Particle Techniques Exercises

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1 Exercise 1: Estimating Momentum with GEANT4

Code for this first assigment along with installation instructions stored in this repository.

1.1 Brief

Having discussed the basic structure of the code, and how to run/modify it, the first exercise is the following:

(1) For a magnetic field of 0.5 T, an angle of the spectrometer of 0 degrees (i.e. the axes of the tow arms are on the same line), send 1000 antimuons (mu+) along z+ with a momentum of 100 GeV. The precision coordinate is "x", and the second coordinate is "y". Assume a precision along x of 100μm, and along y of 1cm. Reconstruct the track of the muons along the x-z plane and estimate their momentum. Draw the distribution of reconstructed momentum, and find the resolution.

(2) Estimate the momentum resolution if the magnetic field was 0.25 T or 1T.

(3) For a magnetic field of 0.5 T, estimate the momentum resolution for 50 GeV and 200 GeV muons.

(4) What is the momentum resolution for 100 GeV muons if you assume perfect knowledge of the position along x? why?

Please create a git hub account where you keep your simulation and analysis code, and put the results in an overleaf document.

1.2 Breakdown of Exercise

Using position information from hits in geant4, will be able to calculate the curvature of tracks and therefore velocity and momentum. Potentially will need to account for relativistic effects! Precision values will allow an error to be propagated. Only reconstruct in 2D (xz) plane. Assume error on mag field of 0? Propagate an equation and draw into latex to answer (4).

1.3 Theory/Equations

The relativistic momentum equation is is defined as

$$\vec{p} = \gamma m \vec{v} \tag{1}$$

where symbols have usual meanings. The rate of change of momentum is equal to force, $\vec{F}$ which, in a magnetic field, is

$$F = q\vec{v} \times \vec{B} = q|v||B|\sin(\theta)\vec{\hat{n}} \tag{2}$$

where $q = +e$ (anti-muons), θ the angle between B and v and $\vec{\hat{n}}$ the unit vector perpendicular to both B and v. Hence, with centripetal acceleration

$$\frac{d\vec{v}}{dt} = \frac{v^2}{r} \tag{3}$$

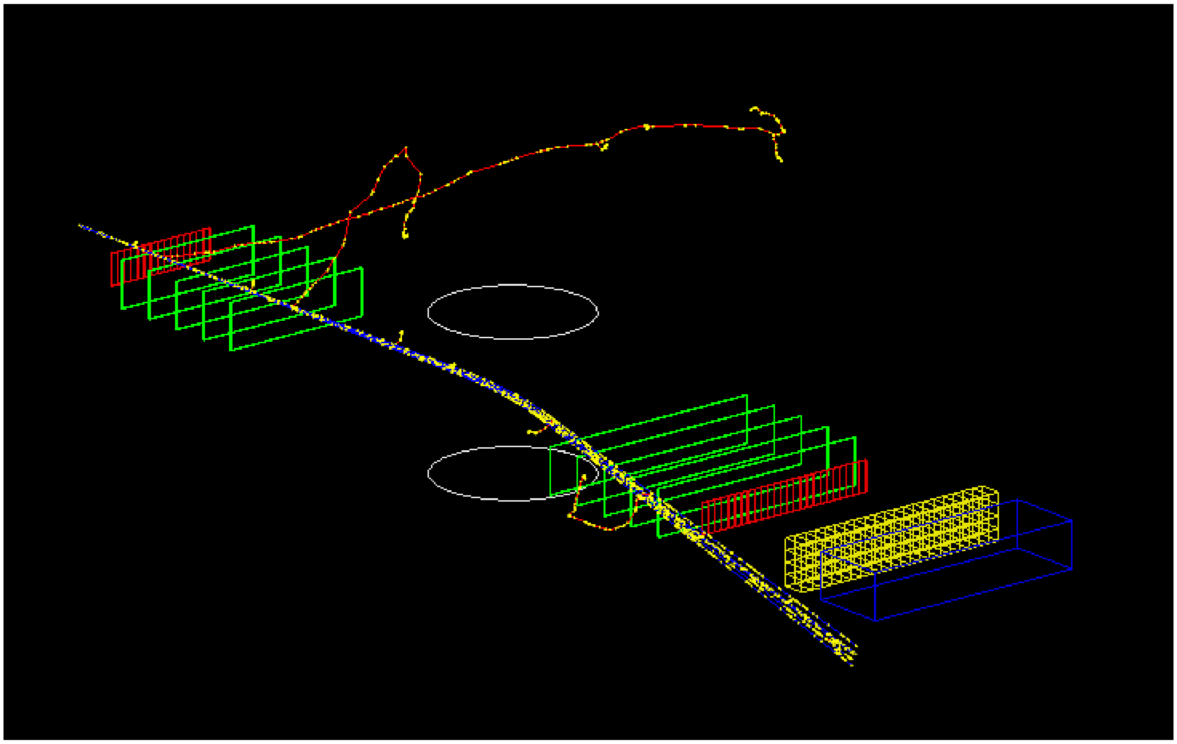

Figure 1: Screenshot of visualiser for 5 1 GeV anti-muons, to show pronounced curve which won't neccesarily be obvious for higher energy muons.

