CS 222 Homework 5 [100 Points Total]

Online Submission via Canvas Only! If you are not able to produce a PDF version, you can scan or take picture of your homework for submission. No paper submission will be accepted.

Write your name on this sheet. No name or cover sheet will miss 2 points

1. (50 pts) For the following problems, consider a group of 50 students. There are 8 Computer Engineering (CE) majors, 12 Computer Science (CS) majors, 20 Electrical Engineering (EE) majors, and 10 Software Engineering (SE) majors. There are no dual major students.

   (a) (15 pts) Select 7 of the 50 students and line them up from left to right. How many ways can this be done when we consider their individual names, not their majors? Note that the "left to right" order matters, e.g, ABC is different as BAC.

   Choose 7 of 50 individual students:

   n Perm r $= \frac{n!}{(n-r)!} = \frac{50!}{(50-7)!} = \frac{50*49*48*47*46*45*44*43!}{43!}$

   $= 50 * 49 * 48 * 47 * 46 * 45 * 44 = \mathbf{5.034 \times 10^{11}}$

   (b) (15 pts) Select 5 of the 50 students and line them up from left to right. How many ways can this be done if we consider only their majors, and not their names?

   Every major has more than 5 people. Choose 5 people each from 1 of 4 majors.
   $\mathbf{4^5 = 1024}$

   (c) (20 pts) The department chair will pay for 12 students to go to a conference. In how many ways can the 12 students be selected if exactly 3 are selected from each major?

   n Choose r for all 4 majors: $\frac{n!}{(n-r)!*r!}$

   $\binom{8}{3} * \binom{12}{3} * \binom{20}{3} * \binom{10}{3} = \frac{8!}{(8-3)!*3!} * \frac{12!}{(12-3)!*3!} * \frac{20!}{(20-3)!*3!} * \frac{10!}{(10-3)!*3!}$

   $= 56 * 220 * 1140 * 120 = \mathbf{1685376000}$

2. (20 pts) Suppose an event e has the probability $p(e) = 0.2$ for one trial. What is the probability that e will occur exactly four times in six trials?

   Binomial Distribution! $nCr \times p^r q^{n-r}$
   $\binom{6}{4} \times (0.2)^4(0.8)^2 = 15 \times 0.0016 \times .64 = \mathbf{0.01536}$

3. (15 pts) Suppose an event e has the probability $p(e) = 0.2$ for one trial. Given that e has already occurred four times in a row, what is the probability that e will happen six times in a row?

   $P(E|F) = \frac{P(E \cap F)}{P(F)}$ $\quad P(E) = 0.2$
   $P(F) = (0.2) \times (0.2) \times (0.2) \times (0.2) = 0.0016$
   Only one possible outcome for e happening 6 times in a row; $(0.2)^6$
   $P(E|F) = \frac{(0.2)^6}{(0.2)^4} = (0.2)^2 = 0.04$

4. (15 pts) Suppose there is a roulette with 2 green, 18 red, and 18 blue. The payout for blue and red are 1:0.5, and the pay out for green is 1:2. For example, if you bet \$X on red and the roulette lands on red, you get \$0.5X reward. If each bet for the roulette is \$10, what is the expected payout (profit) for your bet?

   $\frac{2}{38}$ chance to get green, $\frac{18}{38}$ chance to get red, and $\frac{18}{38}$ chance to get blue

   0.0526 chance to get \$2X and 0.9473 chance to get \$.5X
   X = 10

   $.0526 \times 2 * 10 + 0.9473 \times .5 * 10 = 1.0526 + 4.7368 = 5.7895$
   = \$**5.79** expected payout for a \$10 bet.