Particle Techniques Exercises

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1 Exercise 1: Estimating Momentum with GEANT4

Code for this first assigment along with installation instructions stored in this repository.

1.1 Brief

Having discussed the basic structure of the code, and how to run/modify it, the first exercise is the following:

(1) For a magnetic field of 0.5 T, an angle of the spectrometer of 0 degrees (i.e. the axes of the tow arms are on the same line), send 1000 antimuons (mu+) along z+ with a momentum of 100 GeV. The precision coordinate is "x", and the second coordinate is "y". Assume a precision along x of 100μm, and along y of 1cm. Reconstruct the track of the muons along the x-z plane and estimate their momentum. Draw the distribution of reconstructed momentum, and find the resolution.

(2) Estimate the momentum resolution if the magnetic field was 0.25 T or 1T.

(3) For a magnetic field of 0.5 T, estimate the momentum resolution for 50 GeV and 200 GeV muons.

(4) What is the momentum resolution for 100 GeV muons if you assume perfect knowledge of the position along x? why?

(5) Add 5cm of Pb immediately before the magnetic and, subsequently immediately after the magnet. What is the effect?

(6) (Optional) Perform a complete fit to the track that takes as input the measurements in the x-z plane for both DC1 and DC2.

Please create a git hub account where you keep your simulation and analysis code, and put the results in an overleaf document.

1.2 Breakdown of Exercise

Using position information from hits in geant4, will be able to calculate the curvature of tracks and therefore velocity and momentum. Potentially will need to account for relativistic effects! Precision values will allow an error to be propagated. Only reconstruct in 2D (xz) plane. Assume error on mag field of 0? Propagate an equation and draw into latex to answer (4).

1.3 Theory/Equations

The relativistic momentum equation is is defined as

$$\vec{p} = \gamma m \vec{v} \tag{1}$$

where symbols have usual meanings. The rate of change of momentum is equal to force, $\vec{F}$ which, in a magnetic field, is

$$F = q\vec{v} \times \vec{B} = q|v||B|\sin(\theta)\vec{\hat{n}} \tag{2}$$

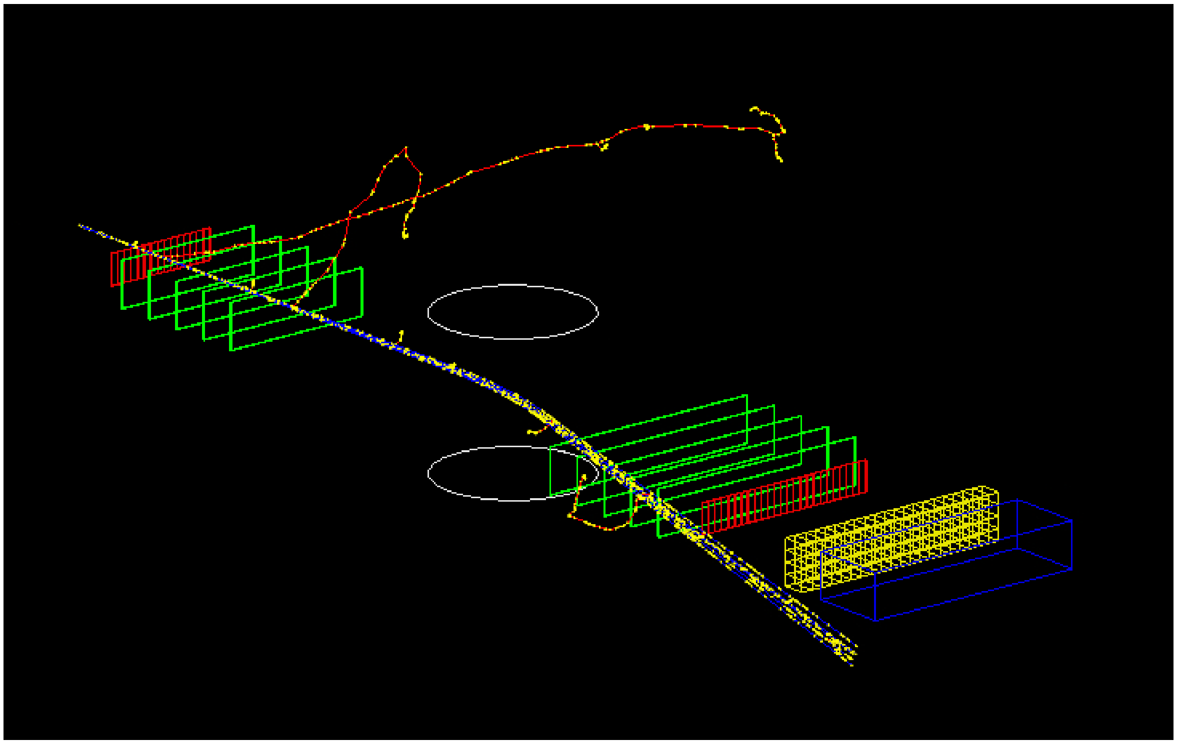


Figure 1: Screenshot of visualiser for 5 1 GeV anti-muons, to show pronounced curve which won't neccesarily be obvious for higher energy muons.

where $q = +e$ (anti-muons), θ the angle between B and v and $\vec{n}$ the unit vector perpendicular to both B and v. Hence, with centripetal acceleration

$$\frac{d\vec{v}}{dt} = \frac{v^2}{r} \tag{3}$$

where r is the radius of curvature, a simple equation for momentum, with gamma factor cancelled out, is derived:

$$\vec{p} = eBr\sin(\theta)\vec{n}. \tag{4}$$

Figure 1 shows an example where the curvature is more well pronounced hence the field can be determined to be in the $+y$ direction, with the muons fired in the $+z$ direction and curving towards the $+x$ direction. Hence, the magnetic field is always perpendicular to the velocity of the muon so equation 4 reduces to

$$p = eBr \tag{5}$$

where the vector notation has been dropped for now. Radius is difficult to measure and would require a parabola fit, possible but there will be very little curvature at the high energies shown. Instead s, the sagitta, will be measured. For an arc with small curvature ($r >> L$, where L is the width of the arc) the momentum equation becomes

$$p = \frac{eL^2B}{8s}. \tag{6}$$

This is a useful equation for track reconstruction when there are hits within the magnetic field, but for this case there are no hits in the magnetic field and we must calculate the momentum from the deviation of the track. Figure X is annotated to show the geometry of the detector and the diagrams required to calculate the radius of curvature for the muon allowing equation 5 to be used to extract the momentum. By linear fitting the beams before and after the deviation we obtain line gradients and intercepts m_1,m_2,c_1 and c_2 where index 1 refers to before the deviation (negative z values and index 2 to after the deviation. The two gradients allow an angle of interception θ between the two lines to be calculated;

$$\tan(\theta) = \frac{m_2 - m_1}{1 + m_2 m_1}. \tag{7}$$

The centre of the magnetic field is set as the coordinate $(0, 0)$ for simplicity in deriving the circle equation which details the position of the magnetic field $x^2 + z^2 = (r^B)^2$, where r^B is the radius of the magnetic field derived exactly in GEANT4. Knowing the circle equation and 2 line equations, (x_1^B, z_1^B) and (x_2^B, z_2^B), the intercepts between the lines and magnetic field, can be calculated and hence $L = \sqrt{(x_1^B - x_2^B)^2 + (z_1^B - z_2^B)^2}$ is obtained. Using circle geometry L and θ, can be combined to calculate the radius of curvature for the muon path as

$$r = \frac{L\sqrt{2}}{2}\left[1 - \left(1 + \tan^2(\theta)\right)^{-\frac{1}{2}}\right]^{-\frac{1}{2}}, \tag{8}$$

and hence p can be extracted. Examining the complexity of this equation and the L and $\tan(\theta)$ equations required to calculate it, I will not show the error propagation in full. But the calculation is entirely derived from m_1,m_2,c_1 and c_2 which are in turn fitted from the initial data points. I will not need this analytical error on momentum as I should be able to derive a standard deviation from the gaussian produced from the momentum values. However I do need to understand how the error will scale with momentum. Taking equation 5, the momentum error is proportional to error the on the measurement of r, specifically

$$\sigma_p = \frac{p}{r}\sigma_r. \tag{9}$$

Propagation of σ_r finds it is proportional to r and $f(\theta, L)$ hence the overall momentum resolution should scale with p as the ignored $f(\theta, L)$ is derived from the error on the linear fit parameters which in turn are only affected by the x and z resolutions

Hence for question (2), as the momentum is fixed, the error will not change (an increase in B will reduce the radius of curvature) and for (3) the answers will be 0.5x and 2x the resolution calculated for the data-taking case. For (4), perfect knowledge of the x momentum, a perfect linear fit will be made resulting in the linear fit parameters having 0 error and hence the resolution on momentum being 0. This would equate to a Dirac delta distribution for momentum, with the muons exactly 100 GeV.

Due to the high velocity of the particles passing through the magnetic field, the curvature will be very small. Hence the angle that the centre of the curve makes with the intersections with the magnetic field is small enough that $\sin(\phi) = \phi$. Also, with a square, the deviation angle is so small that you can assume that $L = S$, the side length of the square. Hence equation 5 reduces to

$$p = \frac{0.3qBS}{\phi}, \tag{10}$$

considerably simpler to calculate, where p is in GeV and q is the number of e.

1.4 Plan

Will output the root file to NumPy arrays using overleaf, use the z values/the geant4 code to understand which layers are before and after the mag field. 0.5m between layers, start of each chambers are 10m apart (at -5 and +5). Remember x values are in mm! going to use fitting randomiser to find best chi2 for cases with back-scattering. will check the y values to check that it is flat as it should be!

(1) Extract data from drift chambers 1 & 2 into NumPy arrays using uproot.

(2) Find the geometry of the detector in the geant code, use this to create an equation for the magnetic field and positions for each DC (a map for z to position). Remember: centre of mag field is (0,0), x,y in mm and z chambers are about 0.5m apart with 10m between the first chamber in each arm.

(3) Make a linear fit to 5 hits in each arm, one from each chamber. If there are multiple hits per chamber, fit each permutation and keep min χ^2. Remember to use uncertainty in fit and also to fail/report on an diverged fit! Multiple hits are from back scattering. Maybe only keep events with reduced chi2 between 0.5 and 1.5, see how many that deletes, can check affect on result.

(4) Work out whether y should be considered by calculating a χ^2 for each event to the straight line $y = 0$ (in y-z plane) and potentially discarding events that are bad if this is a minority or investigating why if not. Of fitted points will calculate a χ^2 check about y=0.

(5) With this best fit can get the intersects with the magnetic field of the two beams (x_1^B, z_1^B) and (x_2^B, z_2^B) which allows a calculation of L.

(6) Using the gradients to get θ and some geometry, the radius and hence momentum can be calculated.

(7) If I find a package to propagate the uncertainty, use that and store results! But either way plot the Gaussian and get uncertainty from that!

Note: If make a function that returns the intersects with the magnetic field, the magnetic field geometry can be changed easily, will have to deal with choosing the correct intersects as more than one will be available! That might need manual work!

1.5 Schematics

From the source file "B5DectectorConstruction" the schematics of the detector can be extracted. Without precision information these schematics are assumed to be perfectly measured (unrealistic for a real detector which would, for example, use magnetic probes to measure the geometry of the magnetic field produced). All coordinates/sizes quoted in (x,y,z) order and probably in m unless specified.

(a) Cavern: 20x6x20 m of air.

(b) Magnetic Field: Cylinder of radius r_B=1m and height 1m with centre at (0,0) in x,z. Orientated in the positive y direction (shown by direction of curve in the negative x direction in figure 1).

