# Circuits Lab 2

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## 1 Experiment 1: Diode-Connected Transistor Characteristics

Our goal for experiment 1 was to setup up a 2N3904 transistor as a diode and measure its current-voltage and voltage-current characteristics. From this data we modeled our transistor with the ideal diode equation to gain insight on its saturation current and thermal voltage values.

### 1.1 Circuit

In order to simulate a diode with our 2N3904 transistor, we had to attach the collector and base of the transistor together. We then used the SMU to both supply a given current through channel 1 and measure the voltage across channel 1: which due to KVL is the same voltage across the transistor. The schematic for this experiment is shown below.

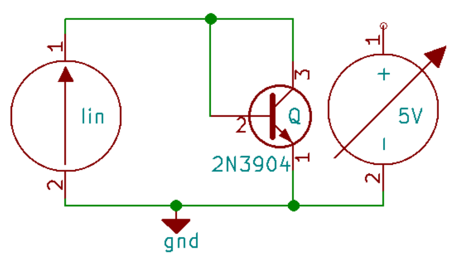


Figure 1: Schematic for the circuit used to measure the voltage across the transistor with respect to the current supplied by the SMU.

The schematic used for measuring the current through the transistor is similar to the one used to measure voltage except that channel 1 is now used as a voltage source instead of a current source.

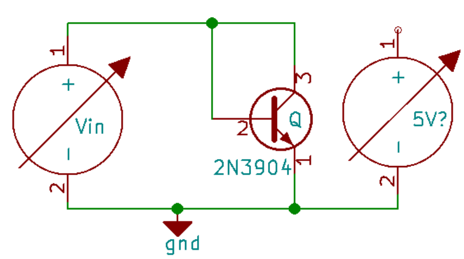


Figure 2: Schematic for the circuit used to measure the current through the transistor with respect to the voltage applied by the SMU.

## 1.2 Observations

Our first graph shows both the current-voltage (I-V) measurements and voltage-current (V-I) measurements. Both sets of data are plotted with the x-axis being the voltage across the diode-connected transistor and the y-axis as the current through the transistor. In order to obtain the V-I characteristic, we supplied a current at logarithmic intervals ranging from 1nA to 10mA using the logspace command and measured the resulting voltage difference across the transistor. Similarly, to obtain the I-V characteristic, we applied a voltage at intervals ranging from 0.75V to 0.3V using the linspace command and measured the resulting current through the transistor. When plotted together, the V-I and I-V measurements are very similar. We then used Matlab's polyfit function to obtain approximations for the slope and y-intercept that we needed to compute $U_T$ and $I_s$ before fitting an exponential curve to each of our measurements using the ideal diode equation, shown in Equation 1. The best fits closely matched our data when the current is on the order of $\mu$A or greater but diverges slightly from the measured values at currents lower than a $\mu$A. This is most likely due to the inaccurate current readings from the SMU when reading small current values.

