MA448 – Homework (Set 4)

do by hand, with a calculator

Due: 10/22/2019

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1. Solve these BVPs by adjusting the general solution of the differential equation.

a. $x'' = x$, $x(0) = 0$, $x(\pi) = 1$ **b.** $x'' = t^2$, $x(0) = 1$, $x(1) = -1$ **c.** $x'' = -x$, $x(0) = \alpha$, $x(\pi) = \beta$

2. Determine the function $F(s) = \beta - \phi(s)$ (as described in the shooting method) explicitly in the case of the following BVP.

$$\begin{cases} x'' = -x \\ x(0) = 1,\ x(\frac{\pi}{2}) = 3 \end{cases}$$

3. Use the shooting method on this BVP and explain what happens:

$$\begin{cases} x'' = -x \\ x(0) = 3,\ x(\pi) = 7 \end{cases}$$

Note: This is to be solved analytically, not by computer or calculator.

4. Boundary-value problems may involve differential equations of higher order than 2. For example,

$$\begin{cases} x''' = f(t, x, x', x'') \\ x(a) = \alpha,\ x'(a) = \gamma,\ x(b) = \beta \end{cases}$$

Discuss the ways this problem can be solved using the shooting method.

5. If standard finite-difference approximations to derivatives are used to a two-point boundary-value problem with

$$x'' + x' - 2x = \cos t,$$

what is the typical equation in the resulting linear system of equations?

6. Write down the system of equations $\mathbf{Ax} = \mathbf{b}$ that results from using the usual second order central finite-difference approximation to solve the following BVP.

$$\begin{cases} x'' = (1 + t)x \\ x(0) = 0,\ x(1) = 1 \end{cases}$$

7. What finite difference equation should be a suitable replacement for the following equation in numerical work?

$$u_{xx} = u_x + u_t$$