Application of Conservation Laws Using Ballistics Test to Determine the Velocity of a Projectile

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Abstract

The goal of this two-method experiment was to show multiple ways to find the launch velocity of a projectile based on either its final height or horizontal range. Then, use these two calculated values and calculate the discrepancy to determine whether the experiment was a success or failure. If the experiment was a failure, then the ultimate goal of this experiment was to find the sources of error and determine ways to mitigate these factors in the future. Every test in this experiment used a spring-loaded-launcher mechanism to fire the initial projectile and consisted of five trials. Part One of this experiment used a pendulum designed to capture the projectile and then record the height it reached. The height differential between the initial and final measurements, the mass of the projectile and pendulum bob, and the acceleration due to gravity at the experiment location of $9.79262 m/s^2$ [2] were used to determine the initial velocity, this velocity was averaged to 6.20m/s [1]. The objective for Part Two was to calculate the velocity using a horizontally-launched projectile instead of a pendulum. Initially the velocity from Part One was used to estimate the distance the projectile would travel. Five trials were performed to determine the average initial velocity for this new method of launching. The Part Two mean velocity was 5.718 m/s [1].

Background

The Law of Conservation of Energy is the physical law that states that the total energy in any isolated system will remain constant. The energy present in the initial state will be the same magnitude as the energy measured in the final state. Similarly, the Law of Conservation of Momentum states that the total momentum in a closed system will remain constant and unchanged. These laws are based upon the discoveries made by Newton in his force laws, and this two-part experiment was a test of these laws. Ideally this experiment would be performed in a perfect environment with equipment that could measure every gain or loss of energy. For example, in Part One the ball created a sound when it connected with the pendulum which was a representation of a "loss" of energy when the only calculable energies were kinetic and potential energy. There were no other deviations from the expected procedure.

Theory & Methods

This experiment used two different methods to calculate the muzzle velocity of a launched projectile from a spring-loaded ballista. The first method launched this projectile directly into a pendulum bob that could capture the projectile. Once captured, the pendulum preserved its momentum and then continued to move until it stopped on a grooved track that the pendulum could stick to. When it stopped, we compared the initial height of the pendulum to the final height it reached on the track to get Δh, the change in height of the pendulum. The mass for the projectile (m in Figure 1 [3]) was measured with a triple-beam-balance to be 62.2±.1g [1] and the mass for the pendulum (M in Figure 1 [3]) was given by the pendulum itself at 274.5±.05g. State One for the projectile system was the projectile, spring, and pendulum in motionless states with the spring fully compressed and set up to fire the projectile. Once the spring was released the projectile was fired directly towards the pendulum with a velocity $\vec{v}_1$ and entered State Two as shown by Figure 1.

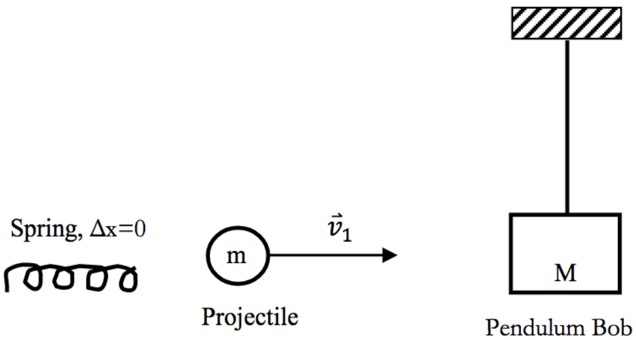

Figure 1: State Two, a representation of the launcher-projectile-pendulum system after the projectile was launched but before it was embedded within the pendulum bob. Here the projectile has a velocity of $\vec{v}_1$ equivalent to the muzzle velocity. [3]

In State Three the projectile was embedded in the pendulum bob and the two masses travelled horizontally together with a different velocity from the initial muzzle velocity. State Four, shown in Figure 2, represented the final stage of the projectile launch where the velocity reached 0 and was stopped on the set track.

