IIR FILTERING OF SURFACE MESHES FOR THE REGULARIZATION OF DEFORMABLE MODELS

Jérôme Velut, Hugues Benoit-Cattin, Christophe Odet

CREATIS, CNRS UMR 5220, Inserm U 630 INSA, Bt. B. Pascal, 69621 Villeurbanne, France

ABSTRACT

A new adaptive IIR filtering of surface meshes by smoothing B-spline is proposed. We define some ad hoc processing of borders and singular points for genus 0 meshes. By adapting the filter cut-off frequency to the local mesh sampling, it enables homogeneous smoothing of non-uniform meshes. We present genus 0 and genus 1 meshes filtering as well as volume segmentations. These segmentations have been obtained using the IIR filtering of deformation forces within a smoothing B-Spline active surface.

Index Terms— Adaptive signal processing, surfaces, geometric modeling, image segmentation.

1. INTRODUCTION

Deformable models are intensively used in many varied domains. In his seminal paper [1], Terzopoulos gave an energy minimizing formulation based on physical deformation. A minimal energy, which involves elasticity of the model and external forces depending on the application, defines the stable state of the deformable model. Kass [2] applied this formulation to image segmentation and proposed the well-known snake where external forces are derived from image features.

Several methods extend the concept of deformable model to 2D image segmentation. In [3], Precioso et al. took advantage of a smoothing B-Spline filtering as a regularization process. Internal forces are not needed anymore, the elastically constraint being embedded in the filter kernel. In [4], Velut et al. extended the Precioso approach by implementing the regularization through an IIR filtering of the deformation forces. Such a strategy solves the range limitation of the regularization parameter as well as the shrinking effect of regularization often mentioned in the literature, while retaining complete freedom in the choice of external forces.

For 3D segmentation, The 1D contour becomes a 2D surface that deforms in a volume. Existing works propose several representations of surfaces to implement this 3D extension known as active surface.

Parametric surface. Snakes are represented through a 1D parametric curve $g(s) = (g_x(s), g_y(s))$, where s is the curvilinear abscissa and x and y are the two directions of the plane - i.e the image - where the contour evolves. A natural 2D extension of a contour is then the parametric surface [5], built from three components g_x, g_y, g_z depending on two parameters u, v such as $g(u, v) = (g_x(u, v), g_y(u, v), g_z(u, v))$, where u and v are the directions of the parametric domain. In [6, 7], they developed the parametric model and noticed some issues, essentially topological. The continuous function is indeed not defined on singular points, such as the poles of a sphere. This motivates the use of discrete surfaces also called surface meshes.

Surface mesh. The representation of a surface through a mesh is the most usual one in computer graphics. This overcomes the topological constraint of parametric deformable models. The surface is no more defined by two parameters, but instead by a set of vertices and the neighborhood of each vertex [6, 8]. These 3D deformable models are efficient as long as the required regularization level is not too high. Indeed, the regularization process is comparable to a FIR filtering of the mesh and thus implies a large filtering window - i.e neighborhood - in case of high regularization level. The smoothing B-Spline active surface [9] addressed this point by extending the smoothing B-Spline IIR filtering over a parametric mesh.

Parametric mesh. A parametric mesh $g(k, l)$ is the discretization of a parametric surface $g(u, v)$. The parameter k defines the index of the point along the direction u, and so does l along direction v. This a 4-valenced almost everywhere mesh and is represented by a grid except at the singular points. The representation of a surface through a parametric mesh allowed us to extend the efficient IIR filtering of external forces in [9]. Nevertheless, several problems remain. First, as we mentioned and it is noticed in the literature, the parametric definition is unusable on singular points. Second, the 4-valenced connectivity induces a necessary non-uniform sampling. Consequently, it introduces different regularization effects over the mesh. Note that in 2D, such a problem was solved by a uniform resampling of the contour [4].

