MA345 Differential Equations & Matrix Method

Lecture: 02

Date: 8/28/2018

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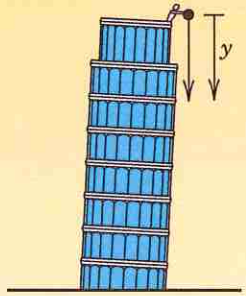
COAS.301.12

$y'' = 2$

Definition 1.1.1 Differential Equation

An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a **differential equation (DE)**.

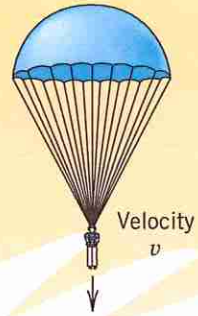
In order to talk about them, we will classify a differential equation by **type**, **order**, and **linearity**.



Falling stone

$y'' = g = const.$

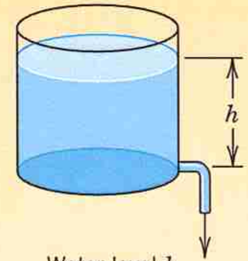
(Sec. 1.1)



Parachutist

$mv' = mg - bv^2$

(Sec. 1.2)

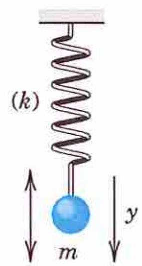


Water level h

Outflowing water

$h' = -k\sqrt{h}$

(Sec. 1.3)

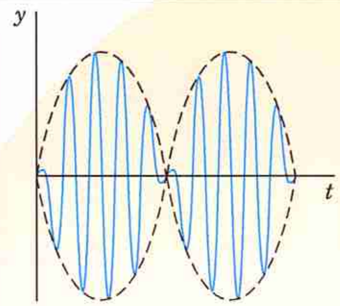


Displacement y

Vibrating mass on a spring

$my'' + ky = 0$

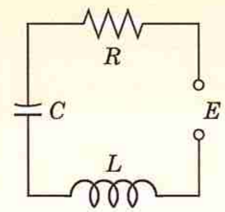
(Secs. 2.4, 2.8)



Beats of a vibrating system

$y'' + \omega_0^2 y = \cos \omega t, \quad \omega_0 \approx \omega$

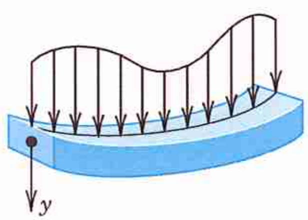
(Sec. 2.8)



Current I in an RLC circuit

$LI'' + RI' + \frac{1}{C}I = E'$

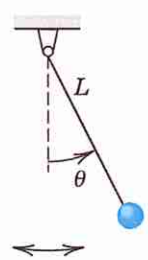
(Sec. 2.9)



Deformation of a beam

$EIy^{iv} = f(x)$

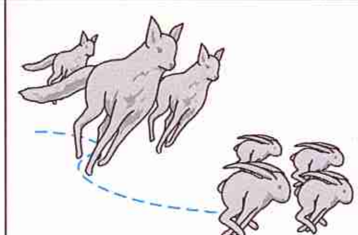
(Sec. 3.3)



Pendulum

$L\theta'' + g \sin \theta = 0$

(Sec. 4.5)



Lotka–Volterra predator–prey model

$y_1' = ay_1 - by_1y_2$

$y_2' = ky_1y_2 - ly_2$

(Sec. 4.5)

To begin our study of differential equations, we need some common terminology. If an equation involves the derivative of one variable with respect to another, then the former is called a **dependent variable** and the latter an **independent variable.**

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + kx = 0$$

$a, k \to$ constant coefficient

$$\frac{dy}{dx} e^x + \cos x \frac{d^2y}{dx^2} = 0$$

$\to$ coefficients variable

Definition 1.1.1 Differential Equation

An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a **differential equation (DE)**.

In order to talk about them, we will classify a differential equation by **type**, **order**, and **linearity**.

Classification by Type If a differential equation contains only ordinary derivatives of one or more functions with respect to a *single* independent variable it is said to be an **ordinary differential equation (ODE)**. An equation involving only partial derivatives of one or more functions of two or more independent variables is called a **partial differential equation (PDE)**.

Leibniz notation: $\frac{dy}{dx}, \frac{d^2y}{dx^2}$

prime notation: y', y'' $\qquad y'+6y=e^{-x}$

Newton's notation: $\frac{d^2s}{dt^2}=-32 \qquad \ddot{s}=-32$

$\frac{\partial^2 u}{\partial x^2}=u_{xx}$ $\qquad u_{xx}=u_{tt}-u_t$ / $u_{xx}+u_{yy}=0$

an ODE can contain more than one dependent variable

$$\frac{dy}{dx} + 6y = e^{-x}, \quad \frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 0, \quad \text{and} \quad \frac{dx}{dt} + \frac{dy}{dt} = 3x + 2y$$

(b) The equations

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - \frac{\partial u}{\partial t}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}. \tag{3}$$

are examples of partial differential equations. Notice in the third equation that there are two

Classification by Order The **order of a differential equation** (ODE or PDE) is the order of the highest derivative in the equation.

The differential equations

$$\overset{\text{highest order}}{\downarrow}\qquad\qquad\qquad\qquad\qquad\qquad \overset{\text{highest order}}{\downarrow}$$

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x, \qquad 2\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0$$

are examples of a second-order ordinary differential equation and a fourth-order partial differential equation, respectively.

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$(y-x)\,dx + 4x\,dy = 0 \qquad\qquad y' = \frac{dy}{dx}$$

$$4xy' + y = x$$

$$(y-x) + 4x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y-x}{4x}$$

General form:

$$F(x, y, y', \ldots, y^{(n)}) = 0$$

Definition 2.3.1 Linear Equation

A first-order differential equation of the form

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \qquad (1)$$

is said to be a **linear equation** in the dependent variable y.

When $g(x) = 0$, the linear equation (1) is said to be **homogeneous**; otherwise, it is **nonhomogeneous**.

A second-order ODE is called **linear** if it can be written

$$(1) \qquad y'' + p(x)y' + q(x)y = r(x)$$

and **nonlinear** if it cannot be written in this form.

The distinctive feature of this equation is that it is *linear in y and its derivatives,* whereas the functions p, q, and r on the right may be any given functions of x. If the equation begins with, say, $f(x)y''$, then divide by $f(x)$ to have the **standard form** (1) with y'' as the first term.

$y' + 2y = e^x$ → linear first order ODE

$\underbrace{y''} + (\cos x)\underbrace{y'} + x\underbrace{y} = 0$ → linear second order ODE

$\underbrace{(1-y)}\,\underbrace{y'} + 3\underbrace{y} = x^3$ → non linear first order ODE

non linear

$\frac{d^2y}{dx^2} + \underbrace{\sin y}_{\text{nonlinear function of } y} = 0$ → nonlinear 2nd order ODE