Circuits Lab 3

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1 Experiment 1: Bipolar Transistor Characteristics

Our goal for experiment 1 was to characterize the behavior of a 2N3904 bipolar transistor by measuring the base and emitter currents as we swept the base voltage over a range of values.

1.1 Circuit

In order to characterize the bipolar transistor, we connected the 5V DC power supply of the SMU to the transistor's collector, channel 1 to the base, and the channel 2 to the emitter, as shown in Figure 1. We fixed the emitter voltage at 0V and swept the base voltage from 0.4V to 0.8V in order to measure the emitter and base currents. Because we fixed the emitter voltage at ground, the transistor was actually drawing current from the emitter; this meant that we needed to negate the value we obtained for the emitter current in order to obtain the value with the correct sign.

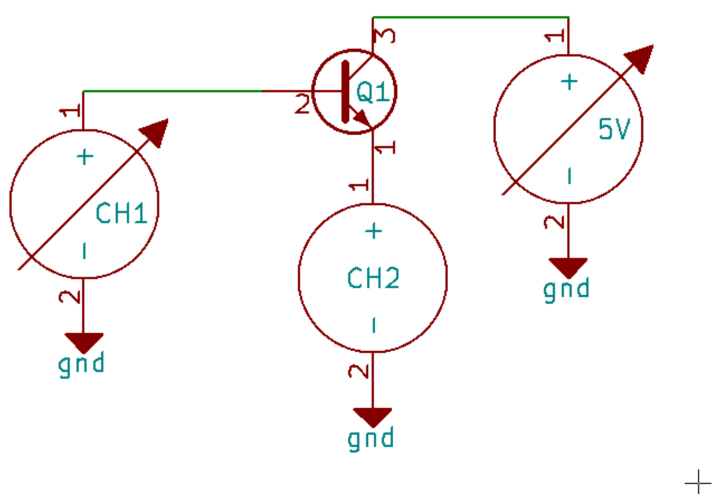

Figure 1: Simple 2N3904 NPN Transistor setup. Our SMU was setup to source to voltage and measure current through channel 1 and 2.

1.2 Observations

In this experiment, we utilized the emitter current as a proxy for measuring the collector current as a result of instability in the SMU when configuring the circuit to measure collector current directly.

In order to compute the collector current at each base voltage value, we used the relationship shown in Equation 2.

$$I_e = I_c + I_b \tag{1}$$

$$I_c = I_e - I_b \tag{2}$$

Our first graph, shown in Figure 2, shows the collector and base currents as a function of base voltage on a semilog-y scale. On a log scale, we can clearly see that the base and collector currents are related exponentially to the base voltage. To further confirm this, we utilized the Matlab polyfit function in order to obtain approximations for the slope and y-intercept of the exponential portion, shown as a straight line on a semilog-y scale, of the I-V characteristics. We then calculated the points on our best-fits using Equation 3 and plotted the results over our data.

$$y = e^{mx+b} \tag{3}$$

The collector current best fit closely matches our data over the entire range until the current becomes linear when the base voltage becomes slightly greater than 0.75V. For the base current, the measured current values were too low for the SMU to obtain accurate data when the base voltage was below approximately 0.5V and the base current was approximately $10^{-9}A$. Above this lower limit, however, the best fit closely matches the data until the current becomes linear.

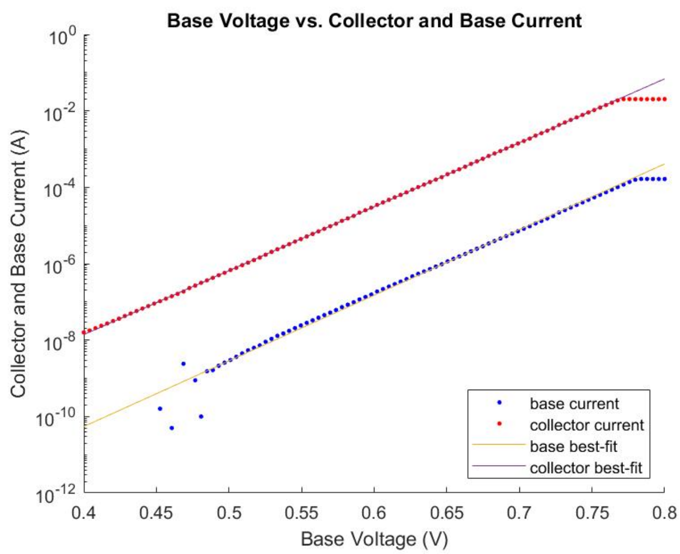

Figure 2: Semilog-y plot showing the collector and base currents as a function of base voltage.

