plumeAnalyserInversion

May 31, 2019

In [1]:
```
from bentPlumeAnalyser import *
from fumarolePlumeModel import *
from scipy.io.matlab import loadmat
from itertools import product

import pandas
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mpl
import os, sys
import json

# Set numpy options, notably for the printing of floating point numbers
np.set_printoptions(precision=6)

# Set matplotlib options
mpl.rcParams['figure.dpi'] = 300
```

In [2]:
```
def integrator(V0, p):
    # Initialise an integrator object
    r = ode(derivs).set_integrator('lsoda', nsteps=1e6)
    r.set_initial_value(V0, 0.)
    r.set_f_params(p)

    # Define state vector and axial distance
    V = []     # State vector
    s = []     # Axial distance
    V.append(V0)
    s.append(sexp[0])

    # Define the individual variables - these will be calculated at run time
    Q, M, F, theta = 0., 0., 0., 0.
    Q = np.float64(Q0)
    M = np.float64(M0)
    F = np.float64(F0)
    theta = np.float64(theta0)
```

```
    ######################################

    # Integrate, whilst successful, until the domain size is reached
    ind = 0
    while r.successful() and r.t < t1 and M >= 0.:
        dt = dsexp[ind]
        r.integrate(r.t + dt)
        V.append(r.y)
        s.append(r.t)
        Q, M, F, theta = r.y
        ind += 1
    s = np.array(s)
    V = np.float64(np.array(V))
    return s, V
```

In [3]:
```
exptNo      = 3
plotResults = True

# Read analysed experimental data from file
fname    = './data/ExpPlumes_for_Dai/GCTA_plumeData.xlsx'
# exptData = pandas.read_excel(fname, sheet_name='exp%02d' % exptNo)

# # Display the first 5 lines of the data frame
# exptData.head()
```

In [4]:
```
## DEFINE THE DISTANCE ALONG THE AXIS, THE ANGLE AND THE WIDTH OF THE PLUME
# sexp  = exptData.distAlongAxis
# thexp = exptData.plumeAngle
# bexp  = exptData.plumeWidth

path = './data/ExpPlumes_for_Dai/exp%02d/' % exptNo

skip = 10  # How many points to skip

xexp = loadmat(path + 'xcenter.mat')['xcenter'][0]
zexp = loadmat(path + 'zcenter.mat')['zcenter'][0]
Ox, Oz = (xexp[0], zexp[0])
pPixels = np.array([xexp, zexp]).T[::skip]

data, xexp, zexp, extent = loadExptData(exptNo)
if data.mean() < .5:
    data = 1. - data
p = np.array([xexp, zexp]).T[::skip]

# # # Uncomment if user guesses are to be used for the plume trajectory
# # with open(path + 'exp%02d_initGuess.json' % exptNo) as f:
# #     exptData = json.load(f)
# # p = np.array(exptData['data'])
```

```
# pPixels = p.copy() * scaleFactor
# pPixels[:,0] += Ox
# pPixels[:,1] -= Oz
# pPixels[:,1] *= -1

# Calculate angle, width and distance along plume and errors
thexp, sig_thexp = plumeAngle(p[:,0], p[:,1], errors=[1/scaleFactor]*2)
_, bexp, sig_p, sig_bexp = trueLocationWidth(pPixels, data, errors=[1/scaleFactor])
sexp      = distAlongPath(p[:,0], p[:,1])
bexp     /= scaleFactor
sig_bexp /= scaleFactor
bexp[0]   = 0.55 / 2
thexp[0]  = np.pi / 2
```

In [5]:
```
# Import table of experimental conditions
GCTA = pandas.read_excel('./data/ExpPlumes_for_Dai/TableA1.xlsx', sheet_name='CGSdata',
                         names=('exptNo', 'rhoa0', 'sig_rhoa0', 'N', 'sig_N', 'rho0', 's
                                'gp', 'sig_gp', 'Q0','sig_Q0', 'M0', 'sig_M0', 'F0', 'si

# Extract densities of ambient and plume, and calculate g' at the source
expt  = GCTA[GCTA['exptNo'] == 3]
rhoa0 = expt['rhoa0']
rho0  = expt['rho0']
g = 981 #cm/sš
gp0   = (rhoa0 - rho0) / rhoa0 * g

parameters = pandas.read_excel('./data/ExpPlumes_for_Dai/TableA1.xlsx', sheet_name='CGSp
b0theoretical = parameters[parameters['property'] == 'nozzleSize']['value'].values[0]
u0theoretical = expt['U0'].values[0]
```

In [13]:
```
# Load initial conditions for a given experiment, run the model for those conditions
# and then compare model and experimental data
# fig, ax = plt.subplots()
# ax.plot(exptData.plumeWidth, exptData.distAlongAxis, 'g.', label='natural')
V0, p = loadICsParameters(pathname, exptNo, alpha=0.05, beta=0, m=2)

#p = (0.05, .5, .012, 2., 4.)
nGrid = 51    # Number of grid points
b0Vec = np.linspace(.05, 2, nGrid) #cm
u0Vec = np.linspace(5, 30, nGrid) #cm/s
Q0Vec = u0Vec * b0Vec**2 #cm3/s
M0Vec = Q0Vec * u0Vec #cm4/s2

theta0 = np.pi / 2

objFn, initialConds = [], []
# Set integration domain and step size
t1    = sexp.max()     # Domain of integration
```

