To Reload or Not To Reload?

Screening Risk-Averse Executives Using Employee Stock Options With An Enhanced Reload Feature

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Summary

1 Introduction

To be written.

2 Literature Review

In this introductory chapter, we first present a brief historical overview of the evolution of Employee Stock Options (ESOs), the reasons behind their adoption, and the main issues related to their valuation and incentive alignment. We complete with a short description of the most relevant terminology we are using in the paper. We then focus on the reload feature, which allows the holder to receive a new option when exercising the original one, with a special focus on the more recent Dynamic ESOs, which allow the holder to receive a mix of stock and new options at exercise. We conclude with a brief overview of the empirical literature on executives' risk aversion and the modeling of ESOs as a principal-agent problem with adverse information and moral hazard.

2.1 Employee Stock Options

Employee (or Executive) Stock Options are compensation contracts granted from an employer to its executives. They entitle the holder to buy a certain number of shares of the company's stock at a predetermined price (the strike price) within a certain timeframe (the maturity), usually starting from a certain date (the vesting date). The holder is not however obliged to exercise the option, and will do so when it gives her positive utility (i.e., when the stock price is above the strike price). Together with cash and other benefits (e.g., restricted stock, performance bonuses), they form the compensation package of an executive. They usually come with some other features, such as non-transferability — the holder cannot sell them as would usually do with traditional options — and restrictions on short-selling the underlying stock, which limit the possibility of hedging the option. Historically, they have been used to align the incentives of the executive with those of the shareholders, as the executive's wealth gets closely tied to the stock price of the company. Moreover, they may appear as tying the executive to the firm with "golden handcuffs" because the vesting and nontransferability features reduce the likelihood of the executive leaving the firm. They are liked by companies also for their cash flow benefits, as they do not require immediate cash outflows, and for their ability to attract and retain talent, especially motivated and entrepreneurial employees ([34]).

2.1.1 Historical overview

The use of ESOs as a relevant form of compensation in the US has been very low up until the 1970s, where we start to observe an important regime change. Before, the levels of total pay and dispersion were much lower, while after the 1970s the levels of pay started to increase, together with the use of equity-related compensation. This rapid acceleration can be clearly seen in figure **??** from [29], which shows the evolution of pay for the three highest-paid executives in the 50 largest U.S. firms from 1936 to 2005. We can identify a J-shaped curve, which however seems to flatten out in the recent years. The raw numbers are also quite impressive: in 1992, firms in the S&P500 were granting employees options for a total of $11 billion at the time of grant; by 2000, option grants in S&P 500 firms increased to $119 billion ([34]). We refer to [29] or [25] for a more detailed historical overview of the evolution of CEO pay in the US.

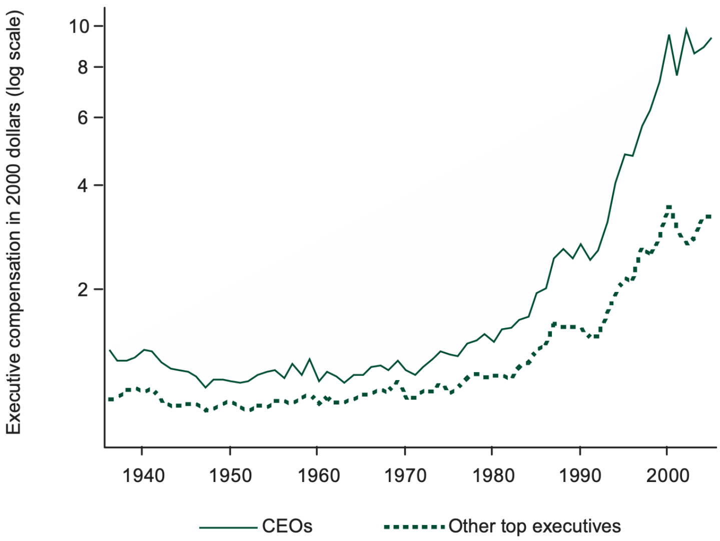

Figure 1: Median compensation of CEOs and other top officers, 1936-2005 (Fig. 1 in [29])

The reasons for the regime change are still debated. On the one hand, we have the competitive hypothesis, which argues that the rise in CEO pay is the result of competitive forces, which have increased the demand for talented executives and thus their pay. On the other hand, we find the managerial power or rent extraction hypothesis, which argues that the rise in CEO pay is the result of powerful managers setting their own compensation.

Both the exogenous and the endogenous hypotheses are consistent with empirical evidence, but neither is sufficient to explain the rise in CEO pay on its own ([29]). [25] highlights the importance of institutional forces, such as legislation, taxation, accounting policies and accounting standards in driving the executive pay up; the latter factors were already emphasized by [34] as a key driver of the rise in ESOs in the 1990s.

Nevertheless, the last years have marked a modest trend-reversal in the use of ESOs. [29] shows that stock options made almost half of the total compensation package at the end of last century, with the percentage decreasing to 25-30% in the 2010s, in favor of other means of performance-based compensation. This trend is confirmed also by figure **??**, which shows the evolution in the number of firms in S&P500 granting options, together with their average vesting period and time to maturity. The answer to whether stock or options are better means of compensation remains still open: as an example, [51] argues that stocks are desirable only when the nonviability (bankruptcy) risk is high, otherwise options are preferable for incentive purposes. More recently, the debate has however focused on the increasingly high levels of CEO pay, especially in the US, where the median CEO in the S&P 500 earned $10.1 million per year by 2014, a sixfold increase from 1980 ([25]), which some argues has also been contributing to a rising economic inequality ([57]).

Year	% of S&P500 companies granting ESOs	Avg. vesting period (years)	Avg. maturity (years)
2000	92.98%	2.00	9.24
2001	94.35%	2.22	9.28
2002	93.56%	2.18	9.53
2003	89.59%	2.18	10.17
2004	88.09%	2.03	8.66
2005	75.34%	2.16	8.61
2006	81.30%	2.12	7.86
2007	77.33%	2.18	8.14
2008	76.31%	2.38	7.35
2009	75.30%	2.16	8.41
2010	72.69%	2.32	8.71
2011	73.39%	2.16	8.07

[a] *Source*: Thomson Reuters Insider Trading database and Compustat ExecuComp database.

Figure 2: Overview of ESO compensation in S&P 500 firms, 2000-2011 (Table 1.1 in [53])

2.1.2 Incentive alignment

The main reason for the adoption of ESOs is to align the incentives of the executive with those of the shareholders. The misalignment between the two started with the separation of corporate ownership and corporate control at the beginning of the past century, which led to the need for a direct link between executive's realized compensation and the firm's performance. Indeed, the shareholders capture most of the benefits while the employee bears most of the costs, and thus the need to link the employee's compensation to some performance measure, such as the stock price for publicly traded firms. The sensitivity of executive wealth to the underlying stock price — delta, in the case of options — is seen as a measure of the incentive alignment between the executive and the shareholders. The higher the delta, the more the executive's wealth is tied to the stock price, and the more the executive will be incentivized to take actions that increase the stock price. However, [16] argues this effect may not be linear, as the inclusion of more and more performance-based and equity-based compensation in the executive's pay package has led to a higher sensitivity of the executive's wealth to the stock price, which in turn has led to a higher risk-taking behavior by the executive. Indeed, there is strong empirical evidence of a causal relation between manager compensation and investment policy, debt policy, and firm risk. Higher sensitivity of executive wealth to stock volatility — vega, in the case of options — leads to taking riskier policies, and, also when controlling for the delta, it "implies significantly higher R&D expenditures, less investment in property, plant, and equipment, and an increased focus" ([16]). However, the relation between vega and risk-taking may be less evident than it may appear *prima facie* and be mainly a matter of accounting ([35]). A more recent study by [5] suggest that risky policies, as measured by investment and return in R&D, vary concavely instead of convexly with vega, meaning that the marginal returns diminish with increasing vega, and so incentives may not be aligned after a given threshold. Therefore, higher delta and vega seem to expose the executive to more risk — be it convexly or concavely — which may not be desirable from the executive's point of view, at least after a certain threshold. This is especially true for risk-averse executives, who may prefer to have a more stable compensation package, even if it means sacrificing some upside potential. But not all risk is equal: some suggest that executives prefer systematic to idiosyncratic risk because it can be hedged against ([1]), while others suggest that executives prefer the idiosyncratic uncertainty because they believe they can better predict or influence it ([39]). This distinction of risks affects also the difference in valuation between cost of option and executive value ([56]), as we will later see. Therefore, striking a balance between risk and incentives is desirable both for the executive and the firm. From the firm's perspective, it is desirable that risk-averse executives undertake more risk than they would otherwise, but executives may engage in undesired behaviors, such as taking too much risk for the sake of increasing the value of the option, or, on the contrary, taking too little risk to preserve the remaining value of the option.[32].

