Programming with

L ogic

I nheritance

F unctions

E quations

(An Introduction)

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Outline

- History
- Generalities
- ullet LIFE's Basic Data Structure: $\psi ext{-Terms}$
- Predicates
- Functions
- Sorts
- Toy Programming Examples
- Conclusion

LIFE was originally conceived *ca.* 1986 by Hassan Aït-Kaci and colleagues at MCC, in Austin, Texas.

• Idea:

Reconcile programming with predicate logic, functions, and structured object inheritance.

• Key:

Use a universal, simple, but powerful, formal data structure called ψ -term.

How?

By solving equational and entailment constraints over order-sorted feature graphs.

Still LIFE was the first prototype of LIFE, done at MCC in 1987-88 by David Plummer in Quintus Prolog.

• Idea:

To experiment with the prototype to have a feel of worth.

• Plus:

Fun to see it work, surprising convenience, fixed the syntax.

• Minus:

Incomplete, slow, and MCC proprietary.

Wild LIFE is the successor of Still LIFE, done by Richard Meyer at Digital's Paris Research Laboratory in C.

- Idea:
 Full independent reimplementation in C.
- Plus:
 Much more complete, reasonably fast, good user conveniences, convincingly solid to support serious applications.
- Minus: Interpreted, big, still incomplete (not by much).

A compiler is currently in the works...

Wild LIFE Contributors:

- Richard Meyer
 90% of Wild LIFE, and the compiler
- Peter Van Roy the missing 10%, and the compiler
- Bruno Dumant graphics toolkit, term expansion, static analyzer
- Jean-Claude Hervé basic X interface
- Kathleen Milsted LIFE shell
- Andreas Podelski theorems, theorems, theorems...
- Hassan Aït-Kaci
 watching the rest work and taking all the credit!

Generalities

LIFE is a generalization of Prolog: most Prolog program run under LIFE.

Same syntactic conventions:

- variables are capitalized (or start with _)
- other identifiers start with a lower-case letter
- the unification predicate is =
- defining Horn clauses uses :-
- the cut control operator is !
- etc.

Except for these differences:

- queries are terminated with a ?
- assertions are terminated with a .

*Ψ***-Terms**

- 42
- int
- −5.66
- real
- "a piece of rope"
- string
- foo bar
- '%* PsyCH(a)oTic**SyMboL!'
- date(friday, 13)
- date(1 => friday, 2 => 13)
- freddy(nails => long, face => ugly)
- [this, is, a, list]
- cons(this, cons(too,[]))

Sorts

Sorts are the data constructors of LIFE.

Sorts are partially ordered by < in a sort hierarchy.

 $\mathbf{@}$ is the most general sort (\top) .

 $\{\}$ is the least sort (\bot) .

values are sorts like all others.

Variables and Tags

Like Prolog, LIFE's variables start with _ or an upper case letter,

Unlike Prolog, LIFE's variables are not restricted to appear only as leaves of terms.

Thus, variables can be used as (reference) tags within a ψ -term's structure.

They are used as explicit handles for referencing the part of ψ -term they tag.

These references may be cyclic; that is, a variable may occur within a ψ -term tagged by it.

Variables and Tags

Tagging of a ψ -term \mathbf{t} by a variable \mathbf{x} is of the form $\mathbf{x}:\mathbf{t}$.

If a variable occurs not as a ψ -term's tag but as a simple isolated variable, it is implicitly be tagging \top , exactly as if it had been written \mathbf{x} : @.

If the same variable needs to be constrained to be the conjunction of two terms, it is written using the connective, as in x:t1&t2. This is equivalent to writing x=t1, x=t2.

Disjunctive terms

A disjunctive term is an expression of the form:

```
{t1; ...; tn}
```

 $n \geq \textit{0}$, where each ti is either a ψ -term or a disjunctive term.

In Wild LIFE, disjunctive terms are enumerated using a left-right depth-first backtracking strategy, exactly as Prolog's (and LIFE's!) predicate level resolution.

