RC Circuits

PS253 — Physics III Laboratory for Engineers
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The diagram below (not drawn to scale) shows a very simple circuit consisting of a resistor and a capacitor along with a DC power supply, Digital Multimeter (DMM) and a computer equipped with a Pasco data acquisition interface. The DC power supply is used to charge the capacitors. The computer records the voltage from the control of Capstone software. Capstone allows the user to select the time interval between voltage measurements and stores the data. The data can then be imported into an Excel spreadsheet as a text file. Excel provides the means for the computational and graphical analysis of the voltage and time data.

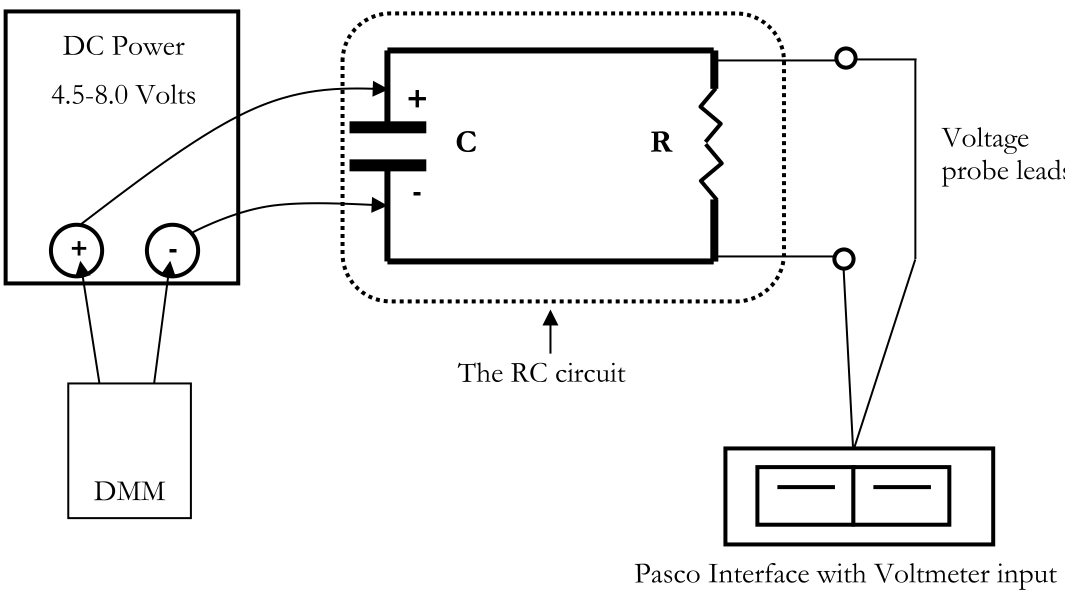

Figure 1: Schematic diagram for charging a capacitor as part of a simple RC circuit. The DMM at the power supply can be used to more precisely monitor the supplied voltage. Once charged the power supply is quickly removed from the circuit leaving the capacitor to discharge across a resistor while a voltage probe monitors and records the decaying electrical potential.

After starting to record voltage using *Capstone*, the DC power supply is disconnected and the charge on the capacitor 'drains' through the resistor. As the charge equilibrates, the capacitor voltage continually decreases. Theoretically, this process follows an exponential decay function:

$$\boxed{V = V_o e^{-t/RC}} \qquad (1)$$

V_0 is recorded at t=0, defined in our experiment as the first voltage recorded *after the power was disconnected*, *R* is resistance, and *C* is capacitance. The combination of *RC* is a special item known as the time constant, many times represented by the Greek letter Tau: $\tau = RC$. The time constant is defined as the time it takes for a capacitor to discharge until approximately 37% of V_0 is remaining. The time constant mathematically comes from the special case when the time *t* is equal to *RC*, so Equation (1) then has $V_o e^{-1}$ which is $V=0.368V_0$. In physical terms we can think of the time constant as a rough measure of how fast the capacitor will charge or discharge.

