

Problem

There are 3 urns labeled X, Y, and Z.

- Urn X contains 4 red balls and 3 black balls.
- Urn Y contains 5 red balls and 4 black balls.
- Urn Z contains 4 red balls and 4 black balls.

One ball is drawn from each of the 3 urns. What is the probability that, of the 3 balls drawn, 2 are red and 1 is black?

Solution

Probability of drawing a red ball:

- Urn X has a $4/7$ probability of drawing a red ball.
- Urn Y has a $5/9$ probability of drawing a red ball.
- Urn Z has a $1/2$ probability of drawing a red ball.

Probability of drawing a black ball:

- Urn X has a $3/7$ probability of drawing a black ball.
- Urn Y has a $4/9$ probability of drawing a black ball.
- Urn Z has a $1/2$ probability of drawing a black ball.

$$P(A) = P(\text{Red}, \text{Red}, \text{Black}) + P(\text{Red}, \text{Black}, \text{Red}) + P(\text{Black}, \text{Red}, \text{Red})$$

$$\begin{aligned} &= \frac{4}{7} \frac{5}{9} \frac{1}{2} + \frac{4}{7} \frac{4}{9} \frac{1}{2} + \frac{3}{7} \frac{5}{9} \frac{1}{2} \\ &= \frac{20}{126} + \frac{16}{126} + \frac{15}{126} = \frac{51}{126} = \frac{17}{42} \end{aligned}$$