Our second plot, shown in Figure 3, is a semilog-x plot of β as a function of the base current. β can be calculated using Equation 4.

$$\beta = \frac{I_{collector}}{I_{base}} \tag{4}$$

In order to obtain measured data points for β, we simply plugged our measured values of the base and collector currents into Equation 4. We then used Matlab's polyfit function to calculate the slope and y-intercept of a line of best fit. After plotting this line of best-fit and finding its slope to be $0.1510\frac{1}{A}$, we approximated β to be constant and equal to the y-intercept of the line for purposes of the next experiments, yielding $\beta = 194.093$. Though the current gain is not actually constant with base current, we can reasonably assume it to be constant over the interval $[10^{-8}A, 10^{-4}A]$ for the base current.

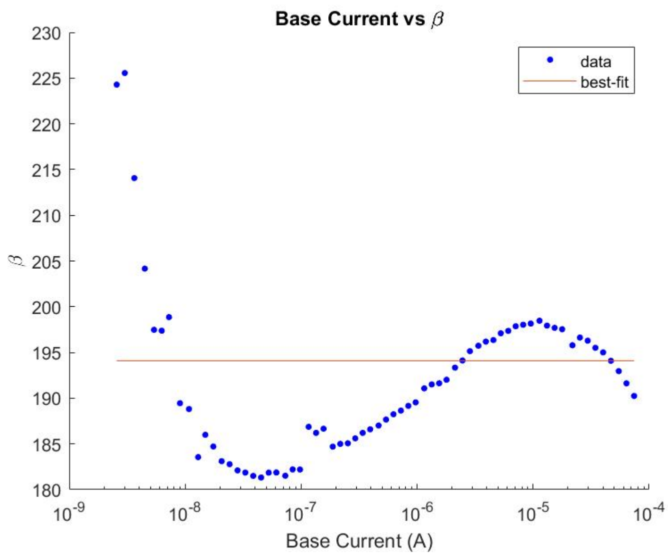

Figure 3: Semilog-x plot showing the value of beta as a function of base current.

Our third plot, shown in Figure 4, depicts the incremental base resistance as a function of the base current. The base resistance can be calculated with Equation 5.

$$r_b = \frac{\partial V_b}{\partial I_b} \tag{5}$$

In order to obtain measured values for the incremental base resistance, we utilized Matlab's diff command and dot-divide operator to compute a finite-difference approximation to the partial derivative in Equation 5. We then used the 25.3mV value we had obtained from the base current portion of our I-V characteristic plot in Figure 2 for U_T in conjunction with Equation 6 to fit a straight line to the data on a log-log plot of incremental base resistance against the base current. This theoretical fit matches the data well over the interval $I_b = 10^{-8}A$ to $I_b = 10^{-4}A$. When $I_b < 10^{-8}A$, the SMU was not able to take accurate enough measurements, so our theoretical fit does not trace the more widely distributed data well in that range.