- A={1;2;3}? is like A=1; A=2; A=3? where; signifies "or" in Edinburgh Prolog syntax.
- $p({a;b})$. is like asserting p(a) . p(b) .
- write (vehicle&four_wheels)? first prints car then on backtracking will print truck.

Backtrackable Tag Assignment

The statement **x**<-**y** overwrites **x** with **y**.

The tags \mathbf{x} and \mathbf{y} reference standard (backtrackable) ψ -terms.

Backtracking past this statement will restore the original value of x.

For example:

```
> X=5, (X <- 6; X <- 7), write(X), nl, fail?
6
7
*** No
>
```

This predicate is very useful for building "black boxes" that have clean logical behavior when viewed from the outside but that need destructive assignment to be implemented efficiently.

Sort intersection

```
bike <| two_wheels.
bike <| vehicle.
truck <| four_wheels.
truck <| vehicle.
car <| four_wheels.
car <| four_wheels.
toy_car <| four_wheels.
rolls_royce <| car.</pre>
```

- two_wheels \(\) vehicle = bike
- four_wheels \land vehicle = $\{car; truck\}$
- two_wheels \land four_wheels =\perc \land \text{
- rolls_royce \(\cap \) car = rolls_royce
- truck \wedge @ = truck

*Ψ***-Term Unification**

```
student
                           employee
                     / \
staff faculty
/ | \ / | \
bob piotr pablo workstudy art judy don john sheila
             simon elena
student < person.
employee < | person.</pre>
staff < employee. workstudy < student.</pre>
faculty < | employee. workstudy < | staff.</pre>
bob < student. don < faculty.
piotr < | student. john < | faculty.</pre>
pablo < | student. sheila < | faculty.
elena < workstudy. art < staff.
simon < | workstudy. judy < | staff.</pre>
```

person

*Ψ***-Term Unification**

```
X = student
     (roommate => employee(rep => S),
      advisor => don(secretary => S)),
Y = employee
     (advisor => don(assistant => A),
      roommate => S:student(rep => S),
      helper => simon(spouse => A)),
X = Y?
X = workstudy
       (advisor => don
                    (assistant => _A,
                     secretary => _B: workstudy
                                         (rep => _B)),
        helper => simon(spouse => _A),
        roommate => _B),
Y = X.
```

Predicates

LIFE's predicates are defined exactly as Prolog's, except that terms are replaced by ψ -terms.

They are executed using ψ -term unification.

```
> truck < | vehicle.

*** Yes
> mobile(vehicle).

*** Yes
> useful(truck).

*** Yes
> mobile(X), useful(X)?

*** Yes
X = truck.
```

Compatibility with Prolog

A difference with Prolog is that LIFE terms have no fixed arity.

```
pred(A,B,C) :- write(A,B,C).

In (SICStus) Prolog:

?- pred(1,2,3).
123
?- pred(A,B,C).
_26_60_94
?- pred(A,B,C,D).
WARNING: predicate 'pred/4' undefined.
?- pred(A,B).
WARNING: predicate 'pred/2' undefined.
```

Compatibility with Prolog

In Wild LIFE:

```
> pred(1,2,3)?
123
*** Yes
> pred(A,B,C)?
999
*** Yes
A = 0, B = 0, C = 0.
--1>
*** No
> pred(A,B,C,D)?
aaa
*** Yes
A = 0, B = 0, C = 0, D = 0.
--1>
*** No
> pred?
999
*** Yes
```

User interaction

Interaction with user is more flexible than Prolog's: Once a query is answered, a user can extend it in the current context by entering:

