

# Demonstration of Tests of Conversion of PM Dot Notation to Parentheses

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January 16, 2025

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## SECTION 0. VERIFICATION TESTS (of dot to paren dot icn)

For each proposition is given:

- 1: the PM notation with dots.
  - 2: the notation with parentheses
  - 3: the Polish (with Lukasiewicz symbols) notation
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\*2•06†  $\vdash p \supset q . \supset : q \supset r . \supset . p \supset r$

Version with parentheses

\*2•06†  $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$

Polish Lukasiewicz notation

\*2•06†  $CCpqCCqrCpr$

\*3•47†  $\vdash p \supset r . q \supset s . \supset : p . q . \supset . r . s$

Version with parentheses

\*3•47†  $(p \supset r) \wedge (q \supset s) \supset ((p) \wedge (q) \supset (r) \wedge (s))$

Polish Lukasiewicz notation

\*3•47†  $CKCprCqsKCKpqrs$

\*4•22†  $\vdash p \equiv q . q \equiv r . \supset . p \equiv r$

Version with parentheses

\*4•22†  $(p \equiv q) \wedge (q \equiv r) \supset (p \equiv r)$

Polish Lukasiewicz notation

\*4•22†  $CKEpqEqrEpr$

\*4•41†  $\vdash p . \vee . q . r : \equiv . p \vee q . p \vee r$

Version with parentheses

\*4•41†  $((p) \vee (q) \wedge (r)) \equiv (p \vee q) \wedge (p \vee r)$

Polish Lukasiewicz notation

\*4•41†  $KEKApqrApqApr$

\*4•43†  $\vdash p . \equiv : p \vee q . p \vee \sim q$

Version with parentheses

\*4•43†  $(p) \equiv ((p \vee q) \wedge (p \vee \sim q))$

Polish Lukasiewicz notation

$$*4\cdot43\vdash EpKApqApNq$$

$$*4\cdot44\vdash \vdash p.\equiv\vdash p.\vee.p.q$$

Version with parentheses

$$*4\cdot44\vdash (p) \equiv ((p) \vee (p) \wedge (q))$$

Polish Lukasiewicz notation

$$*4\cdot44\vdash EpKAppq$$

$$*4\cdot87\vdash \vdash p.q.\supset.r\equiv\vdash p.\supset.q\supset r\equiv\vdash q.\supset.p\supset r\equiv\vdash q.p.\supset.r$$

Version with parentheses

$$*4\cdot87\vdash ((p) \wedge (q) \supset (r)) \equiv ((p) \supset (q \supset r)) \equiv ((q) \supset (p \supset r)) \equiv ((q) \wedge (p) \supset (r))$$

Polish Lukasiewicz notation

$$*4\cdot87\vdash EEECKpqrCpCqrCqCprCKqpr$$

$$*4\cdot88\vdash \vdash p.q.\supset.r\equiv\vdash p.\supset.q\supset r\equiv\vdash q.\supset.p\supset r\equiv\vdash q.p.\supset.r$$

Version with parentheses

$$*4\cdot88\vdash (p) \wedge (q) \supset (r) \equiv ((p) \supset (q \supset r)) \equiv ((q) \supset (p \supset r)) \equiv ((q) \wedge (p) \supset (r))$$

Polish Lukasiewicz notation

$$*4\cdot88\vdash EEECKpqrCpCqrCqCprCKqpr$$

$$*5\cdot33\vdash \vdash p.q.\supset.r\equiv\vdash p:p.q.\supset.r$$

Version with parentheses

$$*5\cdot33\vdash (p) \wedge (q \supset r) \equiv (p) \wedge ((p) \wedge (q) \supset (r))$$