$$r_b = \frac{U_T}{I_b} \tag{6}$$

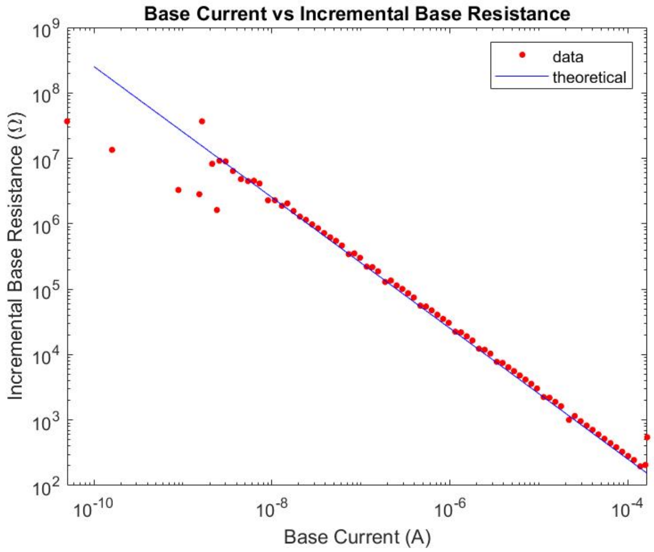

Figure 4: Log-log plot showing incremental base resistance as a function of base current.

For our final plot, shown in Figure 5, in Experiment 1, we showed the incremental transconductance gain of the bipolar transistor as a function of collector current on a log-log scale. The transconductance of the transistor can be calculated using Equation 7.

$$g_m = \frac{\partial I_c}{\partial V_b} \tag{7}$$

Like the previous plot, we utilized Matlab's diff command and dot-divide operator to compute a finite-difference approximation to the partial derivative in Equation 7 in order to obtain measured values for the incremental base resistance. To find a theoretical fit to the data, we combined the 25.9mV value we had obtained from the collector current-base voltage characteristic plot in Figure 2 for U_T with Equation 8. The resulting theoretical straight line fit does not quite match the data when $I_c < 10^{-8}A$, where the SMU cannot accurately gather data, but closely matches our data over the range $I_c = 10^{-8}A$ to $I_c = 10^{-4}A$.

$$g_m = \frac{I_c}{U_T} \tag{8}$$

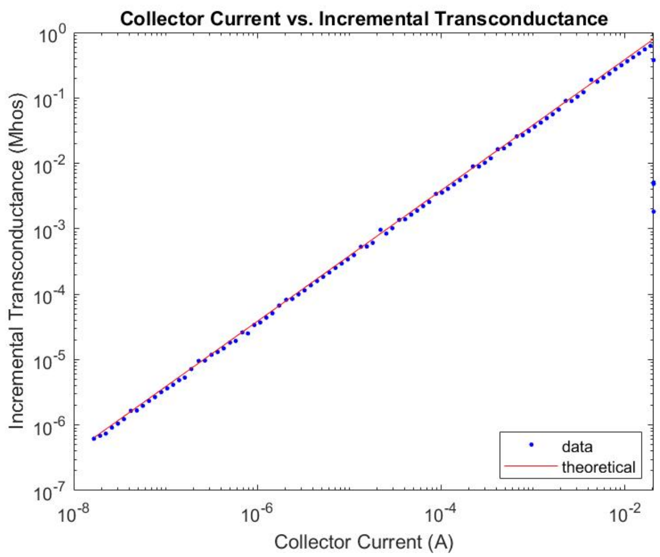

Figure 5: Log-log plot showing transconductance as a function of collector current.

2 Experiment 2: Emitter-Degenerated Bipolar Characteristics

2.1 Circuit

For this experiment, we setup a resistor in series with the emitter of our transistor as seen in Figure 10. We then measured the current through the resistor as we swept the base voltage from 0V to 5V. This process was repeated two more times with a resistor value that was an order of magnitude greater than the last. To extract the values for the collector current we used Equation 2.

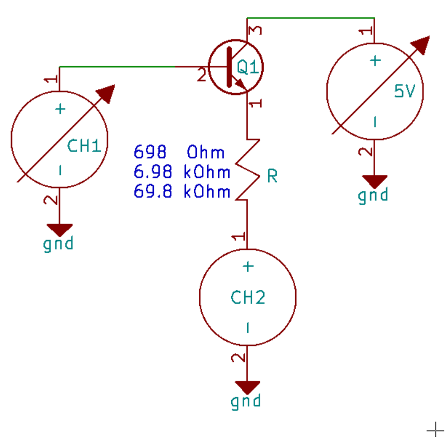

Figure 6: This NPN transistor is setup in a Emitter-Degenerated Bipolar circuit.