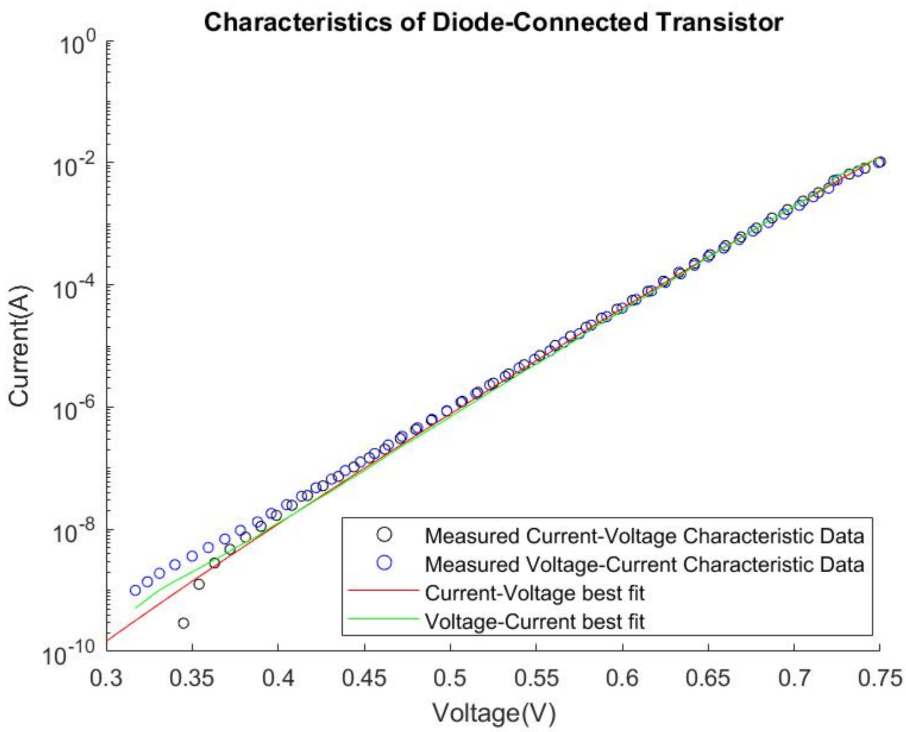


Figure 3: The current-voltage and voltage-current characteristics as measured by the SMU, along with our best fit lines to the curve. The slope of our best-fit line for the I-V characteristic is 39.03 e-folds/V.

By plotting the data on a semilog-y plot, we were able to keep the data linear. This means that we can use linear regression to find the values of thermal voltage ($U_T$) and saturation current ($I_s$) of our transistor, using Equations 1 and 2.

$$I = I_s * e^{\frac{V}{U_T}} \tag{1}$$

$$\ln I = \ln I_s + \frac{V}{U_T} \tag{2}$$

From Equation 2, we can find $U_T$ by taking the reciprocal of the slope of our I-V characteristic. The saturation current ($I_s$) can be found by setting the y intercept of our line equal to $\ln I_s$. In other words, $I_s = e^{y-intercept}$. From our data, we found that $U_T = 25.4mV$ and that $I_s = -1.67\text{e}{-15}A$. Because we cannot in practice attain a negative current value for $I_s$, this error could be coming from any number of places, including changes in the temperature of the room or the inaccuracies of the SMU when the current is on the order of femtoamps.

Our second graph is a log-log plot of the incremental resistance ($r_d$) versus current through the transistor. The incremental resistance can be calculated with Equation 3.

$$r_d = \frac{\partial V}{\partial I} \tag{3}$$

In order to obtain measured values for the incremental resistance of the diode, we used Matlab's diff command and dot-divide operator in order to compute a finite-difference approximation to the partial derivative in Equation 3. We then used the 25.4mV value we had obtained from our characteristic plots for $U_T$ in conjunction with Equation 4, found by taking the derivative of Equation 3, in order to fit a straight line to the data on a log-log plot of incremental resistance against the current flowing through the diode.

$$r_d = \frac{U_T}{I} \tag{4}$$

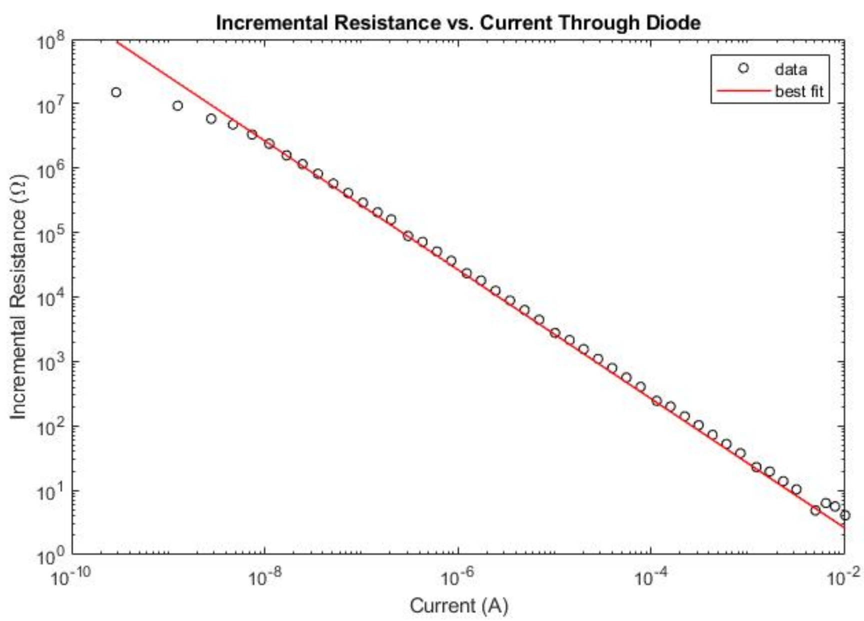


Figure 4: Incremental resistance shown with respect to the current through our diode-connected transistor.