Among the factors that drive the adoption of ESOs, we find ownership concentration, liquidity, CEO and institutional ownership, investment intensity, and historical market return ([59]). Some add also the ESO's convexity of payoff, which has however been strongly criticized by many others. [61] takes a strong standpoint arguing that no incentive scheme will make all agents more or less risk averse uniformly. He argues that it cannot be assumed *a priori* that a convex fee schedule will make an agent less risk averse and that a concave one will induce greater risk aversion: they are both necessary conditions, but each insufficient on its own. Therefore, we need to consider also whether the fee schedule moves the executive into the more or less risk averse domain of her utility function, and the possible marginal magnification effects which may result from this ([61]). [35] also finds no strong evidence on the convexity of ESOs. Nevertheless, it remains incomplete talking about incentives without considering the whole compensation package, which influences heavily the executive's risk taking behavior but also the value she puts on the options. [11] was the first to provide a solid foundation for the difference between firm cost and executive value for an ESO; a difference that was later confirmed also by [56] and [34]. In particular, [56] argues that this difference originates in the difference to risk exposure between the two parties, since undiversified managers are exposed to total firm risk but rewarded only for the systematic portion of it. This would then lead to a higher cost of the option for the firm than the value of the same for the executive. Moreover, she finds that "managers at the average NYSE firm who have their entire wealth invested in the firm value their options at 70% of their market value, while undiversified managers at rapidly growing, entrepreneurially-based firms, such as Internet-based firms, value their option-based compensation at only 53% of its cost to the firm" ([56]). This is consistent with the idea that the executive's wealth and risk aversion play a key role in the valuation of the ESOs. Along these lines, [33] presents the divergence between firm (opportunity) cost and executive value, arguing that the former is the value if the firm were to sell the tradable and hedgeable option to an outside investor, while the latter is the value for the executive, who is not able to hedge the option and is exposed to the total risk of the firm. These differences would thus lead to a higher cost of the option to the firm than the corresponding value for the executive. Finally, early exercise has been a widely studied topic in the literature. [42] analyze early exercise behavior and show that early exercise is strongly related to recent stock price movements, the market-to-strike ratio, proximity to vesting dates, time to maturity, volatility, and the employee's level within the company. [39] and [48] also find that the volatility of the stock is inversely related to early exercise, and this seems to trace back to the executive's valuation of the ESO. On the contrary, [31] argue that utility-based ESO models do not predict early exercise, while market frictions (e.g., costly exercise) or liquidity needs (for this, see also [58]) may instead be better predictors of early exercise. They suggest that these findings could imply that utility-based models assuming block exercise result in an underestimate of the value of ESOs.

2.1.3 Valuation issues

Valuation of ESOs has been a controversial topic already starting from the 90s. At first, companies were required to disclose them in the annual statements, but they were not obliged to expense them. This changed slightly in 1995, when the Financial Accounting Standards Board (FASB) published the FASB123 ([27]), which encouraged companies to adopt a fair-value-based method of accounting for stock options instead of the intrinsic-value-based method, without however making this mandatory. Indeed, most companies continued using the intrinsic-value-based method, which was less costly for them ([44]). Appendix B of FASB123 proposed a three-step valuation method for the fair valuation: (i) estimate the expected life of the ESO, (ii) use Black-Scholes or CRR binomial method to estimate the value, (iii) adjust ex-post to account for possibility of the employee leaving the company when vested. Therefore, two key parameters drive this valuation: the estimated expected life of the option and the employee exit rate during the vesting period. The proposed methodology has been heavily criticized, most notably by [44], because it fails to account for some key distinctive elements of ESOs, such as the vesting period and the restrictions usually entailed with the grant of these options. They thus proposed an "Enhanced FASB123" model, which accounts also for the possibility of the employees leaving the company after the vesting period — and hence, exercising only if the option is in-the-money when she leaves — and incorporates the employee's early exercise policy as a multiple of the strike price, so that when the current stock price is above such threshold the employee will exercise. This, along with other critics, spurred an important revision of the standard in 2004, by which it "eliminates the alternative to use Opinion 25's intrinsic value method of accounting that was provided in Statement 123 as originally issued. Under Opinion 25, issuing stock options to employees generally resulted in recognition of no compensation cost. This Statement requires entities to recognize the cost of employee services received in exchange for awards of equity instruments based on the grant-date fair value of those awards (with limited exceptions)" ([28]). The controversy around the valuation of ESOs has also been fueled by the fact that the Black-Scholes-Merton model, which was the most widely used model for valuing options, has been criticized for overstating both the executive value and the cost of the option, as some of its underlying assumptions are not applicable to ESOs ([11], [56], [34], [45], [13] [29]). A different approach has then been to model the exercise decision of the executive as part of a more comprehensive maximization problem of the executive, taking into account also other decision-relevant features, such as optimal portfolio allocation, optimal stopping time, exit from the firm, choices of consumption and effort, and other factors (e.g., [42], [45],

[31], [13]). Others have tried to combine theory and data to estimate option values to calibrate the model ([11]).

2.1.4 Basic terminology

Before delving into further topics, we provide a very short overview of the main terminology used in option pricing. A call option is a financial instrument known as a derivative that conveys to the purchaser (the holder) the right, but not the obligation, to buy a set quantity or dollar value of a particular asset at a fixed price by a set date (maturity). Employee Stock Options are thus non-transferable long-term call options on the stock of their employer. It is clear that such call option appreciates when the stock price goes up, and loses (intrinsic) value when the stock price goes below the initial strike price. The opposite is true for put options, which give the right to sell the asset at a fixed price by a set date, which thus appreciate when the stock price goes down. ESOs are American options, since they can be exercised before maturity. On the other hand, we have European options, that can be exercised only at maturity. We say that an option is in-the-money when the stock price is above the strike price, out-of-money when the stock price is below the strike price, and at-the-money when the stock price is equal to the strike price. Most ESOs are usually granted at-the-money or slightly out-of-money. The difference between the stock price and the strike price is called the intrinsic value of the option, while the difference between the option price and the intrinsic value is called the time value of the option. The time value is thus the value of the optionality of the option, i.e., the value that accounts for the possibility of further appreciation of the option. If the option is granted at-the-money, its value at the grant date is entirely time value (since the intrinsic value equals zero when the option is at-the-value); at expiry, the value is entirely intrinsic value, as there is no more time value left. As stated before, ESOs are a special type of American call options, since they come with some restrictions, such as non-transferability and restrictions on short-selling the underlying stock. Non-transferability means that the holder cannot sell the option to somebody else, as she could usually do with traditional options. Restrictions on short-selling of the stock imply that the holder cannot (fully) hedge against the firm-specific risk of the option. Moreover, ESOs are usually granted with a vesting period, which is the time the employee has to wait before being able to exercise the option. Therefore, she can exercise from when the option has been vested up to the maturity date. We show a visual example in figure ??.

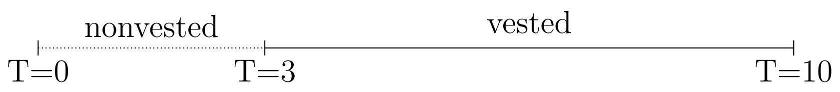

Figure 3: Visual representation of an ESO with 3y vesting period and 10y time to maturity.

The vesting period is usually fixed, but may otherwise depend on some performance metric or some other relevant event (e.g., the IPO of the company). Or, options may not be all vested at once (cliff vesting), but they may be vested steadily in a uniform or non-uniform progression (graded vesting). Finally, we have the Greeks, which are the sensitivities of the option price to different factors. The delta of an option is the sensitivity of the option price to the stock price, while the vega is the sensitivity of the option price to the volatility of the stock. The gamma is the sensitivity of the delta to the stock price, while the theta is the sensitivity of the option price to the time to maturity. The rho is the sensitivity of the option price to the interest rate. The delta is usually positive for call options and negative for put options, while the vega is always positive. The gamma is always positive, while the theta is always negative. Finally, the rho is usually positive for call options and negative for put options.

2.2 Reload options

In its standard formulation, a reload option (also called restoration or replacement option) is a call option that, at exercise, grants a new option for every share tendered to exercise the original option. Therefore, if the holder exercises the option between expiry using previously owned company shares, she will receive, in addition to one share for each option exercised, a new option. Each new option has strike price set to the market price at time of exercise — so it is granted at-the-money — and the same maturity as the original option. We will consider reload options in the specific case of ESOs, so that both the original reload option and the new option granted at exercise are non-transferable and non-hedgeable.

2.2.1 Reload feature

Reload options were first developed in 1987 by Frederic W. Cook and Company for the Norwest Corporation, and they rapidly gained traction: 14% of new stock option plans in 1996 and 17% in 1997 were already including them ([24]). The reload provision is nothing more than an option enhancement, whereby the exercise automatically triggers the award of new at-the-money options. This is indeed where the idea of recouping some time value comes into play: the holder exercises the option, receives the stock, and receives a new option at-the-money, which allows her to recover some time value sacrificed when exercising early the original option. Indeed, receiving an additional option allows to lock in a portion of the gain which could be possible, and of which time value is a manifestation. Another advantage is that the exercise does not change the manager's total equity holding but only its composition, which ties the executive to the firm's longer-term performance and is thus a desirable feature from an incentive perspective. Moreover, it encourages stock ownership, which is needed to enjoy the benefits of the reload provision at exercise. [37] find that the optimal exercise policy is to exercise as soon as the option is in-the-money, which holds also when the option features a vesting period, a result confirmed later also by [24]. Reload options encourage early exercise as this allows to lock in a portion of the gain while retaining the same upside potential of the options ([36]). This is why the value of an option with a reload option is strictly greater than the value of the option without. But the same reason has been criticized for being a possible instrument of wealth transfer from the shareholders to the executives, a view that for some has not been supported by empirical evidence ([24]). Numerous modifications of the original reload option have been proposed and implemented in the stock option plans of different companies. For example, [4] studies a setting in which the reload option may be granted at a strike price higher than the stock price at exercise by a small percentage, and finds that both the exercise policy and the option value are very sensitive to small changes in this percentage increase. Similarly, [24] studies the case of an infinite reload option, which is a reload option that grants a new option at exercise for every share tendered, for infinitely many exercises, and finds that the value of the reload option lies between the value of the American option and the stock price, notwithstanding the number of reloads or the (possibly infinitely lived) time horizon of the option. Indeed, a holder could follow the American call's optimal exercise strategy and never exercise the reload option, which would make the value of the reload option equal to the American option. On the other hand, the stock price is an upper bound for the value of the reload option, as the holder could exercise the option and receive the stock directly. In particular, the value of the option is strictly smaller than the stock price when the stock grants dividends, as the dividends would not be enjoyed by the reload option holder.