 $\langle CR \rangle$ to abandon this query and go back to the previous level,

; to force backtracking and look for another answer,

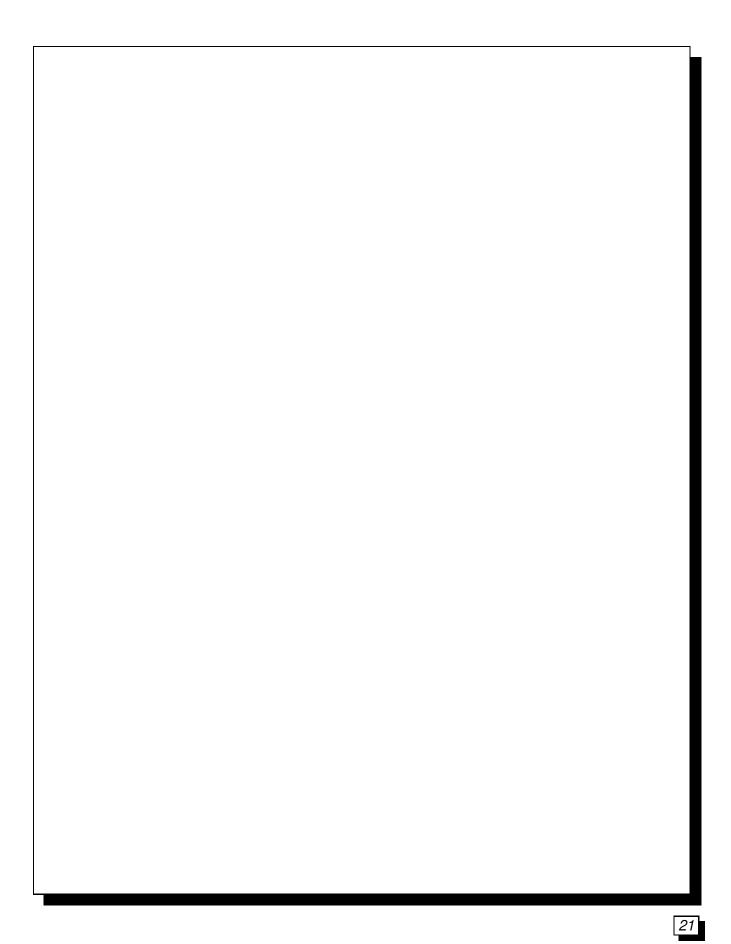
a goal followed by ? to extend this query,

. to pop to top-level from any depth.

Example:

User interaction

```
> grandfather(A,B)?
*** Yes
A = john, B = michael.
--1> father (A, C)?
*** Yes
A = john, B = michael, C = harry.
----2> ;
*** Yes
A = john, B = michael, C = mike.
----2> ;
*** No
A = john, B = michael.
--1> father(C,B)?
*** Yes
A = john, B = michael, C = harry.
----2> father (A, C)?
*** Yes
A = john, B = michael, C = harry.
----3>
*** No
A = john, B = michael, C = harry.
----2> .
>
```



Functions

LIFE's function are defined exactly as rewrite rules transforming ψ -terms into ψ -terms.

They are executed using ψ -term matching, NOT unification.

```
fact(0) -> 1.
fact(N:int) -> N*fact(N-1).

> write(fact(5))?
120
*** Yes
```

Residuation

```
> A=fact(B)?
*** Yes
A = 0, B = 0.
--1> B=real?
*** Yes
A = 0, B = real \sim.
----2> B=5?
*** Yes
A = 120, B = 5.
----3>
*** No
A = 0, B = real \sim.
----2> A=123?
*** Yes
A = 123, B = real \sim.
----3> B=6?
*** No
A = 123, B = real \sim.
----3>
```

Functions

Functions are deterministic---no value guessing nor backtracking.

Calling f(foo,bar) skips definition f(x,x) -> ... if foo and bar are non-unifiable; otherwise, it residuates. It will use it only if, and when, the two args are unified by the context.

Arithmetic functions are inverted---e.g., the goal 0=B-c causes B and c to be unified.

```
> A = F(B), F = /(2=>A), A = 5?
*** Yes
A = 5, B = 25, F = /(2 => A).
```

Note that here / (division) is curryed before being inverted.

Currying

Currying is not the same as residuation, because the result of currying is a function, not \top .

In curryed form, $f(a \Rightarrow X,b \Rightarrow Y)$ is:

$$f(a => X) & @(b => Y)$$

but also:

$$f(b => Y) & @(a => X)$$

Argument order is irrelevant!

```
> f(X,Y,Z) -> [X,Y,Z].
*** Yes
> A=f(a,3 => c)?
*** Yes
A = f(a,3 => c).
--1> A=f(2 => b)?
*** Yes
A = [a,b,c].
```

Functional variables

Functional variables are allowed.

