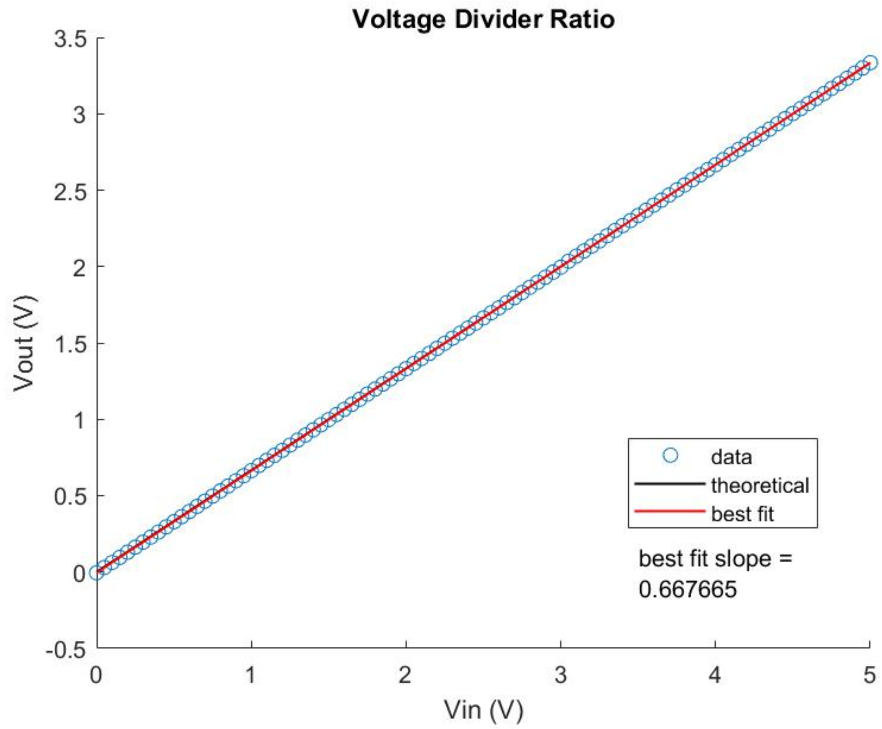

Figure 4: Plot showing the voltage divider ratio, both theoretically and from our measurements. The slope of our best fit line agrees with our data over the whole range from 0V to 5V.

2.2 Observations

For this circuit, we fit a straight line to our data using Matlab's polyfit function to attain a slope of 0.668 for $\frac{V_{out}}{V_{in}}$. This line agreed with our data over the whole range from 0V to 5V. Our measured $\frac{V_{out}}{V_{in}}$ ratio was .15% away from the theoretical ratio. Using the equation below we calculated that the theoretical worst case error is 1.3%: far greater than our .15% measured error. By choosing the three resistors that had the smallest difference between them on the Bourns chip, we tried to increase the proximity of our measured ratio to the theoretical ratio.

$$\left|\frac{\delta\gamma}{\gamma}\right| = (1-\gamma)\left(\left|\frac{\delta R_1}{R_1}\right| + \left|\frac{\delta R_2}{R_2}\right|\right)$$

Figure 5: While this equation is for a simple voltage-divider with two resistors in series, we combined our first two resistors in parallel into a single resistor with a tolerance equal to the tolerance of the chip (2%)

3 Experiment 3: Resistive Current Division

For this experiment, we validated the current-divider ratio by recreating a current-divider circuit and comparing our I_{out} with the theoretical I_{out}.

3.1 Circuit

Instead of having two resistors in parallel and one in series as in the voltage divider, we put all three resistors in parallel, as shown in Figure 6. With three identical resistors in parallel, the current through the third resistor should be equal to $1/3 * I_{in}$ by Ohm's law.