where r is the radius of curvature, a simple equation for momentum, with gamma factor cancelled out, is derived:

$$\vec{p} = eBr\sin(\theta)\vec{n}. \tag{4}$$

Figure 1 shows an example where the curvature is more well pronounced hence the field can be determined to be in the $+y$ direction, with the muons fired in the $+z$ direction and curving towards the $+x$ direction. Hence, the magnetic field is always perpendicular to the velocity of the muon so equation 4 reduces to

$$p = eBr \tag{5}$$

where the vector notation has been dropped for now. Radius is difficult to measure and would require a parabola fit, possible but there will be very little curvature at the high energies shown. Instead s, the sagitta, will be measured. For an arc with small curvature ($r >> L$, where L is the width of the arc) the momentum equation becomes

$$p = \frac{eL^2B}{8s}. \tag{6}$$

This is a useful equation for track reconstruction when there are hits within the magnetic field, but for this case there are no hits in the magnetic field and we must calculate the momentum from the deviation of the track. Figure X is annotated to show the geometry of the detector and the diagrams required to calculate the radius of curvature for the muon allowing equation 5 to be used to extract the momentum. By linear fitting the beams before and after the deviation we obtain line gradients and intercepts m_1,m_2,c_1 and c_2 where index 1 refers to before the deviation (negative z values and index 2 to after the deviation. The two gradients allow an angle of interception θ

between the two lines to be calculated;

$$\tan(\theta) = \frac{m_2 - m_1}{1 + m_2 m_1}. \tag{7}$$

The centre of the magnetic field is set as the coordinate $(0,0)$ for simplicity in deriving the circle equation which details the position of the magnetic field $x^2 + z^2 = (r^B)^2$, where r^B is the radius of the magnetic field derived exactly in GEANT4. Knowing the circle equation and 2 line equations, (x_1^B, z_1^B) and (x_2^B, z_2^B), the intercepts between the lines and magnetic field, can be calculated and hence $L = \sqrt{(x_1^B - x_2^B)^2 + (z_1^B - z_2^B)^2}$ is obtained. Using circle geometry L and θ, can be combined to calculate the radius of curvature for the muon path as

$$r = \frac{L\sqrt{2}}{2}\left[1 - \left(1 + \tan^2(\theta)\right)^{-\frac{1}{2}}\right]^{-\frac{1}{2}}, \tag{8}$$

and hence p can be extracted. Examining the complexity of this equation and the L and $\tan(\theta)$ equations required to calculate it, I will not show the error propagation in full. But the calculation is entirely derived from m_1,m_2,c_1 and c_2 which are in turn fitted from the initial data points. I will not need this analytical error on momentum as I should be able to derive a standard deviation from the gaussian produced from the momentum values. However I do need to understand how the error will scale with momentum. Taking equation 5, the momentum error is proportional to error the on the measurement of r, specifically

$$\sigma_p = \frac{p}{r}\sigma_r. \tag{9}$$

Propagation of σ_r finds it is proportional to r and $f(\theta, L)$ hence the overall momentum resolution should scale with p as the ignored $f(\theta, L)$ is derived from the error on the linear fit parameters which in turn are only affected by the x and z resolutions

Hence for question (2), as the momentum is fixed, the error will not change (an increase in B will reduce the radius of curvature) and for (3) the answers will be 0.5x and 2x the resolution calculated for the data-taking case. For (4), perfect knowledge of the x momentum, a perfect linear fit will be made resulting in the linear fit parameters having 0 error and hence the resolution on momentum being 0. This would equate to a Dirac delta distribution for momentum, with the muons exactly 100 GeV.

1.4 Plan

Will output the root file to NumPy arrays using overleaf, use the z values/the geant4 code to understand which layers are before and after the mag field. 0.5m between layers, start of each chambers are 10m apart (at -5 and +5). Remember x values are in mm! going to use fitting randomiser to find best chi2 for cases with back-scattering. will check the y values to check that it is flat as it should be!

(1) Extract data from drift chambers 1 & 2 into NumPy arrays using uproot.

(2) Find the geometry of the detector in the geant code, use this to create an equation for the magnetic field and positions for each DC (a map for z to position). Remember: centre of mag field is (0,0), x,y in mm and z chambers are about 0.5m apart with 10m between the first chamber in each arm.