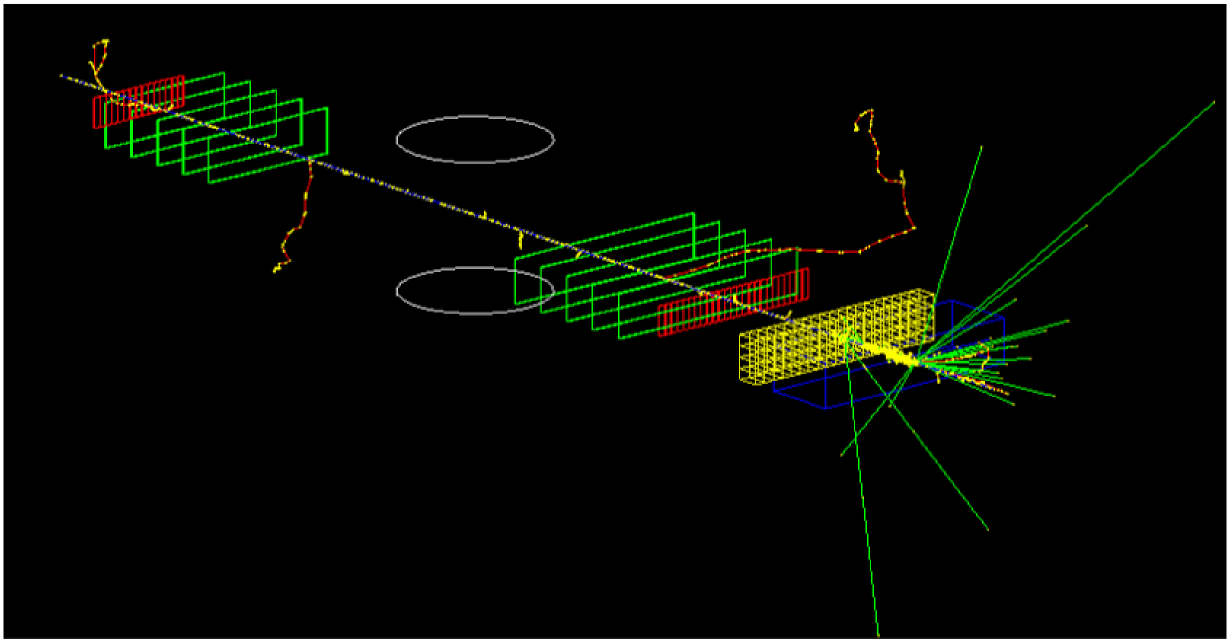


Figure 2: Screenshot of visualiser for 5 100 GeV anti-muons.

(c) First Arm: Placed at (0,0,-5) on z axis, with geometry 3x2x6

(d) Second Arm: Placed at (0,0,5) on z axis (for arm angle of 0), with geometry 4x4x7. I think the difference doesn't matter, it all depends where the drift chambers are. Not sure whether the placement is centre or front of arm! I think centre.

(e) First arm drift chambers: all 2x0.6x0.02, within there is a virtual wire at centre which is 2x0.6x0.0002. The drift chambers are placed at (0,0,(i-2.5)x0.5) in the arm where i runs 0-4. So -1.25, -0.75, -0.25, 0.25, 0.75 relative to the centre of the arm (0,0,-5).

(f) Second Arm Drift Chambers: Very similar setup however placed at (0,0,(i-2.5)*0.5-1.5) so above values -1.5 relative to centre of second arm at (0,0,5). This does seem correct, looking at figures 1 and 2, with the last DC in the first arm being at -4.25 from centre of mag field and the start of the second arm at +2.25. This can be seen more clearly in the top down schematic 3.

02/02: Update, the magnetic field shape has changed to a cube of side length 2m centred on (0,0,0). I will continue with the circle but add an option to switch to square. The only difference is the intercepts! New schematic shown in figure 4.

1.6 Plan Update

Since making the plan, during development of the code, the following changes have been made.

(a) Realised the x,y measurements are in mm, recursively updated the doc as this is important.

(b) Discard events with >30 hits in one of the drift chambers. This could lead to >7776 possibilities of straight lines, difficult to be sure you have correct trajectory, will check efficiency hit this brings.

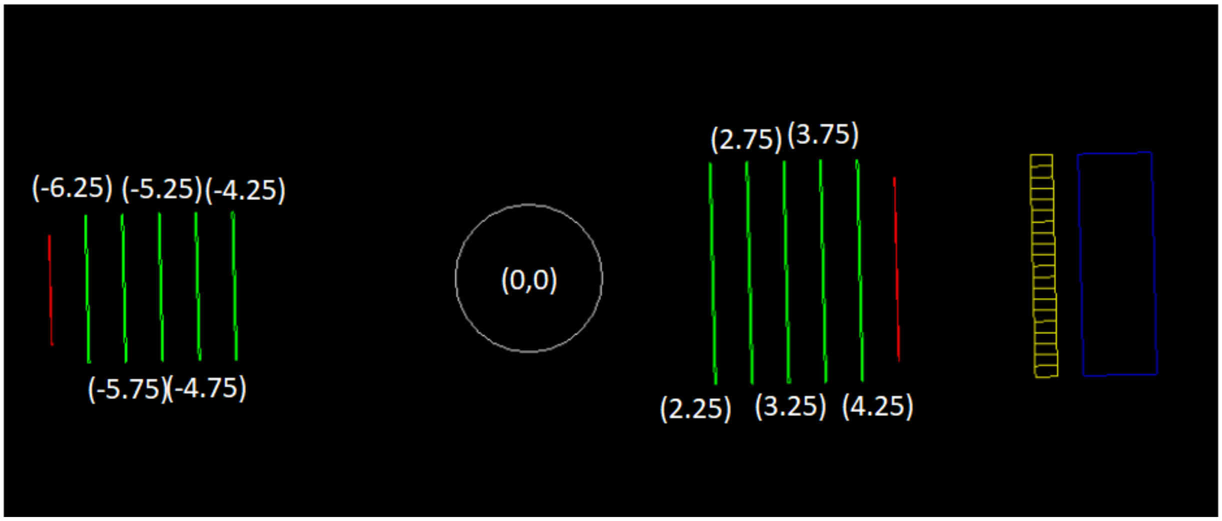


Figure 3: Top Down view of detector schematic in geant4 visualiser. With annotated measurements in m showing the z positions of the drift chambers. The central circle is the magnetic field tube

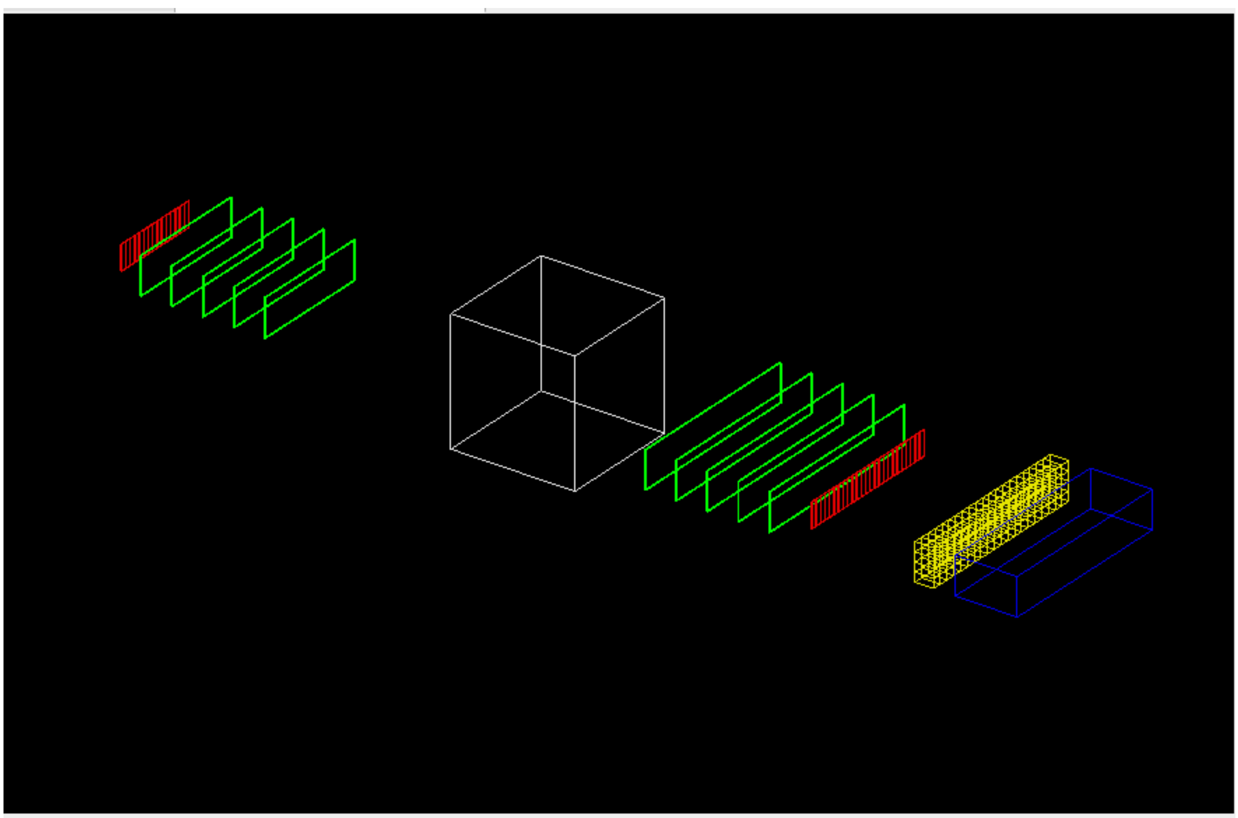
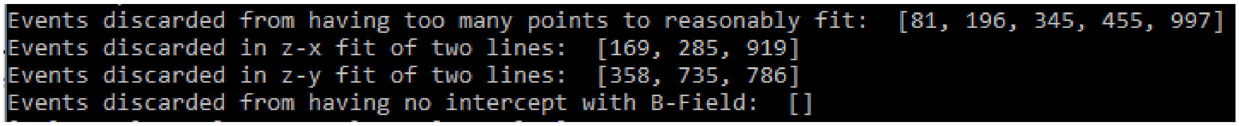


Figure 4: Schematic view of detector with square field.



```
Events discarded from having too many points to reasonably fit:  [81, 196, 345, 455, 997]
Events discarded in z-x fit of two lines:  [169, 285, 919]
Events discarded in z-y fit of two lines:  [358, 735, 786]
Events discarded from having no intercept with B-Field:  []
```

Figure 5: Screenshot of ipython console showing discarded events.

(c) When fitting the two straight lines, fit as $x = mz + c$ (and invert at the end). Split the hits into those in different sections of each drift chamber and cycle through combinations of one hit from each section. If any section has no hits (e.g 5 hits, one doubled up, one empty) discard event. Again will study efficiency hit. Do minimum χ^2 fit to choose which fit is best.

(d) Check reduced χ^2 of these fits, if they are greater than 1.5, discard. Wanted to make lower cut too, but large uncertainties on x (compared to deviations) mean all of the reduced χ^2 are very small(order 10^{-6})! (maybe we should've looked at the width of the wire as the z uncertainty!)

(e) For y=0 fit, discard events with reduced $\chi^2 < 1$. This removes some bad events but due to large y uncertainty again the reduced χ^2 are extremely small (order 10^{-5}). Do not feel confident cutting on reduced χ^2 less than 1. Need to understand why this is.

(f) Intercept calculation return "none" if there is no intercept or the intercepts are in entirely the wrong place geometrically. Some errors could fall through but should never have collided with drift chambers so shouldn't even be possible

(g) Calculate momentum store, plot and fit a normal distribution.

1.7 Results

Choosing the settings (choose initial momentum, B-field and magnetic field shape) and running the analysis produces a histogram of the momentum with a gaussian fit over it. The gaussian is fit around the central peak. I worked with a few intervals but decided on: $p_{init} \pm 3 * p_{init}/100$ as this seemed to fit the gaussian well (minimised χ^2) and not try and over-fit any fluctuation beyond the peak.

For efficiency, figure 5 shows that, for the example of 100 GeV and square mag field, an efficiency of 99%. Making plots of these discarded events often show deviations from y=0 that have led to a bad fit in x-z.

Figures 6,7,8,9,10, show a series of histograms with gaussian fits to different detector scenarios. The mean, standard deviation and reduced χ^2 from the fits are displayed on each plot. The χ^2 imply that these are good fits, but the circle has produced a considerably better fit. Work on the fitting interval could potentially improve these. The means are all correct to the initial momentum which implies the method used is plausible.

Figure 6 shows the variation that is possible from this method, with fluctuations ranging from <5 GeV to nearly 200 GeV. These seem to be due to bad y-fits but I cannot remove them without making harsh cuts on χ^2_y that damage the quality of the Gaussian. More analysis on these later.

