CS 222 Test 3 Name: David Jeffs

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This is an open book and open notes test, but **no "e-format" resources are allowed**. You can use calculators if necessary, but all other electronic devices, such as Computers, pad, phones, etc. are not allowed.

**Write your name on each sheet you turn in**. Do not staple your sheets or otherwise attach your sheets together.

**Do not write on the backs of sheets. Problems written on the backs of sheet will not be graded. This is a hard rule without any exception!**

1. (20 pts) Let set $A = \{$ a, b, c, d$\}$ and $\mathbb{R}$ is the relation from $A$ to $A$, where $\mathbb{R}= \{$(a, a), (a, b), (b, c), (c, c), (c, d)$\}$

   (a) What is the reflexive closure of $\mathbb{R}$?

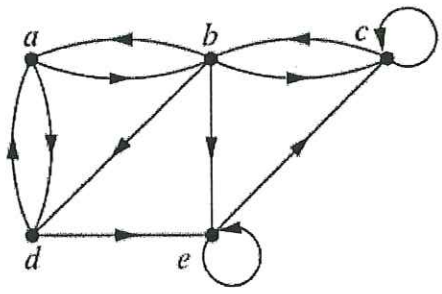
   (b) What is the symmetric closure of $\mathbb{R}$?

   (c) What is $\mathbb{R}^3$?

2. (15 pts) Draw the **undirected graph** represented by the left adjacency matrix and the **directed graph** represented by right adjacency matrix.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \end{array}$$

3. (10 pts) Represent the following graph using adjacency list and adjacency matrix.




4. (35 pts)

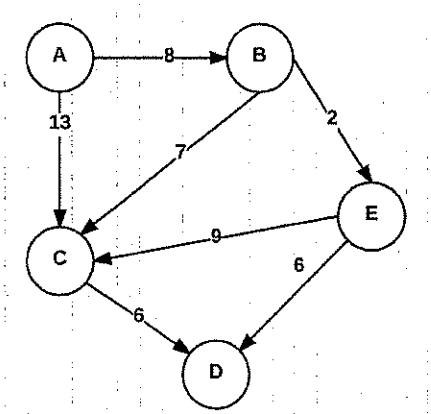
   (a) Construct the call graph using **directed graph** for a set of seven telephone numbers {A,B,C,D,E,F,G}. There were three calls from A to D and two calls from D to A, two calls from E to F, two calls from B to each of the other numbers, and one call from C to each of A, B, and G.

(b) What is the in-degree and out-degree of the vertex for A?

(c) Is this graph strongly connected? If not, is this graph weakly connected? You need to justify your answer.

(d) Is there any directed cycle in this graph? If yes, list one of them.

5. (20 pts)Determine Shortest Path from vertex A to others using the Dijkstras Algorithm.




| Iteration | Init | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| Cur Node | -- | | | | | |
| A | 0 | | | | | |
| B | ∞ | | | | | |
| C | ∞ | | | | | |
| D | ∞ | | | | | |
| E | ∞ | | | | | |

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1) $R = \{(a,a)\ (a,b)\ (b,c)\ (c,a)\ (c,d)\}$

(a) Reflexive closure: R and $\{(b,b)\ (d,d)\}$

$\{(a,a)\ (a,b)\ (b,b)\ (b,c)\ (c,c)\ (c,d)\ (d,d)\}$

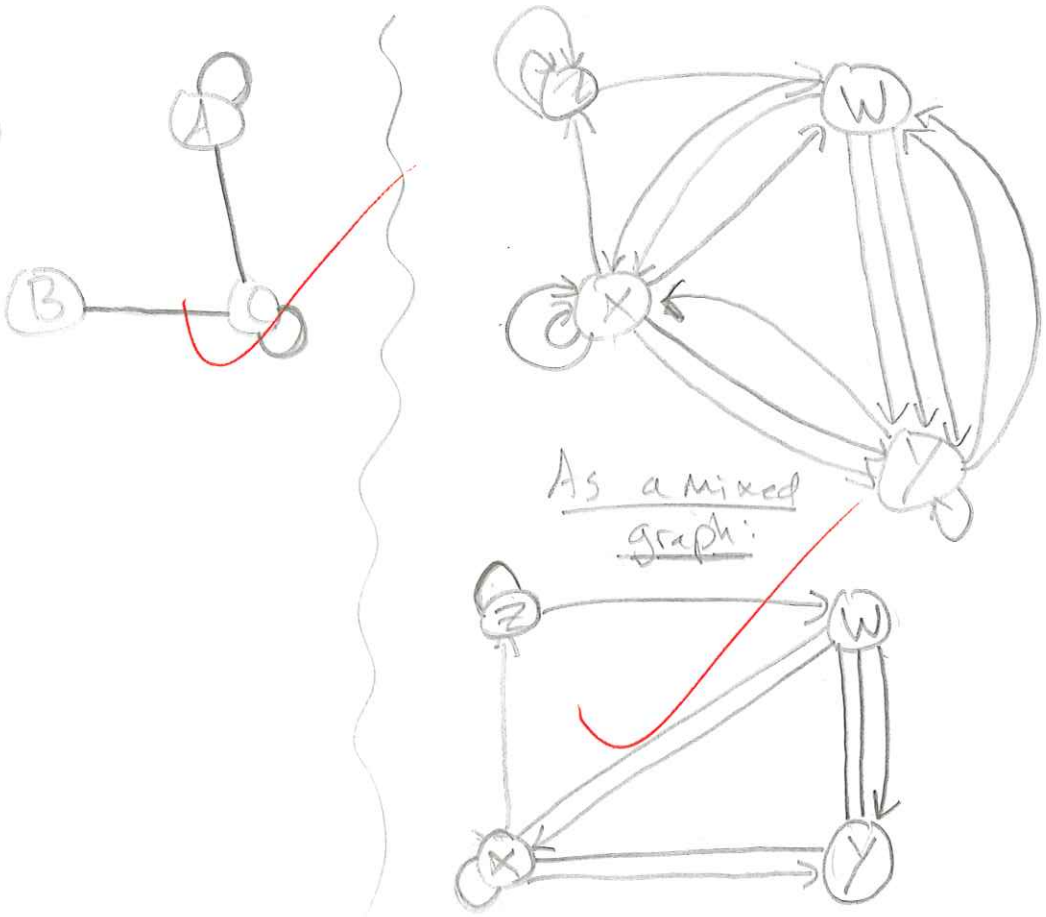
(b) Symmetric closure: R and $\{(b,a)\ (c,b)\ (d,c)\}$