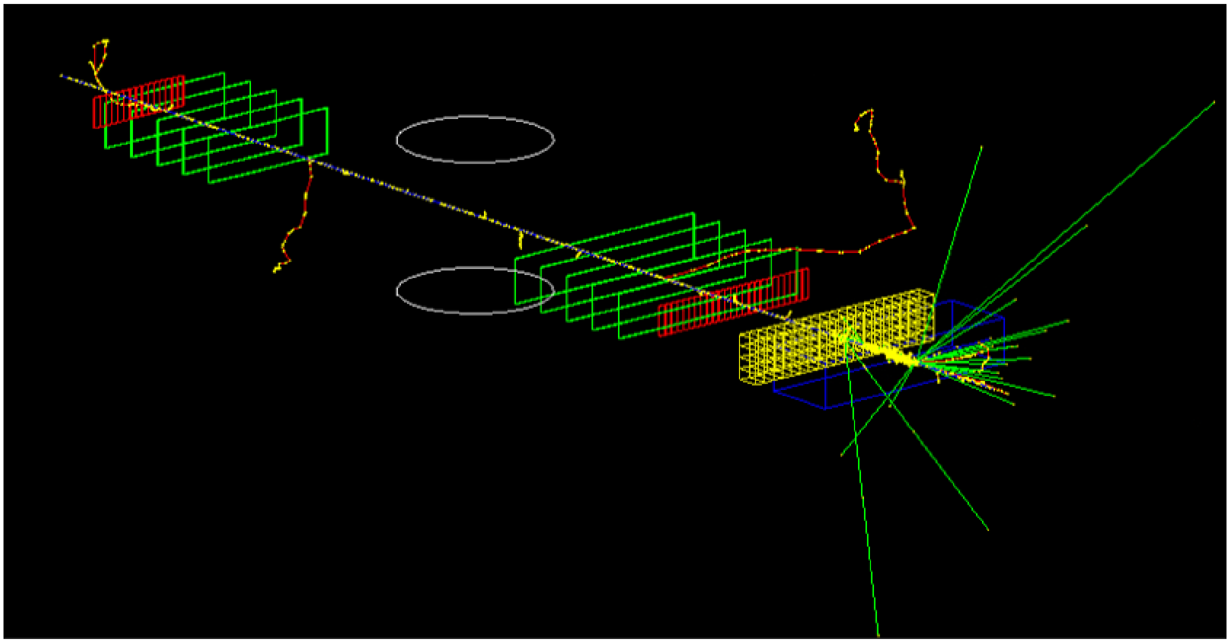

Figure 2: Screenshot of visualiser for 5 100 GeV anti-muons.

(3) Make a linear fit to 5 hits in each arm, one from each chamber. If there are multiple hits per chamber, fit each permutation and keep min χ^2. Remember to use uncertainty in fit and also to fail/report on an diverged fit! Multiple hits are from back scattering. Maybe only keep events with reduced chi2 between 0.5 and 1.5, see how many that deletes, can check affect on result.

(4) Work out whether y should be considered by calculating a χ^2 for each event to the straight line $y = 0$ (in y-z plane) and potentially discarding events that are bad if this is a minority or investigating why if not. Of fitted points will calculate a χ^2 check about y=0.

(5) With this best fit can get the intersects with the magnetic field of the two beams (x_1^B, z_1^B) and (x_2^B, z_2^B) which allows a calculation of L.

(6) Using the gradients to get θ and some geometry, the radius and hence momentum can be calculated.

(7) If I find a package to propagate the uncertainty, use that and store results! But either way plot the Gaussian and get uncertainty from that!

Note: If make a function that returns the intersects with the magnetic field, the magnetic field geometry can be changed easily, will have to deal with choosing the correct intersects as more than one will be available! That might need manual work!

1.5 Schematics

From the source file "B5DectectorConstruction" the schematics of the detector can be extracted. Without precision information these schematics are assumed to be perfectly measured (unrealistic for a real detector which would, for example, use magnetic probes to measure the geometry of the