Comparing figures 7, 9 and 10 shows that, as mentioned in section 1.3, σ_p scales with momentum as the 50 GeV case has σ_p half that of 100 GeV and a quarter of 200 GeV. I need to make a

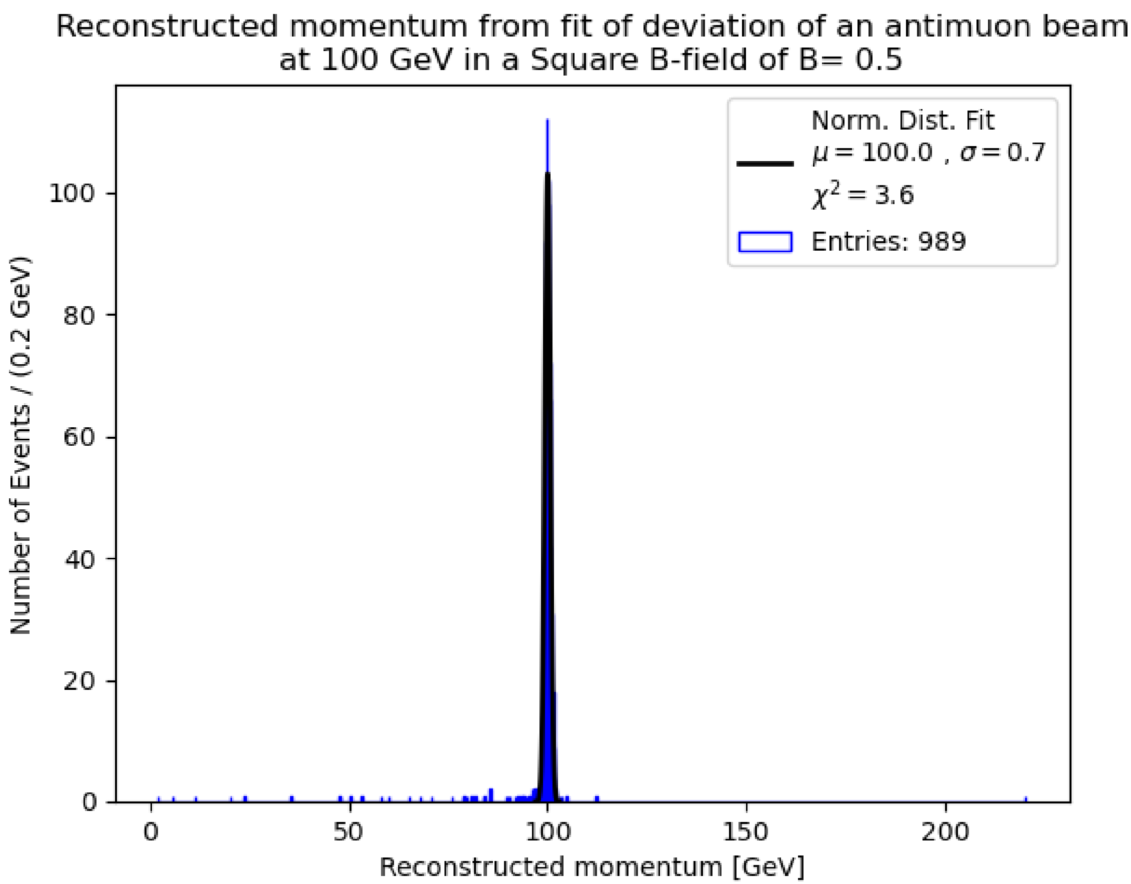


Figure 6: Histogram with Gaussian fit for a square mag field and $p_{init} = 100\text{GeV}$. The plot has not been zoomed to show fluctuations throughout p.

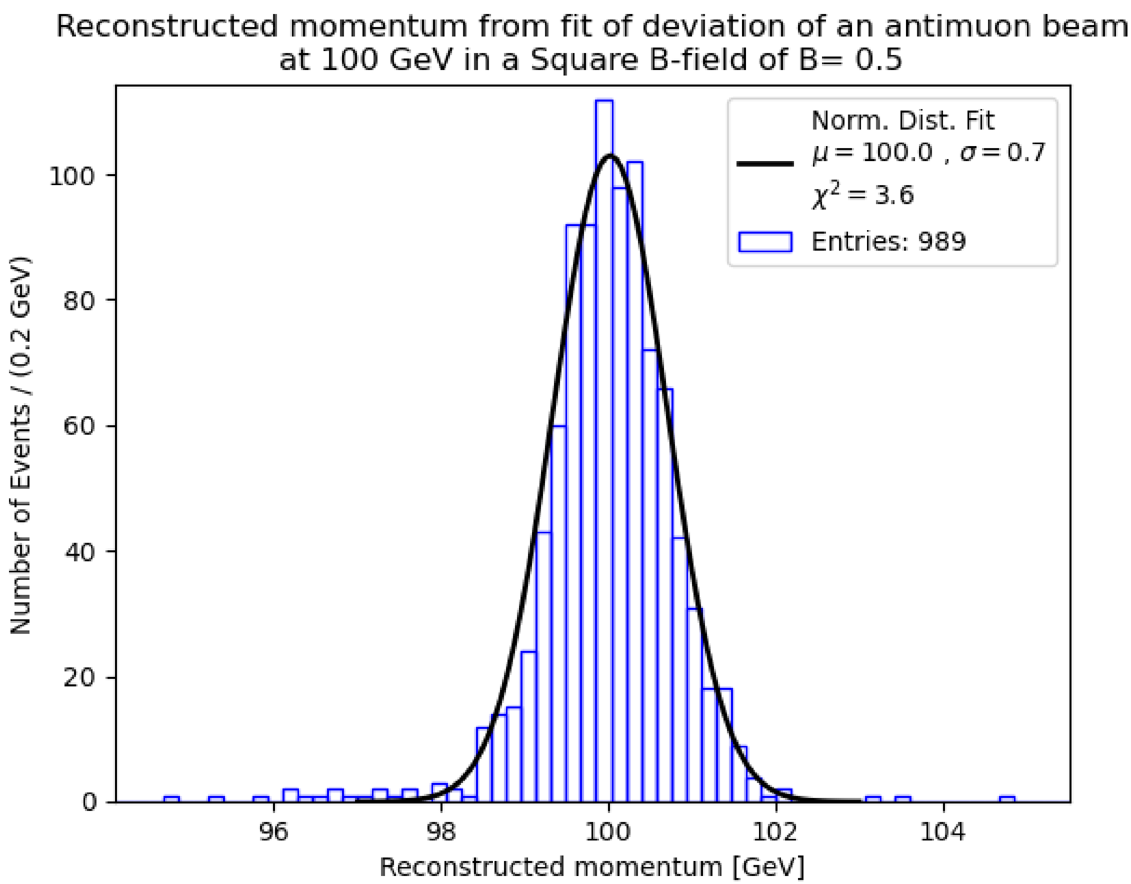


Figure 7: Histogram with Gaussian fit for a square mag field and p_{init} = 100GeV. The plot has been zoomed to show central fitted region.

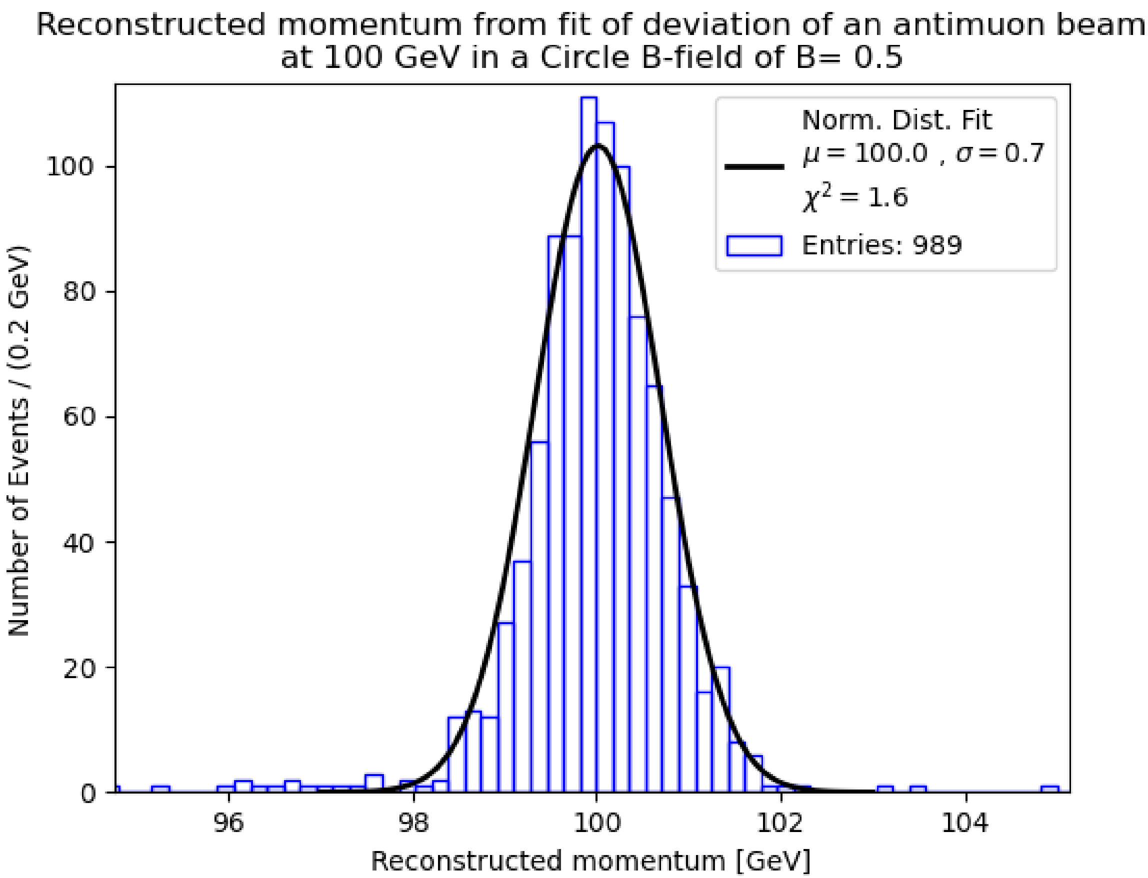


Figure 8: Histogram with Gaussian fit for a circular mag field and p_{init} = 100GeV. The plot has been zoomed to show central fitted region.

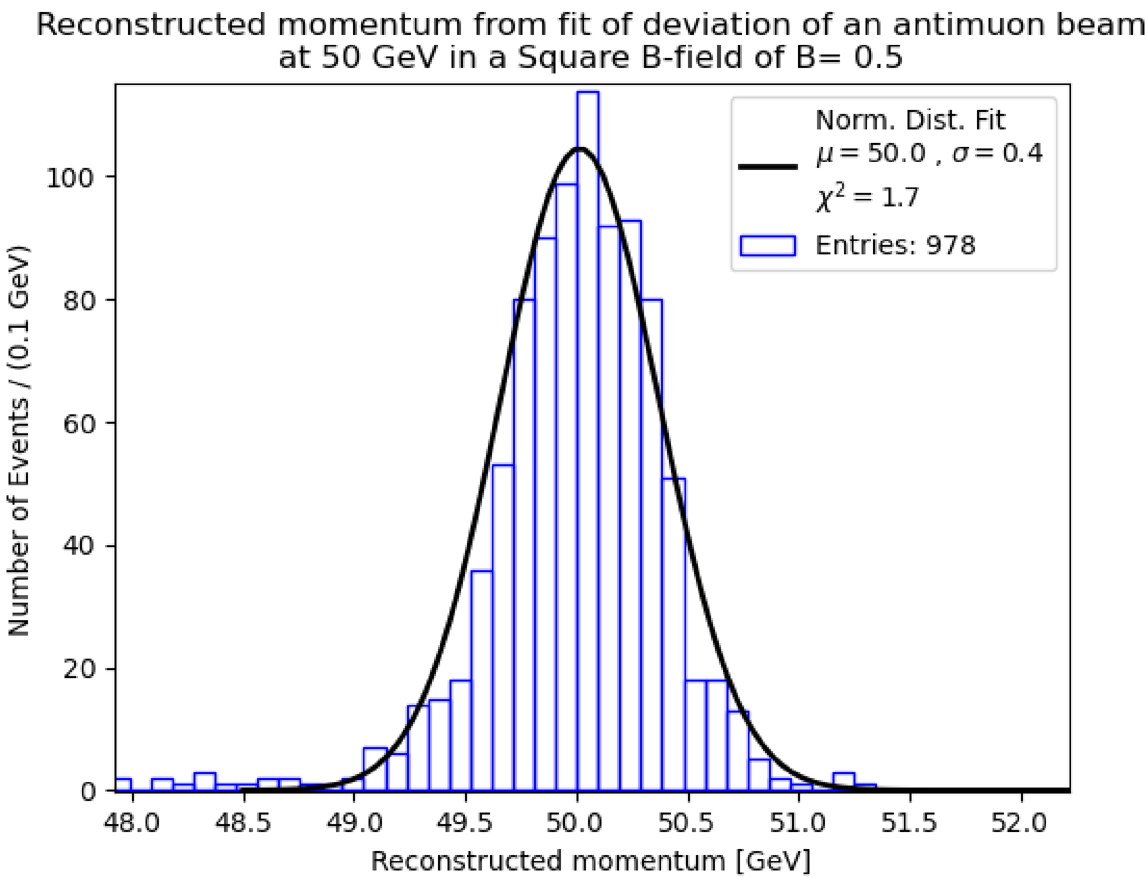


Figure 9: Histogram with Gaussian fit for a square mag field and p_{init} = 50GeV. The plot has been zoomed to show central fitted region.

