Sliding Mode Control - Simulink Instructions

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The Sliding Mode Controller (SMC) library utilises previous work completed in [1] to create a graphic interface for rapid construction of SMCs. Some of the key features of the library include:

- Quadratic Minimisation for sliding surface design
- Integral Action for reference tracking
- Adaptive schemes for when the bound of uncertainty is unknown

1 Instructions

1.1 Controller Configuration

- *Integral Action* - Toggles the use of Integral Action and relevant parameter options
- *Show s(t)* - Outputs the value of the function $s(t)$
- *Adaptive* - Selects the value of rho adaptively and enables relevant parameters
- *Show Adaptive Parameters* - Outputs the value of the adaptive parameters in a vector $\begin{bmatrix} c & k & \rho \end{bmatrix}^T$.

1.2 Controller Tuning

The hyperplane $\mathcal{S}$ (and therefore the closed loop performance) is designed using quadratic minimisation to minimise the cost function

$$J = \frac{1}{2}\int_{t_s}^{\infty} (x^T Q x)dt$$

where t_s is the time sliding occurs and Q is the state weighting matrix. When adaptive mode is on, the value of ρ is chosen such that $\rho = c + k$ and where the adaptive laws for c and k are chosen as

$$\begin{aligned} \dot{c} &= c_0 + c_1\|s(t)\| \\ \dot{k} &= k_0 + k_1\|s(t)\|\|x(t)\| \end{aligned} \tag{1}$$

- *State Weighting Matrix* - Weighting matrix used to determine $\mathcal{S}$, the resulting eigenvalues are printed to the diagnostic viewer.
- *Rho* - The selected value of ρ. Only used when adaptive mode isn't active
- *Adaptive Parameters* - Initial values of parameters c_1 and k_1 with associated learning rates. These are used to adaptively select ρ when in adaptive mode.

1.3 Chattering Avoidance

Chattering is a high frequency motion through the hyperplane S [1]. This controller has to ways to help mitigate this motion

- *Sigmoidal Approximation* - The discontinuity in the control law $u(t)$ is smoothed into a sigmoid of the form $\frac{s(t)}{\|s(t)\|+\delta}$ where δ is chosen in the parameter box
- *Boundary Layer Thickness* - A layer is placed around the hyperplane $\mathcal{S}$ in which the system is considered to be on the plane (i.e $s(t) = 0$)

1.4 Integral Action

Integral action adds a tracking feature to the controller by augmenting the system states to $\bar{x} = col(x_r, x)$ where

$$x_r(t) = r(t) - Cx(t)$$

a differentiable reference signal is given by $r(t)$ and C represents the systems output matrix. In the event that the reference signal is not differentiable the controller can be equipped with a built in filter.

- *Integral Action Weighting Matrix* - Used for the design of $\mathcal{S}$ to weight the states x_r
- *Pre-Filter Reference Signal* - When toggled provides a filter for the reference signal
- *Filter* - Provides the value of the filter, must be a stable design matrix
- *Show Filtered R(t)* - Outputs the value of the filtered reference signal

2 Sliding Mode Control

Consider the n^{th} order linear time invariant system with m inputs given by

$$\dot{x}(t) = Ax(t) + Bu(t) + f_m(u, x, t) \tag{2}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $x(t)$ represents the system state and where $f_m(t, x, u) : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathcal{R}(B)$ is the unknown but bounded matched uncertainty and Consider a 'switching function' [1] of the form

$$s(t) := Sx(t) \tag{3}$$

where $S \in \mathbb{R}^{l \times n}$ and is chosen using quadratic minimisation.

Choosing a control law of the form:

$$u(t) = -Lx(t) - \rho \frac{s(t)}{\|s(t)\|} \tag{4}$$

where L is chosen as a function of the system matrices and S. It can be verified that if ρ is chosen such that

$$\rho > \|f_m(t, x, u)\|$$

then choosing $u(t)$ in (2) as (4) guarantees that the system states reach the hyperplane $\mathcal{S}$ in finite time and remain on $\mathcal{S}$ for all further time, where

$$\mathcal{S} := \{x \in \mathbb{R}^n : Sx(t) = 0\}$$

Here it is clear that the choice of $\mathcal{S}$ is used to tune the systems performance in contrast to other control methods where the choice $u(t)$ is the main freedom.

3 Copyright

References

[1] C. Edwards and S. Spurgeon, *Sliding mode control: theory and applications.* Taylor & Francis, 1998.

[2] H. Alwi and C. Edwards, "Fault tolerant control using sliding modes with on-line control allocation," *Automatica*, 2008.