Adiabatic Processes: Sensor Calibration, Work, and the Adiabatic Index γ

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In these experiments a gas in a clear plastic cylinder is smoothly compressed by hand in half a second. Over this short period the gas (being a good insulator) in the cylinder has little time to come to equilibrium with the surrounding room, so the data closely approximates the behavior of an adiabatic process. During compression the pressure of the gas goes up, the temperature increases as well, and of course the volume decreases. The apparatus employed in this experiment has three sensors that provide voltage signals at a very high sampling rate; these can be converted to obtain temperature, pressure, and volume. Thus the system provides a means for recording P, V, and T values at a rate that matches the required speed to explore in detail the characteristics of the gas compressed under nearly adiabatic conditions.

Theoretical Analysis of C_P/C_V

The first law of thermodynamics states that the change in internal energy of any system is equal to the sum of the heat added/subtracted and the work done by/on the system. If we define our system such that the standard change in internal energy is to add heat to the system and to let work be done by the system then the first law can be stated as:

$$dU = dQ - dW \tag{1}$$

Note that in this case the number of particles inside the system must remain constant; if the particle count changed an extra term *dn* would need to be included. Solving Eq. 1 for the change in heat and applying it to a system in adiabatic conditions ($dQ=0$) we obtain:

$$dQ = dU + dW = 0 \tag{2}$$

By knowing the following simple thermodynamic definitions we can reformat Eq. 2 so that it contains only variables that we may observe and record directly in lab:

$$dW = PdV \tag{3}$$

$$dU = C_v dT \tag{4}$$

$$PdV + VdP = d(PV) = nRdT \tag{5}$$

Eq. 5 is the differentiated form of the ideal gas law with *n*, particles in moles, held constant.

First, Equations 3 and 4 are substituted into Eq. 2. Then Eq. 5 is solved for dT and substituted in as well:

$$dQ = C_v dT + PdV = C_v \frac{1}{nR}(PdV + VdP) + PdV = 0 \quad (6)$$

Simplifying further and knowing that $C_p = C_v + nR$,

$$(C_v + nR)PdV + C_v VdP = C_p PdV + C_v VdP = 0 \quad (7)$$

Finally, separating the variables and integrating yields:

$$\int \frac{1}{P} dP = -\frac{C_p}{C_v} \int \frac{1}{V} dV$$

$$\boxed{\ln P = -\gamma \ln V + C'} \quad (8)$$

Where C' is an integration constant and gamma (γ) is the adiabatic index of the gas defined as the ratio between a gas's heat capacity at constant pressure and its heat capacity at constant volume.

$$\gamma = \frac{C_p}{C_v} \quad (9)$$

Heat capacity is a measure of how much heat is needed to raise a given material's temperature by 1°C. To understand why the two heat capacities have slightly different values here is an example. Say a closed cylinder holds an ideal gas and there is a piston on one end of the cylinder. Case 1: Heat the gas in the cylinder while keeping the piston locked in place- constant volume. All the heat applied to the gas in the cylinder goes directly into an increase in temperature. Case 2: Now, heat the same gas in the same cylinder but allow the piston to move freely. Following the ideal gas law, the applied heat causes the gas to both increase in temperature and increase in volume. The gas will increase in volume to keep the contents of the cylinder at the same pressure as the external room pressure- constant pressure.

However, for the gas to expand it must perform work on the piston to make the cylinder volume larger, this consumes some of the applied heat. In this second case, extra heat must be applied to raise the gas the same 1°C as in the constant volume case, since under constant pressure it must use up some of the heat as work on the piston. In this way $C_p > C_v$ for all cases and $\gamma > 1$ always.

Equation 8 is in the standard $y=mx+b$ format and allows for experimental determination of γ. A set of P and V values collected while a gas sample is compressed/expanded results in a plot of ln(P) versus ln(V) with a slope equal to -γ.

Since we have temperature data we can use it to obtain a quasi-independent second estimate of γ. The ideal gas law is used to eliminate P and introduce T as a variable. The result is:

$$\boxed{\ln T = -(\gamma - 1)\ln V + C''} \quad (10)$$

where $C''=C' - ln(nR)$. Then a plot of ln(T) versus ln(V) has a slope equal to 1-γ. Note that with linear fits to data for both Eq. 8 and 10 you could estimate n, the moles of gas present.