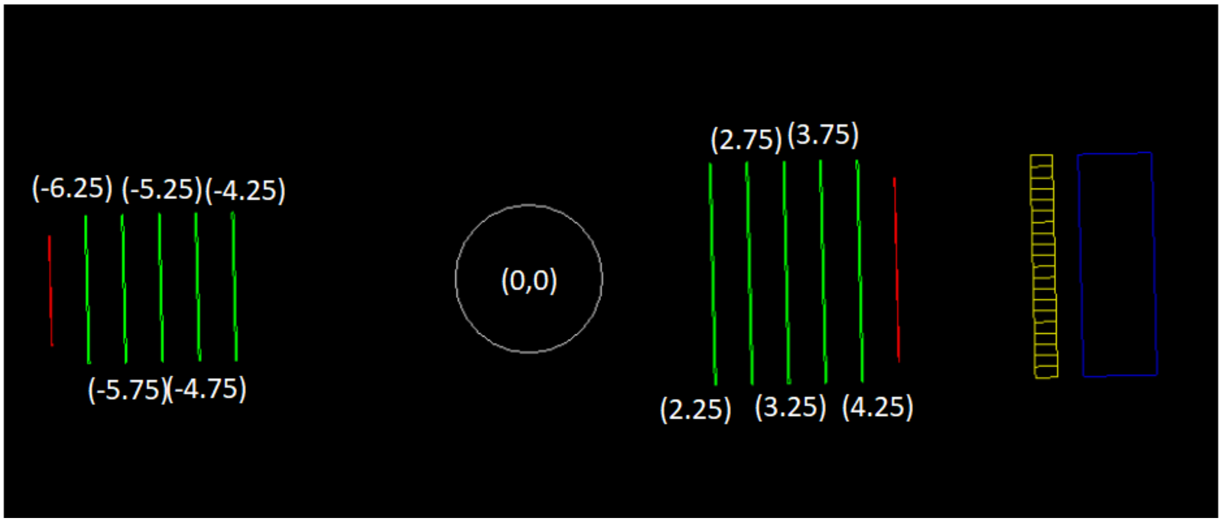

Figure 3: Top Down view of detector schematic in geant4 visualiser. With annotated measurements in m showing the z positions of the drift chambers. The central circle is the magnetic field tube

magnetic field produced). All coordinates/sizes quoted in (x,y,z) order and probably in m unless specified.

(a) Cavern: 10x3x10 m of air.

(b) Magnetic Field: Cylinder of radius r_B=1m and height 1m with centre at (0,0) in x,z. Orientated in the positive y direction (shown by direction of curve in the negative x direction in figure 1).

(c) First Arm: Placed at (0,0,-5) on z axis, with geometry 1.5x1x3

(d) Second Arm: Placed at (0,0,5) on z axis (for arm angle of 0), with geometry 2x2x3.5. I think the difference doesn't matter, it all depends where the drift chambers are. Not sure whether the placement is centre or front of arm! I think centre.

(e) First arm drift chambers: all 1x0.3x0.01, within there is a virtual wire at centre which is 1x0.3x0.0001. The drift chambers are placed at (0,0,(i-2.5)x0.5) in the arm where i runs 0-4. So -1.25, -0.75, -0.25, 0.25, 0.75 relative to the centre of the arm (0,0,-5).

(f) Second Arm Drift Chambers: Very similar setup however placed at (0,0,(i-2.5)*0.5-1.5) so above values -1.5 relative to centre of second arm at (0,0,5). This does seem correct, looking at figures 1 and 2, with the last DC in the first arm being at -4.25 from centre of mag field and the start of the second arm at +2.25. This can be seen more clearly in the top down schematic 3.

02/02: Update, the magnetic field shape has changed to a cube of side length 2m centred on (0,0,0). I will continue with the circle but add an option to switch to square. The only difference is the intercepts!

```
Events discarded from having too many points to reasonably fit:  [81, 196, 345, 455, 997]
Events discarded in z-x fit of two lines:  [169, 285, 919]
Events discarded in z-y fit of two lines:  [358, 735, 786]
Events discarded from having no intercept with B-Field:  []
```

Figure 4: Screenshot of ipython console showing discarded events.

1.6 Plan Update

Since making the plan, during development of the code, the following changes have been made.

(a) Realised the x,y measurements are in mm, recursively updated the doc as this is important.

(b) Discard events with >30 hits in one of the drift chambers. This could lead to >7776 possibilities of straight lines, difficult to be sure you have correct trajectory, will check efficiency hit this brings.

(c) When fitting the two straight lines, fit as $x = mz + c$ (and invert at the end). Split the hits into those in different sections of each drift chamber and cycle through combinations of one hit from each section. If any section has no hits (e.g 5 hits, one doubled up, one empty) discard event. Again will study efficiency hit. Do minimum χ^2 fit to choose which fit is best.

(d) Check reduced χ^2 of these fits, if they are greater than 1.5, discard. Wanted to make lower cut too, but large uncertainties on x (compared to deviations) mean all of the reduced χ^2 are very small(order 10^{-6})! (maybe we should've looked at the width of the wire as the z uncertainty!)

(e) For y=0 fit, discard events with reduced $\chi^2 < 1$. This removes some bad events but due to large y uncertainty again the reduced χ^2 are extremely small (order 10^{-5}). Do not feel confident cutting on reduced χ^2 less than 1. Need to understand why this is.

(f) Intercept calculation return "none" if there is no intercept or the intercepts are in entirely the wrong place geometrically. Some errors could fall through but should never have collided with drift chambers so shouldn't even be possible

(g) Calculate momentum store, plot and fit a normal distribution.