Equation (1) may be derived by integrating the expression for Ohm's law considering the time varying voltage provided by the capacitor to a constant resistance. Let $q(t)$ represent the charge on the capacitor as it drains out through the resistor, let $V(t)$ represent the voltage across the capacitor, and let $i(t)=dQ/dt$ represent the current flowing through the resistor during this process. The rate at which the capacitor loses charge is equal in magnitude but opposite in sign to the current through the resistor. This is because each unit of charge lost from the capacitor contributes to the current flowing through the resistor.

Then consider the definition of capacitance in terms of the charge it holds q and the voltage across it V:

$$q(t) = CV(t) \tag{2}$$

Next, we write Ohm's law as applied to the resistor solving for the current through the resistor and equate this with the definition of current, which is a change in charge over time at a specific location:

$$i(t) = \frac{V(t)}{R} = \frac{dQ}{dt} \tag{3}$$

Since the capacitor charge decreases to produce current for the resistor we have:

$$\frac{dQ}{dt} = -\frac{dq(t)}{dt} = -\frac{d}{dt}CV(t)$$

and substituting this into Equation (3) we obtain:

$$\frac{V(t)}{R} = -\frac{d}{dt}CV(t)$$

Assuming the capacitance stays constant with time we then separate the voltage and time variables.

$$\frac{1}{V(t)}dV(t) = -\frac{1}{RC}dt \tag{4}$$

It is left to the student to integrate Equation (4) from voltage V_0 to V and from time 0 to t to directly obtain Equation (1). You should also use different substitutions but a similar method and an additional step to find current (I) as a function of time.

$$\boxed{I = I_o e^{-t/RC}} \tag{5}$$

Safety Precautions

⚠ **WARNING** Be very careful when connecting the DC power supply to the capacitor. The large round capacitor is a polarized capacitor and must be charged in a certain way. The *Positive terminal* of the capacitor has white tape across it and MUST be connected to the power supply terminal marked positive (**+**). The *Negative terminal* of the capacitor must be connected to the terminal of the power supply or battery marked negative (-). Attempting to charge with the connections reversed can lead to catastrophic failure of the capacitor.

⚠ **WARNING** Always treat a capacitor as if it is CHARGED when you handle it! There is no way to tell how much charge a capacitor has stored just by looking at it, so to be safe assume it is fully charged. They can hold charge for varying lengths of time if no load is applied to them. To test you can try discharging it across a light bulb or resistor with the proper power rating for the given capacitor.

⚠ **Caution** Some smaller capacitors are also polarized just like the large disk capacitor. These will look something like the one in **Figure 2**. Notice the positive (+) sign on the right side of the capacitor, this indicates the wire lead below it and to the right is the *positive lead*. This lead MUST connect towards the positive terminal of the power supply. Attempting to charge with the connections reversed can lead to catastrophic failure of the capacitor.

The top number indicates the capacitance in micro-Farads [μF] while the bottom number gives the maximum operating voltage in Volts [V]. Other polarized capacitors might indicate the negative terminal rather than the positive terminal, these will generally have a large dark negative (-) sign on one side. If there are no clear markings indicating a (-) or (+) polarized terminal you can assume it is safe to connect the capacitor in any orientation.

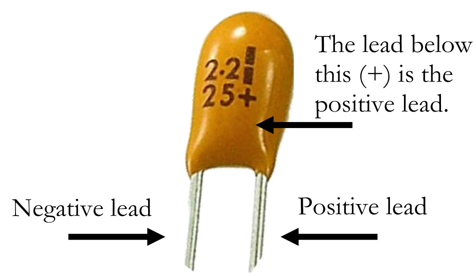

Figure 2: Example 2.2μF polarized tantalum capacitor with max 25V operating voltage.

⚠ **WARNING** Always read the labels on capacitors before inserting them into a circuit, especially to determine their safe operating voltage. If a capacitor is pushed beyond its labeled operating voltage it will degrade over time and without warning could explode. Polarized capacitors will explode if they are connected in reverse and experience voltages anywhere from 30-70% of their designed forward voltage. Capacitors come in many different sizes, operating voltages, and materials- some far more dangerous than others.