The line of best fit matches our data well except when the current is on orders of a nanoamp or smaller, which can be attributed to the measurement error from the small current and voltage values.

# 2 Experiment 2: Characteristics of a Resistor and Diode in Series

Similar to the previous experiment, this experiment focused on the measuring the voltage across and current through our diode-connected transistor with an additional resistor added in series with the transistor. For this experiment, we used three resistance values: $402\Omega$, $4.02k\Omega$ and $41.2k\Omega$.

## 2.1 Circuit

The new schematic used to measure the voltage across and current through our transistor in series with a resistor is shown in Figure 5.

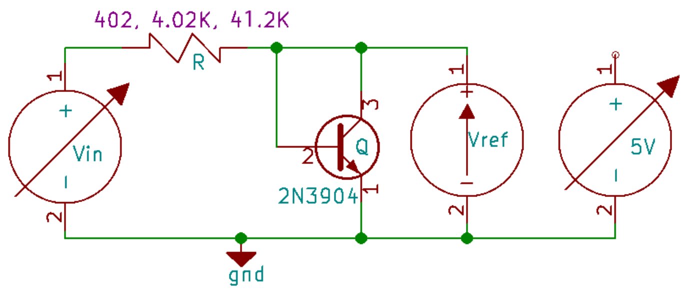


Figure 5: Resistor and diode-connected transistor in series. SMU is setup to measure the voltage across and current through the transistor.

## 2.2 Observations

For each of the resistance values, we measured the voltage across the transistor ($V_t$) with respect to the input voltage of the system ($V_{in}$). We plotted these data points, along with a straight line fit to the linear region of the data obtained using Matlab's polyfit function, in Figure 6. The slope of this line reflects the ratio $\frac{\delta V_t}{\delta V_{in}}$ which in our case is equal to .999. This implies that in the linear region, i.e. when $V_{in}$ is less than roughly $0.5V$, any change in the applied input voltage is equal to the change in transistor voltage with almost no change in voltage occurring across the resistor. In other words, the transistor is contributing nearly all of the change in input voltage in this region. In contrast, when $V_{in}$ is greater than roughly $0.5V$, the slope begins to level off, implying that the transistor is contributing very little of the change in input voltage at this point. To be more rigorous in our interpretation of the results, we learned in prelab 2, question 2 that the voltage across the transistor transitions from increasing linearly with increases in $V_{in}$ to levelling off when the incremental voltage across the resistor is equal to the incremental voltage across the transistor. This voltage is known as $V_{on}$ and we can see empirically that it is around 0.5V in Figure 6. We will find a more precise value for the turn-on voltage later.