1.7 Results

Choosing the settings (choose initial momentum, B-field and magnetic field shape) and running the analysis produces a histogram of the momentum with a gaussian fit over it. The gaussian is fit around the central peak. I worked with a few intervals but decided on: $p_{init} \pm 3 * p_{init}/100$ as this seemed to fit the gaussian well (minimised χ^2) and not try and over-fit any fluctuation beyond the peak.

For efficiency, figure 4 shows that, for the example of 100 GeV and square mag field, an efficiency of 99%. Making plots of these discarded events often show deviations from y=0 that have led to a bad fit in x-z.

Figures 5,6,7,8,9, show a series of histograms with gaussian fits to different detector scenarios. The mean, standard deviation and reduced χ^2 from the fits are displayed on each plot. The χ^2

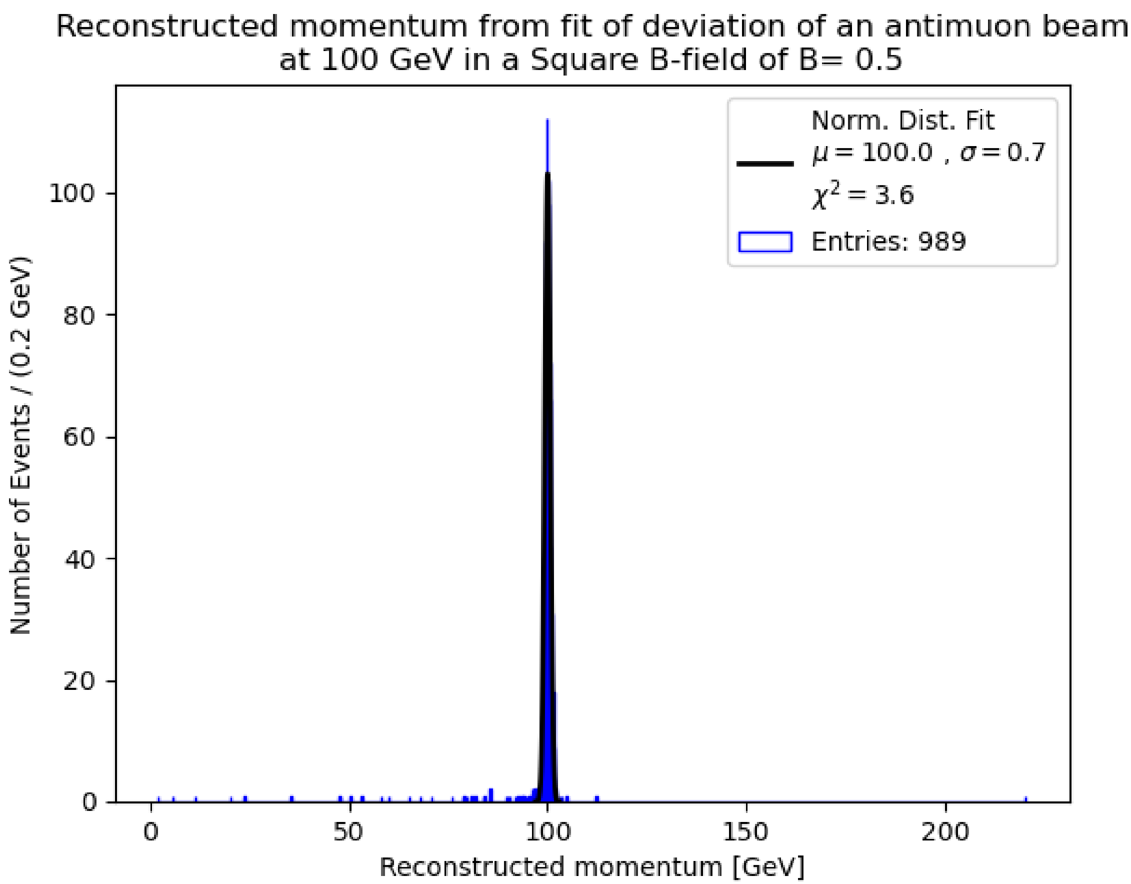

Figure 5: Histogram with Gaussian fit for a square mag field and p_{init} = 100GeV. The plot has not been zoomed to show fluctuations throughout p.

