Demonstration of Tests of Conversion of PM Dot Notation to Parentheses

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SECTION 0. VERIFICATION TESTS (of dot to paren dot icn)
For each proposition is given:
1: the PM notation with dots.
2: the notation with parentheses
3: the Polish (with Lukasiewicz symbols) notation

$\ast 2{\cdot}06 \vdash :. p \supset q . \supset : q \supset r . \supset . p \supset r$
Version with parentheses
$\ast 2{\cdot}06 \vdash (p \supset q) \supset ((q \supset r) \supset (p \supset r))$
Polish Lukasiewicz notation
$\ast 2{\cdot}06 \vdash CCpqCCqrCpr$
$\ast 3{\cdot}47 \vdash :. p \supset r . q \supset s . \supset : p . q . \supset . r . s$
Version with parentheses
$\ast 3{\cdot}47 \vdash (p \supset r) \wedge (q \supset s) \supset ((p) \wedge (q) \supset (r) \wedge (s))$
Polish Lukasiewicz notation
$\ast 3{\cdot}47 \vdash CKCprCqsKCKpqrs$
$\ast 4{\cdot}22 \vdash : p \equiv q . q \equiv r . \supset . p \equiv r$
Version with parentheses
$\ast 4{\cdot}22 \vdash (p \equiv q) \wedge (q \equiv r) \supset (p \equiv r)$
Polish Lukasiewicz notation
$\ast 4{\cdot}22 \vdash CKEpqEqrEpr$
$\ast 4{\cdot}41 \vdash :. p . \vee . q . r : \equiv . p \vee q . p \vee r$
Version with parentheses
$\ast 4{\cdot}41 \vdash ((p) \vee (q) \wedge (r)) \equiv (p \vee q) \wedge (p \vee r)$
Polish Lukasiewicz notation
$\ast 4{\cdot}41 \vdash KEKApqrApqApr$
$\ast 4{\cdot}43 \vdash :. p . \equiv : p \vee q . p \vee \sim q$
Version with parentheses
$\ast 4{\cdot}43 \vdash (p) \equiv ((p \vee q) \wedge (p \vee \sim q))$

Polish Lukasiewicz notation

$\ast 4{\cdot}43 \vdash EpKApqApNq$

$\ast 4{\cdot}44 \vdash :. p . \equiv : p . \vee . p . q$

Version with parentheses

$\ast 4{\cdot}44 \vdash (p) \equiv ((p) \vee (p) \wedge (q))$

Polish Lukasiewicz notation

$\ast 4{\cdot}44 \vdash EpKAppq$

$\ast 4{\cdot}87 \vdash :. p . q . \supset . r : \equiv : p . \supset . q \supset r : \equiv : q . \supset . p \supset r : \equiv : q . p . \supset . r$

Version with parentheses

$\ast 4{\cdot}87 \vdash ((p) \wedge (q) \supset (r)) \equiv ((p) \supset (q \supset r)) \equiv ((q) \supset (p \supset r)) \equiv ((q) \wedge (p) \supset (r))$

Polish Lukasiewicz notation

$\ast 4{\cdot}87 \vdash EEECKpqrCpCqrCqCprCKqpr$

$\ast 4{\cdot}88 \vdash :. p . q . \supset . r . \equiv : p . \supset . q \supset r : \equiv : q . \supset . p \supset r : \equiv : q . p . \supset . r$

Version with parentheses

$\ast 4{\cdot}88 \vdash (p) \wedge (q) \supset (r) \equiv ((p) \supset (q \supset r)) \equiv ((q) \supset (p \supset r)) \equiv ((q) \wedge (p) \supset (r))$

Polish Lukasiewicz notation

$\ast 4{\cdot}88 \vdash EEECKpqrCpCqrCqCprCKqpr$

$\ast 5{\cdot}33 \vdash :. p . q \supset r . \equiv : p : p . q . \supset . r$

Version with parentheses

$\ast 5{\cdot}33 \vdash (p) \wedge (q \supset r) \equiv (p) \wedge ((p) \wedge (q) \supset (r))$

Polish Lukasiewicz notation

$\ast 5{\cdot}33 \vdash KEKpCqrpCKpqr$

From Landon D. C. Elkind's Paper in Russell: Vol. 43, no. 1, page 44

$\ast 431{\cdot}441 \vdash . p \vee q . \equiv . r \supset s$

Version with parentheses

$\ast 431{\cdot}441 \vdash (p \vee q) \equiv (r \supset s)$

Polish Lukasiewicz notation

$\ast 431{\cdot}441 \vdash EApqCrs$

$\ast 431{\cdot}442 \vdash : p . \vee . q \equiv r : \supset : s$

Version with parentheses

$\ast 431{\cdot}442 \vdash ((p) \vee (q \equiv r)) \supset ((s))$

Polish Lukasiewicz notation

$\ast 431{\cdot}442 \vdash CApEqrs$

$\ast 431{\cdot}443 \vdash : p \vee q . \equiv . r : \supset : s$

Version with parentheses

$\ast 431{\cdot}443 \vdash ((p \vee q) \equiv (r)) \supset ((s))$

Polish Lukasiewicz notation

$\ast 431{\cdot}443 \vdash CEApqrs$

$\ast 431{\cdot}444 \vdash : p : \vee : q \equiv r . \supset . s$

Version with parentheses

$\ast 431{\cdot}444 \vdash (p) \vee ((q \equiv r) \supset (s))$

Polish Lukasiewicz notation

$*431\cdot444 \vdash ApCEqrs$

$*431\cdot445 \vdash : p : \vee : q \,.\equiv.\, r \supset s$

Version with parentheses

$*431\cdot445 \vdash (p) \vee ((q) \equiv (r \supset s))$

Polish Lukasiewicz notation

$*431\cdot445 \vdash ApEqCrs$

From same, page 54

$*431\cdot54 \vdash : p \,.\, q : r \,.\, s : \supset : p \,.\, s : r \,.\, q$

Version with parentheses

$*431\cdot54 \vdash ((p) \wedge (q)) \wedge ((r) \wedge (s)) \supset ((p) \wedge (s)) \wedge ((r) \wedge (q))$