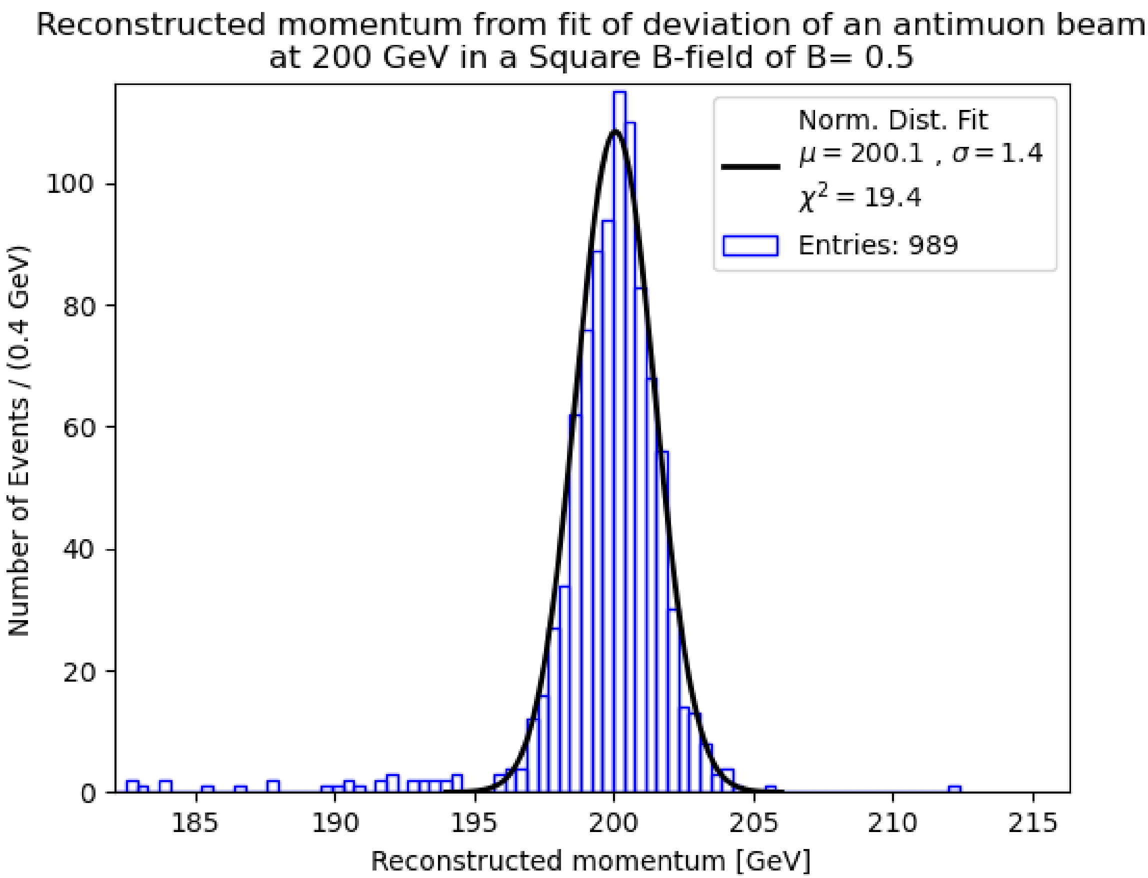


Figure 10: Histogram with Gaussian fit for a square mag field and $p_{init} = 200$GeV. The plot has been zoomed to show central fitted region.

varying mag field sample to test the B-field hypothesis. I cannot reasonably test the zero error scenario without entering geant4 and understanding where and how the variation is introduced.

I have not propagated errors through yet, but that is quite easily done. I am not sure how this affects the fitting of the Gaussian or how you even calculate errors on bins! I need to research this. Update: of course, Poisson distributed sampling, error just the sqrt of the number in the bin!

To do for task 1:

1. Run with different mag fields with antimuon to confirm theory about how resolution doesn't scale with mag field
2. Make table of values
3. Add some lead to the detector and see how that effects results.
4. Try and smear the incoming positions.
5. Maybe remake samples without randomised which was making other particles other than just antimuons.
6. Apply weights to fit, maybe try using iminuit rather than just scipy norm fit.
7. Full fit.

Before beginning exercise 2, realised that randomPrimary was set to true in the default settings for GEANT. This meant that, despite specifying the particle gun requested, the actual particle produced was randomised between muons, protons, pions and electrons. This likely lead to the large spread in momentum measurements shown on the plots as electrons could lose energy through Bremstrahlung and protons/pions are more likely to undergo multiple scattering in the trackers. If it was a purely muonic beam, these effects would've been reduced. I will correct the code to remove the randomiser and use the new settings for the magfield test with 0.25, 0.5, 1 Tesla.

Figures 11, 13 & 12 show the distributions for the fit when changing the magnetic field strength. Expect greater B field to lead to smaller radius of curvature and hence no change in momentum, this is shown in the plots. We expected changing B field to not change the resolution but our gaussian fits imply that, as you increase the b-field the resolution gets smaller almost linearly with 0.25T to 1T quartering the resolution. This is against what we expect.

However, for a greater B field, the muons will follow a more defined curve which will lead to a more accurate measurement of the angle and hence the radius of curvature. Potentially the observed increase in precision for higher B field is due to this. Looking again at equation 9, with σ_r dependent on r and function in theta and L, it makes sense that improving the precision on the theta measurement (as a result of greater B field) will increase the precision on the radius and hence momentum measurements.

Another note-able point from the plots is the lack of quality of the gaussian fit for these distributions, visually they fit considerably worse than the other plots. It must be noted that these new plots are with random primary set to false and you can see by comparing figures 11 and 6,7 the difference in shape of the distribution. Another point is that the lack of randomised particles has removed the large anomalies at low and high momenta. This makes sense as muons scatter less than pions or electrons.

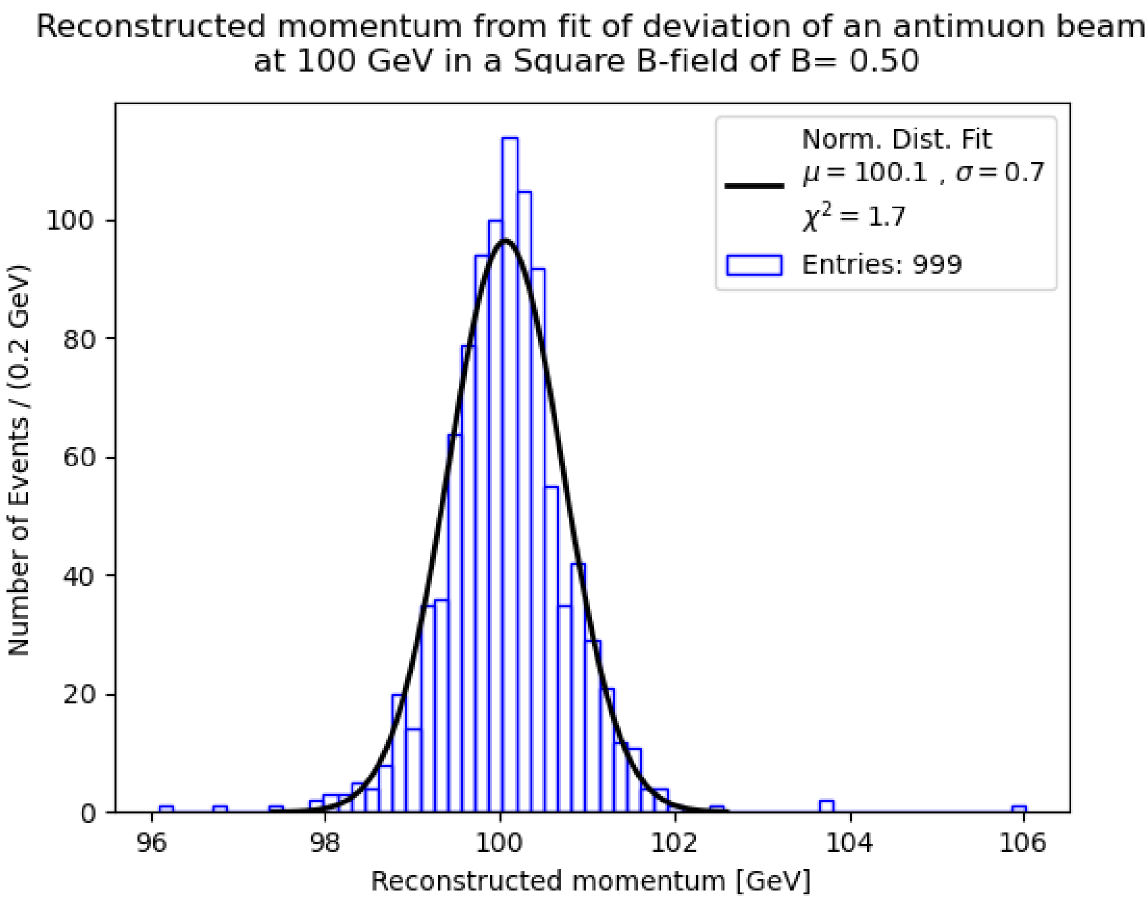


Figure 11: Histogram with Gaussian fit for a square mag field of 0.5T and p_{init} = 100GeV. The randomise primary particle has been removed, removing the random effects at high and low p.

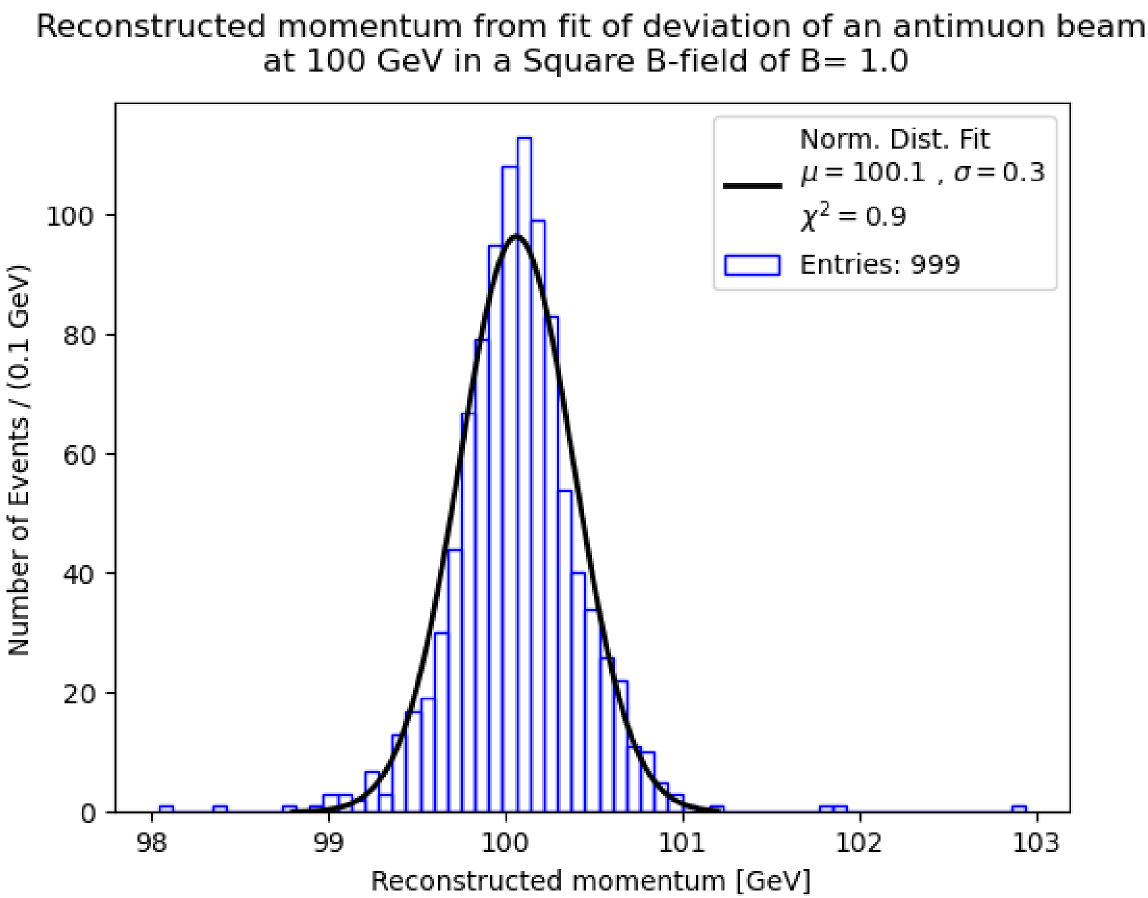


Figure 12: Histogram with Gaussian fit for a square mag field of 1T and p_{init} = 100GeV. The randomise primary particle has been removed, removing the random effects at high and low p.