In this paper, we propose to address these two problems by a specific management of singular points and by a local adaptation to the mesh sampling of the filter cut-off frequency. In the next section, we define the IIR mesh filtering method. In section 3, we give results on mesh filtering and present some volume segmentation by smoothing B-Spline active surface that implements this new sampling-adaptive IIR filtering.

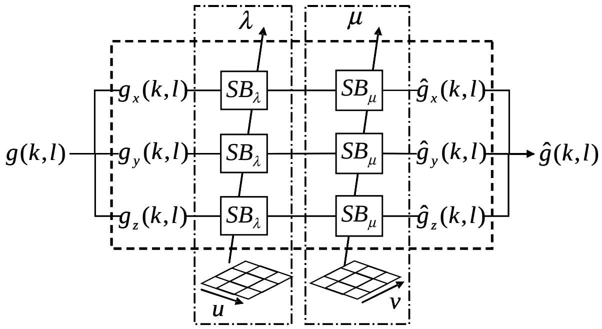


***Fig. 1**. IIR filtering of a parametric mesh $g(k,l)$ through monodimensional IIR filtering of each component g_x, g_y, g_z in both directions u, v. The filter is SB_λ, the smoothing B-Spline filter which cut-off frequency is tuned according to λ on u and to μ on v.*

2. IIR MESH FILTERING

The main advantage of parametric meshes is their grid-like structure. Almost every vertice $V_{k,l}$ is characterized by its grid coordinates that are the couple of parameters k and l of the surface $g(k,l)$. A surface g is thus defined by three bi-dimensional functions g_x, g_y and g_z such that:

$$g(k,l) = (g_x(k,l), g_y(k,l), g_z(k,l)) \quad (1)$$

where g_x, g_y and g_z are called the components of the surface. This representation allows a trivial extension of an image filtering to a parametric mesh filtering [9]. The components are indeed filtered through the bi-dimensional smoothing B-Spline filter. As this filter is a separable one [10], a parametric mesh filtering restricts to a mono-dimensional smoothing B-Spline SB_λ filtering of each component in both directions (See figure 1).

We focus in the following on the 1D signal extensions as the considered geometric objects are built from 1D signals. The mesh is a grid-like almost everywhere and thus we have to define exactly the 1D signals on the border of the parametric domain to be able to initialize the IIR filtering process. Three types of signal extension are considered.

2.1. Signal extensions

2.1.1. Periodic extension

A 1D periodic signal is associated to closed curves. For 2D surfaces, the genus 1 meshes are the one built from closed curves, thus from periodic 1D signals. These meshes are homeomorphic to a torus.

2.1.2. Mirror extension

When a 1D signal is not periodic, the IIR filter needs a coherent initialization at the border points. The typical extension is an anti-mirror with pivot point [11]. Open curves are the counterparts of non-periodic signals, as well as open surfaces. Finite planes are open surfaces built from open curves on both directions.

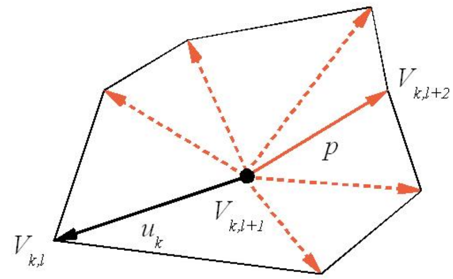


***Fig. 2**. Extension of a meridian through a singular point $V_{k,l+1}$. The extended meridian is defined by the current processed vertice $V_{k,l}$. The chosen extension p aims to choose a proper $V_{k,l+2}$.*

2.1.3. Singular point extension

The implementation of IIR filtering of the singular points requires knowing what information is to be considered after such points. On a sphere, poles are singular. In this case, periodic and mirror extensions are not usable. Figure 2 presents the proposed method to extend a signal at singular points. Let $V_{k,l}$ be the current filtered vertice. $V_{k,l+1}$ is on a singular point. Consequently, existing extension methods fail to determine which vertice is $V_{k,l+2}$. Let u_k be the vector defined by $V_{k,l+1} - V_{k,l}$. Let p be a vector belonging to the set of edges starting from $V_{k,l+1}$. We choose as a signal extension the point belonging to a meridian directed by p such that the scalar product $< u_k \cdot p >$ is minimal.