Polish Lukasiewicz notation

$*431\cdot54 \vdash KCKKpqKrsKpsKrq$

check longer prop name

Version with parentheses

Polish Lukasiewicz notation

Propositions involving quantifiers

$*9\cdot2 \vdash : (x) \,.\, \psi x \,.\supset.\, \psi y$

Version with parentheses

$*9\cdot2 \vdash (((x))\psi x) \supset (\psi y)$

Polish Lukasiewicz notation

$*9\cdot2 \vdash C(x)\psi x\psi y$

$*9\cdot21 \vdash \therefore (x) \,.\, \psi x \supset \phi x \,.\supset : (x) \,.\, \psi x \,.\supset.\, (x) \,.\, \phi x$

Version with parentheses

$*9\cdot21 \vdash (((x))\psi x \supset \phi x) \supset (((x))\psi x) \supset ((x))\phi x$

Polish Lukasiewicz notation

$*9\cdot21 \vdash CCC(x)\psi x\phi x(x)\psi x(x)\phi x$

$*9\cdot22 \vdash \therefore (x) \,.\, \psi x \supset \phi x \,.\supset : (\exists x) \,.\, \psi x \,.\supset.\, (\exists x) \,.\, \phi x$

Version with parentheses

$*9\cdot22 \vdash (((x))\psi x \supset \phi x) \supset (((\exists x))\psi x) \supset ((\exists x))\phi x$

Polish Lukasiewicz notation

$*9\cdot22 \vdash CCC(x)\psi x\phi x(\exists x)\psi x(\exists x)\phi x$

$*9\cdot31 \vdash \therefore (\exists x) \,.\, \phi x \,.\vee.\, (\exists x) \,.\, \phi x : \supset .\, (\exists x) \,.\, \phi x$

Version with parentheses

$*9\cdot31 \vdash (((((\exists x))\phi x) \vee ((\exists x))\phi x)) \supset ((\exists x))\phi x$

Polish Lukasiewicz notation

$*9\cdot31 \vdash CA(\exists x)\phi x(\exists x)\phi x(\exists x)\phi x$

$*9\cdot401 \vdash :: p : \vee : q \,.\vee.\, (\exists x) \,.\, \psi x \therefore \supset \therefore q : \vee : p \,.\vee.\, (\exists x) \,.\, \psi x$

Version with parentheses

$*9\cdot401 \vdash (((p) \vee ((q) \vee ((\exists x))\psi x))) \supset ((q) \vee ((p) \vee ((\exists x))\psi x))$

Polish Lukasiewicz notation

$*9\cdot401 \vdash CApAq(\exists x)\psi xAqAp(\exists x)\psi x$

$*10\cdot35 \vdash \therefore (\exists x) \,.\, p \,.\, \psi x \,.\equiv : p : (\exists x) \,.\, \psi x$

Version with parentheses

$*10{\cdot}35 \vdash (((\exists x))p) \wedge (\psi x) \equiv (p) \wedge (((\exists x))\psi x)$

Polish Lukasiewicz notation

$*10{\cdot}35 \vdash KEK(\exists x)p\psi xp(\exists x)\psi x$

$*11{\cdot}2 \vdash : (x,y) . \phi[x,y] . \equiv . (y,x) . \phi[x,y]$

Version with parentheses

$*11{\cdot}2 \vdash ((x,y) . \phi[x,y]) \equiv ((y,x) . \phi[x,y])$

Polish Lukasiewicz notation

$*11{\cdot}2 \vdash E(x,y) . \phi[x,y](y,x) . \phi[x,y]$

One Step in proof of 11.55 I wanted example of 2 adjacent quantifiers - hard to find.

$*11{\cdot}551 \vdash :. (x) :. (\exists y) . \psi x . \phi[x,y] . \equiv : \psi x : (\exists y) . \phi[x,y]$

Version with parentheses

$*11{\cdot}551 \vdash ((((x))(\exists y))\psi x) \wedge (\phi[x,y]) \equiv (\psi x) \wedge (((\exists y))\phi[x,y])$

Polish Lukasiewicz notation

$*11{\cdot}551 \vdash KEK(x)(\exists y)\psi x\phi[x,y]\psi x(\exists y)\phi[x,y]$

From same, page 46

$*431{\cdot}46 \vdash : (x) . \psi x . \phi x . \supset . (x) . \psi x$

Version with parentheses

$*431{\cdot}46 \vdash (((x))\psi x) \wedge (\phi x) \supset ((x))\psi x$

Polish Lukasiewicz notation

$*431{\cdot}46 \vdash CK(x)\psi x\phi x(x)\psi x$

Other Tests

$*99{\cdot}99 \vdash :. \sim(\exists x) : \sim\psi x . \supset . (x) . \sim\psi x$

Version with parentheses

$*99{\cdot}99 \vdash ((\sim(\exists x))\sim\psi x) \supset ((x))\sim\psi x$

Polish Lukasiewicz notation

$*99{\cdot}99 \vdash CN(\exists x)N\psi x(x)N\psi x$