Qualitative Demonstration of the RC Circuit

1. Connect the power supply and the 1.0F POLARIZED capacitor together to charge the capacitor to 4.5V.

⚠ **WARNING** Do not exceed 4.8V.

2. Disconnect the power supply and then carefully connect a low voltage light bulb, similar to a flashlight bulb, to the capacitor and observe how the bulb intensity decays fast at first and then glows as a small red filament for a much longer time. This qualitatively

demonstrates the characteristic exponentially decaying voltage of the capacitor as the light bulb, acting as a load resistor, bleeds charge off the capacitor.

Quantitative tests of the RC circuit

1. Select a ***22kΩ*** resistor and use a DMM to measure and record the exact resistance of the resistor. The resistor must not be connected to the power supply or to the capacitor yet! Why?... This resistance will be used in all your calculations, the nominal manufacturer rating is a less reliable estimate.
2. The following procedures are written using ***3.3μF*** capacitors as examples. The actual capacitors at your lab station may have other ratings. Look at the labels on the capacitors and record the manufacturer ratings printed on them both for maximum operating voltage and reference capacitance.

⚠ **Caution** Always be very mindful of the maximum voltage rating as you should stay well below this value at all times. Also identify if the capacitor is polarized or not and remember all the safety precautions. If you are at all unsure, please ask your instructor for help.

3. Use a DMM set to the 20V measurement dial position to monitor the voltage of the power supply. Connect the circuit as shown in **Figure 1**. While constructing, note that you will need to quickly physically disconnect the power supply from the circuit. See your instructor if you need help in properly completing the circuit.
4. Calculate the expected time constant, τ_{ref} of your RC circuit.
5. In this lab, you will use a Voltage sensor plugged into Channel A of the Pasco box. Open the application *Capstone*. When *Capstone* opens, in the left-side menu, click on the *Hardware Setup* button to open a window where you can add your sensor.
6. Click on *Channel A*. Scroll down and select *Voltage Sensor* from the menu that appears and select it. Close the *Hardware Setup* window.
7. In the bottom-menu bar of *Capstone*, set the *Sample Rate* to at least 200 Hz. This will take trial and error depending on the time constant of your RC Circuit and how fast it discharges. The data will be used for graphical analysis; at least 9-11 data points would make a good minimum sample.
8. In the central display area select the option *Table & Graph*. Select the *Table* and set it to display Voltage and Time columns. Select the *Graph* and set it to display Voltage on the y-axis and Time on the x-axis.
9. Turn on the power supply and slowly increase the voltage until it reaches about 4.8V; monitor the power supply voltage with a DMM. The Pasco interface and software can only record a maximum input of 5.0V so there is nothing gained by adding more voltage.

⚠ **WARNING** Do not exceed the operating voltage of your capacitors.

10. Click on the *Record* button at the bottom of the screen to start recording voltage data.

    Now, physically disconnect the power supply from the capacitor so that the power supply is no longer a component of the remaining complete circuit. If you just turn the power supply off, your data will be dramatically incorrect. The voltages are displayed on the computer screen as they are recorded. When the voltage drops to less than ~0.5 V, click on the *Stop* button.

11. Compare your τ from Step 4 to the discharge time of your capacitor. Remember that the time constant is the amount of time it takes for voltage to drop 63% from some initial value. If they differ by orders of magnitude there was a problem and you need to redo the trial. Consult your instructor if you are having problems.
12. For your report's *Theory and Methods* section make a schematic circuit drawing for the circuit after the power supply is disconnected but with the voltage probe included. Indicate the rated capacitance and the measured resistance values printed neatly upon it.
13. Repeat Steps 3-12 for a circuit with *two identical capacitors in series* and again for *two identical capacitors in parallel*. Each trial will save as a different "run" in *Capstone* under the relevant data areas.
14. In your dataset, locate the first voltage that shows a drop after the power supply was removed. It will be selected as V_0 for the experiment. You only want the data values that are changing. Points prior to V_0 will be thrown out as well as all values lower than 0.5V. Eliminate the same points in your "time" column, and subtract the first time t_0 from all time measurements so that t=0 corresponds with the first voltage drop. Data can be quickly selected and deleted in *Capstone* from the *Graph* window using the selection tool in the *Graph* menu.