We now turn to a different modification of reload options, the so-called Dynamic Employee Stock Options.

2.2.2 Dynamic Employee Stock Options

Dynamic Employee Stock Options (DESOs) were first proposed by [41] in 2013. The difference with the traditional reload options is that the holder has more flexibility in the choice of the composition of the new options granted at exercise: for example, she can choose to receive 100% stock (i.e., a traditional ESO), or to receive less than 100% in stock but also some positive amount of ESO or restricted stock. The rationale behind the latter choice is that, similarly to the traditional reload options, this choice allows her to recover some time value at early exercise. They argue that this type of option has many advantages, both for the executive and the firm. The former avoids forfeiture of all time value when exercising early, and decreases her overall risk at exercise by locking part of the gain, reducing thus the need for hedging. The firm instead enjoys cash flows from tax credits and tax deductions flowing in sooner, the executive is incentivized to take riskier projects sooner due to reduced overall delta risk, and attracts better talent due to an overall better compensation scheme. They run a simulation with a 10-year DESO with a 3-year vesting period and a 10-year time to maturity, granting 75% in stocks and 25% in new ESOs at exercise, and compare it with a traditional ESO. They find that such DESO costs only 4% more than a traditional ESO, while providing the abovelisted benefits to both parties. Therefore, the firm has a limited higher accounting cost while providing a better compensation scheme for the executive and a better alignment with the shareholders' interests. However, they do not elaborate on the subjective values of the two options to the executive.

2.2.3 Valuation issues

[52] shows that also for reload options the values obtained via classical valuation and utility-based approaches show important differences. What is more, [46] finds that the deltas obtained via the utility-maximizing approach — the so-called subjective deltas — appear to be lower than the deltas obtained via binomial pricing — the so-called objective deltas — at least for infinite reload options. Finally, [75] propose a valuation model that is able to account also for Knightian uncertainty. As expected, they find that the value of the reload option to the Knightian-averse executive is lower as the coefficient of Knightian uncertainty increases. Valuation of reload options is difficult due to the vast heterogeneity in the structure of the reload provision. [27] states that the optimal valuation would be to consider the reload option when the option is granted at its fair value, but that no valuation method seemed able to do so. Therefore, "the Board concluded that the best way to account for an option with a reload feature is to treat both the initial grant and each subsequent grant of a reload option separately" ([27]). However, this methodology was later shown to overstate the cost of the option ([46]). [63] shows that it is possible to properly account for the reload feature using a binomial option pricing model, which was later confirmed also by [37]. Moreover, the latter showed also that the exercise policy is to exercise whenever the option is in-the-money, independently of the degree of risk aversion of the agent. However, [62] highlights that this is true for infinite reload options, but may not be true when the number of reloads is finite. Later, [21] proposed a model which also accounts for the time vesting requirement.

2.3 (Executive's) Risk aversion

Risk aversion is one of the fundamental factors driving the rationale behind the use of ESOs. As we have seen, it motivates ESOs as incentive alignment mechanisms for shareholders that want to induce their risk-averse executives to take more risk. ESOs encourage risk taking, which compensates thus for the risk aversion of the executive. Therefore, it is expected that with ESOs, executives will take more risk than they would otherwise, which is desirable for the firm (up to some degree of risk taking, beyond which it could be detrimental). However, risk aversion also plays an important role in the valuation of ESOs, as the executive's risk aversion (and non-diversification) creates a wedge between the firm cost and the executive value of the ESO.

Generally, risk aversion is understood as the tendency for people to avoid or dislike risk. Going more formal, a preference $\succsim$ on a space of lotteries $\mathbb{L}$ over some space of consequences $\mathbf{C}$ is called risk averse if, for each lottery $l \in \mathbf{L}$, it holds $\mathbb{E}[l] \succsim l$. This means that the expected value of the lottery is preferred to the lottery itself. A risk-neutral agent would have $\mathbb{E}[l] \sim l$, while for a risk-loving agent it would hold $\mathbb{E}[l] \precsim l$. In the case of a vN-M preference, the preference allows for a utility representation u, whose concavity operationalizes risk aversion: if u is concave, then the agent is risk averse; if u is linear, then the agent is risk-neutral; if u is convex, then the agent is risk-loving. This allows also to rank different degrees of risk aversion, as greater risk aversion is associated with greater concavity of the utility function, so that the utility function of a more risk-averse agent will be more concave than that of a less risk-averse agent, and in particular a strictly increasing transformation of the latter's utility function. Nevertheless, maybe the most famous characterization of risk aversion is by means of Jensen's inequality, stating that for a risk-averse agent it holds that $\mathbb{E}[u(X)] \leq u(\mathbb{E}[X])$ for some random variable X. Risk aversion is relevant in this context because it affects block ([31]) and early exercise of options ([48], [58]): the exercise policy of American options prescribes indeed that a risk-neutral agent would not exercise earlier, if not for receiving some additional benefits of possessing the stock (e.g., dividends, voting rights, etc.). However, non-diversification and (more recently) liquidity constraints may lead risk-averse agents to exercise earlier, even though the two effects are unclear. [13] argues that wealthier or less risk-averse executives exercise later and create greater option cost, while [58] uses an exogenous appreciation of executive's home prices as proxy for increased wealth, showing that in this circumstance early exercise is driven by liquidity concerns rather than diversification purposes: in-the-money vested ESOs options provide thus easy access to cash to make major purchases. The liquidity hypothesis stands ground also because of implausibly high levels of risk aversion needed to justify early exercise and block exercise. [48] consider also the role of ambiguity or Knightian uncertainty and finds opposing effects of risk and ambiguity on the incentive of early exercise of vested options: risk — measured in terms of equity volatility — causes executives to hold the option to preserve its time value (consistently with [39]), while ambiguity — defined as attitude towards mean-preserving spreads in probabilities — is positively related to early exercise. For the former, convexity of option payoff dominates concavity of utility function. The consideration of the role of ambiguity may be a promising venue for future research, as it may provide a more comprehensive understanding of the executive's behavior in exercising options and how it intertwines with risk aversion. Clearly, risk aversion coefficients of executives cannot be observed, but some studies have tried to estimate them from representative data on observable behavior. [8] finds some heterogeneity in risk aversion coefficients among senior managers of boards of directors and non-senior executives, which appears to be strongly correlated with sector membership and firm-level variables such as size, performance, and capital structure. On gender differences, [14] finds that female executives hold significantly lower equity incentives and demand larger salary premiums for bearing a given level of compensation risk, suggesting thus higher risk aversion. But [47] find that male executives engage in higher diversification-related stock sales than the female executives. Nevertheless, the literature on this remains scarce and more research is surely needed.

2.4 Continuous-time principal-agent problems

Principal-agent problems have been one of the most studied topics in the economic literature. The basic principal-agent problem arises when one party (the principal) delegates some decision-making authority to another party (the agent), who may have more information about the decision to be made. The agent's incentives may not be aligned with the principal's, and the principal may not be able to observe the agent's actions or characteristics. The principal's problem is then to design a contract that aligns the agent's incentives with her own, so that the agent will take the actions that are best for the principal. The agent's problem is to choose the actions that maximize her own utility, given the contract that the principal has offered.

The first principal-agent models were static, one-period models, where the agent's action was observable by the principal. Future models incorporated the possibility of hidden actions or hidden characteristics, leading to the development of the extensive literature on moral hazard and adverse selection models. Adverse selection is usually modeled in terms of productivity or preferences, while moral hazard in terms of effort or risk choice ([19]). However, despite the simplifications that can be made on static models which render them a convenient modeling tool, some features cannot be well accounted for in a static setting, for example firms' dynamics leading to cessation, private savings, and discounting. Therefore, dynamic models were developed to account for these features, but also for other possible dynamics, such as modeling rewards over time, studying the variation of the level and sensitivity of pay, and employee exit dynamics [25]. The first dynamic models were treated as a series of one-period models, with (continuous) time being thus discretized in a series of time steps. A continuous-time generalization for the model in discrete time was first given by [40]. They propose a model where the risk-averse and exponential utility agent controls the drift rate of some output process with built-in random fluctuations. They show that the optimal contract is linear in aggregate profits, which is consistent with the empirical evidence. The model has since then been extensively studied and extended in different directions: for example, [66] develop a first-order approach to the principal-agent model to an optimal stochastic control problem and relax the exponential utility requirement, while [69] limit the model to CARA preferences but extend it by allowing the agent to control the diffusion-rate process, showing that the optimal contract remains linear.

We focus on the application of dynamic models to the case of ESOs, but different approaches not mentioned in this stream of literature have been proposed (for example, [60] suggested using Markov decision processes). Also here, the continuous-time approach has many advantages over the discrete-time one, among which "tractability (which stems from the differential equation that characterizes the optimal contract), clarity (discrete-time models become messy very quickly), and computing power" ([65]).

[64] proposes a landmark infinite horizon model where payments are made continuously as function of past effort, rather than as a lump-sum payment at terminal date. The output is a diffusion process whose drift is determined by the agent's unobserved effort. The agent is risk-neutral, while the principal is risk-averse and enjoys consumption that is influenced by the agent's costly effort. The agent has the possibility of retirement. The model is solved by a system of coupled forward-backward stochastic differential equations, which are then solved numerically. The novelty of the paper is that the complete information case can be solved using the continuation variable as the only state variable; with moral hazard and adverse selection, the continuation value is not the only state variable anymore. [64] solves this case for finite time horizon, while [20] solves for infinite time horizon where the agents are subject to an initial private shock. [74] proposed a more generic method, for which only a partial characterization is however possible.