That is, a functional expression may have a variable where a root symbol is expected.

```
map(F,[]) -> [].
map(F,[H|T]) -> [F(H)|map(F,T)].

> L=M(F,[1,2,3,4])?
*** Yes
F = @, L = @, M = @~.
--1> M=map?
*** Yes
F = @~~~~, L = [@,@,@,@], M = map.
---2> F= +(2=>1)?
*** Yes
F = +(2 => 1), L = [2,3,4,5], M = map.
----3>
```

Functions

Residuation, currying, and functional variables give functions extreme flexibility:

Functions

```
> test?
function *(2 => 4) applied to 5 is 20
function fact applied to 5 is 120
function *(2 => 4) applied to 3 is 12
function fact applied to 3 is 6
function *(2 => 4) applied to 7 is 28
function fact applied to 7 is 5040
*** No
```

Quote and eval

LIFE's functions use eager evaluation. This can be prevented using a quoting operator \.

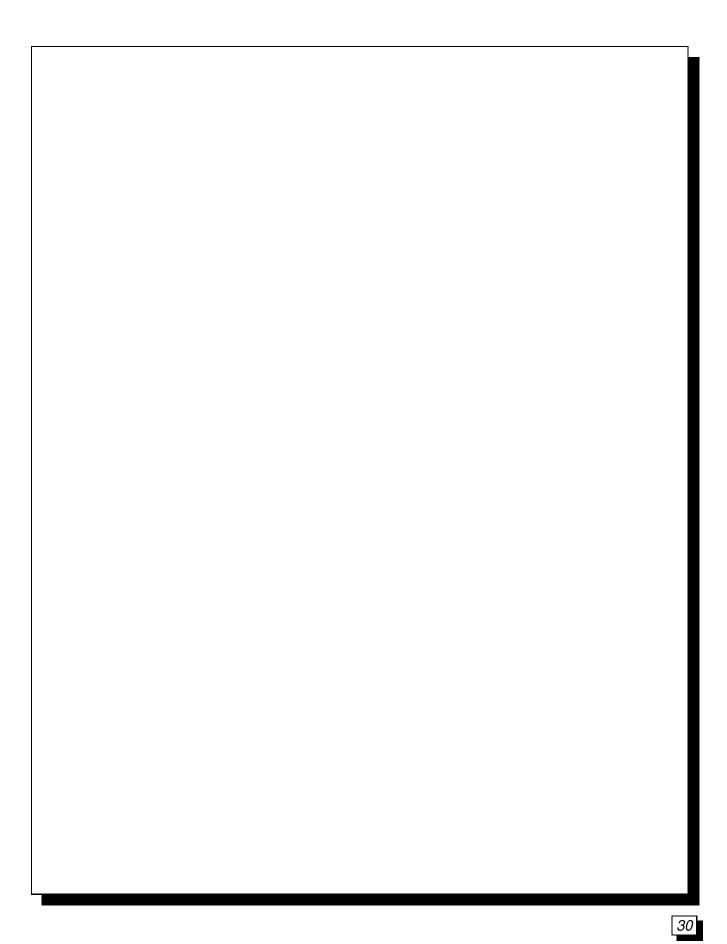
```
> X =1+2?
*** Yes
X = 3.
--1> Y='(1+2)?
*** Yes
X = 3, Y = 1 + 2
```

Dually, a function called **eval** may be used to compute the result of a quoted form.

```
----2> Z=eval(Y)?
*** Yes
X = 3, Y = 1 + 2, Z = 3.
```

Note that eval does not modify the quoted form.

Another function called **evalin** works like **eval** but evaluates the expression side-effecting it "in-place."



Arbitr-Arity

In LIFE everything is a ψ -term!