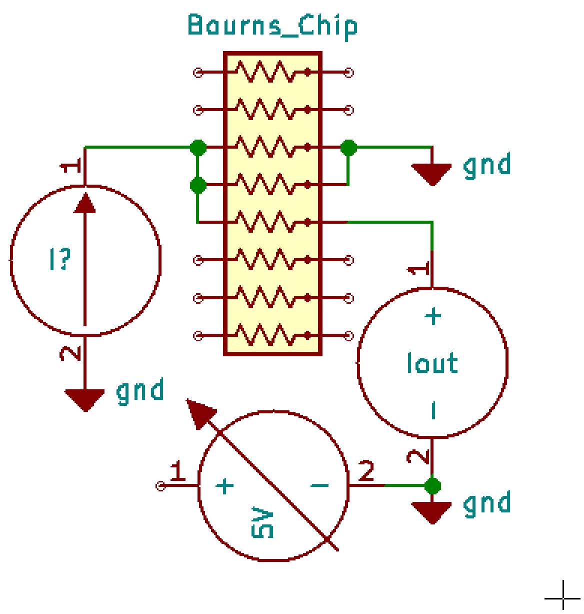

Figure 6: A circuit schematic for Experiment 3, showing our current divider using three resistors in parallel from the Bourns chip. The first two had values of 9931Ω and 9932Ω, respectively. The third resistor, which we measured the current through, had a value of 9932Ω.

3.2 Observations

After collecting our data, we fit a straight line to the data using Matlab's polyfit function. The resulting best-fit line agreed with our data over the range 0mA to 10mA. For this circuit, we calculated our $\frac{I_{out}}{I_{in}}$ ratio to be .64% away from the theoretical ratio using Equation 1, as the slope of the best-fit line we plotted is 0.336. Compared to the variation in the resistors within the Bourns chip, which span a range of 1.6%, the difference between the slope of our best-fit line and the theoretical slope, a difference of .64%, is appropriate.

$$PercentError = \frac{Experimental - Theoretical}{Theoretical} * 100 = \frac{0.335478 - 0.33}{0.33} * 100 = 0.64\% \quad (1)$$

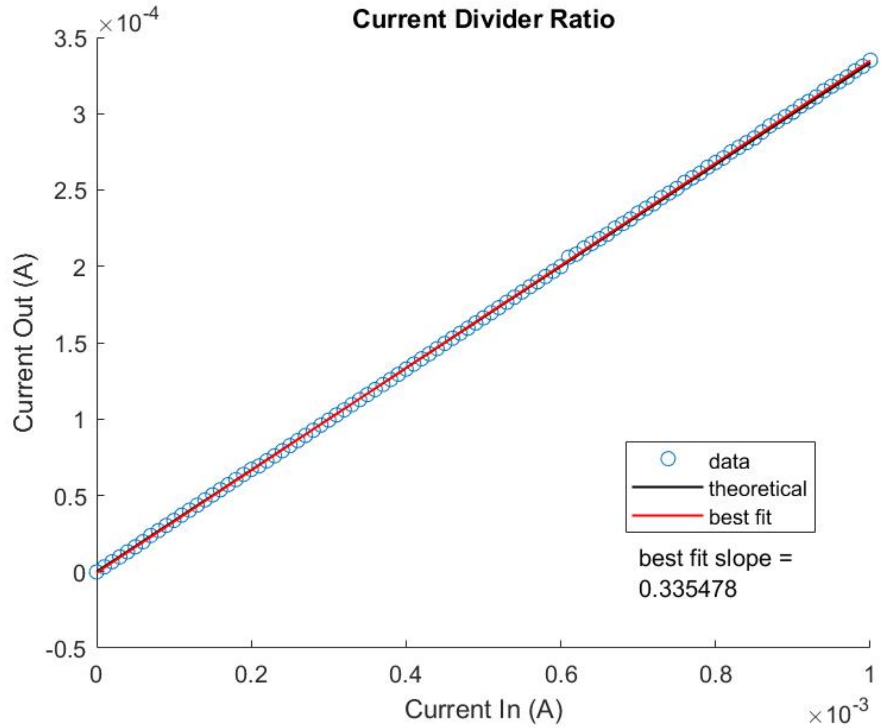

Figure 7: Plot showing the current divider ratio, both theoretically and from our measurements. The slope of our best fit line agrees with our data over the whole range from 0mA to 5mA.

4 Experiment 4: R-2R Ladder Network

For this section, we created a R-2R ladder network using two Bourns resistor chips. We then measured the output current through each 2R resistor.

4.1 Circuit

To recreate the R-2R Ladder Network using resistors of the same value, we combined a resistors in a combination of series and parallel branches. The parallel branches represented the "R" branch and the series branches the "2R" branches.

