CS 222 Homework 1 [100 Points Total] David Jefts

Online Submission via Canvas Only! If you are not able to produce a PDF version, you can scan or take picture of your homework for submission. No paper submission will be accepted.

Write your name on this sheet. No name or cover sheet will miss 2 points
Write clearly, show all work, partial credits are your friend!

1. (30 pts) Given **P**: $\neg(\neg p \vee \neg q \vee \neg(r \vee \neg r)) \equiv p \wedge q$

   (a) Prove **P** by truth table.

p	q	r	$\neg p$	$\neg q$	$\neg(r \vee \neg r)$	$(\neg p \vee \neg q \vee \neg(r \vee \neg r))$	$\neg(\neg p \vee \neg q \vee \neg(r \vee \neg r))$	$p \wedge q$
T	T	T	F	F	F	F	T	T
T	T	F	F	F	F	F	T	T
T	F	T	F	T	F	T	F	F
T	F	F	F	T	F	T	F	F
F	T	T	T	F	F	T	F	F
F	T	F	T	F	F	T	F	F
F	F	T	T	T	F	T	F	F
F	F	F	T	T	F	T	F	F

   (b) Prove **P** by using the Laws of Logic.

$$\begin{aligned} \neg(\neg p \vee \neg q \vee \neg(r \vee \neg r)) & \quad \text{Negation Law} \\ \equiv \neg(\neg p \vee \neg q \vee \neg T) & \quad \text{DeMorgan's Law} \\ \equiv \neg(\neg p \vee \neg q \vee F) & \quad \text{Identity Law} \\ \equiv \neg(\neg p \vee \neg q) & \quad \text{DeMorgan's Law} \\ \equiv p \wedge q & \end{aligned}$$

2. (20 pts) Are $(p \implies q) \to r$ and $p \to (q \to r)$ logically equivalent? Show steps to support your conclusion.

$(p \to q) \to r$	1	$p \to (q \to r)$
$(p \to q) \to r$	2	$p \to (q \to r)$
$(\neg p \vee q) \to r$	3	$p \to (\neg q \vee r)$
$\neg(\neg p \vee q) \vee r$	4	$\neg p \vee (\neg q \vee r)$
$(p \wedge \neg q) \vee r$	5	$(\neg p \vee \neg q) \vee r$
$(p \wedge \neg q) \vee r$	$\neq$	$\neg(p \wedge q) \vee r$

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to q) \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

3. (30 pts) Let p and q be the propositions: **p**: I bought a lottery ticket this week; **q**: I won the million dollar jackpot. Express each of these propositions as an English sentence.

   - $\neg p$
     - I did not buy a lottery ticket this week.
   - $p \vee q$
     - I bought a lottery ticket this week or I win the million dollar jackpot.
   - $p \rightarrow q$
     - If I buy a lottery ticket then I will win the million dollar jackpot.
   - $p \wedge q$
     - I bought a lottery ticket this week and won the million dollar jackpot.
   - $p \leftrightarrow q$
     - I will win the million dollar jackpot if, and only if, I buy a lottery ticket.
   - $\neg p \rightarrow \neg q$
     - If I do not buy a lottery ticket this week, then I will not win the million dollar jackpot.

4. (20 pts) Given the truth table, construct the its logical formula using true rows. Can you simplify your formula? If so, how?

p	q	Output
T	T	T
T	F	T
F	T	F
F	F	F

$[(p \wedge q) \vee (p \wedge \neg q)] \equiv p \wedge (q \vee \neg q) \equiv p \wedge T \equiv p$