2.2 Observations

Our first graph, shown in Figure 7, is a comparison of the collector current (I_c) versus the base voltage (V_b). The data gained in experiment 1 regarding the collector current as a function of base voltage was overlaid into this graph to show the similarities of the exponential regions of our

I-V characteristics between the two experiments. Even though there is an additional resistor in series with the emitter in experiment 2, these results are expected because the change in current contributed by the resistor is very small when V_b is less than V_{on}.

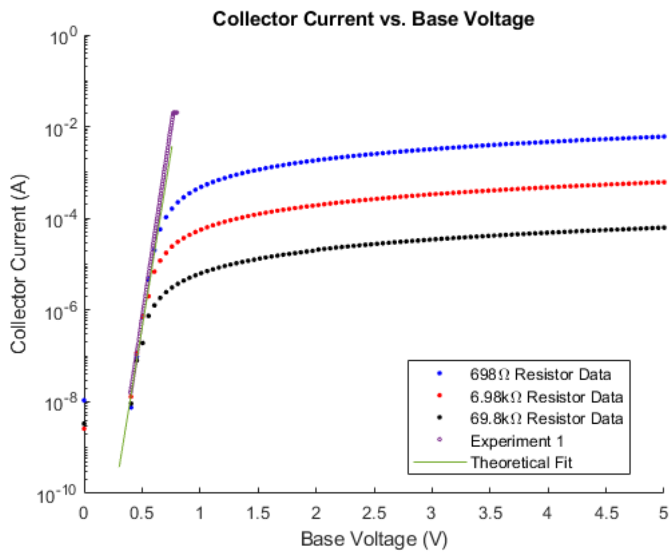

Figure 7: Collector current with respect to base voltage for a Emitter-Degenerated circuit. Each group of data points represents a different resistor placed in series with the emitter. The experiment 1 data reflects the I-V characteristics of a stand alone transistor.

We can extract values for the saturation current and thermal voltage from the graph above by using the equation below. With the following equation, the y-intercept equals the log of the saturation current(I_s) and the reciprocal of the slope equals the thermal voltage (U_T) of our transistor. In the case of the graph above, the calculated I_s and U_T values are 8.161e$-15A$ and $28.2mV$ respectively.

$$When V_b << V_{on}$$
$$V_{be} \approx V_b$$
$$I_c = I_s * e^{\frac{V_b}{U_T}}$$
$$\log I_c = \log I_s + \frac{V_b}{U_T}$$

Our second plot, seen in Figure 8, shows a linear plot of the collector current as a function of base voltage for each of the three resistors. For each resistor, we used Matlab's polyfit function to obtain a straight line fit to our data in the linear region of the plot. These lines of best fit match our data closely over the entire region after $V_b = V_{on}$. From the output of polyfit, we were able to calculate the x-intercepts of the best-fit lines; we approximated V_{on} to equal those x-intercepts. We obtained the values $0.661V, 0.605V$, and $0.555V$ for V_{on} for the $698\Omega, 6.98k\Omega$, and $69.8k\Omega$ resistors, respectively. These values for V_{on} are all roughly consistent with what we have seen in previous experiments, in the range of around $0.5V$ to $0.6V$, which further confirms the accuracy of our data.