Equation 8 can be rearranged to obtain several forms that are commonly seen in texts: $lnPV^{\gamma}=C'$; taking $ln^{-1}(lnPV^{\gamma})=ln^{-1}(C')$; then $\boldsymbol{PV^{\gamma}=C''}$ where $C'' = ln^{-1}C'$

Theory of the Work Done in Adiabatic Compression

The general expression for work is the integral of the dot product between a force vector and a displacement vector from a starting position S_i to a final position S_f, we have:

$$W = \int_{S_i}^{S_f} \vec{F} \cdot d\vec{S} \tag{11}$$

In our apparatus, the force is always parallel to the displacement, which is equal to the volume of the cylinder divided by its area. Furthermore, the force is equal to the pressure in the cylinder multiplied by its area. Taking these into account, the work integral is greatly simplified and takes the following form for our application in this experiment:

$$W = \int_{V_i}^{V_f} PdV \tag{12}$$

The measurement devices we use in this experiment provide us with a hundred or more pairs of pressure and volume values per second as the piston compresses the gas from V_i to V_f. Therefore, with little approximation error, we can use the trapezoidal rule to calculate the work integral directly from the data. The general expression, in terms of the trapezoidal rule, for work in the k^{th} element of the total work integral in compressing the gas from data points (P_k,V_k) to (P_{k+1},V_{k+1}) is:

$$\boxed{W_k = \frac{(P_{k+1} + P_k)}{2}(V_{k+1} - V_k)} \tag{13}$$

Note that all of the work elements will be negative, since the volume shrinks as we compress the gas. <u>From the point of view of the gas, work is being done on it</u>. This makes sense following Eq. 1 as it indicates the internal energy of the gas would increase if we apply work.

The computation of a hundred or more W_k results may be greatly simplified using spreadsheet operators directly on columns of P and V data to automate the computations. Then, using the spreadsheet, it is an even simpler matter to find the sum of the W_k*'s* to find the total work for all the n pairs of pressure and volume data values. Note that n pairs of pressure and volume data yield $n-1$ work terms, <u>NOT n</u>, from $k=1$ to $k=n-1$.

$$\boxed{W_{total} = \sum_{k=1}^{k=n-1} W_k} \tag{14}$$

We will compare W_{total} to a theoretical approximation that was derived assuming perfect adiabatic conditions. Use the following equation for the theoretical estimate of the work done:

$$W_{theoretical} = P_1 V_1^{\gamma} \frac{\left(V_n^{1-\gamma} - V_1^{1-\gamma}\right)}{1-\gamma} \quad (15)$$

In Eq. 15, the subscript "1" refers to the first pressure and volume measurements in the interval that we selected from the data set, and the subscript "n" refers to the last volume measurement in the subset of the data that we selected. Only data collected during an adiabatic process should be included when solving these work equations.

Calibrate the Pressure, Volume, and Temperature Sensors

Locate the Pasco interface box and identify the sockets labeled "A", "B", and "C". Trace the wires from the device to identify the 5-pin DIN "Pressure" sensor plug and connect it to the "A" channel. Identify the "Temperature" sensor plug and connect it to the "B" channel, and connect the Volume sensor plug to the "C" channel.