This can be exploited to great benefit to express that some predicates or functions take an unspecified number of arguments.

```
S:sum -> add(features(S),S).

add([H|T],V) -> V.H+add(T,V).
add([],V) -> 0.

> X = sum(1,2,3,4)?

*** Yes
X = 10.
--1> Y=sum(1,2,3,4,5)?

*** Yes
X = 10, Y = 15.
----2>
```

One can attach properties to sorts: attributes or arbitrary relational or functional dependency constraints.

These properties will be verified during execution, and also inherited by subsorts.

```
> :: person(age => int).
*** Yes
> man <| person.
*** Yes
> A=man?
*** Yes
A = man(age => int).
--1>
```

```
man := person(gender => male).
is sugaring for:
man < person.</pre>
:: man(gender => male).
tree := { leaf ; node(left => tree,
                         right => tree) }.
is sugaring for:
leaf < tree.</pre>
node < tree.
:: node(left => tree, right => tree).
```

```
:: rectangle(long_side => L:real,
             short_side => S:real,
             area => L*S).
square := rectangle(side => S,
                     long_side => S,
                     short_side => S).
> R=rectangle(area => 16,
              short_side => 4)?
*** Yes
R = rectangle(area => 16,
              long_side => 4,
              short_side => 4).
--1> R=square?
*** Yes
R = square(area => 16,
           long_side => _A: 4,
           short_side => _A,
           side => _A).
```

```
:: devout(faith => F, pray_to => X)
 holy_figure(F,X).
holy_figure(muslim, allah).
holy_figure(jewish, yahveh).
holy_figure(christian, jesus_christ).
> X=devout?
*** Yes
X = devout(faith => muslim,
           pray_to => allah).
--1> ;
*** Yes
X = devout(faith => jewish,
           pray_to => yahveh).
--1> ;
*** Yes
X = devout(faith => christian,
           pray_to => jesus_christ).
--1> ;
*** No
```

Impromptu demons:

```
> :: I:int | write(I, " ").
*** Yes
> A=5*7?
5 7 35
*** Yes
A = 35.
--1> B=fact (5)?
5 1 4 1 3 1 2 1 1 1 0 1 1 2 6 24 120
*** Yes
A = 35, B = 120.
---2>
> :: C:cons | write(C.1), nl.
*** Yes
> A=[a,b,c,d]?
d
C
b
a
*** Yes
A = [a,b,c,d].
```

Recursive sorts can also be defined. For example, the (built-in) list sort is defined as:

```
list := {[] ; [@|list]}.
```

But there is a safe form of recursion and an unsafe one:

- safe recursion: the recursive occurrence of the sort is in a strictly more specific sort.
- unsafe recursion: the recursive occurrence of the sort is in an equal or more general sort.

Example of unsafe recursion:

```
:: person(best_friend => person).
```

This loops for ever...

Temporary workaround (hmm... hack!) is to specify:

```
> delay_check(person)?
```

That will prevent checking the definition of **person** if it has no attributes.

```
:: P:person(best_friend => Q:person)
 get_along(P,Q).
*** Yes
> delay_check(person)?
*** Yes
> cleopatra := person(nose => pretty,
                      occupation => queen).
*** Yes
> julius := person(last_name => caesar).
*** Yes
> get_along(cleopatra, julius).
*** Yes
> A=person?
*** Yes
A = person.
--1> A=@(nose => pretty)?
*** Yes
A = cleopatra(best_friend => julius,
              nose => pretty,
              occupation => queen).
```

It is important to relate LIFE's concepts to concepts that are empirically known in O-O programming, like that of class and instance.

Classes are declared by sort definitions:

Like a struct, this adds fields to a class definition.

To say that class1 inherits all properties of class2:

```
class1 < class2.</pre>
```

Instances are created by mentioning the class name in the program. For example, executing:

```
> X=int?
```

creates an instance of the class **int**. Each mention of **int** creates a fresh instance. Therefore, executing:

```
> X=int, Y=int?
```

creates two different instances of the class **int** in **x** and **Y**. We can do:

```
> X=int, Y=int, X=56, Y=23?
```

This would not be possible if **x** and **y** were the same instance.

Wild LIFE assumes that mentioning a class name in the program always creates a fresh instance that is different from all other instances of the class.