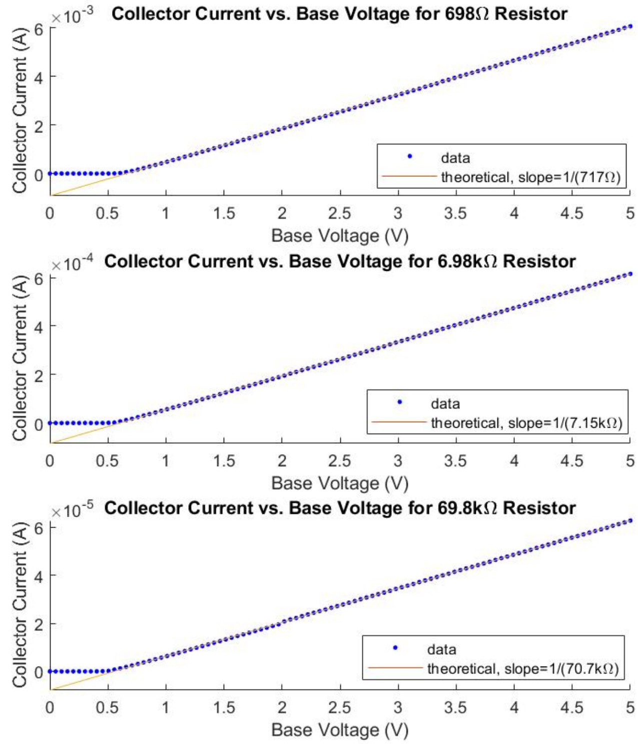

Figure 8: The collector current vs. base voltage characteristic on a linear scale for each of the three resistors we placed in series with the emitter terminal of the transistor.

Moreover, in Figure 8, the slope of the linear portion of the graph is equal to the reciprocal of the resistor we put in parallel with the emitter of the transistor. As we can see in the legend of each plot, the reciprocals of the slopes are fairly close to the resistances of the resistors used in their respective experiments.

Figure 9 shows the incremental base resistances of our Emitter-Degenerated circuits with respect to the base current. The equation used to calculate our data-set is simply:

$$R_b = \frac{\delta V_b}{\delta I_b} \tag{9}$$

We then produced a theoretical fit for this dataset by using the following equations:

$$R_b = r_b + (\beta + 1) * R \tag{10}$$

$$R_b = \frac{U_T}{I_b} + \beta * R \tag{11}$$

In Equations 10 and 11, R is the value of the resistor connected to the emitter, $U_T = 28.2mV$, and $\beta = 194.093$. The theoretical fit is more closely related to the data when the base current is relatively high. In general, the data is fairly scattered at low base currents because the SMU collects less accurate readings at lower current values.

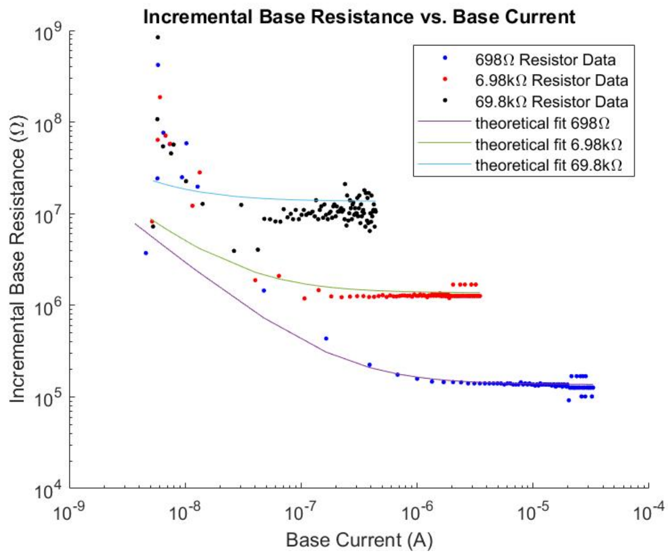

Figure 9: The incremental base resistance of our Emitter-Degenerated transistor over a range of base currents. Each dataset represents a different valued resistor placed in series with the emitter.

We can also find the transconductance(G_m) of our Emitter-Degenerated circuits with respect to the base currents using the following equation.

$$G_m = \frac{\delta I_c}{\delta V_b} \tag{12}$$

We again matched the experimental values with a theoretical fit of G_m. The theoretical equation we used to calculate the theoretical transconductance is:

$$G_m = \frac{1}{R} * \frac{1}{1 + \frac{U_T}{I_c * R}} \tag{13}$$

Overall, the data-sets in Figure 10 are more closely aligned to their respective theoretical fits than the plot of the incremental resistance (R_b) in Figure 9. While the experimental calculation for R_b relies on δV_b and I_b, the calculations for the experimental G_m rely only on V_b and I_c. Because V_c is twice the order of magnitude of I_b, the SMU can record more accurate I_c values than I_b. This will lead data in the transconductance graph to be more accurate than our incremental resistance graph.