$\{(a,a)\ (a,b)\ (b,a)\ (b,c)\ (c,b)\ (c,c)\ (c,d)\ (d,c)\}$

(c) $R^3 = R^2 \circ R = \{(a,a)(a,b)(a,c)(a,d)(b,c)(c,c)(c,d)\}$ (b,d) -1

$R^2 = R \circ R = \{(a,a)(a,b)(a,c)(b,c)(c,c)(c,d)\}$

2)



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3)

R: {(a,b) (a,d) (b,a) (b,c) (b,d) (b,e) (c,b) (c,c)
(d,a) (d,e) (e,c) (e,e)}

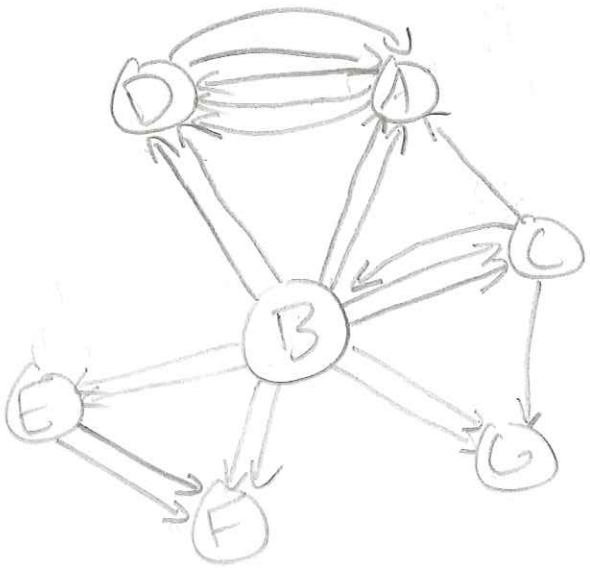
Matrix:

| | a | b | c | d | e |
|---|---|---|---|---|---|
| a | 0 | 1 | 0 | 1 | 0 |
| b | 1 | 0 | 1 | 1 | 1 |
| c | 0 | 1 | 1 | 0 | 0 |
| d | 1 | 0 | 0 | 0 | 1 |
| e | 0 | 0 | 1 | 0 | 1 |

Adjacency List

| initial | terminal |
|---|---|
| a | b, d |
| b | a, c, d, e |
| c | b, c |
| d | a, e |
| e | c, e |

4) (a)




| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 3 | 0 | 0 | 0 |
| B | 2 | 0 | 2 | 2 | 2 | 2 | 2 |
| C | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| D | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| E | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| F | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(b) $d^-(A) = 5$

$d^+(A) = 3$

(c) The graph is not strongly connected, for example there is no path from E to B. or from G to B

The graph is weakly connected though

(d) There is a directed cycle A ... D ... A

also B ... C ... B

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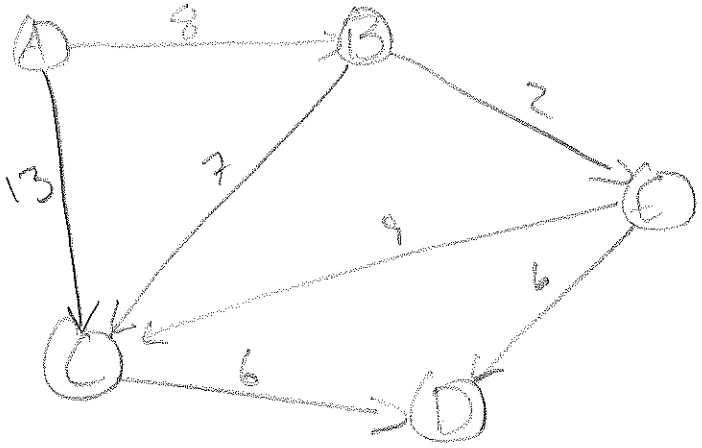
5)




| Iteration | Initial | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Cur Node | -- | A | B | E | C |
| A | 0 | | | | |
| B | ∞ | 8 | | | |
| C | ∞ | 13 | ~~13~~ 13 | ~~15~~ 13 | |
| D | ∞ | ∞ | ∞ | 10+6 = 16 | ~~13+6~~ 16 |
| E | ∞ | ∞ | 8+2 = 10 | | |