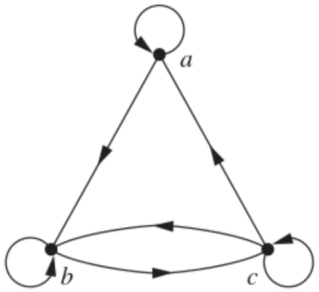
CS 222 Homework 6 [100 Points Total]

Online Submission via Canvas Only! If you are not able to produce a PDF version, you can scan or take picture of your homework for submission. No paper submission will be accepted.

Write your name on this sheet. No name or cover sheet will miss 2 points

1. (35 pts) What are the ordered pairs in the relation R represented by the directed graph shown in the figure? Determine whether the relation is reflexive, symmetric, antisymmetric, transitive.



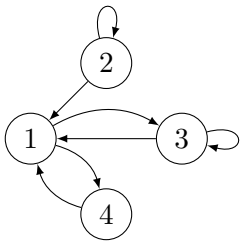
R is reflexive if all elements point to themselves. $\{(a,a),(b,b),(c,c)\}$ all exist in R so R is reflexive.
R is symmetric if all pairs point to each other. $\{(b,a),(a,c)\}$ do not exist in R so R is not symmetric.
R is antisymmetric if all pairs are unmatched and there is all or none of the $a = b$ pairs. R is not symmetric, but it also contains all three of $\{(a,a),(b,b),(c,c)\}$ meaning that R is antisymmetric.
R is transitive if $\{(a,b)$ and (b,c) then $(a,c)\}$. (c,a) and (a,b) exist and so does (c,b) $\therefore$ R is transitive.

2. (25 pts) Let $R = \{(1,2),(2,3),(3,4),(2,1)\}$ on set $A = \{1,2,3,4\}$. What is R^2 and R^3
   $R^2 = \{(1,3),(2,4),(1,1),(2,2)\}$
   $R^3 = \{(1,4),(2,3)\}$

3. (40 pts) Let $R = \{(1,3),(1,4),(2,1),(2,2),(3,1),(3,3),(4,1)\}$. on set $A = \{1,2,3,4\}$.
   - What is the matrix representation of R?

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

- What is the digraph representation of R?



- What is the reflexive closure of R?

  All of R plus the values that make it reflexive:
  $\{(1,3),(1,4),(2,1),(2,2),(3,1),(3,3),(4,1),(1,1),(4,4)\}$

- What is the symmetric closure of R

  All of R plus the values that make it symmetric:
  $\{(1,3),(1,4),(2,1),(2,2),(3,1),(3,3),(4,1),(1,2)\}$