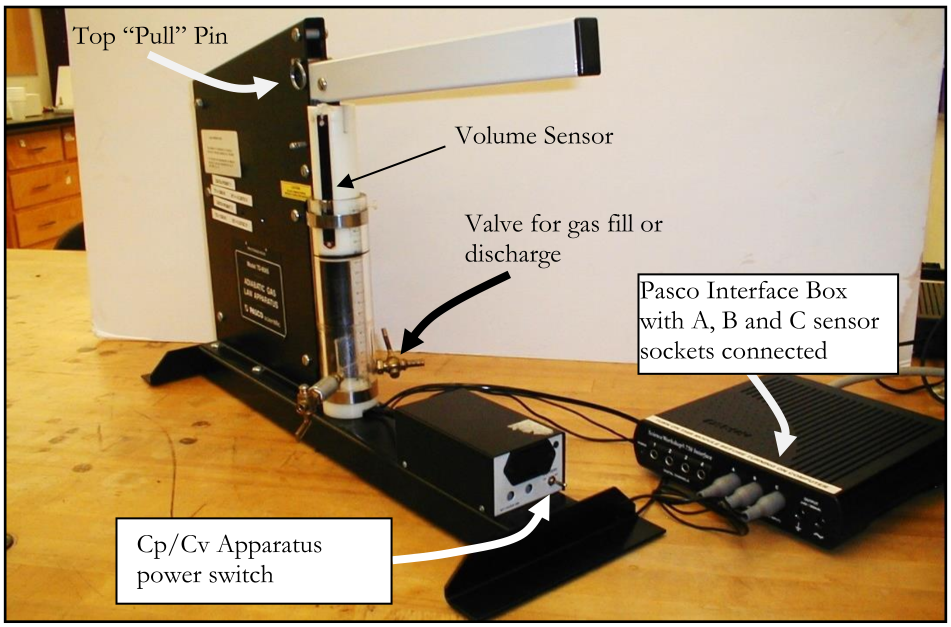

Figure 1: Adiabatic Compression Apparatus and Pasco Interface

1. Make sure your machine is receiving power. Older machines have a 9V battery and power switch on the apparatus; the LED is lit if the power is turned on. Newer machines plug directly into an outlet and are powered on all the time.
2. Using the PC connected to the Pasco interface by USB, download the Capstone file from the course Canvas page for this experiment named *Adiabatic_PS253.cap*. After making sure the compression apparatus is powered, the sensors are connected to the Pasco interface box, the interface is powered, and the interface is connected to the computer, locate the downloaded file and open it with the Capstone software.
3. On the Cp/Cv device, open one of the valves on the clear plastic cylinder so air can freely go in and out as you raise and lower the piston lever. Identify two steel pins with

key rings and verify that they are in place to limit the highest and lowest positions of the lever and piston.

The adiabatic compression apparatus has electronic pressure, temperature, and volume sensors that when connected to the Pasco Interface produce variable voltage signals. The sensors are designed to provide variable resistance or capacitance in a circuit which is measured across an output voltage. The circuit includes a power source that provides a stable voltage and current for the sensors and amplifiers. The amplifiers magnify the output voltages to a level that meets the input power requirements for the Pasco Interface which converts the varying voltages into binary inputs for the computer. One of the benefits of this design is that the amplified sensor outputs will overwhelm most electrical noise that is picked up over the connecting cables since these act as antennas in addition to being electrical wires connecting components.

The sensor output voltages are approximately linearly proportional to the actual volume, temperature, and pressure values. Exact proportionality is not achieved over all possible values of the physical property of interest. In this work, we assume that a single linear equation is sufficiently accurate to translate the sensor outputs to the values of a physical quantity over the ranges that we will measure. We will not explore the accuracy of our linear calibration model due to limited lab time.

For more information on the properties of the individual sensors see Appendix I.

Calibrate the Sensor Voltages to Express Physical Values

Because the sensors are constructed to deliver generic voltage outputs to the interface box, the box and software have no idea what physical value is associated with the voltages it is receiving. You must tell the software what variables our three sensors are actually measuring. This is accomplished by setting up calculation equations in the software that are fed by the incoming voltages.

4. In the Pasco Capstone software, locate the *Calculator* button on the left-side menu and click it to open the *Calculator* window.

There should already be three calculations preset, one for each sensor channel. Each of these equations is a linear equation that converts an input voltage into an output physical variable with the proper indicated units.

5. Check one last time that the proper sensor cables are connected to the correct channels by reading the equations. For example: Pressure is calculated from the Voltage, Ch.A.
6. The Pressure sensor is manufactured and designed in a way that makes it very uniform across all machines. Do not adjust the Pressure calculation.
7. The Temperature and Volume equations must both be adjusted to the specific apparatus at your station. There should be labels on the side of your apparatus that indicate calibration equations for both Temperature and Volume. Use these to change the calculation equation slope ‘m’ and intercept ‘b’ terms.

