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Part 1 – Rotational Inertia

Middle Pulley Radius (cm): 1.61 Pulley mass (g): 6.9

Plate Mass (g): 66.1 Plate side a (cm): 17.7 Plate side b (cm): 17.7

Lg Ring Mass (g): 465.6 Lg Ring R_{outer} (cm): 12.1 Lg Ring R_{inner} (cm): 10.77

Sm Ring Mass (g): 461.9 Sm Ring R_{outer} (cm): 7.64 Sm Ring R_{inner} (cm): 5.46

Table 1: Angular acceleration of objects around a central axis due to string tension from a falling mass. *Friction mass is not necessary for the 'Sensor only' trial.

Trial	Hanging Mass (g)	Angular Acceleration (rad/s^2)	Friction Mass* (g)	Final Angular Velocity (rad/s)
Sm Ring + Plate	29.8 ± .05	9.40 ± .01	0.8	
Lg Ring + Plate	29.8 ± .05	3.82 ± .003	0.4	
Plate + Sensor	29.8 ± .05	33.0 ± .6	0.4	
Sensor only	29.8 ± .05	637 ± 160		

For all calculations assume g=9.79264m/s^2.[1] $9.79264 \frac{m}{s^2} \cdot \frac{100cm}{m}$

1. Using Equations 1-4 from the lab manual, derive an equation for Moment of Inertia *I* of any object that is based on measured variables: hanging mass, pulley radius, and angular acceleration. Show the steps of your derivation.

$F = m(g-a)$ $\quad a = r\alpha \quad \tau = I\alpha \quad \tau = rF$

$F = m(g - r\alpha)$

$$I = \frac{\tau}{\alpha}$$

$$I = \frac{rF}{\alpha} = \frac{r[m(g-r\alpha)]}{\alpha}$$

$$I = \frac{rmg - r^2m\alpha}{\alpha} = \frac{rmg}{\alpha} - r^2m = \boxed{\frac{mr(g-r\alpha)}{\alpha}}$$

2. Using your equation from above, calculate the experimental value of the rotational inertia of the Ring + Plate + Sensor together. Remember to subtract off the mass required to overcome friction from the hanging mass.

   a. Sm Ring + Plate + Sensor:

$$I_1 = \frac{(461.9 + 66.1 - .8)(1.61)[979.264 - (1.61)(9.4)]}{9.4} = 87058.1 \, g \cdot cm^2$$

   b. Lg Ring + Plate + Sensor:

$$I_2 = \frac{(465.6 + 66.1 - .8)(1.61)[979.264 - (1.61)(3.82)]}{3.82} = 217740.3 \, g \cdot cm^2$$

[1] Surface Gravity Prediction made available by the National Geodetic Survey (NGS), an office of the National Oceanic and Atmospheric Administration (NOAA). More NGS tools and calculators can be found at the NGS Geodetic Tool Kit Program.

$\begin{array}{r} 5 \\ 1.6 \\ \hline 477 \end{array}$

3. Calculate the experimental value of the rotational inertia of the Plate + Sensor together. Remember to subtract off the mass required to overcome friction from the hanging mass.

$$I_3 = \frac{(66.1-.4)(1.61)[979.264-(1.61)(33.0)]}{33.0} = 2966.6\,g\cdot cm^2$$

4. Calculate the experimental value of the rotational inertia of the Rotary Motion Sensor alone.

$$I_4 = \frac{(6.9)(1.61)[979.264-(1.61)(477)]}{477} = 4.921\,g\cdot cm^2$$

5. Subtract the rotational inertia of the Rotary Motion Sensor from the rotational inertia of combination of the Plate + Sensor. This will be the rotational inertia of the Plate alone.

$$I_5 = I_3 - I_4 = 2963.7\,g\cdot cm^2$$

6. Subtract the rotational inertia of the combination of the Plate + Sensor from the rotational inertia of the combination of the Ring + Plate + Sensor. This will be the rotational inertia of the Ring alone.

   a. Sm Ring:

   $$I_6 = I_1 - I_3 = 84089.5\,g\cdot cm^2$$

   b. Lg Ring:

   $$I_7 = I_2 - I_3 = 214771.7\,g\cdot cm^2$$

7. Calculate the individual theoretical values of the rotational inertias of the rings and plate using Table 1 in the lab manual and your measurements of their masses and geometries.

   a. Theoretical Sm Ring:

   $$I_6' = \frac{1}{2}M(R_1^2 + R_2^2) = \frac{1}{2}(461.9)(7.64^2 + 5.46^2) = 20365.45\,gcm^2$$

   b. Theoretical Lg Ring:

   $$I_7' = \frac{1}{2}(465.6)(12.1^2 + 10.77^2) = 61087.40\,gcm^2$$

   c. Theoretical Plate:

   $$I_8' = \frac{1}{12}M(a^2 + b^2) = \frac{1}{12}(66.1)(12.7^2 + 12.7^2) = 1776.9\,gcm^2$$

8. Use percent differences to compare the experimental values to the theoretical values.

$$\%\text{difference} = \frac{Experimental - Theoretical}{Theoretical} \times 100\%$$