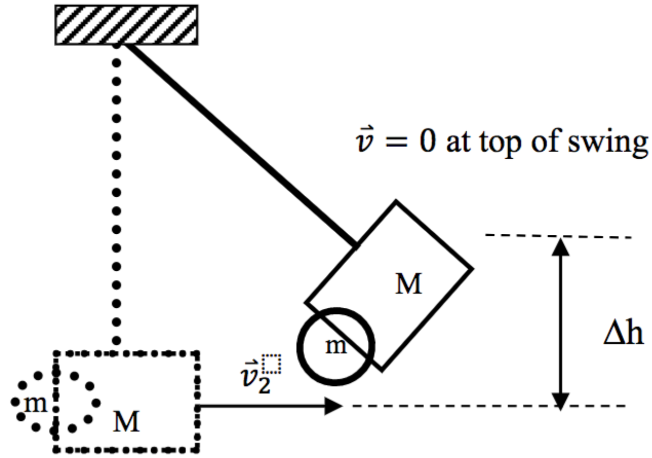

Figure 2: State Four, the final state of the projectile when the velocity reaches 0. Δh was the change in height from the resting position of the pendulum to the final height it reaches on the track. M was the mass of the pendulum and m was the mass of the projectile. [3]

The Conservation of Momentum equation (2) was used to convert the total energy in the system from States Two to Three. Simplifying that equation (3) and plugging it into the Conservation of Energy equation (1) produced the final equation (4) to calculate the muzzle velocity of the projectile.

$$\vec{p}_i = m\vec{v}_1 = (m + M)\vec{v}_2 = \vec{p}_f \qquad (1)$$

$$\frac{1}{2}(m + M)\vec{v}_2^{\,2} = (m + M)\,g\Delta h \qquad (2)$$

$$\vec{v}_2 = \sqrt{2g\Delta h} \qquad (3)$$

$$\vec{v}_1 = \frac{m+M}{m}\sqrt{2g\Delta h} \qquad (4)$$

Method Two was much simpler. The Equations of Motion were the only necessary calculations needed for this method. The projectile was launched horizontally with an initial vertical velocity of 0m/s and only changed due to gravity (5). The horizontal velocity is the value that was supposed to be determined by this experiment, and by assuming that air resistance is negligible the horizontal velocity was constant (6). Solving equation 5 for t (time) and plugging that into equation 6 provided the formula for Method Two's muzzle velocity (7).

$$y = \frac{1}{2}gt^2 \qquad (5)$$

$$\vec{v}_1' = \frac{x}{t} \tag{6}$$

$$\vec{v}_1' = x\sqrt{\frac{g}{2y}} \tag{7}$$

Results

Each Method of this experiment repeated its test five times. The experimental mean velocity determined in Part One was 6.20m/s with a standard deviation of 0.018m/s [1]. This velocity was used to estimate the distance the projectile would travel in Part Two (8).

$$v_0 = x\sqrt{\frac{g}{2y}} \tag{8}$$

This estimated value was 2.552m, and the mean measured value was 2.3532m (standard deviation of 1.75cm) with a percent error of 7.79% [1]. Continuing on to Part Two, five horizontal launch trials demonstrated that the mean muzzle velocity was 5.718m/s with a standard deviation of 0.0425m/s [1]. Comparing this velocity with the initial velocity from Part One gave a discrepancy (9) of 0.482m/s with an uncertainty (10) of 0.00184 [1].

$$d = \left|\vec{v}_1 - \vec{v}_1'\right| \tag{9}$$

$$\delta_d = \sqrt{\sigma_{\vec{v}_1}{}^2 + \sigma_{\vec{v}_1'}{}^2} \tag{10}$$

The expected value for discrepancy and its uncertainty was 0.0 because both parts of this experiment were measuring the same value. This experiment could be determined a success if

$$d \leq 1.96 \times \delta_d$$

The discrepancy, d, was 0.482 and the uncertainty of the discrepancy, δ_d, was 0.00184. Since 0.482 is actually greater than 0.0036 this experiment was determined to be a failure. There were many possible causes for a discrepancy between Methods One and Two. The calculations in Method One ignored any energy lost due to heat and sound in any part of the pendulum launching causing a Systematic Intrinsic Uncertainty. Additionally, the spring catcher was very easily adjustable and could have been bumped, and it was possible to change the spring constant and tension that the spring was placed under once compressed causing a source of Systematic Measurement Uncertainty. This is most likely the major cause of the large discrepancy between Methods. Method Two ignored air resistance while the projectile was flying which was a source of Systematic Intrinsic Uncertainty. Finally, the equipment used to measure the launch distance in Method Two was a large source of Random Measurement Uncertainty. There was not a device readily available to measure distances greater than 2m, so multiple devices with different resolutions and uncertainties had to be placed end-to-end in order to record the full distance the projectile was launched.

A total of ten trials took place over the course of this experiment. Five were with the pendulum in Method One and five were with the horizontal launch in Method Two. Based on the raw data gathered from these two Methods the first method was exponentially more precise. The standard deviation for the velocity in Method Two was over 200% greater than the standard deviation for Method One.