Cvitanić has a good stream of papers of moral hazard and adverse selection in continuous time: for example, with a constant private shock that has a long-lasting effect ([20]), or a dynamic programming solution to a problem with lump sum payment by restriction of the set of feasible contracts and generalization of utility function ([18]). With Cadenillas and Zapatero, he applies this dynamic setting to ESOs, whereby the agent can influence the stock by exerting costly effort and through the choice of projects that influence the volatility. In [9] they solve for the firm's optimal strike price and compute it numerically for the logarithmic case, and in [10] they argue that options may be an effective mechanism to screen out bad executives, as adverse selection requires more leverage than the perfect information case. Finally, the advent of computational power, as well as the need for solving analytically many backward stochastic differential equations, has led to a stream of research focusing on computational solutions when closed-form solutions are not feasible ([2], [23], [22]).

3 Theoretical model

Consider a continuous-time principal-agent model with both moral hazard and adverse selection. The two decision-makers here are the executive (agent) and the firm (principal). The horizon is finite: $T \in [0, \infty)$. We take inspiration from the models in [64] and [9].

We assume that markets are complete and frictionless, i.e., there are no taxes nor transaction costs — the former is a strong assumption but necessary to keep the problem simple.[1]

Consider a risk-neutral firm and a risk-averse executive. Suppose the executive knows her own coefficient of relative risk aversion (type) ρ, while the firm only knows the distribution of types λ. The firm offers two contracts to induce the agent to exert effort. We illustrate the main components of our model and then summarize the problem faced by the agents.

3.1 Stock Price

We consider a publicly listed firm. A standard geometric Brownian motion process $W = \{W_t, \mathscr{F}_t\}_{t \geq 0}$ on a probability space $(\Omega, \mathscr{F}, P)$ drives the stock price. Therefore, the stock price $S(t)$ at time t evolves according to

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

which can be re-written in the more familiar

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

with starting value S_0. The process W is a standard Brownian motion, $\mathscr{F}$ is the filtration generated by the Brownian motion process, μ is the exogenous drift, and σ is the exogenous stock volatility.

When the firm is managed by the executive, assuming $\mu = 0$, the dynamics of the stock

[1] Note that the agents' optimal policies may be otherwise influenced by the incidence of taxes on wealth. For example, an executive may anticipate or postpone the exercise of options due to changes in taxation which, despite being of interest, is out of our scope.

price S in differential form are given by

$$dS_t = a_t dt + \delta \sigma_t S_t dt + \sigma S_t dW_t$$

where $a = \{a_t\}_{t \geq 0}$ and $\sigma = \{\sigma_t\}_{t \geq 0}$ are two adapted stochastic processes and $\delta \geq 0$ is a constant. The process a represents the effort exerted by the executive and σ the choice of volatility level, and they are such that $a_t, \sigma_t \geq 0 \quad \forall t \geq 0$, progressively measurable with respect to $\mathscr{F}$. While the interpretation of the former is straightforward, the latter is justified by the fact that we assume the executive could possibly face a menu of projects with different volatilities — not necessarily with different expected return — so that this choice is ultimately reflected also in the stock price. As we will see later, we assume both a_t and σ_t to be constant $\forall t \geq 0$. The interpretation behind δ is more subtle and taken from [10]: it basically serves as a control for the impact of a project on the firm's overall stock volatility. For example, smaller firms may exhibit higher δ because they have less executives and hence each project matters, while for larger firms the impact of a single project on their stock price may be more limited. Or, we can interpret it as the *relevance* of the executives, that is, when hiring C-level executives the δ would be higher — since they are expected to have a larger impact on the company trajectory — compared to when hiring middle managers, for example.

Few observations are in order. First, it is clear that the stock price is publicly observable by all parties, but its components are not. Therefore, the firm cannot perfectly observe the effort of the executive nor differentiate between the stock's intrinsic volatility and the part of volatility generated by the project's choice. Second, we do not distinguish between systematic and idiosyncratic volatility of the stock, which would be of interest for executive's hedging[2] but not relevant for our case. Finally, we are clearly in a partial equilibrium setting: if this was not the case, the price of the stock would have already incorporated all the possible information in the economy and there would be no room for influence from executives' decisions, making our analysis useless.

[2]Indeed, executives are often prohibited from trading the firm's stock on the market, prohibiting thus a hedge against their ESOs by shorting the stock. Therefore, the executive can only hedge against the idiosyncratic portion, which makes this problem even more interesting

3.2 Two ESOs

We consider as ESO an American call option with maturity T, which we set at 10 years (as in [55]) with strike price K. We assume options are granted at-the-money, hence $K = S_0$. The vesting period is denoted by t_v and fixed at 2 years. Therefore, the executive cannot exercise the option when $t \in [0, t_v)$, but is free to do so whenever $t \in [t_v, T]$, i.e., as soon as the option gets vested.

Now, we assume the firm can only offer two types of ESOs, and she has control over the structure of the second. We call the first one Risk-Neutral (RN) and the second one Risky (R):

1. RN: this is the usual ESO. The option cannot be exercised before t_v. After t_v, the executive can exercise and receives the difference between the stock and the strike price when doing so. The option expires at time T; at expiry, the employee will exercise only if it is in-the-money, otherwise the option simply expires. We denote a generic risk-neutral option as RN.

2. R: this is a modified version of RN, that takes from the literature on ESOs with reload option and DESOs ([41]). Similarly to before, the option cannot be exercised before t_v. However, if exercised at time $\tau \geq t_v$, one unit of R is now converted into (i) the cash equivalent of $\alpha \in (0, 1)$ units of stock, plus (ii) $(1 - \alpha + \gamma)$ units of new RN, with $\gamma \geq 0$. The new RN option will have strike price S_τ. Keeping T, K, t_v fixed, we can thus identify uniquely the R option with (α, γ). Therefore, we denote it as $R_{\alpha,\gamma}$. Note that, for $\alpha = \gamma = 1$, we have a traditional ESO with a single reload option, while for $\alpha = 1$ and $\gamma = 0$ we have the RN option from above, i.e., $R_{1,0} = RN$.[3]

Note that the exercise of ESOs usually involves stocks: the strike price is paid out in stocks, and the firm gives a new stock to the holder for each unit of the option exercised. However, we assume that the executive exercises the option using cash and receives cash

[3] We will use this equality in the next chapter to check the correctness of our algorithm. Indeed, the firm and executive values for the RN option should be the same as for the $R_{1,0}$ options, since they are the same.

in return. This is a simplification that makes the problem easier and does not affect the final result.[4]

Let us illustrate the second option, R, through an example. Consider an R ESO with maturity at $T = 10$ years, strike price $K = S_0 = \$5$, vesting period $t_v = 2$ years, $\alpha = 0.7$, and $\gamma = 0.1$. We denote it with $R_{0.7,0.1}$. This means that the executive cannot exercise the option in the first 2 years. After the 2$^{\text{nd}}$ year, she can choose to exercise it at any point within the 10$^{\text{th}}$ year. Suppose she decides to exercise the option 3 years after the vesting period ended, that is, 5 years from the date of initial issuance. Assume that the stock price when he decides to exercise it is $S_5 = \$10$. Therefore, the executive buys 0.7 of a stock at \$5 when the stock is trading at \$10, netting thus an immediate gain of $0.7 * (\$10 - \$5) = \$3.5$ on the trade. Moreover, she receives $0.3 + 0.1 = 0.4$ worth of new RN, which will thus expire at $T = 15$ and be exercisable starting from year $5 + 2 = 7$ at strike price $S_5 = \$5$. It is clear from this example why the vesting period is key so that the existence of both options is justified. Suppose *by contra* that there was no vesting period in neither option (or, at least, in the RN option). Then, since $\gamma > 0$ and the stock price cannot clearly be negative, no rational executive would choose the RN option. Indeed, the executive could always decide to exercise simultaneously the R and the new RN she obtains, totaling thus $\alpha + (1 - \alpha + \gamma) = 1 + \gamma > 1$, where the right side is the number of stocks they would get from the RN option. But it is exactly this additional vesting period that increases the risk for the executive, which is driven by the stochastic process of the underlying stock. Holding this security is risky because it could as well be that the stock price goes under \$5, at least after the 7th year — when the vesting period of the RN ends — so that the option goes out-of-money and the executive will never exercise it, netting thus only the initial \$3.5. On the contrary, if the executive held an RN option and exercised it at year 5, she would have obtained a payoff of \$5. This loss of \$1.5 is the risk associated with exercising the RN rather than the R option, and represents the maximal

[4] Indeed, this can be imagined as the executive using cash to first buy the stock with which she exercises, and then selling off immediately the received stock to receive cash. Therefore, the only assumption we are making here is the liquidity of the stock, which we can however safely assume to be liquid enough since it is publicly traded. In technical terms, we are assuming these ESOs are cash-settled rather than physically delivered.

downside the executive can expect; clearly, the upside is, at least in theory, infinite. The intuition is that a low risk averse executive would then be willing to take this risk, while a sufficiently risk averse agent would prefer the \$5 payoff of the RN option, foregoing thus the potential of a larger profit. This, and the fact that a traditional ESO contract prohibits the agent both from selling the option and hedging part the firm-specific risk, makes the analysis of executive's risk aversion relevant.

We have focused for simplicity on the case of one option, assuming also that shares can be fractioned, but in reality executives are granted many options all at once — we will account for the latter in our numerical analysis. We will in any case constrain the options to be exercised simultaneously, which simplifies our problem greatly, and is consistent with many empirical findings on block exercise of ESOs.

3.3 Executive

Consider a risk- and effort-averse executive (she). The executive affects stock price dynamics by exerting costly effort — which adds as the drift component of the stock price — and by choosing the project with the desired volatility, which is not costly and affects part of the volatility component of the stock. Therefore, the maximization problem of the executive involves both the choice of effort and volatility.