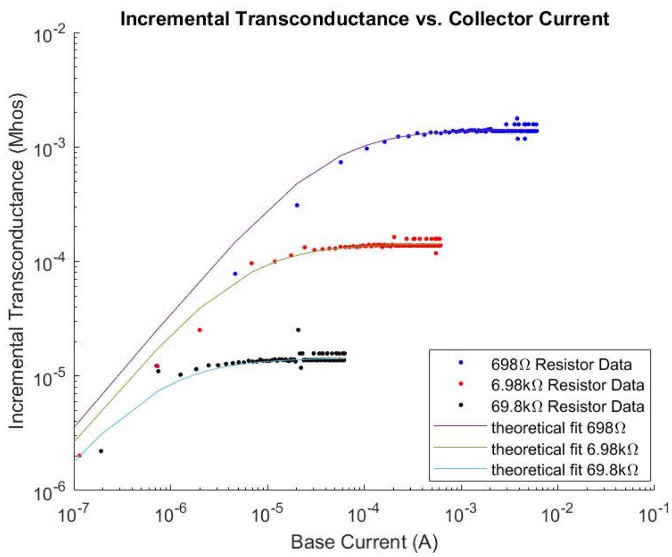

Figure 10: Plot of transconductance (G_m) with respect to base current. Each data set represents a different valued resistor placed in series with our emitter. The theoretical fit is calculated using Equation 13.

3 Experiment 3: Follower Voltage Transfer Characteristics

3.1 Circuit

For this circuit, the input voltage V_{in} is equal to the base voltage V_b and is measured by channel 1. The output voltage of the circuit(V_{out}) is equal to V_e and is measured by channel 2. This circuit is similar to the circuit in experiment 3 but instead of measuring the current through the resistor, we measured the voltage across the resistor. The resistor we choose in this circuit had a resistance of 698Ω.

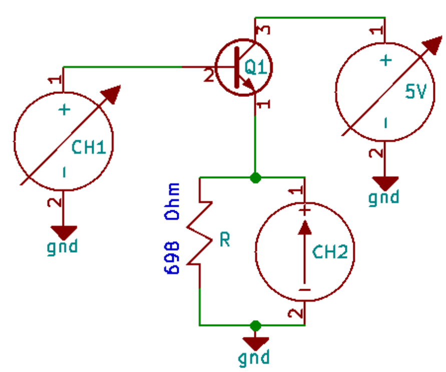

Figure 11: NPN transistor setup as a Voltage Follower between the base and emitter voltages.

3.2 Observations

The Emitter-Follower circuit has two distinct regions of operations. When V_{in} is less than V_{on}, V_{out} increases exponentially and when V_{in} is greater than V_{on}, V_{out} increases linearly. To find the relationship between V_{in} and V_{out} in the linear region as well as the value for V_{on}, we calculated a line of best fit, using Matlab's polyfit function, where the slope is equal to the ratio between V_{out} and V_{in} and the x-intercept is equal to V_{on}. For the following graph, $V_{on} = .68V$.

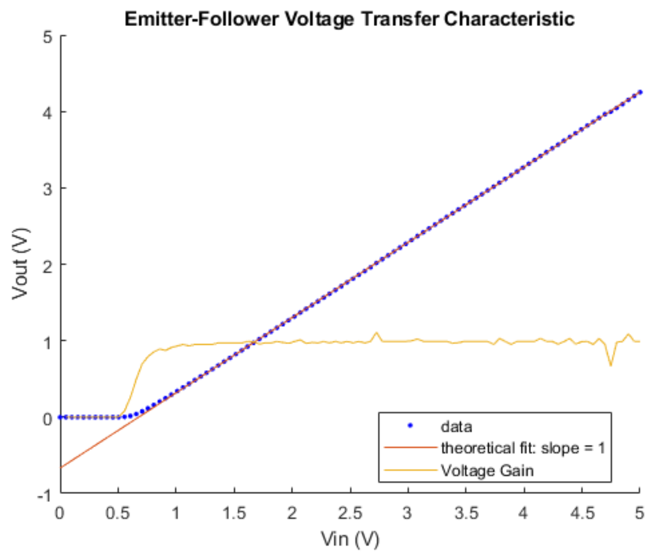

Figure 12: Voltage Transfer Characteristic for Emitter-Follower circuit. The theoretical fit has a slope of 1 with x-intercept equal to V_{on}. The voltage gain or incremental slope relative to V_{in} is also shown. Note how the transition to a voltage gain of 1 coincides with the x-intercept of the line of best fit.

