CS 222 Test 2 **Name:** David Jeffs

92.

This is an open book and open notes test, but **no "e-format" resources are allowed**. You can use calculators if necessary, but all other electronic devices, such as Computers, pad, phones, etc. are not allowed.

Write your name on each sheet you turn in. Do not staple your sheets or otherwise attach your sheets together.

Do not write on the backs of sheets. Problems written on the backs of sheet will not be graded. This is a hard rule without any exception!

1. (20 pts) Given the fact that $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$. What is value of $\sum_{k=10}^{200} k^3$. You need to **list out your steps**, not just a final result.
2. (20 pts) Prove by induction: **P**: $\forall n \in N \rightarrow 0 + 3 + 6 + \cdots + 3n = \frac{3}{2}(n^2 + n)$.
3. (15 pts) Prove by induction: **P**: $\forall n \in Z^+ \rightarrow 11^n - 6$ is divisible by 5.
4. (15 pts) Prove by strong induction: **P**: For all $n \geq 24$, it is possible to produce n cents of postage from 5-cent and 7-cent stamps
5. (30 pts) An online gaming system allows 8 teams of 5 people (40 people total) to play in a single tournament. Players score points individually, and these individual points are combined towards team points. It is not important how the points are calculated. There are never any ties when points are being calculated or compared. At the end of the tournament, the following events happen.
   (a) (15 pts) The top three teams are ranked as first, second, and third place based on total team points. In how many ways can this happen?
   (b) (15 pts) We now need to form a new team called "Team-A". Each team will select 2 people to join "Team-A". Finally, "Team-A" will have 16 people. In how many ways can "Team-A" be formed?

1) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$ $\quad f(n) = \frac{n^2(n+1)^2}{4}$

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$$\sum_{k=10}^{200} k^3 = f(200) - f(10-1) = f(200) - f(9)$$

$\downarrow$

calc says

404007975

match

$$= \frac{(200)^2(200+1)^2}{4} - \frac{9^2(9+1)^2}{4}$$

$$= \frac{(40000)(40401)}{4} - \frac{(81)(100)}{4}$$

$$= (10000)(40401) - (81)(25)$$

$$= 404010000 - 2025$$

$$= \boxed{404007975}$$

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2) $P: \sum_{n=0}^{n} 3n = \frac{3}{2}(n^2+n)$

Basis Step: $P(n=0) = \left[\sum_{n=0}^{0} 3n = \frac{3}{2}(n^2+n)\right] = \left[\sum_{0}^{0} 3n = \frac{3}{2}(0^2+0)\right]$

$$= \left[3(0) = \frac{3}{2}(0)\right] = \left[0 = 0\right]$$

Induction Step:

By induction hypothesis, assume $P(n)$ holds

Prove $P(n+1)$: $\quad 0 + 3 + 6 + \ldots + 3n + 3(n+1) = \frac{3}{2}\left((n+1)^2 + (n+1)\right)$

$$P(n=n+1) = \left\{\sum_{n=0}^{n+1} 3n = \frac{3}{2}\left[(n+1)^2 + n+1\right]\right\} = \left\{\sum_{n=0}^{n} 3n + 3(n+1) = \frac{3}{2}\left(n^2 + 2n + 1 + n + 1\right)\right\}$$

By definition of induction hypothesis

$$= \left[\frac{3}{2}(n^2+n) + 3(n+1) = \frac{3}{2}(n^2+3n+2)\right]$$

$$= \left[\cancel{\frac{3}{2}n^2} + \frac{3}{2}n + 3n + 3 = \cancel{\frac{3}{2}n^2} + \frac{9}{2}n + \frac{6}{2}\right]$$

$$= \left[\frac{3}{2}n + \frac{6}{2}n + \frac{6}{2} = \frac{9}{2}n + \frac{6}{2}\right]$$

$$= \left[\frac{9}{2}n + \frac{6}{2} = \frac{9}{2}n + \frac{6}{2}\right]$$

$P(n)$ is true by induction

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3) $P(n) = [11^n - 6 = 5k]$ where $k, n \in \mathbb{Z}^+$

Basis Step: $P(n=1) = [11^1 - 6 = 5k] = [11 - 6 = 5k] = [5 = 5k]$ ✓

$11^n - 6 = 5k$

Induction Step:

By induction hypothesis, assume $P(n)$ holds

Prove: $P(n+1)$: $11^{n+1} - 6 = 5k$

$P(n = n+1) = [11^{n+1} - 6 = 5k] = [(11)(11^n) - 6 = 5k]$

?

-5

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4) $P: \forall n \geq 24 \quad n = 5x + 7y$

Basis Step: $P(n=24)$: $[24 = 5x + 7y]$ $x=2, y=2 \Rightarrow 24 = 10 + 14 = 24$ ✓

$[25 = 5x + 7y]$ $x=5, y=0 \Rightarrow 25 = 25 + 0 = 25$ ✓

$[26 = 5x + 7y]$ $x=1, y=3 \Rightarrow 26 = 5 + 21 = 26$ ✓

$[27 = 5x + 7y]$ $x=4, y=1 \Rightarrow 27 = 20 + 7 = 27$ ✓

$[28 = 5x + 7y]$ $x=0, y=4 \Rightarrow 28 = 0 + 28 = 28$ ✓

12 = 1,1
13 =
14 = 0,2
15 = 3,0
16 =
17 = 2,1
18 =
19 =
20 = 4,0
21 = 0,3
22 = 3,1
23 =
24 /

Induction Step: For cases above 28, similar solution to $n-5$ but x increases by 1 ∴

$$\underbrace{n-5}\ \underbrace{+5} = n$$

$n \geq 29$:

$n = n$

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5) 8 teams

(a) 1st Place 2nd Place 3rd Place

8 options 7 options 6 options $\Rightarrow 8 \cdot 7 \cdot 6 = 336$

$$_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}} = 8 \cdot 7 \cdot 6 = \boxed{336}$$ ways to pick top 3 teams

(b) 8 teams given 5 choose 2

$$8 \cdot \binom{5}{2} = 8 \cdot \frac{5!}{3! \cdot 2!} = 8 \cdot \frac{120}{12} = 8 \cdot 10 = \boxed{80}$$

Ways to form Team-A

10^8

-3