

CS 222 Homework 3 [90 Points Total]

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Online Submission via Canvas Only! If you are not able to produce a PDF version, you can scan or take picture of your homework for submission. No paper submission will be accepted.

Write your name on this sheet. No name or cover sheet will miss 2 points

1. (20 points) Let $A=[0,2,4,6,8,10]$, $B=[0,1,2,3,4,5,6]$, and $C=[4,5,6,7,8,9,10]$. Find
 - (a) $A \cap B \cap C$
 - i. 4, 6, 8, 10
 - (b) $(A \cup B) \cap C$
 - i. 4, 5, 6
 - (c) $(A \cap B) \cup C$
 - i. 4, 5, 6, 7, 8, 9, 10
2. (30 pts) Determine whether each of these function is a bijection from domain R to R . You need to explain why each function is (or is not) a bijection.
 - (a) $f(x) = 2x + 1$

This only outputs odd numbers therefore it cannot be a bijection due to the function not allowing every real number to be outputted.
 - (b) $f(x) = x^2 + 1$

Similar to part a, this function does not allow every number from R to R to be outputted, as a square function will only allow positive numbers to be the output.
 - (c) $f(x) = x^3$

This function is a bijection, because every single number from $-\infty$ to ∞ can be both inputted and/or outputted, with each input only having one output and visa versa.
3. (15 points) Find a simple formula for a_n if the first 10 terms of the sequence $\{a_n\}$ are 1, 7, 25, 79, 241, 727, 2185, 6559, 19681, 59047. [Hint: these numbers related to value 3]
 $3^n - 2$
4. (25 points) Given the fact that $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, calculate $\sum_{i=50}^{100} i^2$
$$\begin{aligned} \sum_{n=50}^{100} \frac{2n^3+3n^2+n}{6} &\Rightarrow \left. \frac{2n^3+3n^2+n}{6} \right|_{50}^{100} \\ &= \frac{2(100)^3+3(100)^2+(100)}{6} - \frac{2(50)^3+3(50)^2+(50)}{6} = \frac{2000000+30000+100}{6} - \frac{250000+7500+50}{6} \\ &= \frac{2030100}{6} - \frac{257550}{6} = 338350 - 42925 = \mathbf{295425} \end{aligned}$$