Assume that the wealth of the executive at time t is given by:

$$W_t = n_S S_t + n_O (S_t - K)^+ + c(1 + r_f)^t$$

where n_S is the number of shares of the stock, n_O is the number of options, c is the initial cash invested at the risk-free rate r_f. S_t is the value of the stock at time t, while $(S_t - K)^+$ is the payoff of the option at time t.[5]

The executive's utility of wealth is a power utility function:

$$\bar{u}(W_t) = \frac{W_t^{\gamma}}{\gamma}$$

[5]Note that the option value cannot be realized when the option is not vested. However, we assume this is a perceived value of the option, which the executive can use to make decisions.

where $\gamma > 0$ is the coefficient of risk aversion. The coefficient of absolute risk aversion (CARA) at wealth w is defined as:

$$\alpha(w) = -\frac{\bar{u}''(w)}{\bar{u}'(w)} = \frac{1-\gamma}{w}$$

and the coefficient of relative risk aversion (CRRA) at wealth w as:

$$\rho(w) = w\alpha(w) = -\frac{w\bar{u}''(w)}{\bar{u}'(w)} = 1-\gamma$$

Hence, the executive's degree of relative risk aversion is constant and equal to $\rho = 1-\gamma$ at all wealth levels w, which means that the executive's degree of risk aversion does not depend on her wealth level. Hence, we can re-write the utility of wealth as:

$$\bar{u}(W_t) = \frac{W_t^{1-\rho}}{1-\rho}$$

We assume that ρ takes only two values in $\mathrm{P} = \{\rho_L, \rho_H\}$ — we abuse slightly terminology and call executive of type ρ_L risk-lover (she is Low risk averse) and the executive of type ρ_H risk-averse (she is High risk averse). Therefore, $\rho_H > \rho_L$. Clearly, denoting the ρ_L type as risk-lover does not mean she always prefers to be exposed to more risk rather than less, but simply that she is less risk averse than the other type. Indeed, we assume that both types are risk-averse, hence $\rho_H > \rho_L > 1$. The executive knows her own type, but the firm does not. However, the distribution of types in the population $\lambda = \mathbb{P}(\rho = \rho_L)$ is common knowledge. Clearly, $\mathbb{P}(\rho = \rho_H) = 1-\lambda$. Recall that risk aversion is relevant here because the employee cannot sell the ESO nor perfectly hedge against it, hence it remains exposed to at least the portion of firm-specific risk. On the other hand, the agent chooses effort $a_t \in [0, a_M]$ and effort is costly: we denote the cost of effort by $g(a_t)$ such that $g(\cdot)$ is continuous, increasing and convex ($g'(\cdot) > 0$ and $g''(\cdot) < 0$). We normalize it so that the expected output given effort a_t is a_t, and $g(0) = 0$. For simplicity, we can simply assume that g takes a quadratic form. Therefore, if effort is not compensated in some way, the executive would not have the incentive to exert any. As we will see in the next chapter, the incentive for exerting effort has been measured by the delta of the ESO: the intuition is that the delta measures the sensitivity of the option to the changes of the underlying stock, and since the agent affects the drift of the stock with her effort, she will exert it insofar this translates into a better value of her option. For similar reasons, others propose to look also at the option's vega, as the executive also affects the total volatility of the stock. For what concerns the choice of volatility, we could see the projects as comparable in risk, since higher risk yields (proportionally) higher expected return; note that choice of volatility has no cost of effort for the executive, but it affects the expected value of the compensation package. Indeed, volatility can increase the value of the stock and hence the compensation package, but at the cost of exposing the executive to higher risk — both of these features are further magnified by the intrinsic leverage effect of options [10]. Combining the two sides, we obtain that the agent's preferences are represented by an additively separable von-Neumann Morgenstern utility function:

$$u_\rho(W_t, a) = \frac{W_t^{1-\rho}}{1-\rho} - \frac{1}{2}a_t^2$$

The executive's utility is given by the expected utility of wealth, discounted at the rate $r > 0$:

$$U_\rho(a, \sigma) = \mathbb{E}\left[r \int_0^T e^{-rt} u_\rho(W_t, a) dt \right]$$

where a is the effort/output process, σ is the volatility process of the chosen project. We assume that $a_t = a$ and $\sigma_t = \sigma$ until the executive exercises the RN option, or until also the reload feature is exercised for the R option. This assumption is quite strong: while for σ it is easier to be justified, since the choice of the project – hence its volatility — is chosen at time $t = 0$ and cannot be later modified, the same is not true for the (constant) effort a. Indeed, it seems plausible the executive may decide to stop exerting effort later in the ESO contract, as she perceives her impact is more limited towards the expiry of the option. However, we keep this restriction for two reasons: (i) this allows us to avoid modeling effort stopping times and instantaneous incentives at all possible times t, and

(ii) this is intended to be a benchmark model and is thus to be intended as such, with the possible limitations that come with it. Finally, the reservation utility of the executive is denoted by $\hat{U} \geq 0$. Clearly, the executive will accept the contract if the expected utility of the contract is greater than or equal to the reservation utility. Note that we assume the executive is tied forever to the principal once he accepts the contract, or at least until time T, hence we do not model bargaining or exiting dynamics. Moreover, we are considering an extremely simplified model: in practice, the executive usually holds ESOs with different strike prices and vesting periods since they are granted over time as the job contract goes on. In our setting, we are only considering the effect of one grant of ESO, which is the only one the agent receives up to time T.

3.4 Firm

The firm/principal (he) is risk-neutral. The firm knows the distribution of types, but not the type of the agent under adverse selection. Moreover, under moral hazard, he cannot observe the choices of effort and volatility. The firm pays the executive through a comprehensive package that includes three different components: cash, (restricted) stocks, and ESOs. In our analysis, we fix the first two and focus our analysis on ESOs only.[6] The firm offers two contracts to the agent, one including the RN option and the other the R option. The contract space is defined as

$$\Theta = \left(\mathbb{1}_{RN}, \left\{ \mathbb{1}_{R_{\alpha,\gamma}} \right\}_{\alpha \in (0,1), \gamma \in [0,1)} \right) \times \mathbb{Z}^{+}$$

where $\mathbb{1}_{RN}$ is the indicator function for the RN option, and $\mathbb{1}_{R_{\alpha,\gamma}}$ is the indicator function for the R option with parameters α and γ. The firm offers two contracts, one with the RN option and the other with the R option, i.e., $\theta_{RN} = (\mathbb{1}_{RN}, n_{RN})$ and $\theta_R = (\mathbb{1}_{R_{\alpha,\gamma}}, n_R)$. In principle, the firm can offer a different number of options depending on the chosen contract, but for simplicity we set $n_{RN} = n_R = n$. Therefore, $\theta_{RN} = (\mathbb{1}_{RN}, n)$ and $\theta_R = (\mathbb{1}_{R_{\alpha,\gamma}}, n)$. The firm chooses how to structure the R option, that is, he chooses the

[6]In practice, the agent considers the whole compensation package, including also bonus and other benefits, when considering her value of the ESO contract. We will account for this by running some sensitivity analysis.

values of α and γ; this is the control the firm has over the compensation package. The function $C(\cdot)$ denotes the cost of the ESO contract multiplied by the number of options in the contract. We will define this valuation function in the next chapter, but for now we take it for granted.

The time horizon of the firm matches the agent's. The firm is risk neutral, hence the utility is linear and given by:

$$\Pi(\alpha, \gamma; \beta, \mu) = \beta \, \mathbb{E}\left[S_T\right] - \left[\mu C(\theta_{RN}) + (1-\mu) C(\theta_{R_{\alpha,\gamma}})\right]$$

where S_T is the "terminal" stock price at T, μ is the share of executives choosing the RN option, $C(\cdot)$ is the cost of the ESO contract multiplied by the number of options in the contract. Finally, β measures how relevant it is for the firm to maximize the stock price at time T. For example, for firm with large capitalization (i.e., large number of shares outstanding), a small increase in the price of the stock can have a large impact on the firm's value. Differently, for firms with small capitalization, the impact of the stock price on the firm's value is less relevant. The controls α and γ affect the expected terminal stock price through the executive's effort and project choice.

3.5 Problem

So far we have seen the main features of our setting. We have a risk-averse executive that can choose the effort and volatility of the project, and a risk-neutral firm that can structure the R contract. The executive can be of low or high risk aversion, and her utility is given by the expected utility of wealth, discounted at the rate $r > 0$. The firm's utility is given by the expected value of the terminal stock price minus the cost of compensating the agent with the two options. The firm offers two contracts to the agent, one including the RN option and the other the R option. The firm chooses how to structure the R option, that is, he chooses the values of α and γ. The firm's problem is to maximize the expected profit, subject to the executive's participation constraints. In our "realistic" setting, there is both adverse selection and moral hazard: a setting that we call third best. However, for benchmarking, we will consider also the case of complete information (first best), where the firm knows the type of the agent and there is no moral hazard, and the case of moral hazard only (second best). Unless stated differently, we will assume the firm wants to employ both agents.

For computational simplicity, a, σ, and ρ will not take values on a continuum but rather on a binary discrete set, whose values we call low and high. Therefore, the agent solves the following maximization problem:

$$\begin{aligned} \max_{a,\sigma,\theta} \quad & U_\rho(a,\sigma,\theta) \\ \text{s.t.} \quad & a \in \{a_L, a_H\} \\ & \sigma \in \{\sigma_L, \sigma_H\} \\ & \theta \in \{\theta_{RN}, \theta_{R_{\alpha,\gamma}}\} \\ & U_\rho(a,\sigma,\theta) \geq \hat{U} \end{aligned} \tag{1}$$

The firm anticipates Problem **??** and incorporates it in its maximization problem. Therefore, under complete information and observability, the problem of the firm is:

$$\begin{aligned} \max_{\alpha,\gamma} \quad & \Pi(\alpha,\gamma;\beta,\mu) \\ \text{s.t.} \quad & U_\rho(a,\sigma,\theta) \geq \hat{U} \quad \forall \rho \in \{\rho_L, \rho_H\} \end{aligned} \tag{2}$$

Therefore, the only constraint in Problem **??** is that the executive's utility needs to be at least as high as her reservation utility, for both types of agents. The firm thus chooses the contract that maximizes its expected profit, subject to the executive's individual rationality (IR, or participation) constraint.