This ad hoc processing of borders and singular points permits the IIR filtering of meshes representing genus 0 and genus 1 surfaces. The surface only has to be represented by a parametric mesh such as a set of parallels, meridians and a few singular points.

As mentioned in the introduction, another drawback of parametric meshes is the non-uniformity of the sampling. The next subsection deals with this matter through adaptive filtering.

2.2. Sampling-adaptive filter

A non-uniform sampling over a mesh induces a non-uniform filtering effect as illustrated figure 3(b). This is explained by the digital filtering process underlying. Indeed, usually the sampling of the 1D signal is considered uniform and the cut-off frequency of a digital filter is normalized by the sampling frequency. As a consequence, the same filter (i.e the same cut-off frequency) will not have the same effects on two differently sampled signals.

Our proposition is to integrate into the smoothing B-Spline active surface framework the locally adaptive version of the smoothing B-Spline filtering introduced in 2D in [4]. We give in the following a brief overview of the different parameters of the smoothing B-Spline filter and the method to adapt the filter cut-off frequency to the local sampling at each mesh point.

2.2.1. *Smoothing B-Spline Filter*

This filter SB_λ proposed by Unser [10] has a symmetric fourth order IIR transfer function. The cut-off frequency is tuned by a regularization parameter λ. In [4], we characterized the filter and we gave an analytical link between λ and the cut-off frequency f_c:

$$f_c = \phi^{-1}(\lambda) \tag{2}$$

In [4], we showed that λ can be defined locally over a curve with a λ_k value at each point k. As the cut-off frequency f_c is proportional to the sampling frequency f_e, we define the local cut-off frequency $f_{c,k}$ according to the local sampling frequency $f_{e,k}$.

2.2.2. *Locally varying cut-off frequency for global constant smoothing effect*

The local regularization parameter λ_k is thus linked to the local cut-off frequency $f_{c,k}$. We define a global regularization parameter Λ that defines a global smoothing effect. Λ is linked to a global cut-off frequency F_c by:

$$F_c = \phi^{-1}(\Lambda) \tag{3}$$

In case of a varying sampling frequency $f_{e,k}$, we can define the local cut-off frequency $f_{c,k}$ by:

$$f_{c,k} = F_c / f_{e,k} \tag{4}$$

We apply (2) on $f_{c,k}$:

$$\lambda_k = \phi(f_{c,k}) \tag{5}$$

Then from equations (3), (4), and (5) we have:

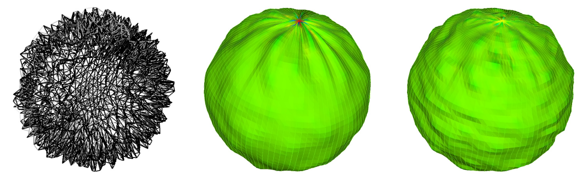
$$\lambda_k = \phi(\phi^{-1}(\Lambda) \cdot f_{e,k}^{-1}) \tag{6}$$

Finally, we define the local sampling step $f_e(k)^{-1}$ by:

$$f_{e,k}^{-1} = \frac{1}{2} \cdot \|g_k - g_{k-1}\| + \frac{1}{2} \cdot \|g_k - g_{k+1}\| \tag{7}$$

where g_k is the k^{th} point on direction u. We pointed out in the previous paragraph that mesh filtering involves a filtering in both directions, u and v. Equations (3-7) describe the sampling-adaptive filtering in direction u. When filtering along direction v, k becomes l and λ becomes μ.

The proposed parametric meshes IIR filtering, that manages properly singular points and that is unaffected by sampling non-uniformity, has been embedded in the smoothing B-Spline active surface framework of [9]. In the next section, we present results on sphere IIR filtering and on external forces IIR filtering for volume segmentation.