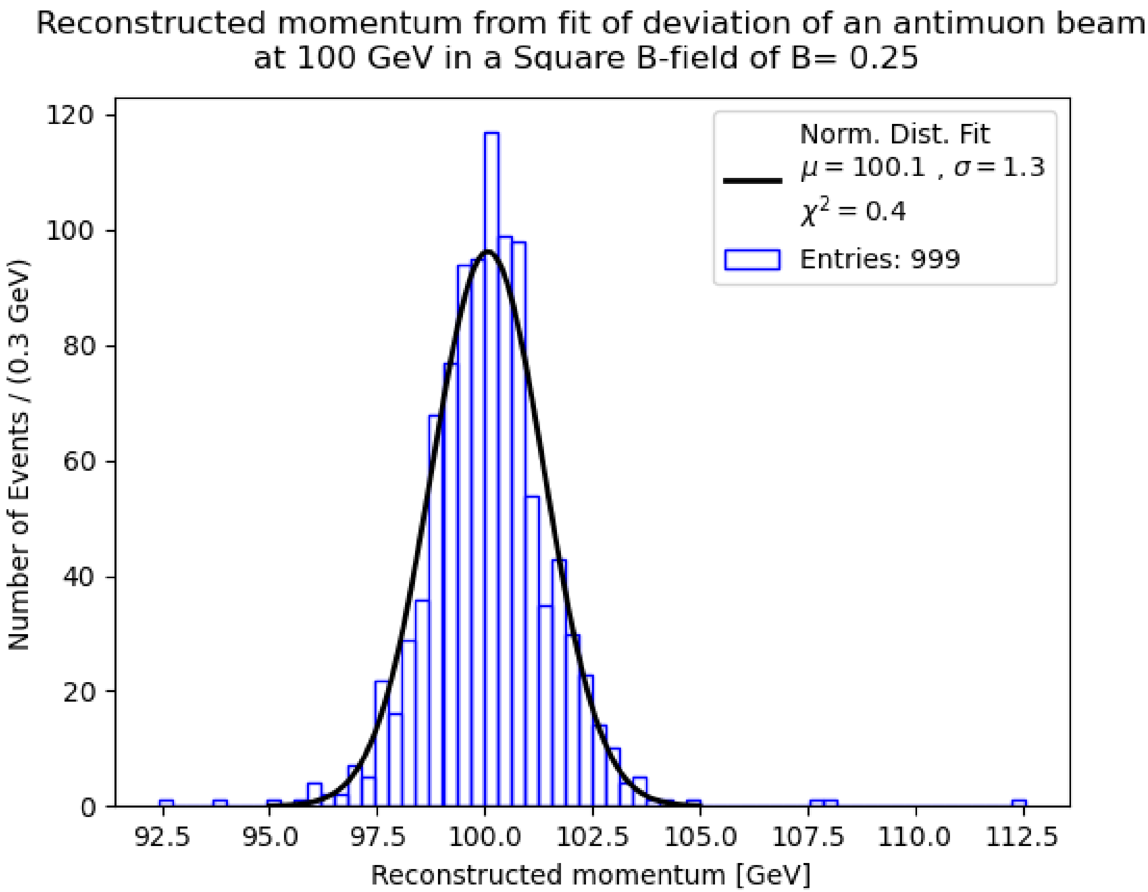


Figure 13: Histogram with Gaussian fit for a square mag field of 0.25T and $p_{init} = 100$GeV. The randomise primary particle has been removed, removing the random effects at high and low p.

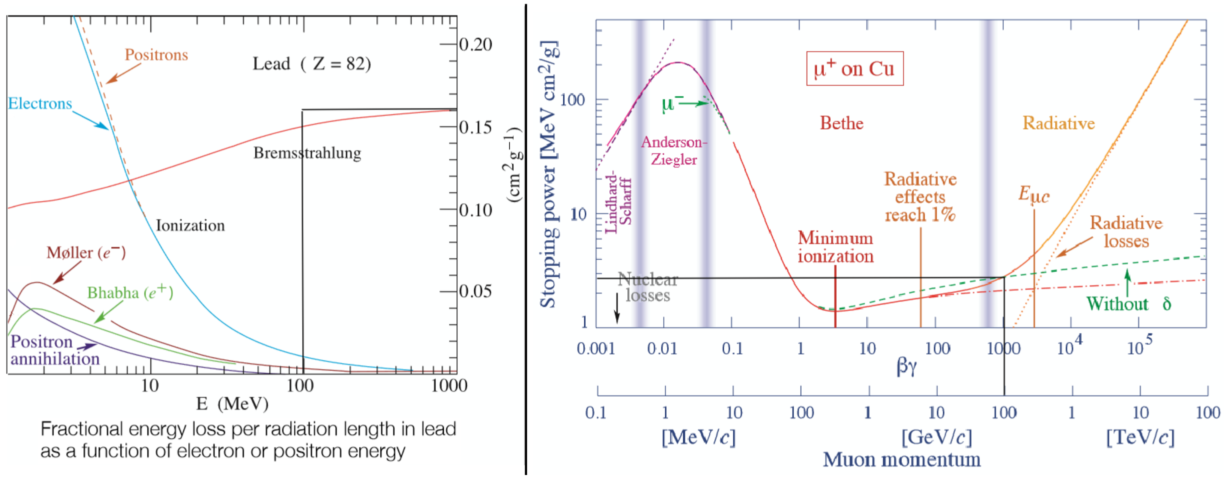


Figure 14: Annotated plots comparing the stopping power for electrons (left) and muons (right). The black lines show the stopping power at 100 GeV

1.8 Lead Blocks

Brief: Add 5cm of lead immediately before the mag field and measure the momentum. Repeat with lead immediately after mag field. 5cm of lead corresponds to a G4Box of size trackerX, trackerY, 2.5cm to ensure same size as the trackers and hence that all muons pass through the lead block. Place 0.5m beyond/before the trackers to ensure no overlap.

1.8.1 Expectations/Plan

I expect the momentum resolution to be worse for both scenarios with the lead block as more multiple scattering will occur and a general smearing added. With the lead block before the magfield, I expect the overall momentum measurement to be reduced as the lead should absorb some energy. For the lead block after the magnetic field, the curvature (and hence the process which measures the momentum) has already occurred so the central momentum value should be the same.

The radiation length of lead is 6.3 gcm^{-2}, 0.56cm with a density of 11.35gcm^{-3} (this rad length is also in output of geant4). So expect electrons in the lead blocks to lose 99.995% of their initial energy in the lead. For muons, this is not the case. Comparing diagrams in figure 14, the stopping power for muons at 100 GeV (in copper) is a factor of roughly 2000 bigger. Scaling up the radiation length accordingly, expect the muons to lose about 0.5% of their energy in the lead block so potentially (for the scenario with a lead block before the mag field) a momentum distribution shifted by a small amount lower in energy. Not convinced by this hypothesis, in plots, one is for muons on copper, significantly different material to lead. Looking further into papers about muonic interaction with matter, the minimum stopping power for muons in lead is 1.122 MeVcm^{2}g^{-1} and critical energy 134 GeV. This means, with 100 GeV muons, we are around where contributions from bremstrhalung and ionisation are roughly equal. But for 5cm of lead, with the large range of muons (order 7000-8000 cm) don't expect any significant loss in energy. (different source states muon range 4000cm [source])

I do expect a spreading out of the distribution, a far worse resolution due to multiple scattering.

The code will see large deviations due to the blocks as different momentum values as the only input data is the positions of the hits in the drift chambers. Due to equation 7 being squared in the final calculation, the direction of the deviation is ignored and large deviations will be seen as lower momentum muons, so expect more contributions at lower energies. In theory, scattering could lead to ”less deviation” and hence a higher momentum measured. Also, remember scattering happens in all directions and planes and a large deviation in y could be seen as a small deviation in x, the code ignores y deviations! Calculating θ_0 for lead, produces 4.4×10^{-4}, implying, on average, very minimal deviation due to very high momentum muons. Overall, expect a low resolution distribution centering just less than 100 GeV. Not sure about how either side will tail.

1.8.2 Method/Detector Setup

Should be able to simply add a lead block following method similar for trackers and calorimeters but without the need to add sensitive sections. Lead is already defined as a material also for the HCAL.

Listing 1: GEANT4 Code to implement lead blocks

```
//lead block in first arm
auto leadBlock1Solid= new G4Box("leadBlock1Box",1.*m,30.*cm,2.5*cm);
auto leadBlock1Logical =
new G4LogicalVolume(leadBlock1Solid,lead,"leadBlock1Logical");
new G4PVPlacement(0,G4ThreeVector(0.,0.,1.25*m),leadBlock1Logical,
"leadBlock1Physical",
  firstArmLogical,false,0,checkOverlaps)
//no rotation, placed at -3.75 compared to world so +1.25 to first arm,
lead block is being placed, inside first arm,only one of them, unique


//lead block in second arm
auto leadBlock2Solid= new G4Box("leadBlock2Box",1.5*m,30.*cm,2.5*cm);
auto leadBlock2Logical =
new G4LogicalVolume(leadBlock2Solid,lead,"leadBlock2Logical");
new G4PVPlacement(0,G4ThreeVector(0.,0.,-3.25*m),leadBlock2Logical,
"leadBlock2Physical",
  secondArmLogical,false,0,checkOverlaps)
//no rotation, placed at +1.75 compared to world so -3.25 to second arm,
lead block is being placed, inside second arm,only one of them, unique
```

Figure 15 and listing 1 shows how the lead blocks are placed in the detector. Now create and study effect this has on beam of muons.

1.8.3 Results for Lead Block

Figures 16 and 17 show the results of the lead studies. Immediately noticeable is that the momentum distributions are no longer gaussian, they look similar to a gaussian with an exponential tail on one side. Initially a landau distribution, (modelled by a moyal random variable) was considered, but as the plots show this doesn’t produce a great fit. A one sided crystal ball was also trialled. Both show a mean less than 100 GeV which is expected but the exponential tail was intially confusing. We expect this kind of landau distribution when measuring energy loss in a calorimeter for example, as a small number of individual collisions can lead to huge transfer of energy which decay in likelihood according to an exponential tail. But this method is measuring

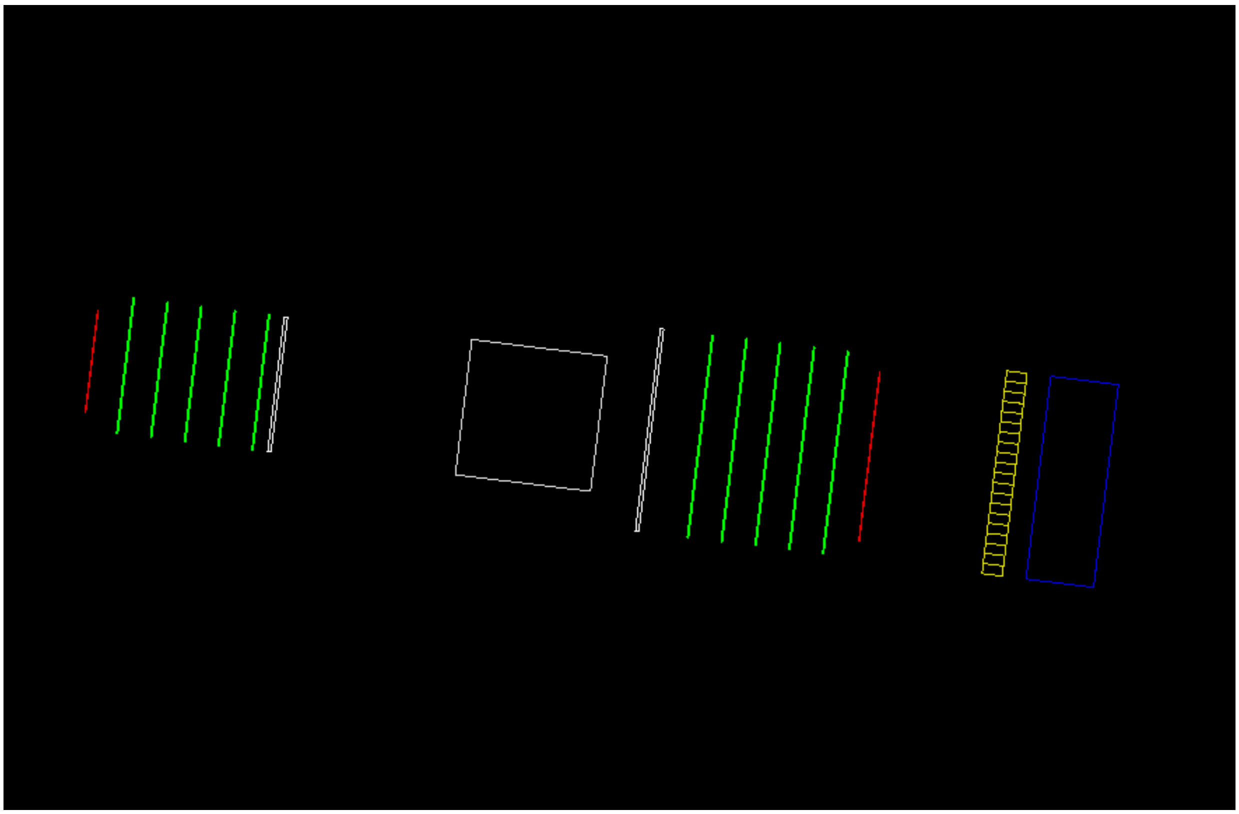


Figure 15: Schematic of both lead blocks in detector (in white). Will remove one when creating samples. Expected blocks to be as far away from gaps in drift chambers but one seems further and the other nearer... Will still work as planned.