References

[1] Jefts, David. "Raw Data." ERAU, Daytona Beach, FL, 5 Sept. 2018

[2] "NGS SURFACE GRAVITY PREDICTION." Home, NOAA, National Geodetic Survey, www.ngs.noaa.gov/cgi bin/grav_pdx.prl.

[3] Schumacher, Donald. "Ballistics: Application of Conservation Laws." ERAU, Daytona Beach, FL, 2 Sept. 2016. Reading.

Calculations

Method One: Ballistic Pendulum Launch

Law of Conservation of Momentum

$$\vec{p_i} = \vec{p_f}$$

Momentum Equation

$$\vec{p} = m\vec{v_1}$$

$$\vec{p_i} = m\vec{v_1} = (m+M)\vec{v_2} = \vec{p_f} \qquad (1)$$

$$m\vec{v_1} = (m+M)\vec{v_2} \qquad (1.1)$$

$$\vec{v_1} = \frac{(m+M)}{m}\vec{v_2} \qquad (1.2)$$

$$\frac{1}{2}(m+M)\vec{v_2}^{\,2} = (m+M)\,g\Delta h \qquad (2)$$

Solve 2 for $\vec{v_2}$

$$\vec{v_2} = \sqrt{2g\Delta h} \qquad (3)$$

Substitute 3 into 1.2

$$\vec{v_1} = \frac{m+M}{m}\sqrt{2g\Delta h} \qquad (4)$$

4 is the initial velocity equation for Method One

Method Two: Horizontal Projectile Launch

Kinematic Equation

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

Vertically: $\boldsymbol{\Delta x = \Delta y, v_0 = 0\text{m/s}, a = g}$

$$\boldsymbol{g = 9.79262\,\frac{m}{s^2}}$$

$$y = \frac{1}{2}gt^2 \qquad (5)$$

$$\frac{2y}{g} = t^2 \qquad (5.1)$$

$$t = \sqrt{\frac{2y}{g}} \qquad (5.2)$$

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$

$$\vec{v}'_1 = \frac{x}{t} \qquad (6)$$

Substitute 5.2 into 6

$$\vec{v}'_1 = x\sqrt{\frac{g}{2y}} \qquad (7)$$

7 is the initial velocity equation for Method Two

Equation 8, calculated on the right and equivalent to equation 7, was also used to find the expected horizontal launch distance

$$v_0 = x\sqrt{\frac{g}{2y}} \qquad (8)$$

Raw Data

Method 1, Pendulum Launch

M of pendulum is 274.5 ± .05 g

m of projectile is 67.2 ± .1 g

h max @ 17 cm

h min @ 14.5

$h_0 = 7.75$ cm

	height (cm)	Δh (cm)	$v_i = \frac{m+M}{m}\sqrt{2g\Delta h}$
T1	15.3 ± .05	7.55 ± .05	6.19 m/s
T2	15.3 ± .05	7.55 ± .05	6.19 m/s
T3	15.4 ± .05	7.65 ± .05	6.23 m/s
T4	15.3 ± .05	7.65 ± .05	6.19 m/s
T5	15.3 ± .05	7.65 ± .05	6.19 m/s
AVG	15.32	7.57	6.20 m/s

$= V_1$

$\delta_{v_1}^2 = 3.25 \times 10^{-4}$ m/s

$\delta_{v_1} = .018$ m/s

Method 2: Range Launch

$y_1 = 86.8 \pm .05$ cm

$y_2 = 3.75 \pm .05$ cm

$\Delta y = 83.05 \pm .05$ cm

$v_0 = x\sqrt{\frac{g}{2y}}$

$x = \frac{v_0}{\sqrt{\frac{g}{2y}}} = v_0\sqrt{\frac{2y}{g}} = 2.552\text{m}$

Expected distance 2.552 m

255.2 cm

Tests

	Actual Distance	Velocity (m/s)
T1	233.2 cm	5.666
T2	233.7 cm	5.679
T3	236.1 cm	5.737
T4	236.6 cm	5.749
T5	237.0 cm	5.759
AVG	235.32 cm	5.718

$\sigma_{v_1}^2 = .001807$

$\sigma_{v_1} = .0425$

Discrepancy: $|v_1 - v_1'| = .482$ m/s

Uncertainty of the Discrepancy: $\delta_d = \sqrt{\sigma_{v_1}^2 + \sigma_{v_1'}^2} = .00184$

Success? $d \leq 1.96 \cdot \delta_d \Rightarrow .482 \not\leq .00359$