CS 222 Homework 2 [125 Points Total]

Write your name on this sheet, and turn it in as a cover sheet with your completed homework. You can turn this assignment in during class or submit a e-copy via Canvas before the deadline. Do not place under the door of the instructor's office.

Write clearly, show all work, and write formulas and assumptions used to solve a problem. Do not staple, dog ear, or otherwise attach your papers together.

1. (25 pts) Show whether the following two arguments are valid or not. You need to justify your answer.

$$\begin{array}{ll} & r \to s \\ & p \wedge q \\ \hline \therefore & p \vee s \end{array}$$

p	q	r	s	$r \to s$	$p \wedge q$	$(r \to s) \wedge (p \wedge q)$	$p \vee s$	$(r \to s) \wedge (p \wedge q) \to (p \vee s)$
F	F	F	F	T	F	F	F	T
F	F	F	T	T	F	F	T	T
F	F	T	F	F	F	F	F	T
F	F	T	T	T	F	F	T	T
F	T	F	F	T	F	F	F	T
F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	F	F	T
F	T	T	T	T	F	F	T	T
T	F	F	F	T	F	F	T	T
T	F	F	T	T	F	F	T	T
T	F	T	F	F	F	F	T	T
T	F	T	T	T	F	F	T	T
T	T	F	F	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	T	T	F	F	T	F	T	T
T	T	T	T	T	T	T	T	T

Valid Argument

$$\begin{array}{ll} & p \to q \\ & q \wedge r \\ \hline \therefore & p \vee r \end{array}$$

p	q	r	$p \to q$	$q \vee r$	$(p \to q) \wedge (q \vee r)$	$p \vee r$	$(r \to s) \wedge (p \wedge q) \to (p \vee s)$
F	F	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	F	**F**
F	T	T	T	T	T	T	T
T	F	F	F	F	F	T	T
T	F	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	T	T	T	T	T	T	T

Not a valid argument

2. (20 pts) Prove by **existence** proof:

   (a) **P**: $\exists\, x, y \in Z,\ x^4 = y^2$.
   Proof: Set x=2 and y=4

   (b) **P**: $\forall\, x \in Z,\ \exists\, y \in Z$ such that, $x^4 = y^2$. (Hint, get y in the format of x).
   Proof: $y^2 = x^4 \implies y = x^2$ which is a valid function.

3. (20 pts) Using **direct proof** to show the statement: An integer number n plus its square n^2 is always even. $n + n^2 = 2k$
   $n = 2k \therefore n^2 = (2k)^2 = 4k^2$
   $2k + 4k^2 = 2k \implies 2(k + 2k^2)$ which is an even number because anything multiplied by 2 is an even number.

4. (20 pts) Using proof by **contraposition** to show that $x + y \geq 2 \rightarrow x \geq 1$ *or* $y \geq 1$, where x and y are real numbers.
   Original Statement: $(x+y \geq 2) \rightarrow [x \geq 1 \vee y \geq 1]$ Contrapositive: $[x < 1 \wedge y < 1] \rightarrow (x+y < 2)$
   If x and y were both 1, then x+y = 2. However, based upon the contrapositive statement, neither x nor y can be equal to or greater than 1, therefore the contrapositive is true.

5. (20 pts) Prove by **contradiction**: The sum of a rational number and an irrational number is irrational.
   Original Statement: rational + irrational = irrational
   Contradiction: rational + irrational = rational
   rational = r; irrational = i

   $r = \dfrac{m}{n} \implies \dfrac{m}{n} + i = \dfrac{m}{n}$
   $i = \dfrac{m}{n} - \dfrac{m}{n}$

   $i = 0 \implies$ 0 is not an irrational number, therefore the original claim is true.

6. (20 pts) Prove that at least one of the real numbers $a_1, a_2, \cdots, a_n$ is greater than or equal to the average of these numbers.
   Claim: $[\dfrac{a_1 + a_2 + a_3 + \cdots + a_n}{n}] \geq a_x$

   Claim v2: $\forall A \in A = \{a_1, a_2, \cdots, a_n\}, a \in \mathbb{R}, \exists a_x, x \in \mathbb{R}; a_x \geq \dfrac{\sum_1^n A}{n}$

   $A = 1, 2, 3, 4, 5 \rightarrow a_x \geq \dfrac{1 + 2 + 3 + 4 + 5}{5}$
   $a_x \geq 3$

   The values 3, 4, and 5 of A all satisfy this claim.