In Problems 1–12, a differential equation is given along with the field or problem area in which it arises. Classify each as an ordinary differential equation (ODE) or a partial differential equation (PDE), give the order, and indicate the independent and dependent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or nonlinear.

1. $5\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 9x = 2\cos 3t$

(mechanical vibrations, electrical circuits, seismology)

2. $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

(Hermite's equation, quantum-mechanical harmonic oscillator)

3. $\frac{dy}{dx} = \frac{y(2-3x)}{x(1-3y)}$

(competition between two species, ecology)

4. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(Laplace's equation, potential theory, electricity, heat, aerodynamics)

5. $y\left[1 + \left(\frac{dy}{dx}\right)^2\right] = C$, where C is a constant

(brachistochrone problem,† calculus of variations)

7. $\frac{dp}{dt} = kp(P-p)$, where k and P are constants

(logistic curve, epidemiology, economics)

8. $\sqrt{1-y}\,\frac{d\,y}{dx^2} + 2x\frac{dy}{dx} = 0$

(Kidder's equation, flow of gases through a porous medium)

9. $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

(aerodynamics, stress analysis)

12. $\frac{d^2y}{dx^2} - 0.1(1-y^2)\frac{dy}{dx} + 9y = 0$

(van der Pol's equation, triode vacuum tube)