The relationship (otherwise known as the voltage gain) between V_{out} and V_{in} changes over the range of V_{in}. The change in gain can be expressed as the incremental voltage gain which is equal following in equation and shown in the graph above.

$$\frac{\delta V_{out}}{\delta V_{in}} \tag{14}$$

In Figure 12, $\frac{\delta V_{out}}{\delta V_{in}} = 1$ when V_{in} is greater than V_{on}. We calculated $\frac{\delta V_{out}}{\delta V_{in}}$ using Matlab's diff command and the dot-divide operator.

The difference between V_{in} and V_{out}, or $V_{in} - V_{out}$, is less than V_{on} when $V_{in} < V_{on}$ and equal to V_{on} when $V_{in} > V_{on}$. This is due to the fact the slope of V_{out} versus V_{in} is practically 0 when V_{in} is smaller than V_{on} and 1 when $V_{in} >= V_{on}$. In other words, the difference between V_{in} and V_{out} increases only when the slope is approximately equal to 0 which is in the range of $0 < V_{in} < V_{on}$. As a result, the maximum difference between V_{in} and V_{out} is V_{on}.

4 Experiment 4: Inverter Voltage Transfer Characteristics

4.1 Circuit

In this circuit we added an additional resistor in series with the collector of our transistor that had a resistance that was roughly a small integer multiple of the emitter resistor from Experiment 3. This circuit is known as a voltage inverter and is shown in Figure 13. We then measured the voltage between the collector node and ground with respect to the base voltage.

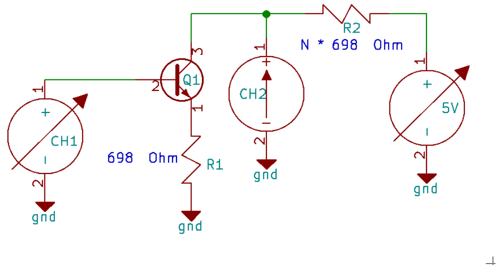

Figure 13: NPN transistor setup as a Inverted Voltage Follower between the base and emitter voltages. The N values for the R2 resistor were roughly 2,3 and 7.

4.2 Observations

Figure 14 shows a plot of V_{out} versus V_{in} for each of the three resistors we tested. The resistors were $1.42k\Omega$, $1.99k\Omega$, and $4.97k\Omega$, which are the equivalent of 2.03, 2.85, and 7.12 times the value of our 698Ω emitter resistor, respectively.