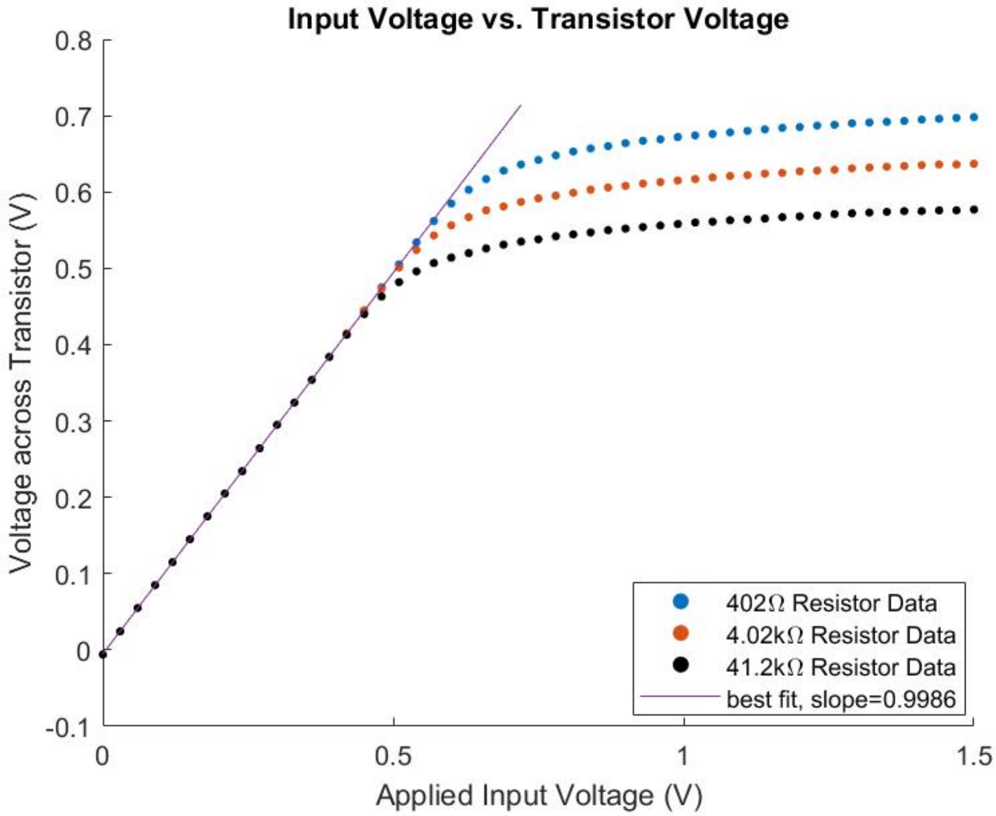


Figure 6: Transistor voltage with respect to input voltage. Three different groups of data are shown for the three different resistance values.

Figure 7 shows the relationship between our measured input voltage ($V_{in}$) and the current through the transistor ($I_t$). This graph also has two distinct regions where the current is increasing exponentially when $V_{in} <\sim .5V$ and increasing linearly when $V_{in} >\sim .5V$. This implies that the circuit follows the ideal diode equation, seen in Equation 1, when $V_{in} <\sim .5V$ and follows Ohm's law when $V_{in} >\sim .5V$. Again, we are using 0.5V as an extremely rough approximation for $V_{on}$ here.

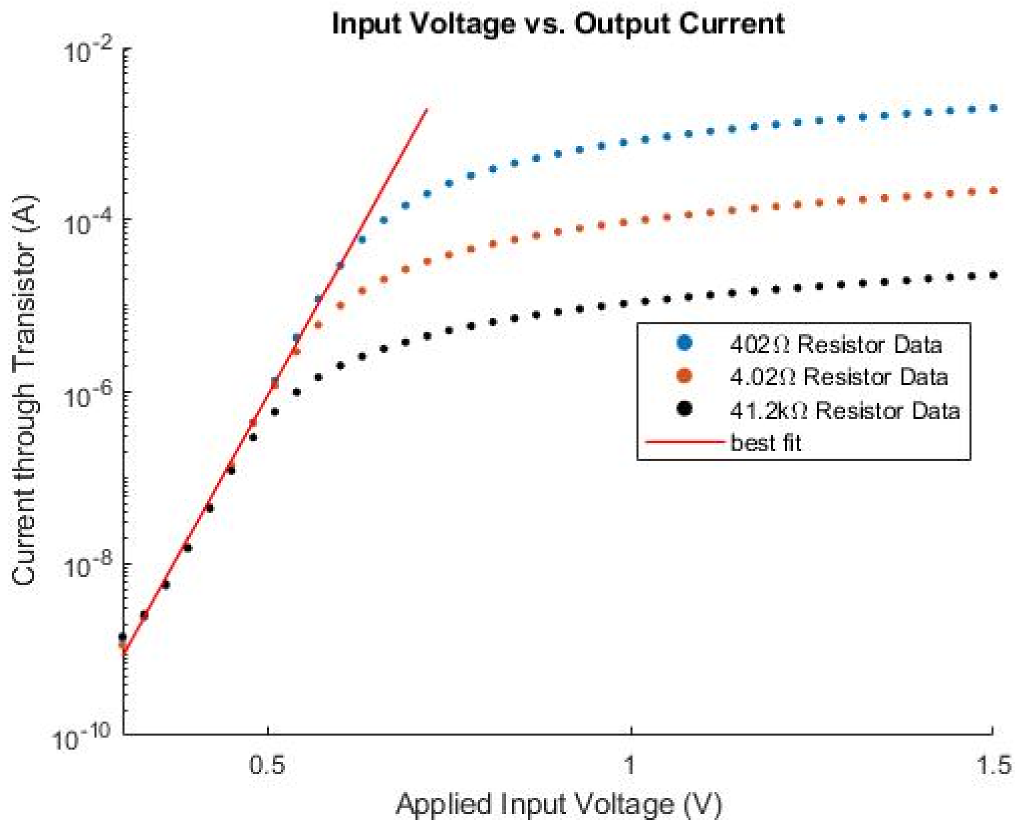


Figure 7: Transistor current with respect to input voltage. Three different groups of data are shown for the three different resistance values. The slope of our best-fit line is equal to 34.91 e-folds/V.