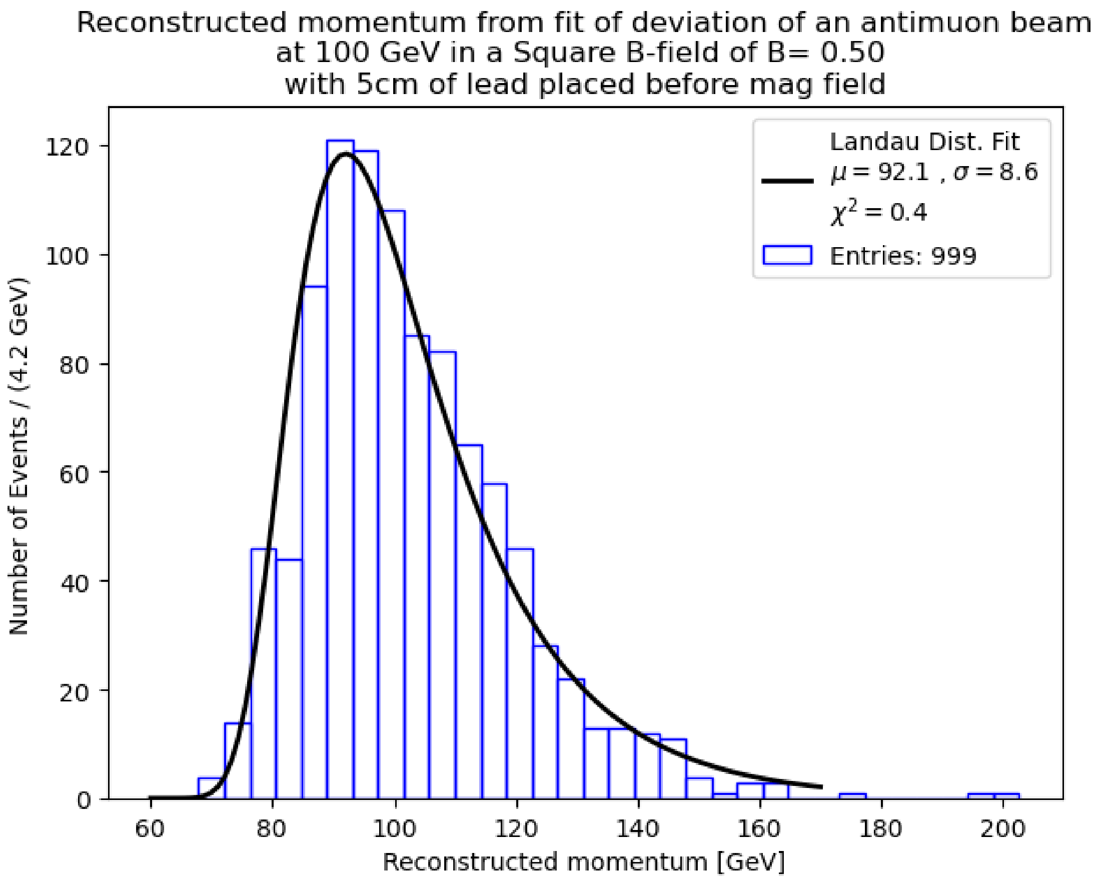


Figure 16: Histogram with landau/moyal fit for a square mag field of 0.5T and $p_{init} = 100$GeV with a lead block of 5cm placed immediately before the magnetic field (0.5m beyond the last drift chamber in arm 1).

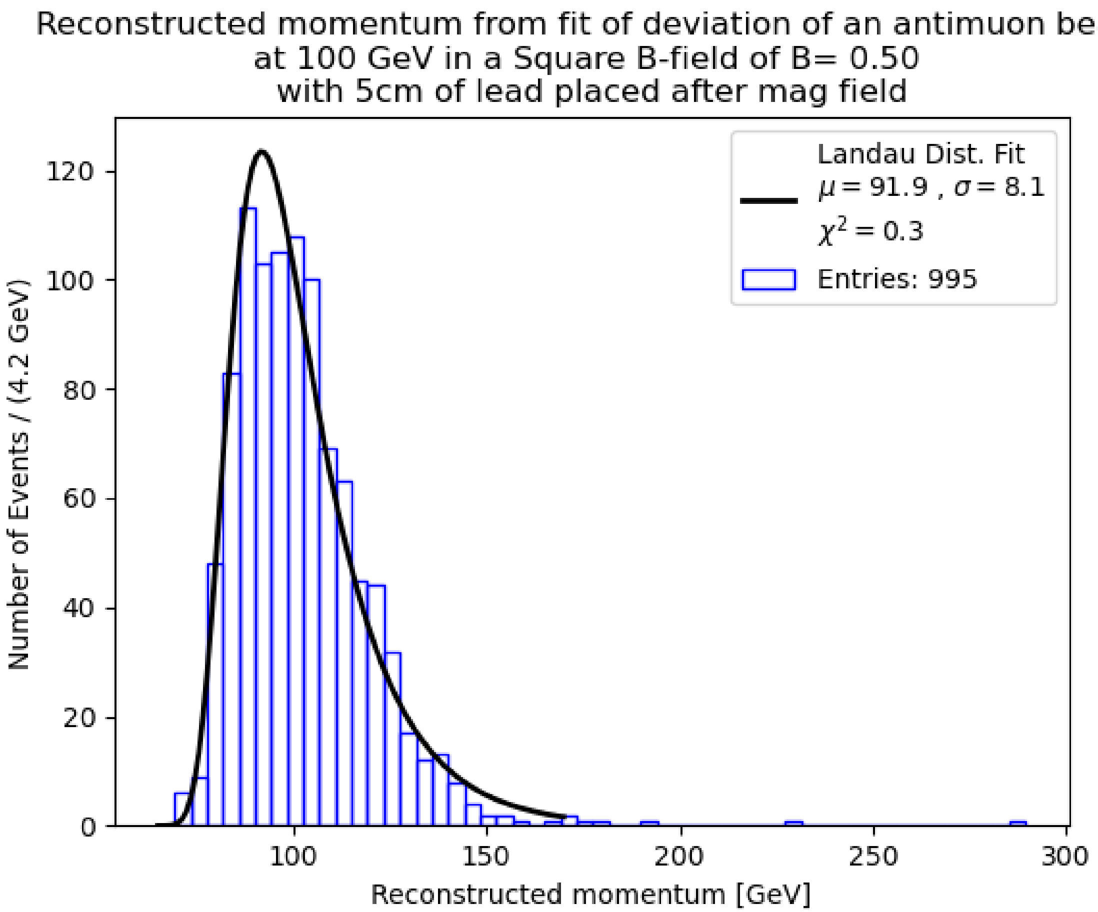


Figure 17: Histogram with landau/moyal fit for a square mag field of 0.5T and $p_{init} = 100$GeV with a lead block of 5cm placed immediately after the magnetic field (0.5m before the first drift chamber in arm 2).

the actual momentum non-destructively, not the energy loss and both leads block examples have this exponential tail. This implies that it is a result of scattering in the lead blocks which is faking the real curvature from the magnetic field itself. High momentum results are from those with less curvature than expected (where expected is the result without a lead block). The fact that the two distributions are very similar is puzzling as we should only be able to "measure" the effect for the lead block before the magnetic field as this change in momentum from ionisation will actually be measured. For the lead block after, the ionization should just lead to scattering and smeared results. **IDEA** what if it is not a gaussian with a long upper exponential tail, but a very wide gaussian with an exponential tail before the peak that has been subtracted due to the landau distribution of energy loss. That would lead to an inverted version of what we had... and also would only make sense for the case with lead before B-field.

The other idea is; the scattering is happening in all directions and the momentum estimated only looks at the x-z plane, hence when projected you will see large deviations in the y direction as small deviations in the x direction and vice versa. This, coupled with the fact that any deflection going in the "wrong" direction will be "reflected" along the z axis in the calculation, leads to a large spread in the momentum results. The exponential tail is likely due to the fact that, as the deviation angle tends to zero, which it will for certain random scatters, the momentum measurement will tend to infinity. Looking at eq 8, if theta is very small, tan(theta) approximates to theta, and by taylor expansion r reduces to $L/\sqrt{\theta}$. An inverse square root relation, which looks similar to an exponential tail on first glance. This further shows that the non-physical measurements are an artefact of the momentum measurement method used. Modelling a gaussian a-top an inverse square root could be interesting.

Going to plot an inverse square relation with random uniform angles ranging from -2 theta0 to +2theta0 either side the expected angle. I will then histogram the data in similar bins as shown above. Will subtract this "background" with the size normalized by a sideband between 125 and 150 GeV.

Figures 18 and 19 show the attempt to subtract a $1/\theta$ background due to deviations originating from multiple scattering. The subtraction seems to account for the exponential tail but, as expected it removes too many contributions from the lower momentum region due to the nature of inverse relationships. The next thought was "only remove background from angles smaller than expected deviation" but that doesn't seem correct as the multiple scattering should happen in all directions and be a spreading of the beam around the expected angle. Another option could've been to create a larger sample which would've created a better defined exponential tail allowing better defintion of what is truly "sideband" and hence can be used for normalising the background curve better. Looking at figure 19, I estimate a true mean of 96 GeV and a FWHM of 30 GeV. This is slightly lower mean as expected due to energy loss and a greater resolution but cannot be taken with any confidence as a gaussian could not be reasonably fit. The values for mean and uncertainty are more reliable from the landau distributions as this fit was reasonable. Overall a landau distribution working is understandable due to the gaussian lower side and "exponential" tail from multiple scattering. The lead before magnetic field had similar resulting plots, which is to be expected as, in the approximation in equation 10, the only contribution to momentum is the gradient of the lines that are fitted and, independent of where the scattering takes place, the resulting gradient would be the same for an identical scatter and transverse of the uniform magnetic field as the deviations from these processes are additive.

