

Interpolation

May 18, 2021

Introduction

We have two arrays of numbers X and Y . Array X contains independent data points. Array Y contains dependent data points $y_i, i = 1, \dots, m$.

We want to find a function $\hat{y}(x)$, which gets the exact same value with given points.

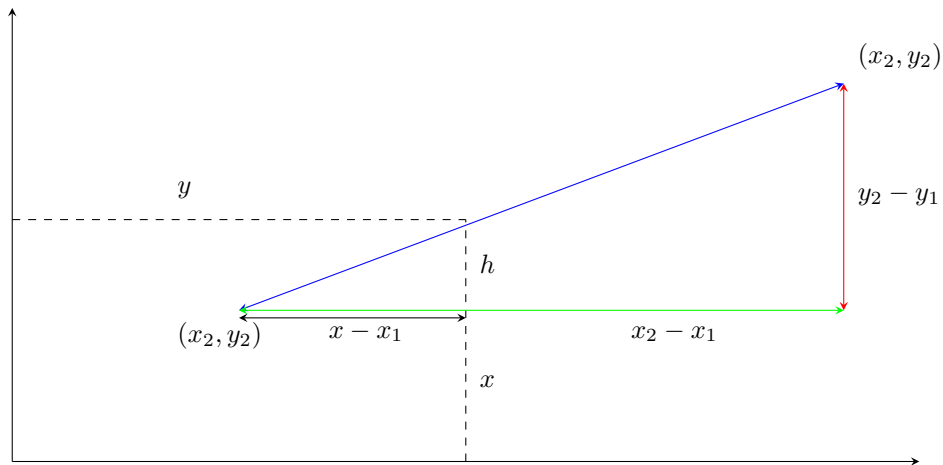
Linear Interpolation

Linear interpolation is achieved by connecting two data points with a straight line.

For $x_i < x < x_{i+1}$:

$$\hat{y}(x) = y_i + \frac{(y_{i+1} - y_i)(x - x_i)}{(x_{i+1} - x_i)}.$$

Derivation



$$\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

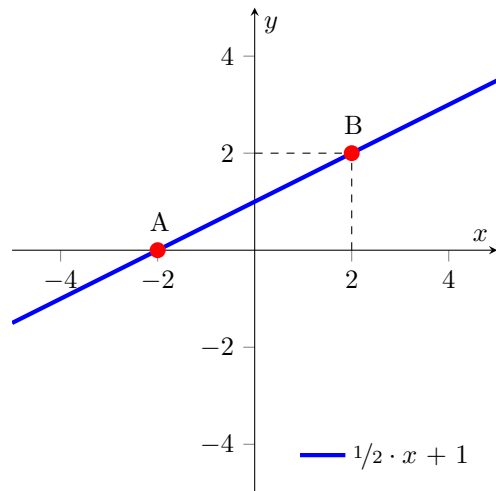
$$h = \alpha \cdot (x - x_1)$$

$$y = y_1 + h$$

$$y = y_1 + (x - x_1) \cdot \frac{y_2 - y_1}{x_2 - x_1}$$

Example

We are given two points A(-2, 0) and B (2, 2).



Let's try to evaluate the value of the function at $x = 1$

$$\hat{y}(x) = y_i + \frac{(y_{i+1} - y_i)(x - x_i)}{(x_{i+1} - x_i)} = 0 + \frac{(2 - 0)(1 - (-2))}{(2 - (-2))} = 1.5$$