Basic Terms and Definitions for Experimental Physics

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- **Uncertainty:** Unavoidable imprecision or variation in a value that can be minimized to some extent depending on available materials, methods, and circumstances. Natural uncertainty can never be completely eliminated. *Uncertainty* and *error* are interchangeable terms, but using the term *error* often leads to confusion in everyday speech. Among scientists the word *error* is always meant to mean an *uncertainty*.
- **Mistake:** Examples can include misinterpreting instructions, forgetting a critical step in procedure, incorrect mathematical calculations, problems with units, etc. *Mistakes* are avoidable and fully correctable once discovered; this is in contrast to *uncertainties*. When most people hear the word "error" in everyday speech they generally take it as *mistake*. It is important to mention mistakes if they occurred and could not be corrected as it helps explain unexpected results.
- **Classification of Sources of Uncertainty**

Random Intrinsic

The quantity itself being investigated changes randomly independent of whether or not it is actually being measured.

Examples: pulse rate, heartbeat, photons per second from a star, microscopic friction varying randomly across a surface

Systematic Intrinsic

The quantity itself being investigated changes predictably between measurements. Such as all equally greater, or equally lower.

Examples: macroscopic friction causes predictable energy loss over time, temperature being physically hotter in direct sunlight versus in the shade.

Random Measurement

The measured value of a quantity being investigated changes randomly due to uncontrollable variability in the measurement method or device.

Examples: Volume of a liquid measuring a meniscus on a graduated cylinder (parallax effect), digital reading bouncing between two values in the last digit, recording from any marked scale in general.

Systematic Measurement

The measured value of a quantity being investigated changes predictably due to the measurement method or device.

Examples: bias offset, a balance scale is not properly calibrated to zero, one thermometer always offset warmer than another.

In most general introductory lab settings only *random measurement* errors are easily accounted for in actual numbers, the other types are much more difficult to discover and account for or minimize. *Systematic measurement* errors, if discovered, can be minimized with improved procedure or finer adjustment of equipment. *Random intrinsic* errors can be estimated if many samples under similar conditions are averaged together to see their variation. *Systematic intrinsic* errors often go unnoticed until deeper levels of physics or more variables are accounted for.

- **Precision:** Describes the consistency or reproducibility of measurements among the measurements themselves, regardless of any expected value. The degree to which measurements show similar results under similar conditions. Can be used to describe data recorded or an instrument itself. Can be quantified by *absolute uncertainties* and *standard deviations*. Large *random errors* will easily destroy any level of precision.

- **Accuracy:** Specifically, how true a measurement is to the ideal or expected value for that quantity. Can be used to describe data recorded or an instrument itself. Can be quantified by *differences* and *percent differences*. Large *systematic errors* or biases will easily destroy any level of accuracy.

  In more general everyday usage something said to be "accurate" is normally considered to be both accurate and precise at the same time, making it the most ideal measurement.

- **Resolution:** The smallest noticeable change in quantity that a measuring device by itself can clearly display or record. Not necessarily equal to a value's *precision* or *uncertainty*.

  Example 1: A meter stick has ruled markings every 1 mm. The *resolution* of the meter stick is 1 mm.

  Example 2: A digital multimeter can display voltage to 0.1 Volts. The DMM *resolution* is 0.1 Volts.

- **Absolute Uncertainty:** δ_x The numerical value signifying the amount of uncertainty in a measurement x. Can be a best estimate, a value based off a measurement scale, or determined from statistical analysis or many measurements in a process.

  Example 1: Using an alcohol thermometer ruled every 1°C, you observe the alcohol column at room temperature to be around 21.5±0.5°C. You feel you can confidently guess between the ruled markings down to every half a ruling, or every 0.5°C, and are confident the column lies somewhere above 21°C but below 22°C. Your *absolute uncertainty* in temperature is then ±0.5°C. Your thermometer *resolution* is 1°C.

  Example 2: Using a digital balance you measure the mass of a block of metal. The digital reading displays in kg and displays xx.xx kg (4 digits, two kg decimals). The display shows 25.15 kg, and every so often it changes to 25.16 kg. You would safely record the mass to be 25.15±0.01kg and your *absolute uncertainty* in mass is ±0.01kg (same as the *resolution*). Since this is a digital scale it is better to be conservative with the uncertainty. You cannot read between digital numbers displayed, and you probably do not know how the manufacturer programed the sensor to round off numbers.

- **Relative Uncertainty:** $\frac{\delta_x}{x}$

  The *uncertainty* in a value with respect to the value itself. Also called *fractional uncertainty*.

- **Percent 'Relative' Uncertainty:** $\frac{\delta_x}{x} \cdot 100\%$

  A percentage form of *relative uncertainty*.

- **Difference, Discrepancy:** $d = x_1 - x_2$ or $d = x_{measured} - x_{reference}$

  Comparing two values by subtraction that ideally you expected to be the same, thus finding the *discrepancy* between them. This could be comparing results from two methods measuring the same thing, or comparing your result to an expected known reference value.

