

Relaxation method

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We are trying to figure out the solutions to the equation:

$$x = f(x)$$

where $f(x)$ is a nonlinear function.

We call the solutions x_0

Iterations i :

$$\begin{aligned} x_i &= f(x_{i-1}) \\ &= f(x_0) + (x_{i-1} - x_0)f'(x_0) + \dots \end{aligned}$$

In each iteration $f(x_0) = x_0$

$$x_i - x_0 = (x_{i-1} - x_0)f'(x_0) + \dots$$

$$\Delta x_i \approx \Delta x_{i-1} f'(x_0)$$

Does Δx_i shrink?

Yes, only if $|f'(x_0)| < 1$

Ex.

$$x = f(x) = 1 - e^{-2x}$$

$$f'(x) = -2e^{-2x}$$

OK:

$$f'(0.797) = -0.406$$

NOT OK:

$$f'(0) = -2$$