The plots in Figure 8 below shows the current passing through the diode-connected transistor with respect to the input voltage of the circuit. Each subplot is associated with a unique resistor. We used Matlab's polyfit function to fit a straight line to the region of the data where the current and voltage varied linearly (i.e. after approximately 0.5V). These best-fits agree with the data over the entire linear range for all three plots. The slopes of these lines of best fit are equal to the conductivities of the circuits, and so the resistances are the reciprocals of the slopes. As a result, we can extract values for the resistance of each circuit when the I-V characteristic is dominated by the resistor. For instance, for the circuit in which we utilized a 402Ω resistor, the output of the polyfit function indicated that the resistance of the circuit was equal to 435.97Ω.

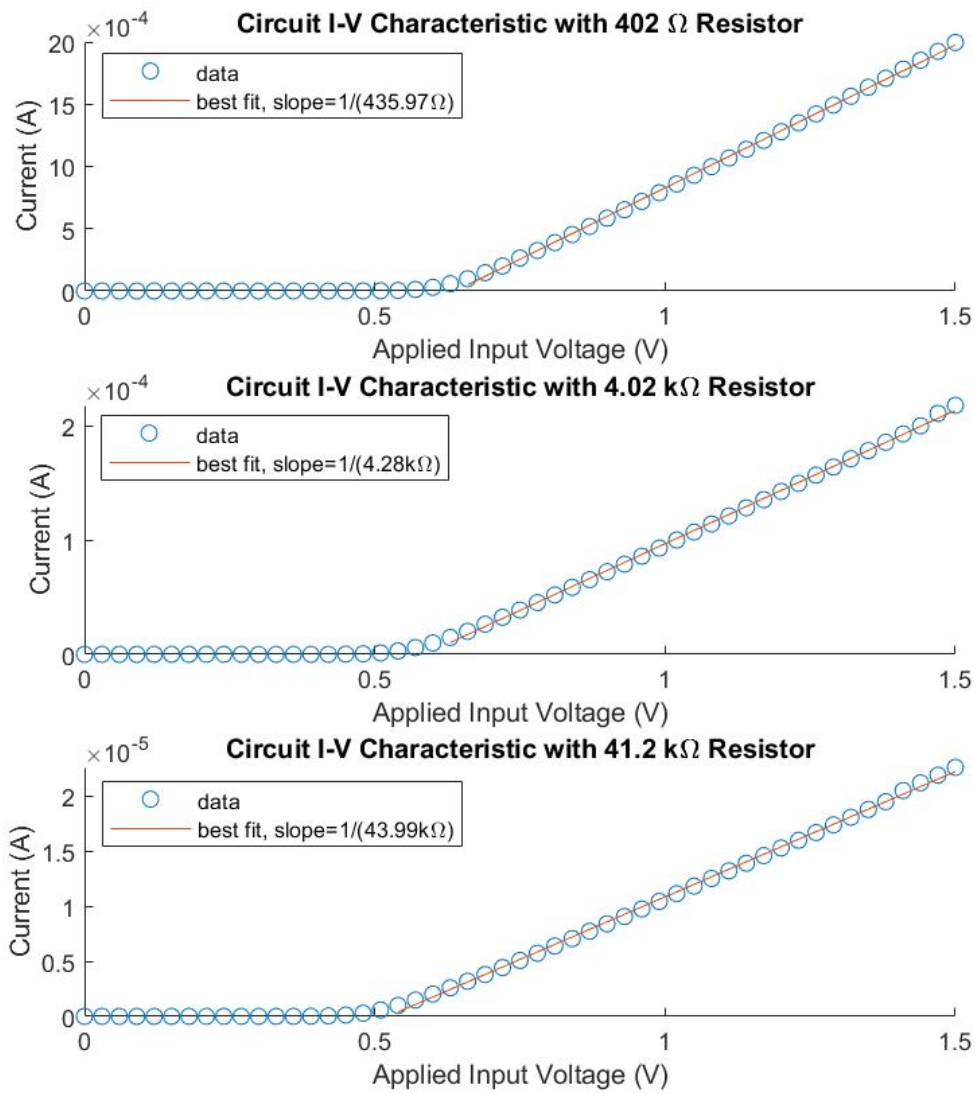


Figure 8: Plots showing the input voltage-current relationship on a linear scale with resistors of three unique resistances. Note the exponential and linear regions of the plots.