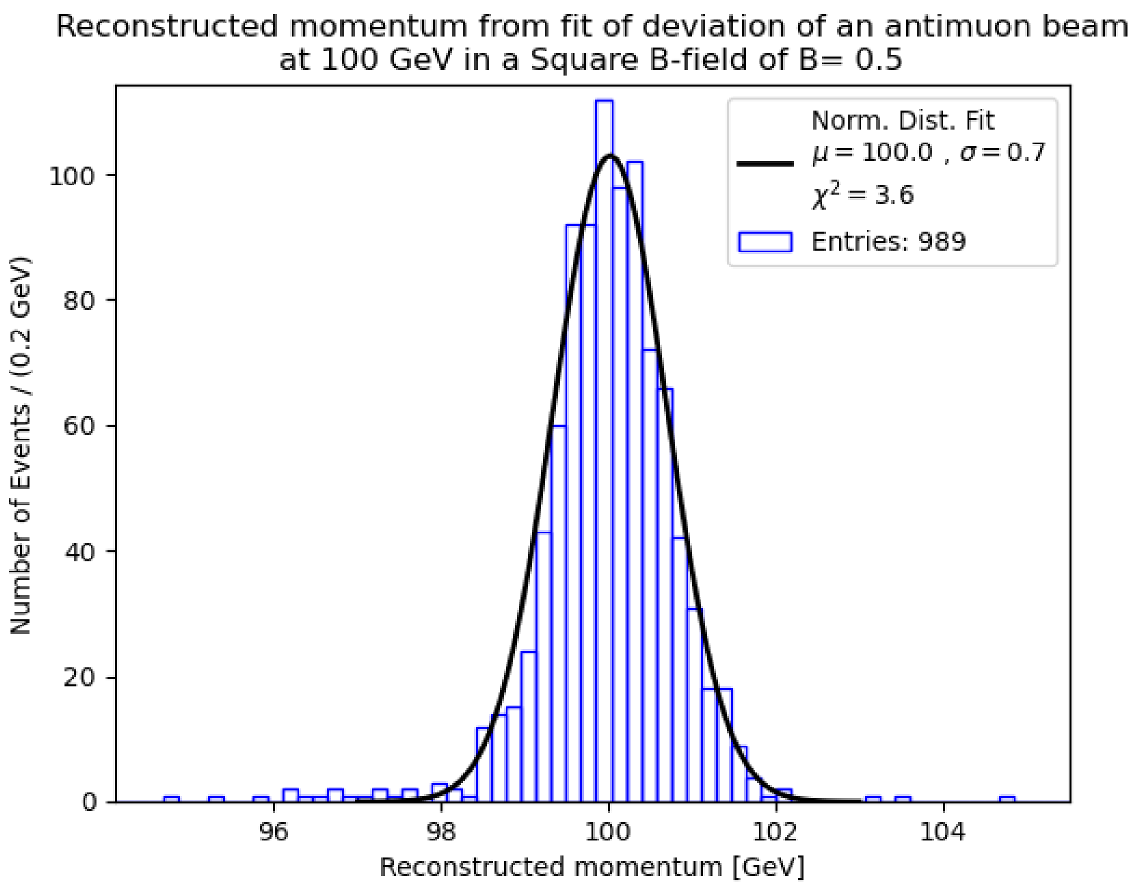

Figure 6: Histogram with Gaussian fit for a square mag field and p_{init} = 100GeV. The plot has been zoomed to show central fitted region.