   Be careful that you are inputting numbers that will result in the proper units as displayed in Capstone. For example, the labels give volume in [m^3] but the software displays [cm^3].
8. Close the Calculator window when you are finished making the adjustments.

Test your Calibrations

9. WITHOUT MOVING THE PISTON and with one of the valves open, click *Record* and after a second or two click *Stop*. Your volume should be ~140-200cm^3, your pressure should read ~101kPa and your temperature should read ~298K.
10. If these are not correct ask the instructor for help. If these are all correct then proceed.

Experimental Procedure

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12. Prime the cylinder by removing the top pin and pulling the piston all the way up to the top. Then, close both valves on the clear plastic chamber. Next, push the piston barely below the top pin, replace the pin, and check to make sure the cylinder holds pressure.
13. In the bottom menu, make sure the sample rate is set to Common Rate: 500Hz.

!Caution! When compressing the piston *DO NOT* slam it down hard, while the compression should be done quickly it only covers a few centimeters distance. Using too much force could damage the sensors or the apparatus or cause it to start leaking.

14. With the cylinder primed and resting against the upper pin:

    Press the *Record* button, then pull/push the handle down *smoothly and uniformly* taking about 0.4-0.8s to complete the compression and continue holding the piston against the bottom pin.

    Do not stop collecting data yet.
15. While you are holding the piston down, watch the live graphs on the computer screen. You should notice the temperature is very slowly returning to the initial room temperature. Continue holding the piston against the bottom pin until the temperature has mostly stabilized (waiting at least 10 seconds after you compressed the gas). Then release the piston so that it returns to resting against the top pin.

    Do not stop collecting data yet.
16. Again, wait for at least 10 seconds for the temperature to stabilize. Then click *Stop* to stop collecting data.
17. Review the data that you just collected. The compression and expansion strokes should take 0.4-0.8 seconds, and the compression/expansion should appear completely smooth and uniform. Also the *Otto Cycle Display* should indicate a complete cycle from start to finish. If all of this is true, then you may proceed to the next step. If not, repeat the cycle until you obtain a dataset meeting the above criteria.

    Wait a minute or two between trials to allow the system to come back into the same starting equilibrium temperature with the room environment. It is also a good idea to prime the cylinder between trials in case gas has leaked out from the previous trial.
18. Change the Capstone active sheet from the Plots sheet to the DataTables sheet (small tabs above main window). Adjust the decimal precision of each column so values are displayed to two decimal places. Export the data by clicking in the Table, clicking *ctrl+A*, then *ctrl+C*. Open Excel and *ctrl+V* paste the data into Excel on an empty page.

19. Save the Excel file.
20. To help ease your analysis later, it is a good idea to record from your Capstone plots the time just after both compression and expansion began as well as the time just before both compression and expansion ended. There is a helpful *Coordinate* button in the Graph toolbar for this.
21. Perform three successful trials repeating Steps 12-20. Each trial should be saved to its own page within a single Excel file. When finished saving new data make sure the Excel file is sent to all of your lab partners.
22. Should you have lots of time remaining in lab, each lab partner should begin their own data analysis using the lab computers and laptops. At the very least, you should edit your data set to separate out the relevant compression/expansion data from the data you will ignore in your calculations.

For your Analysis, Results, and Calculations

The following may seem somewhat arbitrary, but if you understand the theory, then you will realize that it is entirely logical and is a good strategy for minimizing the error due to heat losses or failure to meet the assumed perfect adiabatic condition. For your analysis you are only interested in the data *during* compression/expansion.