Polish Lukasiewicz notation

$$*5\cdot33\vdash KEKpCqrpCKpqr$$

From Landon D. C. Elkind's Paper in Russell: Vol. 43, no. 1, page 44

$$*431\cdot441\vdash p\vee q.\equiv.r\supset s$$

Version with parentheses

$$*431\cdot441\vdash (p \vee q) \equiv (r \supset s)$$

Polish Lukasiewicz notation

$$*431\cdot441\vdash EApqCrs$$

$$*431\cdot442\vdash p.\vee.q\equiv r:\supset:s$$

Version with parentheses

$$*431\cdot442\vdash ((p) \vee (q \equiv r)) \supset ((s))$$

Polish Lukasiewicz notation

$$*431\cdot442\vdash CApEqrs$$

$$*431\cdot443\vdash p\vee q.\equiv.r:\supset:s$$

Version with parentheses

$$*431\cdot443\vdash ((p \vee q) \equiv (r)) \supset ((s))$$

Polish Lukasiewicz notation

$$*431\cdot443\vdash CEApqrs$$

$$*431\cdot444\vdash p:\vee:q\equiv r.\supset.s$$

Version with parentheses

$$*431\cdot444\vdash (p) \vee ((q \equiv r) \supset (s))$$

Polish Lukasiewicz notation

$*431 \cdot 444 \vdash ApCEqrs$   
 $*431 \cdot 445 \vdash p : \forall : q. \equiv . r \supset s$   
 Version with parentheses  
 $*431 \cdot 445 \vdash (p) \vee ((q) \equiv (r \supset s))$   
 Polish Lukasiewicz notation  
 $*431 \cdot 445 \vdash ApEqCr s$   
 From same, page 54  
 $*431 \cdot 54 \vdash p . q : r . s : \supset : p . s : r . q$   
 Version with parentheses  
 $*431 \cdot 54 \vdash ((p) \wedge (q)) \wedge ((r) \wedge (s)) \supset ((p) \wedge (s)) \wedge ((r) \wedge (q))$   
 Polish Lukasiewicz notation  
 $*431 \cdot 54 \vdash KCKKpqKrsKpsKrq$   
 check longer prop name  
 Version with parentheses  
 Polish Lukasiewicz notation  
 Propositions involving quantifiers  
 $*9 \cdot 2 \vdash (x) . psix . \supset . psiy$   
 Version with parentheses  
 $*9 \cdot 2 \vdash (((x))psix) \supset (psiy)$   
 Polish Lukasiewicz notation  
 $*9 \cdot 2 \vdash C(x)psixpsiy$   
 $*9 \cdot 21 \vdash \vdash (x) . psix \supset phix . \supset : (x) . psix . \supset . (x) . phix$   
 Version with parentheses  
 $*9 \cdot 21 \vdash (((x))psix \supset phix) \supset (((x))psix) \supset ((x))phix$   
 Polish Lukasiewicz notation  
 $*9 \cdot 21 \vdash CCC(x)psixphix(x)psix(x)phix$   
 $*9 \cdot 22 \vdash \vdash (x) . psix \supset phix . \supset : (\exists x) . psix . \supset . (\exists x) . phix$   
 Version with parentheses  
 $*9 \cdot 22 \vdash (((x))psix \supset phix) \supset (((\exists x))psix) \supset ((\exists x))phix$   
 Polish Lukasiewicz notation  
 $*9 \cdot 22 \vdash CCC(x)psixphix(\exists x)psix(\exists x)phix$   
 $*9 \cdot 31 \vdash \vdash (\exists x) . phix . \vee . (\exists x) . phix : \supset . (\exists x) . phix$   
 Version with parentheses  
 $*9 \cdot 31 \vdash (((((\exists x))phix) \vee ((\exists x))phix)) \supset ((\exists x))phix$   
 Polish Lukasiewicz notation  
 $*9 \cdot 31 \vdash CA(\exists x)phix(\exists x)phix(\exists x)phix$   
 $*9 \cdot 401 \vdash \vdash p : \forall : q . \vee . (\exists x) . psix : \supset : q : \forall : p . \vee . (\exists x) . psix$   
 Version with parentheses  
 $*9 \cdot 401 \vdash (((p) \vee ((q) \vee ((\exists x))psix))) \supset ((q) \vee ((p) \vee ((\exists x))psix))$   
 Polish Lukasiewicz notation  
 $*9 \cdot 401 \vdash CApAq(\exists x)psixAqAp(\exists x)psix$   
 $*10 \cdot 35 \vdash \vdash (\exists x) . p . psix . \equiv : p : (\exists x) . psix$

Version with parentheses

$$*10\cdot35 \vdash (((\mathfrak{A}x))p) \wedge (psix) \equiv (p) \wedge (((\mathfrak{A}x))psix)$$

Polish Lukasiewicz notation

$$*10\cdot35 \vdash KEK(\mathfrak{A}x)ppsixp(\mathfrak{A}x)psix$$

$$*11\cdot2 \vdash (x, y) \cdot phi[x, y] \cdot \equiv \cdot (y, x) \cdot phi[x, y]$$

Version with parentheses

$$*11\cdot2 \vdash ((x, y) \cdot phi[x, y]) \equiv ((y, x) \cdot phi[x, y])$$

Polish Lukasiewicz notation

$$*11\cdot2 \vdash E(x, y) \cdot phi[x, y](y, x) \cdot phi[x, y]$$

One Step in proof of 11.55 I wanted example of 2 adjacent quantifiers - hard to find.

$$*11\cdot551 \vdash ::(x)::(\mathfrak{A}y) \cdot psix \cdot phi[x, y] \cdot \equiv ::psix::(\mathfrak{A}y) \cdot phi[x, y]$$

Version with parentheses

$$*11\cdot551 \vdash (((x))(\mathfrak{A}y))psix \wedge (phi[x, y]) \equiv (psix) \wedge (((\mathfrak{A}y))phi[x, y])$$

Polish Lukasiewicz notation

$$*11\cdot551 \vdash KEK(x)(\mathfrak{A}y)psixphi[x, y]psix(\mathfrak{A}y)phi[x, y]$$

From same, page 46

$$*431\cdot46 \vdash (x) \cdot psix \cdot phix \cdot \supset \cdot (x) \cdot psix$$

Version with parentheses

$$*431\cdot46 \vdash (((x))psix) \wedge (phix) \supset (((x))psix)$$

Polish Lukasiewicz notation

$$*431\cdot46 \vdash CK(x)psixphix(x)psix$$

Other Tests

$$*99\cdot99 \vdash ::\sim(\mathfrak{A}x)::\sim psix \cdot \supset \cdot (x) \cdot \sim psix$$

Version with parentheses

$$*99\cdot99 \vdash ((\sim(\mathfrak{A}x))\sim psix) \supset ((x))\sim psix$$

Polish Lukasiewicz notation

$$*99\cdot99 \vdash CN(\mathfrak{A}x)Npsix(x)Npsix$$