(a) (b) (c)

Fig. 3. *IIR filtering of a noisy sphere. (a) Initial noisy sphere. (b) IIR filtered sphere with a constant cut-off frequency $\lambda = \mu = 3$. (c) Adaptive IIR filtered sphere with $\Lambda = 3$.*

3. RESULTS

3.1. Global smoothing effect

Figure 3(b) illustrates the consequence of a non-uniform sampling on the smoothing effect due to a constant cut-off frequency IIR filtering. The equatorial zones, being sampled with a lower frequency than the poles, are smoother than the poles regions. More generally such type of filtering implies that the filtered sphere geometry will depend on the poles position.

Figure 3(c) presents the result obtained when using the proposed sampling-adaptive IIR filtering approach. The global regularization parameter is set to $\Lambda = 3$. It can be visually observed that the smoothing effect is similar all over the surface. Indeed, in this case, the cut-off frequency of the filter will vary with the mesh sampling.

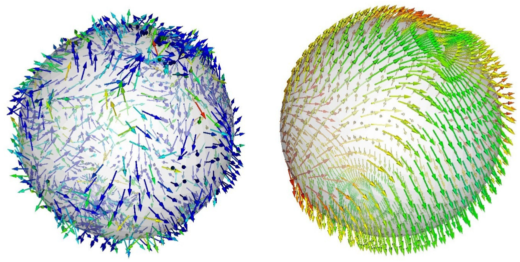
Note that in figure 3, the proposed management of the singularity points (in this case the two poles) is efficient as the smoothing effect is coherent all around the poles.

3.2. Regularization of a smoothing B-spline active surface

The proposed adaptive IIR filtering approach is implemented in the smoothing B-spline active surface algorithm of [9] to filter external forces. These forces form a 3D vector field f define at each vertice $V_{k,l}$ by three components f_x, f_y and f_z. Then $f(k,l) = (f_x(k,l), f_y(k,l), f_y(k,l))$ is comparable to a parametric mesh that can be filtered with the scheme of figure 1.

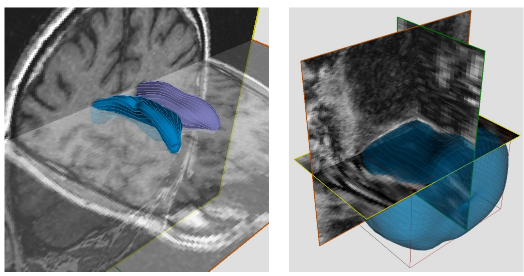
Figure 4 illustrates an external force field filtering with our IIR method. After filtering (figure 4(b)), the forces become homogeneous over the sphere, which traduces a regularization process. And we can note that the filtering effect is still coherent around the poles thanks to the proposed signal extension.

In figure 5, we give some example of segmentation results. In these cases, the external forces are computed from image gradient, yielding a stable state on volume edges. Initial models are chosen to be homeomorphic to a sphere. As the IIR filtering based regularization is independent from the local sampling, the segmentation result is not dependent on



(a) (b)

Fig. 4. IIR Filtering of an external forces field $f(k,l)$ over a sphere. (a) Random vectors are defined at each sphere vertice. (b) f is filtered by a smoothing B-Spline filter with $\Lambda = 500$ and becomes more homogeneous.



(a) Lateral ventricles of MRI human brain. (b) Tibial plate from MRI guinea-pig knee.

Fig. 5. MRI segmentation with closed genus 0 initial model.

the initial model parameterization. Moreover, the uniform sampling constraint becomes not so strong as it is taken into account by the adaptive IIR filtering.

4. CONCLUSION

We propose an adaptive IIR filtering over parametric meshes that is able to process genus 0 and 1, closed or opened surfaces. We extend the varying smoothing B-Spline filter in order to adapt the cut-off frequency to the local sampling of the mesh. This yields a sampling independent and efficient smoothing algorithm of meshes.

We integrate this filtering approach into the smoothing B-Spline active surface framework [4]. The regularization is then no more dependent on sampling and on singularity point position without loss of efficiency.

5. ACKNOWLEDGEMENTS

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