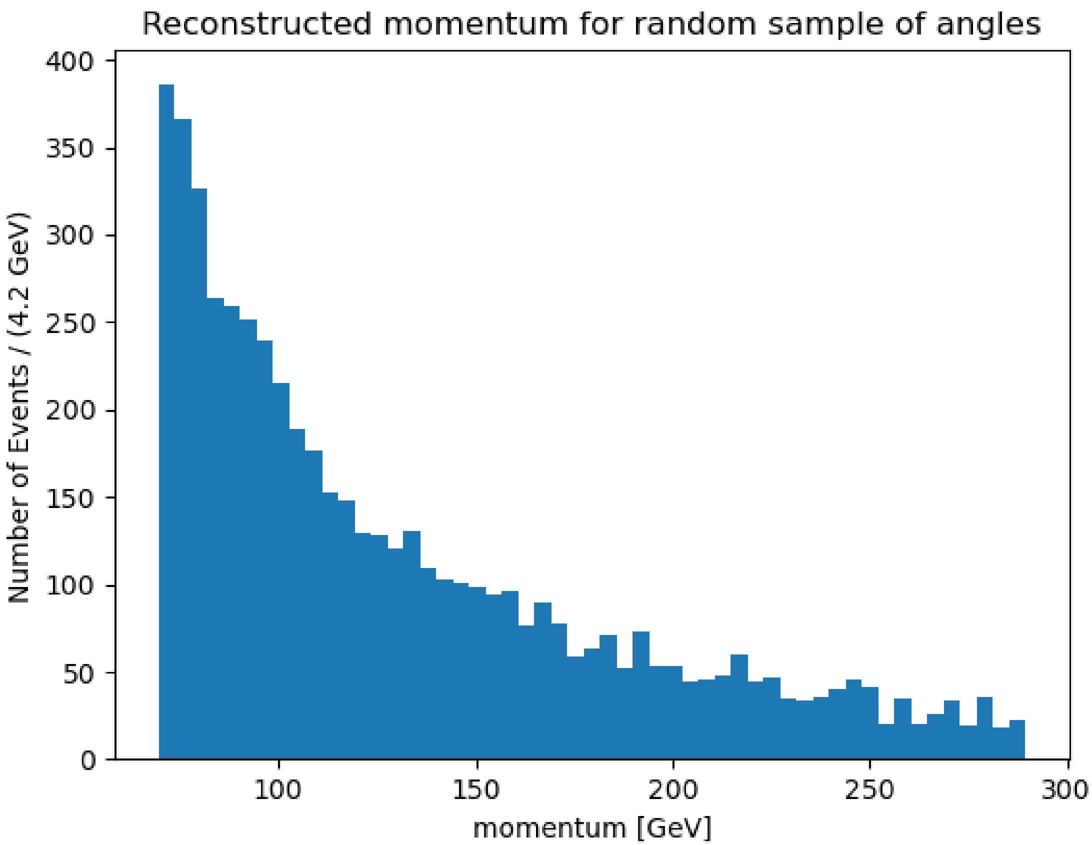


Figure 18: Histogram of a $1/\theta$ background from deviations originating from multiple scattering. Made with 10000 uniform random variables in theta and binned in the same bins as figure 17

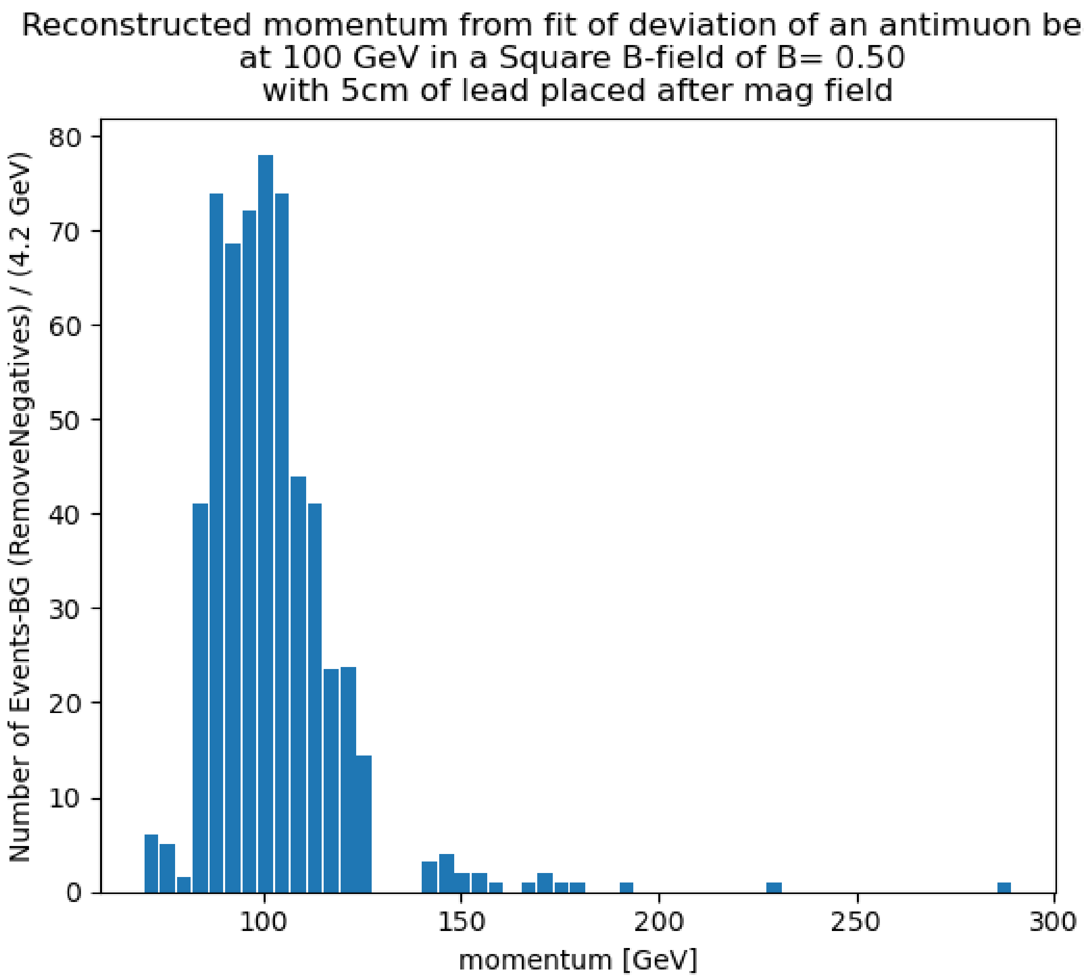


Figure 19: Histogram of figure 17 with the background from figure 18 subtracted in the region $\theta_{expex} \pm 2\theta_0$. No gaussian fit was attempted due to the not ideal shape.

Beam & Detector Settings (B [T], E [GeV])	Mean [GeV]	σ [GeV]	Reduced χ^2
Circle: (0.5, 100) (w/ Random)	100.0	0.7	3.6
(0.5, 100) (w/ Random)	100.0	0.7	1.6
(0.5, 200) (w/ Random)	200.1	1.4	19.4
(0.5, 50) (w/ Random)	50.0	0.4	1.7
(0.5, 100) (w/o Random)	100.1	0.7	1.7
(1, 100) (w/o Random)	100.1	0.3	0.9
(0.25, 100) (w/o Random)	100.1	1.3	0.4
Lead Before: (0.5, 100) (w/o Random)	92.1	8.6	0.4
Lead After: (0.5, 100) GeV (w/o Random)	91.9	8.1	0.3

Table 1: Table of means and resolution for the momentum reconstructed from 1000 antimuons passing through a square magnetic field. w/ random refers to the datasets where the primary particle was randomised. For the lead datasets, the values of mean and resolution are the mean and standard deviation of the landau distribution fit

2 Exercise 2 - Calorimeter

Code for this second assignment stored in same repository.

2.1 Brief

Hi, the second part of the investigation is the following:

(1) For a magnetic field of 0.5 T, an angle of the spectrometer of 0 degrees (i.e. the axes of the tow arms are on the same line), send 1000 antimuons (mu+) along z+ with a momentum of 100 GeV. Study the energy depositions in the Electromagnetic and Hadronic calorimeters.

(2) Repeat with positrons, but also try to reconstruct the positron energy using the calorimeter information.

(3) Repeat with protons, but also try to reconstruct the energy of the protons using the calorimeter information.

2.2 Breakdown

We want to study the EM shower and hadronic showers in the ECAL and HCAL and use our knowledge of processes to extract an energy for the electron and proton. We expect the muon to pass through, depositing very little energy. Maybe a small amount of ionisation in the HCAL and potentially showers in ECAL, but not enough to get a good estimation of energy.

My general idea for electrons: Measure moliere radius, get the critical energy. Measure t_{max}, use crit energy to get E_0. Maybe can just get the critical energy ϵ from a formula. Measuring Moilere radius requires average lateral deflection at one radiation length, possible but that will yield critical energy in the Rossi definiton, which is different to what the t_{max} equation estimates. Could use $t_{95\%}$ which is related to t_{max}, may be easier to estimate.

My plan is to just bin the calorimeter by z position, I will check dimensions, they may already be binned. Figure out calorimeter dimensions and material for help with calculation and then estimate energy. For muons just make a nice 3d plot.

Update, upon investigating ntuples, the energy information is already provided, no fancy conversion will be needed just a summing of energies of sorts!

2.3 Calorimeter Information

From studying the detector construction:

(a) Caesium iodide EM calorimeter, dimensions 3m x 0.6m x 0.3m placed at (0,0,2)m from centre of second arm logical volume. Therefore centred 7m from centre of magnetic field (global 0,0,0).

(b) Within ECAL, 15cmx15cmx30cm cells placed side by side. 80 cells. 20 along the x axis, 4 along the y axis. Therfore good x & y separation but do not know along the z axis where the hit is. This may affect our calculation and prevent us calculating depths. May need to do Moilere radius.

(c) For HCAL, 3m x0.6m x 1m centred 1m beyond the ECAL along z. Made of **lead**

(d) 20 cells, 10 columns split equally along x, 2 rows. So cells of 30cmx30cmx100cm. Within this cell are 20 layers of thickness 5cm along z. Therefore providing some depth information.

(e) Scintillators placed along each cell, 0.5cm thick placed at very end of cell, before exiting.

For ECAL, it is 3m by 0.6m by 0.3m, all of the positions are correct because these reference the centres of the objects, everything is just twice as thick. Each cell is 30cm thick in the ECAL. In the HCAL they are 1m thick with 5cm layers.

This makes sense why layers are 5cm apart etc.

2.4 Theory

To understand the results from the energy deposits in the calorimeters I need to understand the processes inside the calorimeters. In the ECAL, the important quantities are the radiation length X_0, critical energy ϵ, t_{max} and $t_{95\%}$. These are, respectively, the distance electrons travel in a given medium for its energy to reduce by factor $1/e$, the energy at which electron ionization losses and brem. losses are equal (the energy where the shower stops), the shower maximum (distance most secondary particles are produced in units X_0) and the calorimeter thickness where 95% of the shower energy is contained. For HCAL relevance, the nuclear interaction length, λ, analogous to X_0 is stated. These values are calculated using standard equations, and the different values for the corresponding materials in the calorimeters (CsI for ECAL & Pb for HCAL) are displayed in table 2.

2.5 Plan

1. Get information and make some plots.

Value	CsI (ECAL)	Pb (HCAL)
ρ [gcm^{-3}]	4.51	11.35
X_0 [cm]	1.86	0.5613
λ [cm]	39.306	18.248
ϵ [MeV]	11.04	7.33
t_{max} [X_0]	8.61	9.02
$t_{95\%}$ [X_0]	22.53	25.18

Table 2: Table of values that aid in defining the kinematics of the cascades and showers in the ECAL and HCAL. All for a 100 GeV beam of incident positively charged particles. The value of X_0, ϵ, t_{max} & $t_{95\%}$ is specific to electrons.

2. Understand what must be done to sum energies up.

What do we expect? Muon energy reconstruction will be difficult as they just pass through the calorimeter depositing very little energy, can show that.

For protons and positrons expect to be able to recon energy in HCAL and ECAL respectively. Potentially could have energy in HCAL from positrons but shouldn't be much energy in ECAL from protons.

2.6 Analysis

First question: what is diff between EC/HCenergy and the corresponding vector. ECAL vector contains 80 measurements, one for each cell in ECAL (20 along x, 4 along y). Sum of energies does not add up to ECEnergy however.. what is that measure of? (all measured in MeV). For HCAL there are 20 cells referring to the 10 along x and 2 along y.

Can work out which cell the energies are referring to with the GEANT code. But the difference between the sum of the vectors and the one value is confusing. The single value is sometimes larger than the sum, sometimes smaller. Always same order of magnitude, implies they could be calculated from eachother.

Checking GEANT code (remember kDim =2). It seems that the EC/HC energy should just be the sum of the energies. They are for first 49 events... on event number 49 it stops working, and then picks it up at 943! checking 942. Not sure where this error could've come from, code seems to make sense. I will use my own sum of the energies. It looks like from the GEANT code that the energies are listed in ascending x then ascending y. (for ECAL, first 20 values are either lowest y or highest y... not sure.) **Decided to just go with the vector measurements as these are the raw data stored in an ntuple. The other should be a sum according to the genat code so something must've gone wrong with the sum so will ignore.**

Plan

(a) Show separate histograms for ECAL and HCAL.

(b) Calculate sampling fraction for HCAL, scale up energy values accordingly.

(c) Sum ECAL for electron, HCAL for protons, ignore the other.

(d) Remember to include uncertainties on bins.

2.7 Expectations

Expect an energy loss distribution similar to landau as some very large energy losses lead to an exponential tail.

Not necessarily all energy will be lost in calorimeter due to radiation lengths and longitudinal leakage. Expect all to be lost in HCAL but must work out sampling fraction as not all energy loss will be recorded. In HCAL, any photons produced (from pi0 etc) will lead to an EM shower, no longer contributing to the hadronic cascade.