    Incidentally, since it is an arbitrary choice you could choose any voltage, even voltage after it has dropped significantly, and the relationship will still be preserved. However, you do not want to throw away valid data, only data that is irrelevant.
15. Divide each of your voltages by V_0 to obtain V/V_0. After this is done take $\ln(V/V_0)$.
16. Now make a graph of $\ln(V/V_0)$ vs t. On the graph add a trendline, or line of best fit. Remember that in Excel you can use the *Regression* routine to find both the slope and uncertainty in the calculated slope to an appropriate precision. Record the slope and its uncertainty.

    Note: Go back and look at Equation 1, and see how the exponential form can be turned into a linear equation, which can be easily plotted, using natural logs.
17. Repeat Steps 14-16 for the other two runs as well. Those runs correspond to the series and parallel capacitor configurations. Make sure your graphs and data are properly captioned in your *Results* and *Plot Data* sections respectively.

Further Analysis for Your Report

1. Your results section should include both a voltage vs. time (pick one run), and three $\ln(V/V_0)$ vs. time plots (one for each different capacitor circuit).
2. The ultimate check of each test is to compare the slope of the natural log graphs against the theoretical value of $-1/RC$; where R is the value determined from the multimeter ohm measurement and C is based upon the manufacturer's rating (and your calculation if two were used in series or parallel). The shape of the voltage graphs should also be

considered– do they suggest a decaying exponential? Are the data points following a smooth trend, or is there a great deal of scatter in the data? Explain in your report.

3. The capacitance rating given by the manufacturer is only a ballpark estimate (it could easily vary by $\pm 5\%$ and there may be temperature variation on top of that), whereas the resistance value is fairly stable and reliable to $\pm 1\%$. Another way to put this data to good use is to solve $slope = -1/RC$, for *C*. This value of *C* may be much more reliable than the one used from the manufacturer. Find the % difference between the rated capacitance value and the value of *C* from the slope of the graph for each of your trials. Put the results for the three different circuit trials in a summary table in your *Results* section of the report.
4. In your *Theory and Methods* section, state and explain the formulas for equivalent capacitance and explain why *parallel* capacitances are added together and the inverse of the total capacitance is equal to the sum of the inverses of *series* capacitances.

RC Circuit Guidelines

- Include signed raw data sheet, or raw data with your report
- One column abstract, two column for everything else
- Correct units and sig-figs on all numbers
- Cite any item taken from the lab module
- Cite any work taken from anywhere else
- **Abstract:** Very briefly describe the experiment, state goals, summarize results including experimental values, expected values, and percent differences or standard errors, state degree of success
- **Background:** Context, purpose, why doing experiment, state goals, give background and historical info, explain any deviations from procedure, applications
- **Theory & Methods:**

  Theory: Explain the equations for adding capacitors in series or parallel. Explain the voltage discharge equation for an RC Circuit. State any assumed constants including their values and sources. State assumptions or limits of theories applied.

  Methods: Overview of the general process of how data was taken; include circuit diagrams for your 3 constructed circuits. Anything important to point out or clarify about the procedure and equipment's setup/operation *including* sensors. Particular attention to calibrations (if any). Diagrams or photos of setup could help here.
- **Results:** Include and discuss your results to the steps listed under *Further Analysis for Your Report*, discuss at least two likely sources of uncertainty.
- **References:** Proper format as seen in Good lab report and Example Reports.
- **Calculations:** *One complete example of each equation used, should show steps needed to get from data to final result(s).*
- **Plot Data:** Data used for any plot in two column format with correct units and sig-figs

These are just guidelines. You may need to add more items as these are just some of the things expected.