From the pre-lab problem set, we were able to derive two equations to calculate $I_{on}$ and $V_{on}$.

$$I_{on} = \frac{U_T}{R} \tag{5}$$

$$V_{on} = U_T * \ln \frac{I_{on}}{I_s} \tag{6}$$

In order to experimentally calculate the value of $V_{on}$ in the case of each resistor, we had to find the value of the x-intercept from the current-voltage data, again using the output of Matlab's polyfit function to find the y-intercept and slope of the line before solving for the x-intercept. This method of finding $V_{on}$ relies on the idea that the I-V characteristic of our resistor-transistor circuit becomes linear when $V_{in} > V_{on}$ and that the current through the transistor is $0A$ when $V_{in} < V_{on}$.

After finding the experimental values for $V_{on}$ using the x-intercepts, we can calculate the experimental values for $I_{on}$ by taking Equation 6 and solving for $I_{on}$. This equation also required us to calculate the values of $U_T$ and $I_s$ for each resistance value. These values can be obtained be using linear regression on the log(I)-V relationship as shown in Equation 2 from the first experiment. To do this, we used Matlab's polyfit function to fit a straight line to the graphs shown in Figure 8 when the y axis was on a log scale. We chose not to include graphs of these, as they do not aid in our analysis of the turn-on current and turn-on voltage of the circuit. Each $U_T$ was found by finding the reciprocal of the slope of the line of best fit for the linear portion of the semilog-y current-voltage graphs. The $I_s$ values were then extracted by setting the y-intercept of the line of best fit to the power of "E" ($I_s = e^{y-intercept}A$).

To extract theoretical values for $I_{on}$ and $V_{on}$, we first solved for $I_{on}$ using Equation 5. We used this method to find the theoretical fit for $I_{on}$ for each value of resistance. The theoretical fit for $V_{on}$ was then found using Equation 6 and the theoretical value for $I_{on}$.