23. Begin by focusing on the compression stroke for the first trial. Identify the region in your data where the pressure is clearly beginning to increase. Select a point just after compression clearly started and define that as P_1; the Volume and Temperature points taken at the same time will be V_1 and T_1.
    a. Identify the region in your data where the pressure has reached its maximum. Select a measurement slightly before this, where the pressure is clearly increasing but before the maximum. Define this point as P_n, the end of the compression data set. The volume and temperature points taken at the same time will be V_n and T_n. You should have several hundred recorded P, V and T measurements between P_1 and P_n and also between V_1 and V_n and between T_1 and T_n.
    b. Using this data for the compression stroke, include a plot of lnP vs. lnV for the trial's compression stroke. The term "adiabatic" must appear in the caption description. Add a linear trendline to your plot and refer to the earlier theory to compute gamma (γ) from the slope. Show 3 significant figures on your γ.
    c. Using this data for the compression stroke, include a plot of lnT vs. lnV. The term "adiabatic" must appear in the caption description. Add a linear trendline to this plot and refer to the earlier theory to determine a second γ from this plot. Show 3 significant figures in the slope.
    d. Repeat Steps a-c for the expansion stroke of trial one. Note that the expansion stroke should include data starting just after volume is starting to increase and end just before volume reaches its maximum.
24. Repeat Step 23 for the other two trials.

You should now have four plots and four γ values for each of the three trials for a total of 12 plots and 12 experimental γ values. Consider these labelled and captioned plots with learn fits as your *Raw Data* for your report. Make sure to include them with your report; do not include the 100s of data columns from Excel. Due to the sheer amount of data collected, this is a rare case where you are not required to show all data- the plots will stand in for the massive data tables.

25. In your *Results* section, create a summary table that has columns for trial #; experimental γ from compression P,V data; experimental γ from compression T,V data; experimental γ from expansion P,V data; and experimental γ from expansion T,V data. The rows should be Trial #1,2,3, a row for average across all three trials in that column, and a final row for the difference between each average and a reference γ value.

    Each average γ should include an uncertainty, calculated from the standard deviation of the three trial γ's averaged into it. The caption of the summary table should include a properly cited reference value of γ for air. Identify the conditions, if any, which apply to the reference value as stated by the reference source. You must research to find your own appropriate reference source for the adiabatic index of air.

26. Compute the experimental work done during each trial's compression stroke from the sum of the work elements (Equations 13,14 on p.3). Limit your data to the same data you used to make the compression stroke plots in Step 23.a.
27. Compute the experimental work done during each trial's expansion stroke from the sum of the work elements (Equations 13,14 on p.3). Limit your data to the same data you used to make the expansion stroke plots in Step 23.d.
28. Using your experimental average γ values from Step 25, compute theoretical values of the work done in both compression or expansion for each of your three trials (Equation 15 on p.4).
29. For each trial's compression and expansion strokes, calculate the percent difference between your experimental and theoretical work.
30. Similar to Step 25, in your *Results* section, include a summary table for the experimental and theoretical work results and your percent differences between the two.

For your Discussion of Results

In your discussion of your *Results*, comment on how well you determined the adiabatic index γ of air. Did each method, (P,V) and (T,V) data or compression and expansion data, appear to give equally consistent and/or valid results?

In your discussion of your *Results*, comment on how well the amount of work done on/by the gas matched between the two calculation methods. Did the calculations agree better/worse/same for compression and expansion strokes? Did you get the correct sign (-/+) for your work depending on if it was a compression/expansion? One of the work calculations needed to use your already experimental value of γ; taking that into account, do you think your work analysis validates your experimental γ values or puts them into question?

Appendix I: Sensor Physics and Electronics

The actual pressure, volume, and temperature sensor components in our device do not directly produce a voltage. Each sensor is one component in an electrical circuit that responds to the change in the sensor by producing a voltage. The fundamental physics of the sensors is outlined below.

The volume sensor is a variable resistor. Its resistance decreases linearly as the volume decreases. To explain how this works first locate the dark strip running down the side of the piston. That strip is coated with a conductor and there is a very thin vertical line that divides the strip into two parallel conductors, one is much thinner than the other. Observe the strip closely to see the thin line. While operating, electrical current from the Pasco interface box flows down one of the strips, then across a copper tab (on the top of the clear plastic cylinder) and then up the other strip and returning to the interface box. An electrical circuit converts the varying current to a varying voltage and this is the sensor voltage that is recorded.