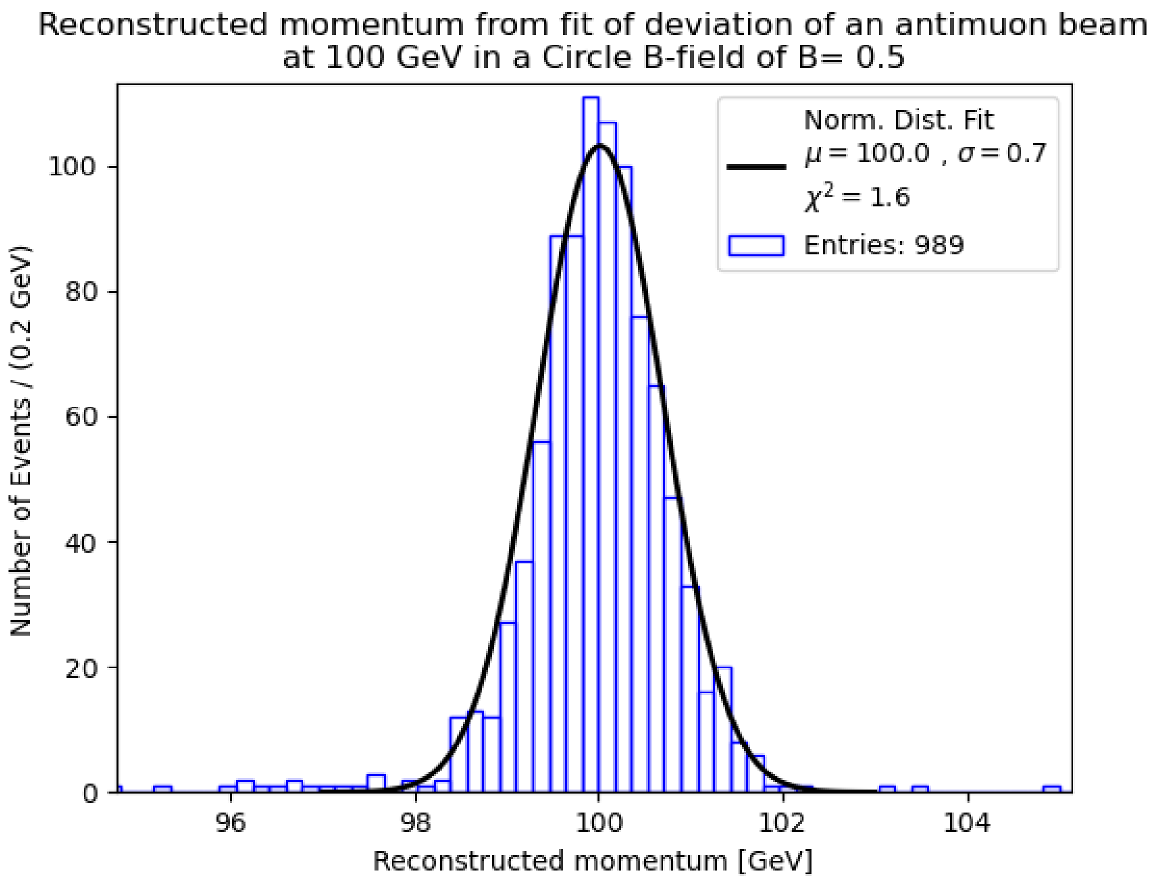

Figure 7: Histogram with Gaussian fit for a circular mag field and $p_{init} = 100\text{GeV}$. The plot has been zoomed to show central fitted region.

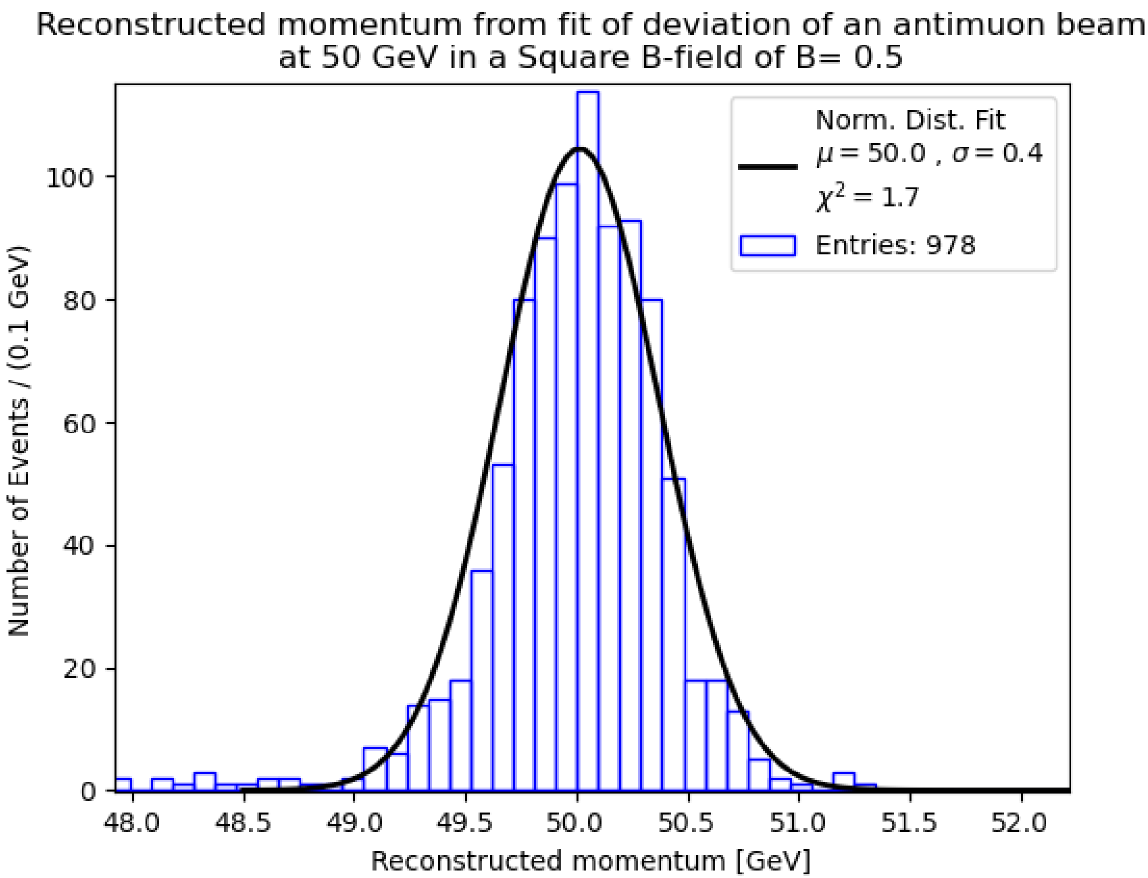

Figure 8: Histogram with Gaussian fit for a square mag field and p_{init} = 50GeV. The plot has been zoomed to show central fitted region.

