CS 222 Homework 4 DAVID JEFTS

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Write your name on this sheet. No name or cover sheet will miss 2 points

1. (25 pts) Prove by induction: **P**: $\forall n \in Z^+$, $(2c+1)^n$ is an odd number, where $c \in Z$
   P(n): $(2c+1)^n$
   Base Step - $P(n=1) = (2c+1)^1 = 2c+1 \Rightarrow$ this is an odd number
   Induction Step - Assume by Induction Hypothesis **P(n)** holds for all n.
   Prove - P(n+1)
   $(2c+1)^{n+1} = (2c+1)^n \times (2c+1)^1$
   $= oddnumber \times oddnumber$ will always produce an odd number.
   $\therefore$ P(n) holds by induction. ✓

2. (25 pts) Prove by induction: **P**: $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$
   P(n): $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$
   Basis Step - P(n=1) $= \frac{1}{2^1} = 1 - \frac{1}{2^1} \Rightarrow 1 = 1$
   Induction Step - Assume by Induction Hypothesis **P(n)** holds for all n.
   Prove - P(n+1)
   $\sum_{i=1}^{n} \frac{1}{2^i} + \frac{1}{2^{n+1}} = 1 - \frac{1}{2^{n+1}}$ $\quad\quad$ $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$

   $1 - \frac{1}{2^n} + \frac{1}{2^{n+1}} = 1 - \frac{1}{2^{n+1}}$

   $1 - \frac{1}{2^n} \times \frac{2}{2} + \frac{1}{2^{n+1}} = 1 - \frac{1}{2^{n+1}} \Rightarrow 1 - \frac{1}{2^{n+1}} + \frac{2}{2^{n+1}} = 1 - \frac{1}{2^{n+1}}$

   $1 + \frac{-1+2}{2^{n+1}} = 1 - \frac{1}{2^{n+1}} \Rightarrow 1 - \frac{1}{2^{n+1}} = 1 - \frac{1}{2^{n+1}}$
   $\therefore$ P(n) holds by induction. ✓

3. (25 pts) Prove by induction: **P**: $\forall n \in Z^+$, $8^n - 3^n \equiv 5k$
   P(n): $8^n - 3^n$ is divisible by 5.
   Base Step - $P(n=1) = 8^1 - 3^1 = 8 - 3 = 5 \equiv 5k$ ✓
   Induction Step - Assume by Induction Hypothesis **P(n)** holds for all n.
   Prove - P(n+1)
   $8^{n+1} - 3^{n+1} = 8^n \times 8 - 3^n \times 3 = (5+3) \times 8^n - 3 \times 3^n$
   $5 + 3(8^n - 3^n) = 5 + 3(5k) \equiv 5k$ $\quad\quad$ $8^n - 3^n \equiv 5k$
   $\therefore$ P(n) holds by induction. ✓

4. (25 pts) Use **strong induction** to show that you can run any number of miles given the following two facts:

   - You can run one mile or two miles.
   - You can always run two more miles once you have run a specified number of miles

   $P(k)$:
   $P(k) : k = 1m + 2n \Rightarrow k = \#$ of miles, $m = 1$ mile and $n = 2$ miles
   Basis Step -
   $P(k = 1) : k = 1, m = 1$ and $n = 0 \rightarrow 1 = 1(1) + 2(0) = 1\checkmark$
   $P(k = 2) : k = 2, m = 0$ and $n = 1 \rightarrow 2 = 1(0) + 2(1) = 2\checkmark$
   Induction Step -
   Assume by induction hypothesis that $P(1)$ to $P(k)$ is true.
   Prove P(k+1):
   Induction
   $(k + 1) > 2$
   $(k + 1) - 2 > 2 - 2 = 0$
   $(k + 1) - 2 > 0$
   Proof:
   $(k + 1) - 2$ is within the range of $P(1)$ to $P(k)$

   $(k + 1) - 2 \leq 1m + 2n$
   $(k + 1) - 2 \leq k$
   $k - 1 \leq k\checkmark$