Numerical differentiation

May 24, 2021

Forward difference:

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Backward difference:

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x) - f(x-h)}{h}$$

Taylor expansion of f(x+h) about x

$$f(x+h) = f(x) + (x+h-x)f'(x) + \frac{1}{2}(x+h-x)^2 f''(x) + ...$$

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + ...$$

$$f'(x) = [\frac{f(x+h) - f(x)}{h}] + \frac{h}{2}f''(x)$$

what is the ideal h? small, but can't be 0
-rounding error is bad if h is too small

Central difference

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x + h/2) - f(x - h/2)}{h}$$