For example:

$$> X=23, Y=23?$$

creates two different instances of the class 23.

If we have the function defined as:

$$f(A, A) \rightarrow 1.$$

then the call f(x, y) will not fire, since x and y are different instances.

To make f(X, Y) fire, X and Y must be the same instance.

In Wild LIFE, the only way to do this is to unify them explicitly:

```
> X=23, Y=23, X=Y, write(f(X,Y))?
```

will write 1 (*i.e.*, the function f will fire).

Hamming numbers

```
mult_list(F,N,[H|T]) ->
         cond(R:(F*H) = < N,
               [R mult_list(F,N,T)],
              []).
merge(L,[]) -> L.
merge([],L) -> L.
merge(L1:[H1 | T1], L2:[H2 | T2]) ->
         cond(H1 = := H2,
               [H1 | merge (T1, T2)],
               cond(H1 > H2,
                     [H2 | merge (L1, T2)],
                     [H1 merge (T1, L2)])).
hamming(N) ->
         S: [1 | merge (mult_list(2, N, S),
                     merge(mult_list(3,N,S),
                            mult_list(5,N,S)))].
> H=hamming(26)?
H = [1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25]
*** Yes
```

Quick Sort

SEND+MORE=MONEY

```
solve :-
      % M=0 is uninteresting:
          M=1,
      % Arithmetic constraints:
          C3 + S + M = O + 10*M
          C2 + E + O = N + 10*C3,
          C1 + N + R = E + 10*C2
               D + E = Y + 10*C1,
      % Disequality constraints:
          diff_list([S,E,N,D,M,O,R,Y]),
      % Generate binary digits:
          C1=carry,
          C2=carry,
          C3=carry,
      % Generate decimal digits:
          S=decimal, E=decimal,
          N=decimal, D=decimal,
          O=decimal, R=decimal,
          Y=decimal,
```

SEND+MORE=MONEY

```
% Print the result:
          nl, write(" SEND ",S,E,N,D), nl,
               write("+MORE +", M, O, R, E), n1,
               write("---- ----"), nl,
               write("MONEY ", M, O, N, E, Y), nl,
      % Fail to iterate:
          fail.
decimal \rightarrow {0;1;2;3;4;5;6;7;8;9}.
carry -> \{0;1\}.
diff_list([]).
diff_list([H|T]) :-
        generate_diffs(H,T),
        diff_list(T),
        H=<9,
        H>=0.
generate_diffs(H,[]).
generate_diffs(H,[A|T]) :-
        generate_diffs(H,T),
        A=\backslash=H.
```

Dictionary

Dictionary

Primes

```
prime := P:int
           | factors(P) = one.
factors (N) \rightarrow cond(N < 2,
               {},
               factors_from(N,2)).
factors_from(N:int,P:int) ->
        cond(P*P > N,
              one,
              cond(R:(N/P) = := floor(R),
                   many,
                   factors_from(N,P + 1))).
primes_to(N:int) :-
        write(int_to(N) & prime),
        nl, fail.
int_to(N:int) ->
        cond(N < 1,
              {},
              \{1; 1 + int_to(N-1)\}).
```

Primes

```
> primes_to(30)?
2: prime
3: prime
5: prime
7: prime
11: prime
13: prime
17: prime
19: prime
29: prime

*** No
>
```

Define the class of activity objects:

Wait until the value is an integer before assigning it:

```
assign(A,B:int) -> succeed A<-B.
```

Pass 1: Calculate the earliest time that A can start.