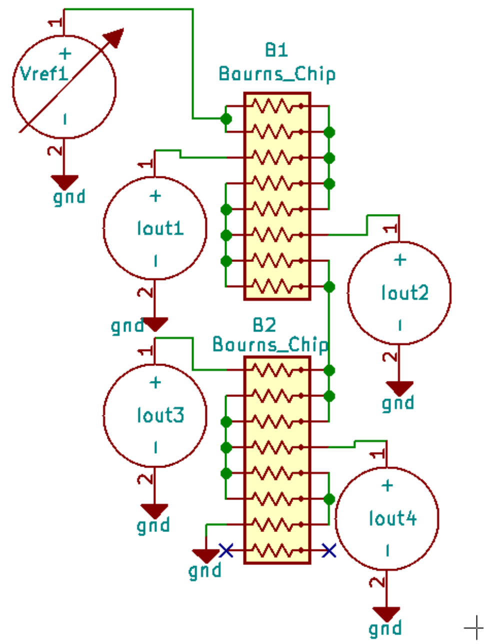

Figure 8: Circuit diagram of R-2R network using two Bourns resistor chips.

4.2 Observations

Most of this circuit worked as expected. For every "2R" branch, the current increased as V_{in} increased. We also measured that the current through the each one of the first three branches, with the first being nearest the voltage source, almost exactly halves the previous branch nearer the source, as shown in Figure 9. These currents vary with position exactly as we expected for an R-2R ladder network. However, for the last branch, we recorded $0A$ for all voltage levels above 0.7V. Although we utilized the Keithley 2400 SourceMeter to check that the current through each 2R branch was half of the current through the 2R branch, including for the fourth and final branch, the SMU failed to detect a current through the last 2R branch. This is likely due to a user error in the lab setup, as either the voltage supply or the measurement probes could have been disconnected from the circuit during the test of the final branch. However, by dividing the current through the third branch by two, we determined that the expected current values for the 4th branch to be at 3.2e$-5A$ for the 2.5V supply and 3.2e$-5A$ for the 5V supply.

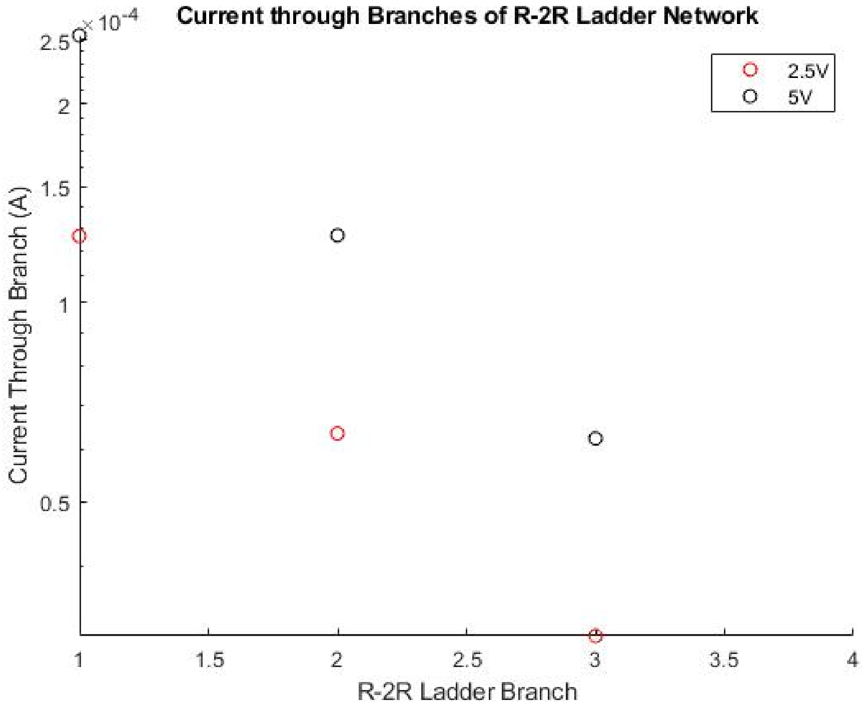

Figure 9: Semilog plot showing the current at each branch of the R-2R ladder. This plot includes values recorded with a 2.5V and 5V voltage source.