

# Root finding

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## Relaxation method

We are trying to figure out the solutions to the equation:

$$x = f(x)$$

where  $f(x)$  is a nonlinear function.

We call the solutions  $x_0$

Iterations i:

$$\begin{aligned}x_i &= f(x_{i-1}) \\ &= f(x_0) + (x_{i-1} - x_0)f'(x_0) + \dots\end{aligned}$$

In each iteration  $f(x_0) = x_0$

$$x_i - x_0 = (x_{i-1} - x_0)f'(x_0) + \dots$$

$$\Delta x_i \approx \Delta x_{i-1}f'(x_0)$$

Does  $\Delta x_i$  shrink?

Yes, only if  $|f'(x_0)| < 1$

## Example

$$x = f(x) = 1 - e^{-2x}$$

$$f'(x) = -2e^{-2x}$$

OK:

$$f'(0.797) = -0.406$$

NOT OK:

$$f'(0) = -2$$