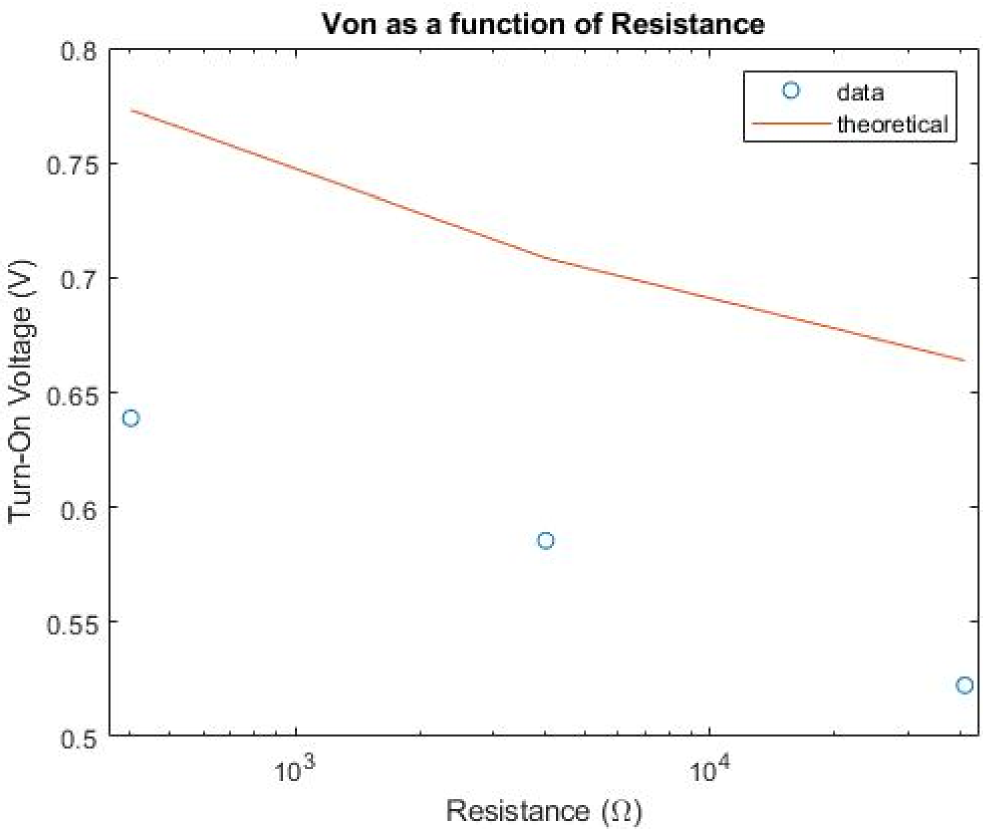


Figure 9: This graph shows $V_{on}$ as a function of resistance. The data is calculated using our experimental method and the theoretical curve is calculated from the theoretical $I_{on}$ values using equation 6.

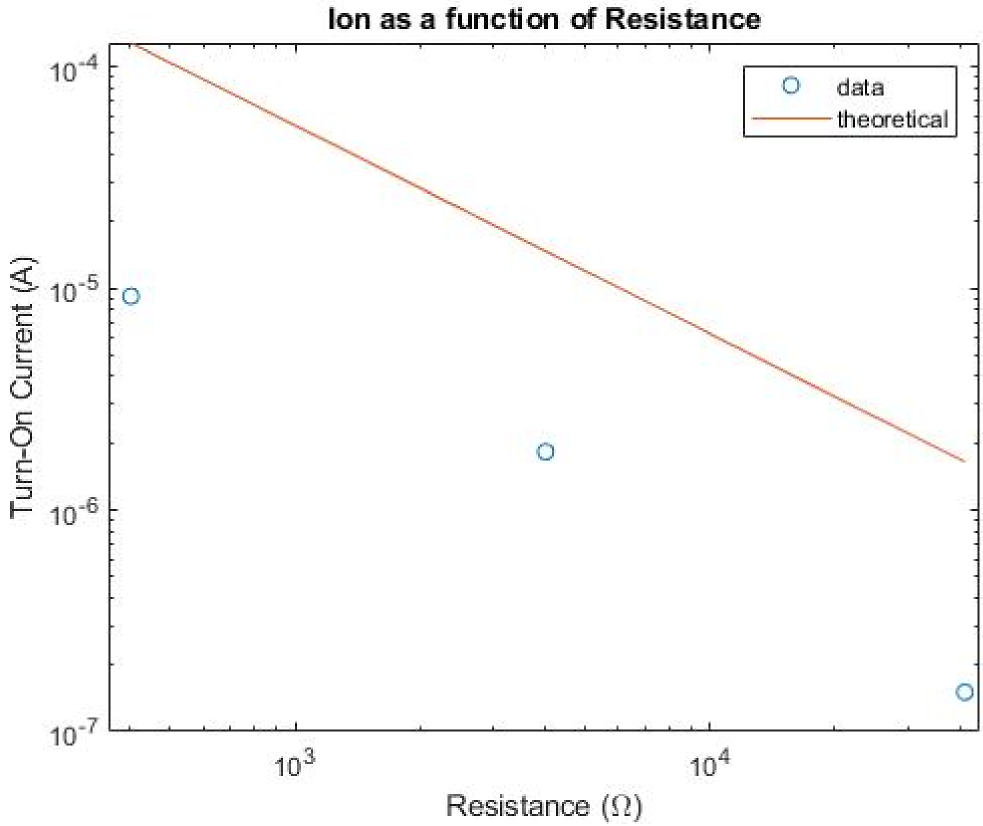


Figure 10: This graph shows $I_{on}$ as a function of resistance. The experimental data was calculated from our experimental $V_{on}$ data while our theoretical data was calculated using equation 5.

In both of these figures, $I_{on}$ and $V_{on}$ decrease with an increase in resistance. This is expected given that $I_{on}$ is inversely proportional with the resistance of the resistor, as shown in equation 5, and that $V_{on}$ is logarithmically related with $I_{on}$. The relatively large offset between our theoretical fit and measured data stem from the utilization of several error prone variables: $I_s$, $U_T$, and $R$. To reiterate, we may consider these error prone as calculating $I_s$ relies on the precision of relatively small measured currents, $U_T$ could change with temperatures in the room over the course of our three tests, and $R$ may vary from the labelled resistances of each resistor.