The pressure sensor is essentially a nano-device. The physically active component is a tiny sealed flat capsule that is made using integrated circuit (IC) technology. The capsule may be described as a very thin empty box made of an insulator (proportions similar to a stick of gum but with dimensions on the order of a human hair). One flat surface is bonded to a thick rigid base, the other surface is free to flex in or out as the ambient pressure increases or decreases. Each flat surface has a thin, few hundred atoms thick, metal coating on the inside of the capsule. The metal surfaces do not touch but since they are separated by an air gap they form a capacitor. When pressure increases the flexible flat surface is pushed towards the rigidly mounted surface. This causes the capacity for the metal surfaces to hold electrical charges to increase, and current flows into the capacitor. On the other hand, when pressure decreases, charge flows out of the capacitor. The charge flows here are extremely tiny, less than a microamp [μA]. The capsule and a tiny amplifier are manufactured as a single IC that is hidden below the white disk at the base of the clear plastic pipe. A tiny hole in the white disk leads to the capsule and the IC. The electrical power to run this system is supplied by a 9 volt battery or a DC power supply. The explanation given above has been simplified to focus on the major concepts. In actual practice the structure of the system may be constructed using a stack of capsules or other strategies to increase the number of conducting surfaces and the magnitude of the current flow when pressure changes.

There are several types of electronic temperature sensors: thermocouples that produce a tiny voltage directly, thermistors that are made of semiconductors, and thermistors made of fine platinum wire. Both types of thermistors react to changing temperature by changing resistance. Platinum wire is the superior choice for this

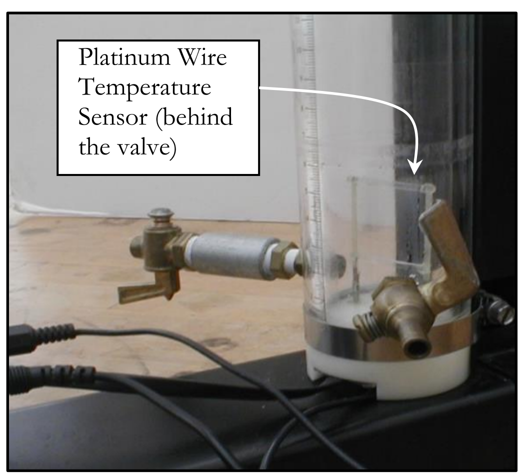

Figure 2: Temperature Sensor. Eight to ten loops of platinum wire stretched over a thin plastic sheet.

application because it can be drawn into very thin wire so it takes very little time for heat to conduct into the center of the wire. Since we are heating a very small amount of air with a low heat capacity, it is also to our benefit that a very thin wire absorbs very little of the heat from the compressed air in the chamber.

Appendix II Adiabatic Compression Guidelines

- Include all the data you took that was asked of you.
- One column abstract, two column for everything else (except very large graphs, figures, tables)
- Correct units and sigfigs on all numbers given.
- Cite anything taken from the lab module.
- Cite any text, values, equations, figures taken from anywhere else.
- **Abstract:** Very briefly describe the experiments, state goals, summarize results including primary experimental values, expected values, and percent differences or standard errors; in general state your degree of meeting the goals. (4-5 sentences max).
- **Background:** Tell Why doing experiment, state goals, give background and historical info, explain any deviations from procedure.
- **Theory & Methods:** State <u>and explain</u> the formulas for first law of thermodynamics; linear equations for lnP and lnT; formulas for experimental work done on the gas, theoretical work done on the gas; definition of adiabatic; definition of adiabatic index γ. Define all parameters or symbols used. Define any constants including their values, any necessary conditions, and cite sources.

  For Methods, very briefly describe the apparatus and experimental procedure
- **Discussion of Results:** Plots of lnP vs. lnV and lnT vs. lnV for compression and expansion strokes of each of three trials, including trendline and fit equation. The summary table of γ results (Step 25). The summary table of Work results (Step 30). Answer all questions and analysis asked in this manual. Identify <u>and describe</u> at least two possible sources of error (generic 'human error' is not acceptable).
- **References:** Properly formatted listing for all references used and cited.
- **Calculations:** Show sample of how to calculate $W_{theoretical}$, W_k, γ from ln T vs ln V plot, a % difference in work
- **Plot/Raw Data:** For this lab only, this is to be your 12 adiabatic plots properly labelled and captioned. You do not need to include the very large raw data tables taken in class.

These are just guidelines. You may need to add more items as these are just some of the things expected.