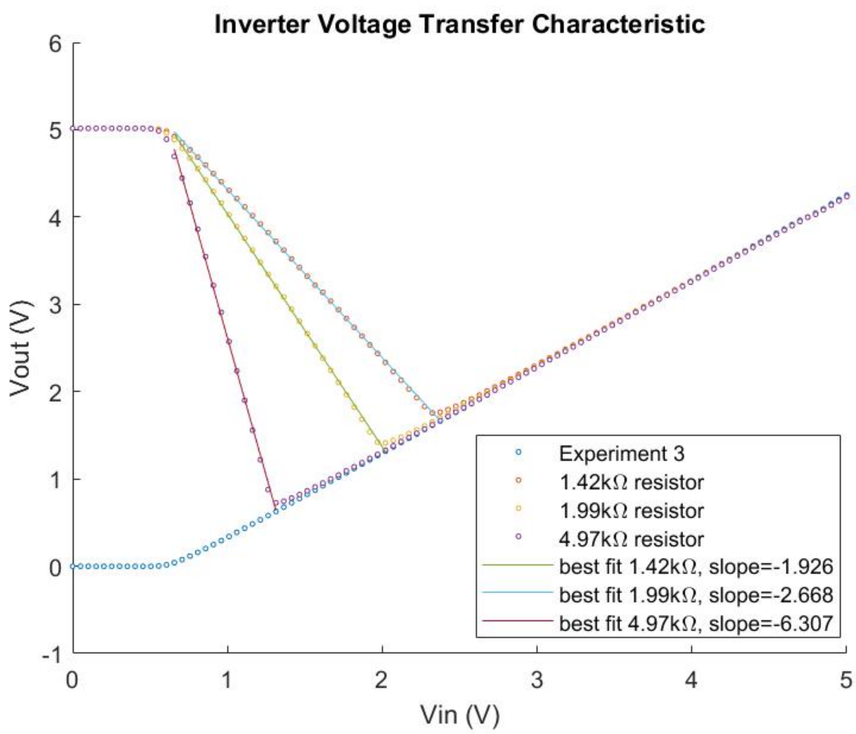

Figure 14: Voltage transfer characteristics for each of our three collector resistors, along with best fits to the forward active region of the data and the voltage transfer characteristic of the emitter-follower from Experiment 3.

As Figure 14 displays, the output voltage for each resistor initially decreases before eventually turning around and increasing again: creating a complex relationship we will explore more in the postlab assignment. For each of the three resistors, we utilized Matlab's polyfit function to fit a line of best fit to the portion of the data when the transistor was in forward active mode, or when the voltage transfer characteristic was steadily decreasing. From these best-fit lines, we obtained slopes that should be equal to negative m, where m is the factor by which our collector resistor is greater than our emitter resistor. The slopes of these polyfits suggested our collector resistors were 1.926, 2.668, and 6.307 times the value of our 698Ω emitter resistor. Though none of these are exactly equal to our expected multiples, they are close enough for us to conclude that the slope of the forward active regions of each plot do, in fact, have slopes roughly equal to negative m.

To help visualize the effects of our inverter circuit, we plotted the incremental voltage gain of each circuit with respect to V_{in} in Figure 15. The equation used to calculate the incremental voltage gain is $gain = \frac{\delta V_{out}}{\delta V_{in}}$. The plot below elucidates the three distinct modes of operation for this circuit. When $V_{in} < V_{on}$ the voltage gain is nearly zero: a characteristic of the cut-off mode of the transistor. The voltage gain then maintains a constant negative value equal to the negative of the ratio between the collector and emitter resistors. This mode is referred to as forward active mode and occurs when $V_e < V_{in} < V_{out}$. Finally, after $V_{in} - V_{out} \approx 4 * U_T$, the transistor reaches deep saturation mode and the voltage gain switches to a steady value equal to 1.

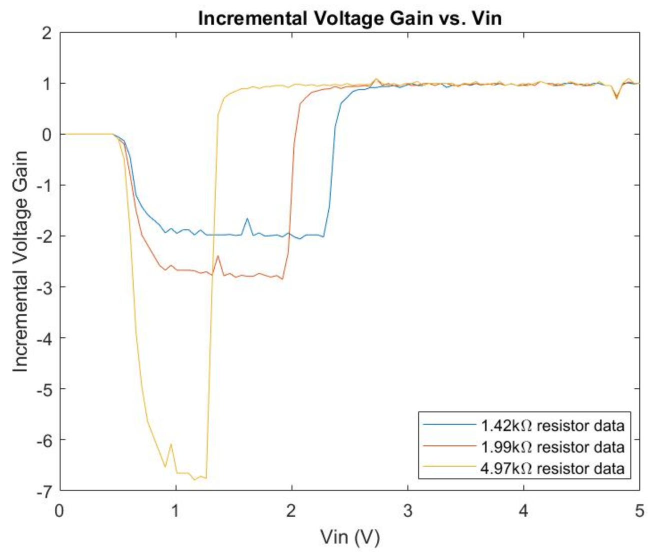

Figure 15: Incremental voltage gain with respect to V_{in}. The incremental voltage gain was plotted for each value of resistor. $Gain = \frac{\delta V_{out}}{\delta V_{in}}$ was used to calculate the incremental voltage gain.