We now allow for moral hazard, i.e., the executive's choices of effort and volatility are not observable anymore by the firm. Therefore, we need to add the incentive compatibility (IC) constraints to the previous setting, which constraints even further the firm's maximization problem. Suppose that the firm wants to implement high effort and high volatility for both agents — her problem now becomes:

$$
\begin{aligned}
\max_{\alpha,\gamma} \quad & \Pi(\alpha,\gamma;\beta,\mu) \\
\text{s.t.} \quad & U_\rho(a,\sigma,\theta) \geq \hat{U} && \forall \rho \in \{\rho_L, \rho_H\} \\
& U_\rho(a_H,\sigma_H,\theta) \geq U_\rho(a,\sigma,\theta) && \forall a \in \{a_L, a_H\}, \\
& && \forall \sigma \in \{\sigma_L, \sigma_H\}, \\
& && \forall \theta \in \{\theta_{RN}, \theta_{R_{\alpha,\gamma}}\}
\end{aligned}
\tag{3}
$$

The IC condition requires that under either the RN or the R option, both agents will be willing to exert high effort and choose high volatility and not deviate to a different combination of the two. Moreover, since the IR constraints imply that both types need to have an IR-compatible contract, the IC-compatible contract will also be IR-compatible, otherwise one of the two conditions fails.

Finally, under both moral hazard and adverse selection, the situation gets trickier. Here, the firm can decide what type of contract she wants to implement — be it separable (screening), pooling, or shutdown (i.e., one type of agent is not employed). As an example, suppose the firm wants to implement our contract of interest, i.e., the screening contract, whereby the high risk averse executive is granted a RN option and the low risk averse receives the R option, so that both agents exert high effort and high volatility. The problem of the firm is:

$$
\begin{aligned}
\max_{\alpha,\gamma} \quad & \Pi(\alpha,\gamma;\beta,\mu,\lambda) \\
\text{s.t.} \quad & U_{\rho_L}(a,\sigma,\theta_{R_{\alpha,\gamma}}) \geq \hat{U} \\
& U_{\rho_H}(a,\sigma,\theta_{RN}) \geq \hat{U} \\
& U_{\rho_L}(a,\sigma,\theta_{R_{\alpha,\gamma}}) \geq U_{\rho_L}(a,\sigma,\theta) && \forall a \in \{a_L, a_H\}, \\
& && \forall \sigma \in \{\sigma_L, \sigma_H\}, \\
& && \forall \theta \in \{\theta_{RN}, \theta_{R_{\alpha,\gamma}}\} \\
& U_{\rho_H}(a,\sigma,\theta_{RN}) \geq U_{\rho_H}(a,\sigma,\theta) && \forall a \in \{a_L, a_H\}, \\
& && \forall \sigma \in \{\sigma_L, \sigma_H\}, \\
& && \forall \theta \in \{\theta_{RN}, \theta_{R_{\alpha,\gamma}}\}
\end{aligned}
\tag{4}
$$

Note that now also λ enters the firm's problem, since the type of the agent is not observable anymore. In principle, μ may either equal or differ from λ. Under the screening contract, $\mu \neq \lambda$ if $\lambda \in (0, 1)$. The first two IR conditions require that under the designed contract, both agents will be willing to accept the contract. The last two IC conditions require that under high effort and high volatility, as well as under the designed contract, both agents will not deviate to a different combination, or lie and pretend to be of the other type to receive the other ESO option. Therefore, both agents will exert high effort and choose high volatility, and the firm will implement the screening contract, with the less risk averse type receiving the R option and the more risk averse type receiving the RN option.

Comment

The formulation of our model has some clear limitations. First, we assume that the executive is a "contract-taker" and does not have any bargaining power. This is a strong assumption, especially at the C-level, where executives may have a lot of bargaining power and hence the ESO-setting procedure could be endogenous. Second, we constrain executives to block exercise all options rather than being able to exercise them at different times. While empirical literature is consistent with some degree of block exercise, this still may not be completely realistic. Third, as already mentioned before, we assume that volatility and effort are constant until the option (and the reload feature, if applicable) have not expired. Finally, we are not modeling other dynamics which may be of interest, such as the optimal stopping time for the agent to exercise the option, or the possibility of the executive to leave the firm. Both are clearly relevant for the optimal exercise of the ESOs, but would add a stochastic machinery to our problem, which is out of our scope.

4 Valuation

In this chapter, we compute the cost of the two options. We first introduce the methods we use to compute them — namely Black-Scholes and lattice-based models — and then show the main snippets of our code. We compute the cost to the employer under the risk-neutral framework and the executive value under the utility maximizing framework. Finally, we compute some comparative statics, which allow us to explore better the relationship between subjective and objective incentives induced by the two options, as well as to compare the objective and subjective valuations, and to analyze how they change with different parameters.

4.1 Valuation models

Black-Scholes

The Black-Scholes model, developed by Fischer Black and Myron Scholes in the early 1970s ([6]), is a well-known mathematical framework used in finance to determine the theoretical value of European-style options. We will just limit ourselves to the minimum requirements and intuition to be able to use the Black-Scholes model in computing the time-value of our ESOs later. A more detailed overview of the model is out of our scope and can be easily found online.

The assumptions are quite standard, so we simply take them from [72]:

Assumption 1 *The rate of return on the riskless asset is constant and denoted as risk-free interest rate r.*

Assumption 2 *The instantaneous log return of the stock price is an infinitesimal random walk with drift: the stock price follows a geometric Brownian motion, and it is assumed that the drift and volatility of the motion are constant.*

Assumption 3 *The stock does not pay any dividends.*

Assumption 4 *The market is arbitrage-free.*

Assumption 5 *It is possible to borrow and lend any amount (even fractional) of cash at the riskless rate.*

Assumption 6 *It is possible to buy and sell any amount (even fractional) of the stock.*

Assumption 7 *There are no transaction fees.*

Note that the first three assumptions are about the underlying asset while the last four are about the market. Moreover, assumption **??** assumes the possibility of short-selling and free trade of the stock, which in the case of ESOs is usually not fully possible. However, the Black-Scholes model provides nevertheless a good approximation and suffices for our (limited) use. Note however that not all of these assumptions are strictly necessary: extensions of this basic model are able to account for dynamic interest rates, transaction costs and taxes, and dividend payout.

Under these assumptions, [6] showed that it is possible to create a hedged position (i.e., a long position in the underlying stock and a long position in the option) whose value is independent of the price of the underlying. This dynamic hedging strategy leads to a partial differential equation governing the price of the option, whose solution is given by the Black-Scholes formula.

However, despite its mathematical rigor and closed-form solution, as well as computational efficiency, Black-Scholes can only be used with European options (options exercisable only at maturity). Therefore, since we are dealing with (American) ESOs, we need to resort to a different valuation methodology for most of our analysis: lattice-based models.

Lattice tree methods

Binomial model

The binomial model was first proposed by William Sharpe in the 1978 ([67]) and later formalized by Cox, Ross and Rubinstein (CRR) in 1979 ([17]). It uses a lattice-based approach to model the price of the underlying stock price. We employ it to value our executive stock options.[7] Similarly to Monte Carlo simulations and finite difference methods,

[7]As said before, Black-Scholes can only be used to value European options. Notwithstanding this, Black-Scholes valuation provides a lower-bound to the value of American options: indeed, giving the

the binomial options pricing model does not yield a closed-form solution, but works quite well with a sufficiently high number n of time steps, i.e., with a sufficiently discretized grid (that is, a small Δt). Clearly, for European options, as $\Delta t \to 0$, the binomial model approaches the Black-Scholes value, which can be thus seen as its limiting case.
We need assumptions **??**, **??**, **??**, **??** from Black-Scholes, and in addition we pose two additional assumptions:

Assumption 8 *Time is divided into discrete periods.*

Assumption 9 *At any time step, the underlying's price can only move one of two ways: either up or down.*

The intuition behind the model is quite simple. We discretize the evolution of the underlying stock price by building a binomial lattice (tree), for a given number of steps between the issue of the option and its maturity. Each node represents thus a possible price of the underlying stock. From each node, the stock price can either go up (u) or down (d), with probability q or $1-q$. The key assumption is thus that the stock moves only in either one of these two directions. Therefore, the generic value of the stock at step t with k upward movements and $t-k$ downward movements is $S_t = S_0 u^k d^{t-k}$, where S_0 is the initial stock price at $t=0$. Figure **??** provides a graphical representation of the first two levels of the stock price tree.

We can divide the procedure in three main steps: (i) build the stock tree, (ii) calculate the option value at each terminal node, (iii) go backwards and decide at each node whether it is optimal to exercise or keep the option.
We use the original CRR method and set $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$, which are the multiplicative factors if the stock price goes up or down, respectively. Note that the price multipliers u and d depend thus only on volatility of the underlying and time step Δt

holder the possibility to exercise before maturity also cannot be worse for her, because she can always simply hold the option up to maturity and then exercise if in-the-money, mimicking thus the behavior under a European option.