- **Percent Difference:** $\frac{|x_{measured}-x_{reference}|}{x_{reference}} \cdot 100\%$ or $\frac{|x_1-x_2|}{x_{min}, x_{max}, x_{mean}} \cdot 100\%$

  A percentage form of the *difference*. For comparing to a reference value use it in the denominator; ideally the reference has much higher precision than your value so it makes sense to normalize to this expected value. In the case of comparing two methods, it is up to the experimenter to decide what works best in the denominator.

- **Mean:** $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

  If measuring the same thing (x) many times (N), and you expect to always get the same or nearly the same result, you would average the many measurements together into one overall recorded value $\bar{x}$ called the *mean*. This mean value should be a more confident, more certain, and lofty result since it combines many similar results into one.

- **Standard Deviation:** $\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$

  If measuring the same thing (x) many times (N), which are only affected by *random sources of uncertainty*, then you would expect to see the many measurements varying randomly but centered around a *mean* value $\bar{x}$. The assumption is that the mean value is the most likely value to occur. The standard deviation σ_x is a definition to assign a numerical value to how far the measurements fall from the mean. This results in the classic "Bell Curve" distribution where most results (68%) are spread within $\pm 1.0\sigma_x$.

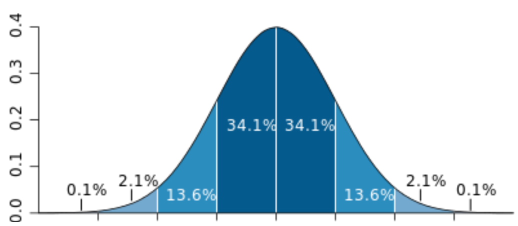

Figure 1: An example "Bell Curve" or Normal Distribution plotting probability distribution.

  The *N-1* term is used for small sample sizes, generally N<20. The standard deviation can be used to describe the *precision* of measured values when many measurements are taken together. It applies to any individual value measured or to the distribution as a whole.

- **Standard Deviation of the Mean:** $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$

  If measuring the same thing (x) many times (N), which are only affected by *random sources of uncertainty*, then all the measurements will be centered around a *mean* value $\bar{x}$ which you can calculate. Since the mean value is based off of many similar individual measurements, all with *uncertainty* σ_x, you would expect to have higher confidence (less uncertainty) in the average of those values. The *standard deviation of the mean* (SDOM) is the mathematical representation of this fact. When averaging together repeated measurements, the end result will always have lower uncertainty than the initial measurements.

- **Propagated Uncertainty:** $\delta_f \cong \sqrt{\left(\frac{\partial f}{\partial x} \cdot \delta_x\right)^2 + \left(\frac{\partial f}{\partial y} \cdot \delta_y\right)^2 + \cdots}$ where $f = f(x, y, \dots)$

In most cases your final result will not be your directly measured values, you will likely have to put your values through an equation or series of equations to obtain a final result of interest. If this is the case you must account for how your initial uncertainties propagate through these equations into an uncertainty on your final result.

Say you make measurements of experimental variables x and y which have experimental *uncertainties* of δ_x and δ_y. These uncertainties can be simple best estimates or formal standard deviations. Then you want to calculate a final result f which is a function of independent variables x and y. The above equation will give you the propagated uncertainty δ_f on result f where $\frac{\partial f}{\partial x}$ is the partial derivative of f with respect to x.

Note that this works for any number of initial measured independent variables. Independent means, for example, that variable y does not itself depend on variable x.

Partial Derivatives: In practice a partial derivative is just like a normal derivative, but you only differentiate with respect to the variable of interest and all the other variables act as temporary constants.

Example 1: $f = x^2 + 3xy + y^2$, $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 + 3xy + y^2) = 2x + 3y + 0$

In this case you differentiate with respect to *x*, so *y* is held constant.

Example 2: Using a meter stick with ruled markings every 1mm, you measure the length of a rod to be from a starting position of 0.0±0.5mm to ending position 55.5±0.5 mm. You feel you can confidently read between the ruled markings to every half a ruling, or every 0.5mm.

Your meter stick *resolution* is 1mm.

Your *absolute uncertainty* in position is ±0.5mm.

The rod length: $l = x_2 - x_1 = 55.5 - 0.0 = 55.5mm$

The uncertainty in rod length:

$$\delta_l \cong \sqrt{\left(\frac{\partial l}{\partial x_1} \cdot \delta_{x_1}\right)^2 + \left(\frac{\partial l}{\partial x_2} \cdot \delta_{x_2}\right)^2} = \sqrt{(1 \cdot 0.5)^2 + (1 \cdot 0.5)^2} = \sqrt{2} \cdot 0.5 = 0.7mm$$

You would state the rod to be of length 55.5±0.7mm.

Note the special circumstance of this example. Here we are measuring something (length) that itself requires us to take *two* simultaneous measurements (positions) on the same scale. Since both position measurements were made using the same method on the same device, they both have the same uncertainty.

This turns out to be a quite common occurrence. Therefore it is good to memorize that in this common, but specific, situation the propagated uncertainty will always be

$\delta_f = \sqrt{2}\delta_x$ when $f = x_1 \pm x_2$ and $\delta_{x_1} = \delta_{x_2}$