Pass 2: Calculate the latest time that A's prerequisites can start and still finish before A starts.

```
lateCalc(A,[]) -> succeed.
lateCalc(A,[B:activity|ListOfActs])
    -> lateCalc(A,ListOfActs)
    | assign(LSB:(B.lateStart),
        min(LSB, A.earlyStart-B.duration)).
```

A sample input for the PERT scheduler: any permutation of the specified order of activities would work, illustrating that calculations in LIFE do not depend on order of execution.

```
schedule :-
A1=activity(duration=>10),
A2=activity(duration=>20),
A3=activity(duration=>30),
A4=activity(duration=>18,prerequisites=>[A1,A2]),
A5=activity(duration=>8 ,prerequisites=>[A2,A3]),
A6=activity(duration=>3 ,prerequisites=>[A1,A4]),
A7=activity(duration=>4 ,prerequisites=>[A5,A6]),
visualize([A1,A2,A3,A4,A5,A6,A7]).
```

Encapsulated programming

Create a routine that behaves like a process with encapsulated data. The caller cannot access the routine's local data except through the access functions ("methods") provided by the routine.

Initialization:

```
new_counter(C) :- counter(C,0).
```

Access predicate:

```
send(X,C) := C=[X|C2], C<-C2.
```

Encapsulated programming

The

counter:

Encapsulated programming

Access to the process is by a logical variable. The internal state of the process is the value of the counter, which is held in the second argument.

```
> new_counter(C)?
*** Yes
C = @~.
--1> send(inc,C)?
*** Yes
C = @~.
---2> send(inc,C)?
*** Yes
C = @~.
----3> send(see(X),C)?
*** Yes
C = @~, X = 2.
-----4>
```

This creates a new counter object (with initial value 0) which is accessed through **c**. The counter is incremented twice and then its value is accessed.

A simple term expansion facility:

The main call is:

A tiny French grammar:

Higher classes of words:

```
adjectif_postfixe_ < | adjectif_.
adjectif_prefixe_ < | adjectif_.
article_indefini_ < | article_.
nom_propre_ < | etre_anime_.
verbe_etre_ < | verbe_transitif_.</pre>
```

A lexicon of word sorts:

```
a < conjonction_.
a < | verbe_transitif_.</pre>
anglais < | adjectif_postfixe_.</pre>
anglais < | nom_commun_.</pre>
animal < | etre_anime_.</pre>
apres < | conjonction_.</pre>
article < | nom_commun_.</pre>
belle < adjectif_prefixe_.</pre>
belle < nom_commun_.
blanc < adjectif_postfixe_.</pre>
blanche < adjectif_postfixe_.
blanche < | femme. % Special!
femme < | personne.</pre>
fille < personne.
francais < | adjectif_postfixe_.</pre>
francais < | nom_commun_.</pre>
garcon < | personne_.</pre>
```

A lexicon of word sorts:

```
la <| article_.
la <| pronom_.
le <| article_.
le <| pronom_.
les <| pronom_.
les <| pronom_.
...
noir <| adjectif_postfixe_.
noir <| homme. % Special!
noire <| adjectif_postfixe_.
...
porte <| nom_commun_.
porte <| verbe_transitif_.
...
voile <| nom_commun_.
voile <| verbe_transitif_.</pre>
```

```
demo :- analyse([la,femme,blanche,porte,le,voile]).
demo :- analyse([richard, est, un, noir, blanc]).
demo :- analyse([richard, est, noir]).
> demo?
phrase([sujet([groupe_nominal
                   ([article_(la),
                    nom_commun_(femme),
                    adjectif_postfixe_(blanche)])]),
        verbe_transitif_(porte),
        complement_d_objet
            ([groupe_nominal
                   ([article_(le),
                    nom_commun_(voile)])])
phrase([sujet([groupe_nominal([nom_propre_(richard)])]),
        verbe_transitif_(est),
        complement_d_objet
            ([groupe_nominal
                   ([article_(un),
                    nom_commun_(noir),
                    adjectif_postfixe_(blanc)])])
phrase([sujet([groupe_nominal([nom_propre_(richard)])]),
        verbe_etre_(est),
        adjectif_(noir)])
```

Conclusion

LIFE is still an experimental language. Nevertheless, it offers conveniences meant to reconcile different programming styles.

It is particularly suited for:

- natural linguistics
- constrained graphics
- expert systems

There are other feature to complement it with like:

- other CLP constraint domains (arithmetic, boolean, finite domains, intervals)
- better language features (extensional sorts, partial features, lexical scoping, method encapsulation, etc...)

This is just a beginning...