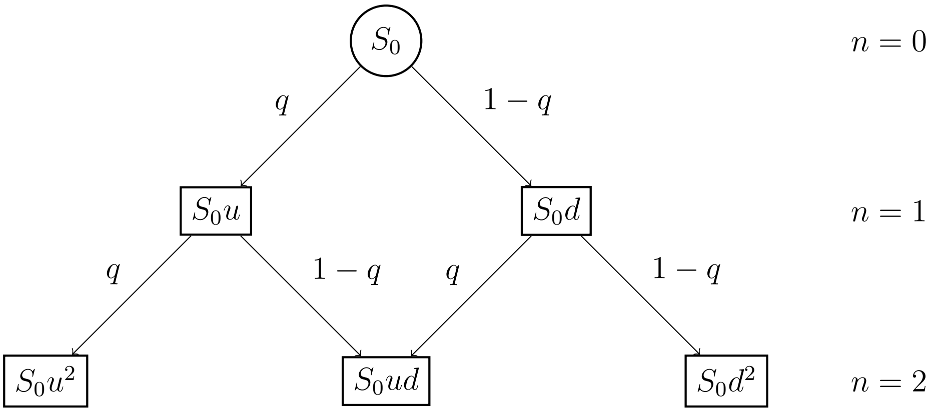

Figure 4: Representation of the binomial tree with n=2

and not on the drift.[8] Therefore, the value of the stock when it goes up or down depends also on its underlying volatility. Moreover, the condition $d = \frac{1}{u}$ ensures that the tree is recombinant, meaning that what matters is the number of ups and downs, not their order. This means that if the stock price moves up and then down, the stock price will be the same as if it instead moved first down and then up. Indeed, as can be seen from figure **??**, S_0ud can be reached by either first going up and down, or first down and then up. This is convenient also because it allows the tree representation of the underlying stock price, ensuring that all the nodes are nicely connected, and it fastens the calculation of the option price (indeed, note that in this way the number of total nodes is reduced).

In the second step, we simply compute the option value at expiration, i.e., its intrinsic value, which in the case of our call option is $\max\{0, S_T - K\}$.

For the third step, we need to introduce the risk-neutral probability $q = \frac{e^{-r\Delta t} - d}{u-d}$ which assesses how likely is that the stock price goes up during each period in a risk-neutral world.[9] In this world, today's fair price of the option is the expected value of its future payoff discounted by the risk-free rate. Therefore, the expected value at time t is calculated using the option values from the subsequent two nodes at $t+1$, weighted by their respective probabilities - q as the probability of an upward move of the underlying stock, $1 - q$ of a downward move. This value is then discounted using r and represents the *binomial value* of the option at a given node. Given q, we can then proceed with our backwards algorithm.

[8] This is the reason for which the probability of going up is usually higher in this model, to compensate for the missing account of the drift component.

[9] This choice of q allows for the related binomial distribution to simulate the geometric Brownian motion of the underlying stock, with parameters r and σ.

We start at each pre-terminal node and evaluate whether it is optimal to exercise (with payoff the stock price at the node minus the strike price) or to not exercise (with payoff the binomial value at the node). We continue this process iteratively going backwards until when the option is vested. When the option is not vested anymore, the holder cannot exercise it and so the value of the option will simply be the binomial value, at that node and all the previous ones. The value of the option at the initial node, obtained with this backwards procedure, is the value of the ESO we obtain using the binomial model. This is the traditional backwards approach; however, we will modify slightly the decision function in the last step to account for the tendency of early exercise, using the multiple technique of [17].

Trinomial model

The trinomial option pricing model was first developed by Phelim Boyle in 1986 as an extension of the binomial model ([7]). Conceptually, the two models are the same: here we are only allowing for the stock to have a third path, the middle path, whereby it does not go down nor up but remains stable over the next time step. Therefore, the differences from the binomial model are very few. We replace assumption **??** with the following:

Assumption 10 *At any time step, the underlying's price can only move one of three ways: either up, down, or remain stable.*

As before, $u = e^{\sigma\sqrt{\Delta t}}$, $d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$, and now $m = 1$. Moreover, we need to change the risk-neutral probabilities (before we had two, now three), which become slightly more complicated:[10]

$$q_u = \left(\frac{e^{r\Delta t/2} - e^{-\sigma\sqrt{\Delta t/2}}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}} \right)^2$$

$$q_d = \left(\frac{e^{\sigma\sqrt{\Delta t/2}} - e^{r\Delta t/2}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}} \right)^2$$

$$q_m = 1 - (p_u + p_d)$$

Therefore, the idea of the procedure is the same: we compute the stock price tree and

[10]Recall that we do not consider dividend yields on our stock, hence the formulas are slightly easier than usually presented.

then proceed backwards to determine the optimal decision at each node. The difference now is that the option at each non-terminal node is computed based on three later nodes — instead of two — and their relative probabilities. The trinomial model produces more accurate results compared to the binomial model when using fewer time steps, but the binomial model is usually preferred due to its faster implementation.

4.2 Firm cost

We compute the cost of the two options to the firm using the classical valuation standards for American ESOs: a simple binomial model for the RN option, and a modified binomial model that accounts for the recovered time premium for the R option. Recall indeed that the theoretical value of an option at any time t is given by (i) the intrinsic value $(S_t - K)$ and (ii) the time premium, i.e., the remaining value forfeited at exercise, which basically accounts for the possibility that the option appreciates in the remaining lifetime. We employ the binomial tree approach from [17]; however, we account for deterministic early exercise introducing a threshold above which the holder will exercise, i.e., always exercise if $S_t \geq K * m$ for some multiple m ([44]), which we set at $m = 2$.

We now comment the relevant code used — the full code can be found in the appendix.

4.2.1 RN option

The function rn_eso is defined as `rn_eso(S0,K,T,v,r,N,sigma,m)` and takes the following parameters as input:

- S_0: spot price of the underlying firm stock
- K: strike price of the option; we always set it equal to S_0
- T: time to maturity (in years)
- v: vesting period of the option (in years)
- $r(/disc)$: risk-free interest rate
- N: height of the tree

- *sigma*: volatility of the underlying firm stock
- *m*: exercise multiple.

We follow the binomial model procedure illustrated before: we first construct the stock price tree, and then we compute the option's value backwards. As mentioned above, we change slightly the decision function at each node, using the multiple technique proposed by [44]. Therefore, we model early exercise as taking place whenever stock price reaches a certain multiple — from here the name — of the strike price. At each step, the relevant decision is the following: if the option is either not vested or is vested but is not above the threshold for early exercise, the value at the node is the binomial value. Otherwise, if it is vested and above the threshold for early exercise, the value at the node is the exercise value. This procedure is illustrated in snippet **??**.

```
if not vested:
    C[j] = disc * (q*C_up + (1-q)*C_down)
elif vested & (S>=K*m):
    C[j] = S - K
elif vested & (S<K*m):
    C[j] = disc * (q*C_up + (1-q)*C_down)
```

Figure 5: Valuation of RN option

4.2.2 R option

We define the function r_eso as `r_eso(S0,K,T,v,r,N,sigma,m,alpha,gamma)`.
We use the same parameters as above, with the addition of:

- *alpha*: amount of option that gets converted to cash at exercise
- *gamma*: *risk premium* granted in terms of new RN option.

The valuation here is trickier because the payoff at exercise is not any more given only by the difference between the stock price and the strike price, but also by the time value that gets recouped with the issuance of the new option. It is hence clear that, holding the same parameters, the cost of rn_eso is a lower bound for r_eso. We follow the procedure proposed by [41] in snippet ??. The difference from above is that, in the case of the option being vested and above the threshold for (early) exercise, the payoff is given by its intrinsic value plus a portion of its time value, computed as the Black-Scholes cost minus the intrinsic value, multiplied by the percentage that gets recouped.

```
if not vested:
    C[j] = disc * (q*C_up + (1-q)*C_down)
elif vested & (S >= K*m):
    C[j] = (S - K) +
             (1-alpha)*(black_scholes(S,K,T,r,sigma)-(S-K))
elif vested & (S < K*m):
    C[j] = disc * (q*C_up + (1-q)*C_down)
```

Figure 6: Valuation of R option with $\gamma = 0$

We run a simulation using the parameters from [41] (S_0=30, K=30, T=10, v=3, r=0.05, N=1, sigma=0.3, m=2) and obtain the following results: \$13.66 for RN, \$14.55 for R. Therefore, the cost of the RN option is the same as the one they find, while the cost of the R option is higher than the one they find (\$14.14). It is unclear where this difference comes from; it could simply be a difference in the choice of the number of time steps per year, which they do not however present explicitly. Nevertheless, running the binomial model with N=1 is too low, as table ?? shows. Indeed, this means allowing the stock to have only one movement per year, which is quite restrictive. The results from table ?? suggest thus that it is safer to run these simulations with at least N=500.

However, this valuation is incomplete, because we are not considering the role played by γ. In their paper, [41] show only one example, where 75% of the option gets exercised and another 25% (i.e., 100% - 75%) gets reloaded. Therefore, it is unclear how they would consider a strictly positive risk premium $\gamma > 0$. We therefore elaborate a novel approach which builds up on their original idea. Note that the value of adding γ is two-fold: first, as a proper exercise compensation since the executive gets stock and

N	RN ($)	R ($)	R-RN ($)
1	13.66	14.55	0.89
10	12.98	14.12	1.14
50	12.79	14.00	1.21
100	12.79	14.00	1.21
365	12.72	13.95	1.23
500	12.66	13.91	1.25
1000	12.69	13.93	1.24

Table 1: Difference in values of RN and R by changing the number of steps per year N

```
if not vested:
    C[j] = disc * (q*C_up + (1-q)*C_down)
elif vested & (S >= K*m):
    C[j] =  (1+gamma)*(S - K) +
            (1-alpha+gamma)*(black_scholes(S,K,T,r,sigma)-(S-
               K))
elif vested & (S < K*m):
    C[j] = disc * (q*C_up + (1-q)*C_down)
```

Figure 7: Valuation of a generic R option

options worth $(1+\gamma)$ the number of options she previously held, and second as time value recovered. Therefore, we propose the valuation methodology in snippet **??** for our R option which embeds this double edge provided by γ: `1+gamma*(S - K)` captures the first effect, `(1-alpha+gamma)*(black_scholes(S,K,T,r,sigma)-(S-K))` the second. Clearly, this methodology equals the previous when $\gamma = 0$. Therefore, it can be seen as a proper extension of **??**.