Looking at the values for the ECAL, with a length of 30cm or $16X_0$, considerably less than 95% of the shower energy will be contained within the ECAL. We therefore expect significant leakage in the ECAL from the positrons, so a mean energy of less than 100 GeV, and less than 95GeV.

For HCAL, need to calculate the sampling ratio f_{samp}, there is an equation

$$f_{samp} = \frac{E_{mip}(\texttt{active})}{E_{mip}(\texttt{active}) + E_{mip}(\texttt{absorber})} \tag{11}$$

that could be used to calculate it using the muon as the MIP, but I am unsure how to get the energy contributed in the absorber. Instead I am going to calculate the fraction of the HCAL that contians the scintilator plates. Looking at the schematics, this equates to 20% of the HCAL, so potentially the mean of the energy deposition in HCAL should be scaled by a factor of 5. WIll check if this value makes sense.

Could take the contribution in the ECAL as the absorber energy value in above equation, and the HCAL value as the active value?

2.8 Analysis

The energy deposits in the cells of the calorimeters were summed to produce event by event energy deposits for ECAL and HCAL. These were plotted side by side as a histogram and fitted with a moyal/landau or 100 GeV - moyal/landau. The results are shown in figures 20, 21, 22. The muon plots imply that $E_{mip}(\texttt{active}) = 0.05$ GeV and $E_{mip}(\texttt{absorber}) = 0.2$ GeV resulting in $f_{samp} = 0.2$, agreeing with the estimation from the fraction of scintilator plates in the detector. Scaling up the muon mean, the muon contributes similar amounts in the two calorimeters, as expected.

The mean positron energy deposition is 90.7 GeV, less than 95 GeV as expected due to longitudinal leakage. I expected that any leakage would be deposited in the HCAL, but even with adjustment only 0.24 GeV is deposited.

For the protons, 20 GeV is deposited on average in the HCAL, a factor of 5 smaller than expected. Initially I thought this could be due to $\frac{e}{\pi}$, the ratio of observable signals in a HCAL, but for lead sampling HCALs this is usually close to 1. The length of the HCAL is about 2.7λ so we expect the protons to exit with about 7 GeV of energy, but this doesn't account for any cascades exiting the HCAL.

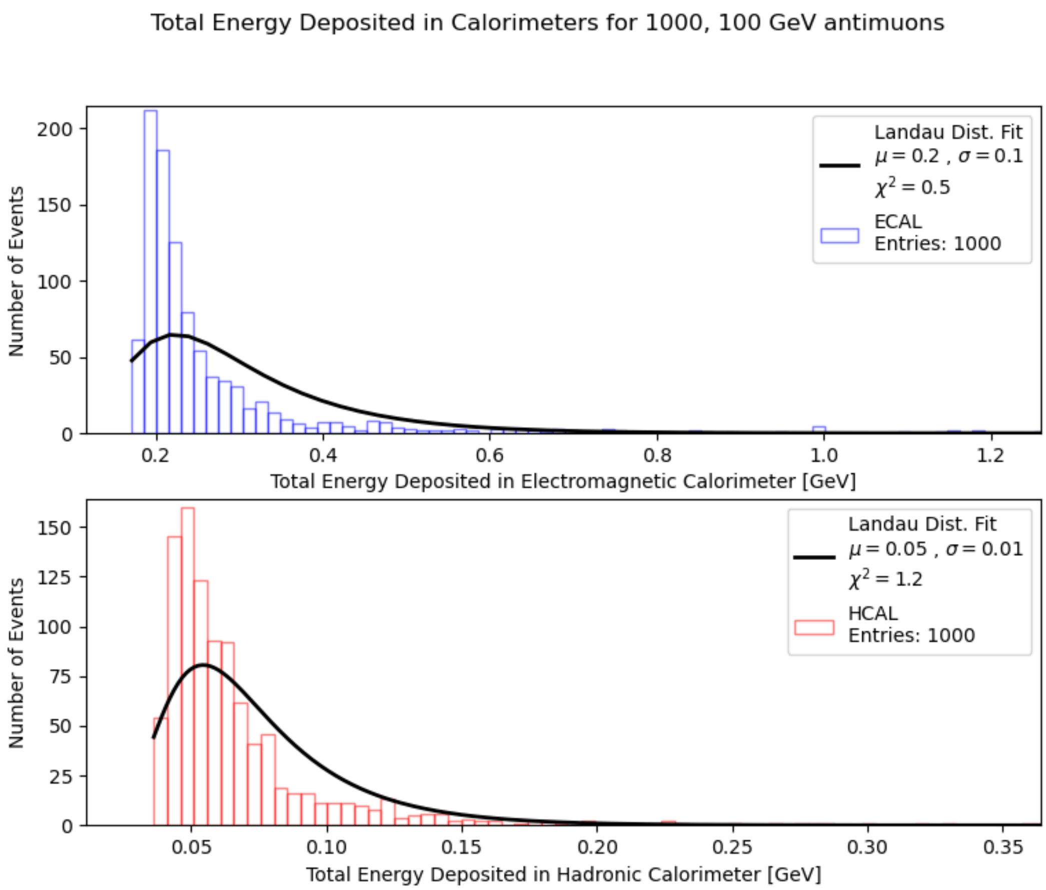


Figure 20: Histograms for energy deposition in ECAL and HCAL for 1000 antimuons at 100 GeV

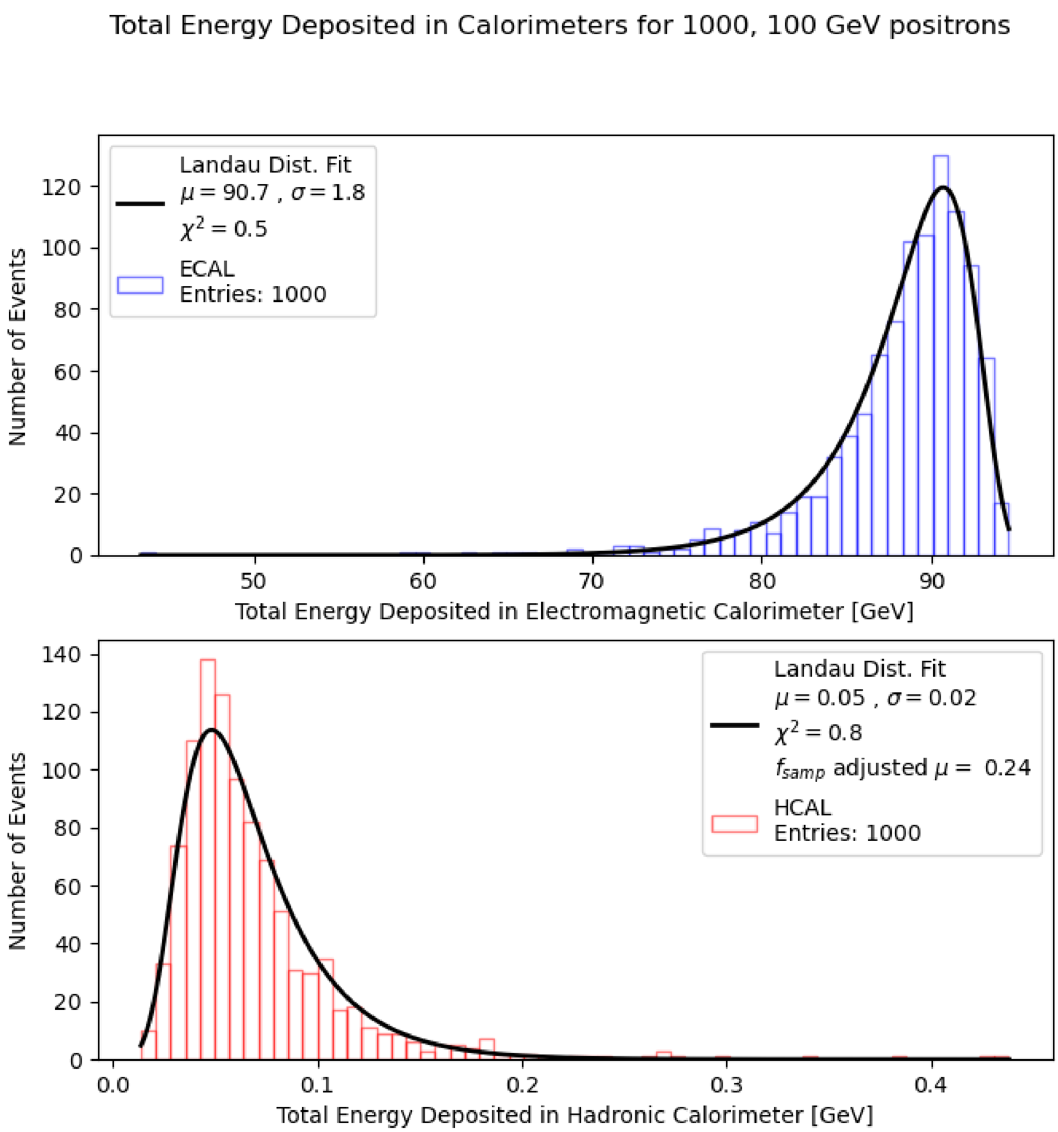


Figure 21: Histograms for energy deposition in ECAL and HCAL for 1000 positrons at 100 GeV

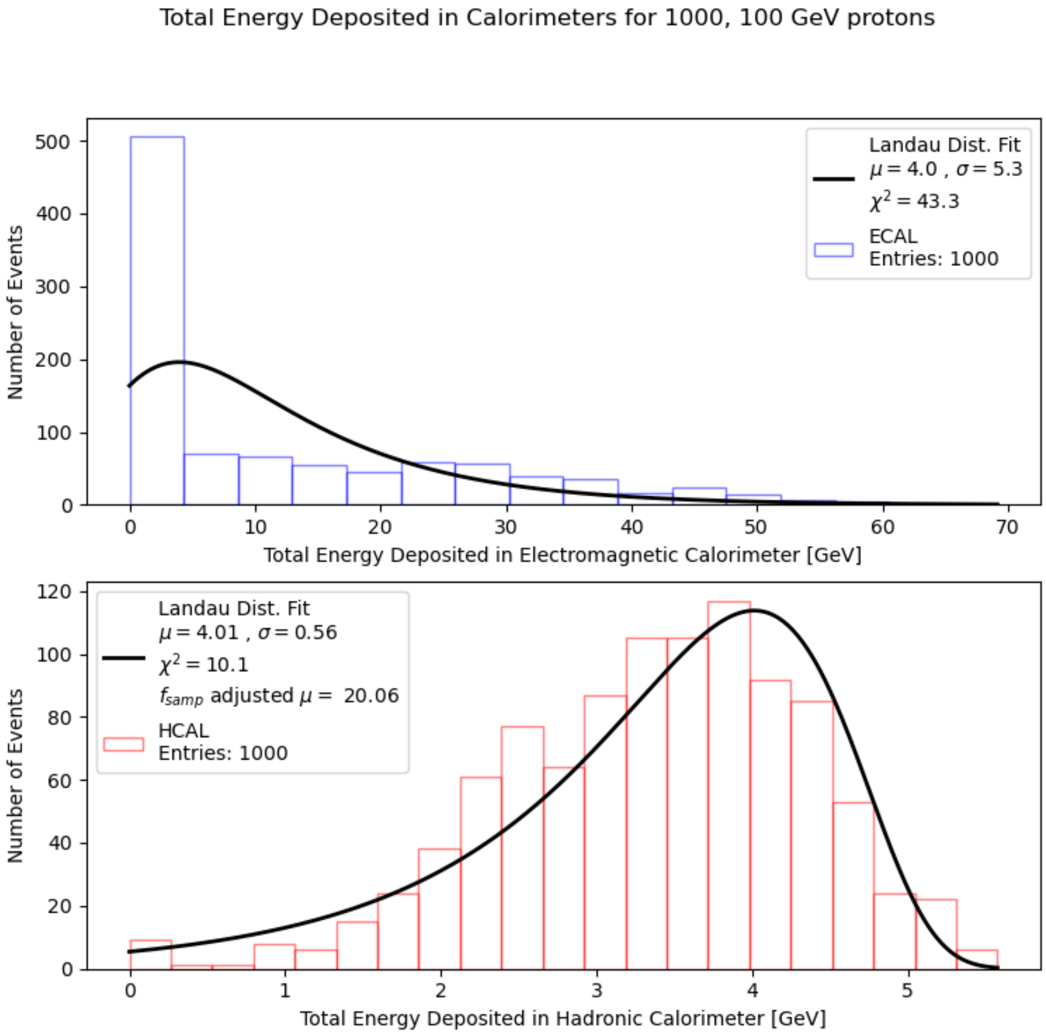


Figure 22: Histograms for energy deposition in ECAL and HCAL for 1000 protons at 100 GeV