Table 2: Summary Moment of Inertia data from angular acceleration experiments

Object	Experimental Rotational Inertia (gcm^2)	Theoretical Rotational Inertia (gcm^2)	Rotational Inertia %Difference
Sm Ring	$I_6 = 84069.5$	$I_6' = 20365.45$	312.9%
Lg Ring	$I_7 = 214771.7$	$I_7' = 61087.40$	251.6%
Plate	$I_5 = 2963.7$	$I_8' = 1776.9$	66.79%
Sensor only	$I_4 = 4.921$		

$\frac{Exp - Theor}{Theor} \cdot 100\%$

9. Which object had the greatest rotational inertia?

Large Ring

10. Which object was hardest to accelerate?

The Large Ring

Part 2 – Conservation of Angular Momentum

Pre-experiment Questions

11. After the ring collides with the plate, will the final angular speed be more than, less than, or the same as the initial angular speed of the plate? Explain your reasoning.

    Less than because the ring will absorb a lot of the angular momentum reducing the speed

12. After the ring collides with the plate, will the final angular momentum be more than, less than, or the same as the initial angular momentum of the plate? Explain your reasoning

    The angular momentum will be conserved

13. Do you think this will be an elastic or inelastic collision? What happens to the rotational kinetic energy of the system?

    Inelastic because the ring stays connected after the collision. The rotational KE stays the same due to conservation of energy. The INERTIA will change.

Data

Pulley Mass (g): 6.9

Outer Pulley Radius (cm): → 5.2 (Inner radius: → .3)

Pulley Inertia (gcm²): 93.6

Plate Mass (g): 66.1

Plate side a (cm): 12.7

Plate side b (cm): 12.7

Plate Inertia (gcm²): 1776.9

Ring Mass (g): 461.9

Ring outer R (cm): 7.64

Ring inner R (cm): 5.46

Ring Inertia (gcm²): 74499.9 + 461.9x^2

Table 3: Angular collision data between a dropped ring and an initially rotating plate.

Trial	I_o Initial Rotational Inertia (gcm^2)	I_f Final Rotational Inertia (gcm^2)	x Ring offset x (cm)	ω_o Initial Angular Velocity (rad/s)	ω_f Final Angular Velocity (rad/s)	L_o Initial Angular Momentum (gcm^2/s)	L_f Final Angular Momentum (gcm^2/s)	%Difference Momentum
1	1870.5	1944.38	.4	44.53	10.204	83292.3	19840.5	76.18
2	1870.5	2651.09	1.3	49.107	10.426	91853.5	27640.3	69.91
3	1870.5	2429.38	1.1	42.706	9.673	79880.6	23449.4	70.58

Table 4: Kinetic Energy before and after an angular collision between a dropped ring and rotating plate.

Trial	Initial Kinetic Energy (gcm^2/s^2)	Final Kinetic Energy (gcm^2/s^2)	%Difference Kinetic Energy
1	3709053.5	202432.5	94.5
2	4510649.8	288177.8	93.6
3	3411380.9	227309.7	93.3

Use the space below to show calculations for Trial 1's angular momenta and kinetic energies.

$$I_o = \frac{1}{12}(M+m)(a^2+b^2)$$

$$I_f = I_o + \frac{1}{2}m(r_1^2 + r_1^2) + mx^2$$

$L_o = I_o\omega_o$

$L_f = I_f\omega_f$

$KE = I\omega^2$

Post-experiment Questions

14. In class, the Capstone file calculated the Inertia's for you for each object. Show that you can correctly calculate the Initial and Final rotational Inertias for Trial 1 based on the initial measurements of each object.

Plate: $I_o = \frac{1}{12} M(a^2+b^2) = \frac{1}{12}(66.1)(17.7^2+17.7^2) = 1776.9 gcm^2$

Plate + Sm. Ring $I_f = I_o + \frac{1}{2} M(R_1^2+R_2^2) = 1776.9 + \frac{1}{2}(461.9)(7.64^2+5.46^2) = 22142.3 gcm^2$

15. What affect should each of the following have on the value you calculate for the final angular momentum? State whether each would cause the final value to be low, high, or unchanged and explain why.
    a. If the Rotary Motion Sensor has a small rotational inertia (in addition to the pulley)?
    The final value will be high because inertia is inversely proportional two angular momentum
    b. If the frictional drag on the bearings during the collision cannot be ignored?
    Frictional drag would reduce the final angular momentum because drag would cause a loss of KE

16. Explain: Does the experimental result support the Law of Conservation of Angular Momentum?
It does not because the final and initial values of our angular momentum were not equivalent.

17. Was Kinetic Energy conserved in the collisions? Explain how you know.
No because of energy released during the collisions due to friction, sound, extraneous vibrations, and heat.

18. Discuss why you might expect to see a negative %Difference for momentum, but be very surprised to see a positive percent difference.
Negative % is expected because final momentum ≤ initial momentum.
However, perfect elastic collisions do not exist in the real world therefore it would be surprising to see [final momentum > initial] which would give positive %

19. How can angular momentum be conserved, but energy not be conserved?
Momentum is conserved in all collisions, while energy can be released due to sound/vibrations/etc and can't be contained 100% within the system.