4.3 Executive value

We now turn to the subjective value of the option to the risk-averse executive, that we compute using the trinomial model under a utility maximization framework. In our setting, the executive is interested in maximizing its expected utility at time $t = 0$; therefore, she will take a forward-looking perspective at the possible stock price grid and, by backwards induction starting from the terminal nodes, she will compute the utility she can expect given the initial parameters. This will then be translated in the amount of cash compensation she would require to give up her options, which results therefore in a certainty equivalence in continuous time.

We build on the model proposed by [52] that computes the value of an option with multiple reloads and fixed expiry date using a trinomial tree under the framework of maximizing the expected utility value of the terminal wealth, where the risk aversion factor is embedded in the coefficient and choice of the utility function. However, differently from them, our R option can be reloaded only once, which allows us to simplify the algorithm. For the RN option, the procedure will be even simpler. The key in both cases is that, at each non-terminal node, the executive is maximizing her utility by deciding whether to exercise or continue (hold), and this will be done as before by dynamic programming starting from the terminal nodes. To be more precise, [52] construct the numerical approximation extending the trinomial model via the forward shooting approach, which basically consists in augmenting the information at each node with a vector of auxiliary variables and modeling the function that describes how the vector evolves from one node to the following. We do not need such formal specification as our model is simpler than theirs; we will instead compute the non-firm related wealth available at each node using the appropriate formula rather than computing it when building the stock price tree. This approach is sensible as it is simpler yet computationally the same.

We use the same parameters from above for the two options. In addition, we need to add the following:

- rho : coefficient of relative risk aversion of the executive
- c : non-firm related initial wealth, that grows continuously at the risk-free rate r
- n_s : number of stocks initially held by the executive
- n_o : number of options initially held by the executive

Note that, at each time step t, the value of the non-firm related wealth c_t is computed as $c_t = c\big(1 + \frac{r}{N}\big)^t$, where N is the number of time steps in a given year and r the annual risk-free rate.

Therefore, we want to find E_c such that

$$U_\rho(n_s, n_o, c) = U_\rho(, n_S, 0, c + E_c)$$

where we included only the most relevant parameters entering the executive's utility function. Therefore, the external wealth of the executive becomes relevant now, both in terms of absolute wealth and portfolio composition (i.e., percentage of total wealth held in firm stocks). As we will see later, it is relevant because the utility function is not separable, hence our results will be sensible to the initial values we will choose. In other words, E_c is the adjustment to the executive's wealth required to keep her on the same utility level while not granting her any option.

Recall that we defined the executive's utility of wealth with the power utility function, that is,

$$\bar{u}_\rho(w) = \frac{w^{1-\rho}}{1-\rho}$$

with $\rho = 1-\gamma$ being the (constant) coefficient of relative risk aversion. The inverse utility is thus given by:

$$w(\bar{u}_\rho) = \Big((1-\rho)\bar{u}_\rho\Big)^{\frac{1}{1-\rho}}$$

We use these two formulas to define the functions `u(w, rho)` and `u_minus(u, rho)` that return u and w, respectively. We will use them to compute the exercise value at each node and to derive then the certainty equivalent at the initial node. Clearly, the executive value is not unique – as was the case with the firm cost – but rather changes as the coefficient of risk aversion (and initial wealth) changes.

Note that we follow the approach used in the literature and assume that only block exercise is possible to keep our algorithm simple. A detailed analysis of separate exercises would be quite complex and computationally intensive. Moreover, we do not discount executive's utility from one time period to the following; in this sense, the estimates we are providing can be considered as an upper bound of the executive value. Finally, a possible modification of the algorithm would be to embed also here an exercise multiple mechanism, that we could make dependent on the risk aversion coefficient of the executive.[11] However, we do not explore this venue here. We will now outline briefly the main components of our algorithm, as done before.

[11] For example, we could set $\frac{m}{1+\rho}$ as the threshold for early exercise for some fixed m, so that it would be lower for more risk averse holders. In other words, more risk averse executives would exercise earlier, which is consistent with the empirical findings.

4.3.1 RN option

We define the function `CE_rn_trinomial(S0,K,T,v,r,N,sigma,rho, n_s,n_o,c)` to compute the executive value of the RN option. At each node, the exercise utility is computed considering the total wealth at the node, which is the sum of the non-firm related wealth c invested at the risk-free rate r, the stock value, and the option value. Snippet **??** shows the relevant part of the code.

```
cont_value = q_u * U[j+1,i+1] + q_d*U[j,i+1] + q_m*U[j,i]
excs_value = util(c*((1+r/N)**n)+ n_s*S[j,n] + n_o*max(0,
    S[j,n]-K), rho)


if vested:
    U[j,i] = max(cont_value, excs_value)
else:
    U[j,i] = cont_value
```

Figure 8: Valuation of RN option under utility maximization framework

Note that the continuation value is the average of three different values, which reflect the possible paths the stock can follow, weighted by the respective risk-neutral probabilities. The exercise value is instead the utility of the wealth at the node, which is the sum of the non-firm related wealth, the stock value, and the option value.

After this backwards iteration, we finally compute the executive value E_c as `RN_ce = (u_minus(U[0,0], rho)-c-n_s*S0)/n_o`. In other words, we find the level of wealth at time $t = 0$ required to reach the same utility as the executive would have with the RN options. From this, we subtract the initial wealth and the value of the stocks, and then divide by the number of options to find the certainty equivalent value per RN option.

4.3.2 R option

We follow a similar approach to the above and define the function `CE_r_trinomial(S0,K,T,v,r,N,sigma,rho, alpha,gamma, n_s,n_o,c)` to compute the executive value of the R option. The main difference is that we now need to compute the exercise value at each node considering also the time value that gets recouped at exercise. The procedure follows a similar approach to that of the R option as firm cost. The relevant part of the code is shown in snippet ??.

```
cont_value = q_u * U[j+1,i+1] + q_d*U[j,i+1] + q_m*U[j,i]
excs_value = util(  c*((1+r/N)**n) +
                    n_s*S[j,n] +
                    max(0,
                        n_o*(alpha*(S[j,n]-K)) +
                        CE_rn_trinomial(S[j,n],S[j,n],T,v
                            ,r,N,sigma,rho,n_s,n_o*(1-alpha
                            +gamma),c*((1+r/N)**n))), rho)


if vested:
    U[j,i] = max(cont_value, excs_value)
else:
    U[j,i] = cont_value
```

Figure 9: Valuation of R option under utility maximization framework

Note that now the exercise value is more complicated than before. Indeed, the option value is now given by the immediate exercise payoff $S - K$, where S is the stock price at a given node, multiplied by α, plus the cash equivalent of the new $(1 - \alpha + \gamma)$ options that the executive receives. Therefore, she may a priori decide also to exercise the option even if the stock price is below the strike price, as the time value that gets recouped may be higher than the intrinsic value. This could happen for example when α is quite low and γ is sufficiently high. After the backwards iteration, we compute the executive value

E_c in the same way of the RN option.

4.4 Comparative statics

We now perform a series of numerical simulations to analyze how the two valuations differ as we change some key parameters. We use the following parameters when not specified differently: $S_0 = 30$, $K = 30$, $T = 10$, $v = 3$, $r = 0.045$, $N = 100$, $sigma = 0.3$, $m = 2$, $rho = 2$, $alpha = 0.75$, $gamma = 0.25$, $c = 1{,}700{,}000$, $n_s = 110{,}000$, $n_o = 300$, if we want a 67-33 portfolio, otherwise we set $c = 2{,}500{,}000$ and $n_s = 87{,}333$ for a 50-50 portfolio.

THE FOLLOWING ARE THE ANALYSIS I WILL RUN - NUMERICAL SIMULATIONS FOR R OPTION IN PROGRESS ...

OBJECTIVE AND SUBJECTIVE DELTA AND VEGA OF OPTIONS: Delta measures the incentive to exert effort, vega the incentive to increase volatility. Subjective/executive values computed for 3 different levels of risk aversion. Show the following:

- Graph of delta change for changes in underlying stock price
- Graph of vega change for changes in underlying volatility
- Table/graph the values of the following formulas:

$$\text{Objective incentive} = \frac{\text{\% increase in firm cost}}{\text{\% increase in stock price}}$$

$$\text{Subjective (/Employee) incentive} = \frac{\text{\% increase in executive value}}{\text{\% increase in stock price}}$$

$$\text{Difference in incentives} = \frac{\text{Objective Incentive - Subjective Incentive}}{\text{Subjective Incentive}}$$

DEADWEIGHT COST: Table/graph the values of the following formula for RN option and R option (for different values of α and γ), for different levels of risk aversion:

$$\text{Deadweight cost} = \frac{\text{Firm cost - Executive value}}{\text{Firm cost}}$$

VALUES' RATIOS: Table/graph the values of the following formulas for R option(for different values of α), for different levels of risk aversion:

$$\text{Executive value ratio} = \frac{\text{Executive value of R } (\alpha, 0)}{\text{Executive value of RN}}$$

$$\text{Firm cost ratio} = \frac{\text{Firm cost of R } (\alpha, 0)}{\text{Firm cost of RN}}$$

IMPACT OF OUTSIDE WEALTH: Re-run and compare the last two simulations for two different mixes of outside wealth: 50-50 portfolio and 67% stock-33% cash portfolio.

5 Numerical Analysis of Equilibria

To be written.

6 Conclusion

To be written.

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References

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