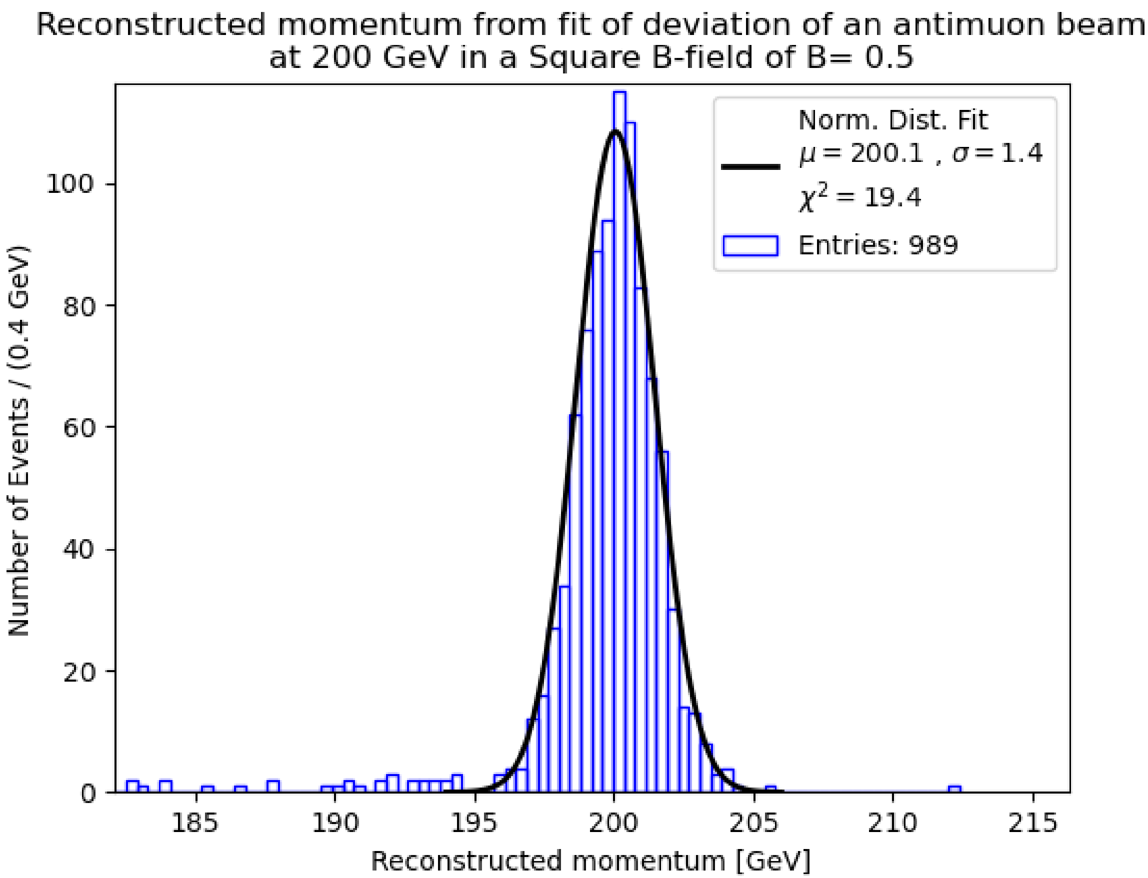

Figure 9: Histogram with Gaussian fit for a square mag field and p_{init} = 200GeV. The plot has been zoomed to show central fitted region.

imply that these are good fits, but the circle has produced a considerably better fit. Work on the fitting interval could potentially improve these. The means are all correct to the initial momentum which implies the method used is plausible.

Figure 5 shows the variation that is possible from this method, with fluctuations ranging from <5 GeV to nearly 200 GeV. These seem to be due to bad y-fits but I cannot remove them without making harsh cuts on χ_y^2 that damage the quality of the Gaussian. More analysis on these later.

Comparing figures 6, 8 and 9 shows that, as mentioned in section 1.3, σ_p scales with momentum as the 50 GeV case has σ_p half that of 100 GeV and a quarter of 200 GeV. I need to make a varying mag field sample to test the B-field hypothesis. I cannot reasonably test the zero error scenario without entering geant4 and understanding where and how the variation is introduced.

I have not propagated errors through yet, but that is quite easily done. I am not sure how this affects the fitting of the Gaussian or how you even calculate errors on bins! I need to research this.