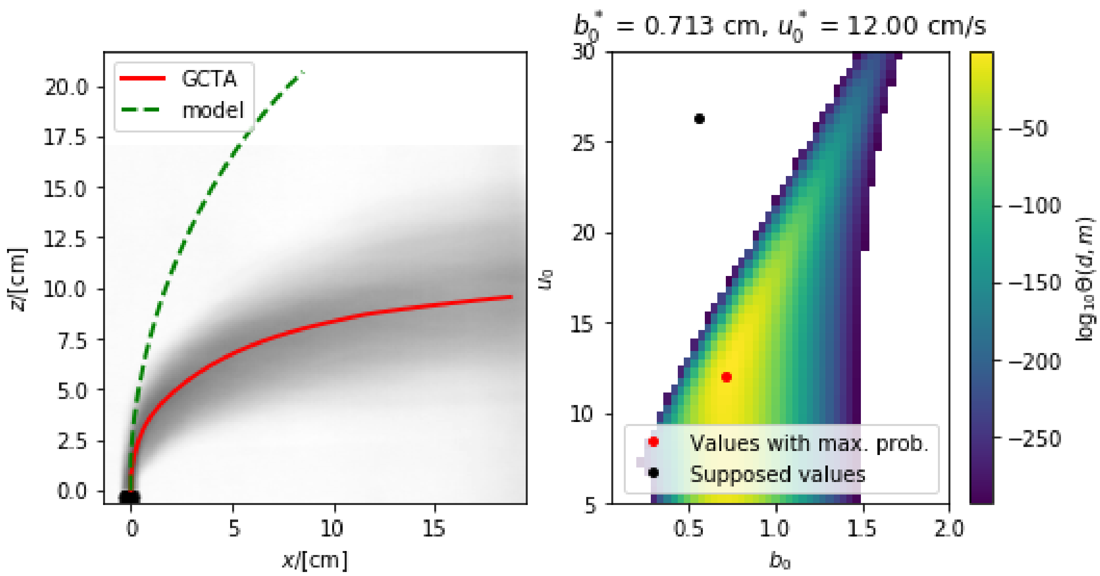
```
    dsexp = np.diff(sexp) # Discretise the distance along plume axis - use as integration s

    sequence = [Q0Vec, M0Vec]

    for (Q0, M0) in list(product(*sequence)):
        F0 = Q0 * gp0
        V0 = [Q0, M0, F0, theta0]

        # Call the 'integrator' function (defined above) to solve
        # the model
        s, V = integrator(V0, p)
        Q, M, F, theta = [V[:,i] for i in range(4)]

        #######################################

        b  = Q / np.sqrt(M) / np.sqrt(2) # Factor of sqrt{2} to correspond with top-hat mod
        u  = M / Q
        gp = F / Q

        Vexp = np.array([thexp]).ravel(order='C')
        Vsyn = np.array([theta]).ravel(order='C')
        sigV = np.array([sig_thexp]).ravel(order='C')
        CovV = np.diag(sigV**2)

        objFn.append(objectiveFn(Vexp, Vsyn, cov=CovV, p=p))
        initialConds.append(V0)
#     ax.plot(b, s, '-', label='Model %.4f %.4f %.4f' % (Q0, M0, F0))

    #ax.legend(loc=5)
    # ax.set_xlim((0, 25))

    # Transform initialConds and objFn from lists to arrays,
    # reshaping the latter
    initialConds = np.array(initialConds)
    objFn = np.array(objFn).reshape((nGrid, nGrid))
```

0.0.1 Plot the optimal model solution and the objective function for the parameter space that we have calculated

```
In [7]: fig, ax = plt.subplots(1, 2, figsize=(8,4))

        ### Plot the model solution over the experimental image and data
        # Model solution trajectory
        xmod, zmod = [0.], [0.]
        ds_ = np.diff(sexp)
        for (ds, th) in zip(ds_, theta):
            xmod.append(xmod[-1] + ds * np.cos(th))
            zmod.append(zmod[-1] + ds * np.sin(th))
```

```
# On an image of the experimental plume, show the plume trajectories for
# 1) GCTA, 2) our initial guess, 3) the model solution
ax[0].imshow(data, extent=extent, cmap=plt.cm.gray)
ax[0].invert_yaxis()
ax[0].set_xlabel(r'$x$/[cm]')
ax[0].set_ylabel(r'$z$/[cm]')
# 1) GCTA
ax[0].plot(xexp, zexp, 'r-', label='GCTA', lw=2)
# 2) our initial guess
# ax[0].plot(exptData.axisLocn_x, exptData.axisLocn_y, 'r--', label='Init. guess', lw=2)
# 3) the model solution
ax[0].plot(xmod, zmod, 'g--', label='model', lw=2)
ax[0].set_xlim((extent[:2]))
ax[0].legend(loc=2)

### Plot optimal values
ui, bi = np.where(objFn == objFn.max())
bOpt = b0Vec[bi[0]]
uOpt = u0Vec[ui[0]]

im = ax[1].pcolor(b0Vec, u0Vec, np.log10(objFn / objFn.sum()))
hX = ax[1].set_xlabel(r'$b_0$')
hY = ax[1].set_ylabel(r'$u_0$')
hL = ax[1].set_title(r'$b^*_0$ = %.3f cm, $u^*_0$ = %.2f cm/s' %
                     (bOpt, uOpt))

ax[1].plot(bOpt, uOpt, 'r.', ms=8, label='Values with max. prob.')
ax[1].plot(b0theoretical, u0theoretical, 'k.', ms=8, label='Supposed values')

cbar = fig.colorbar(im)
cbar.set_label(r'$\log_{10}{\Theta(d,m)}$')

ax[1].legend(loc=4)
fig.savefig('ProbabilityDistribution_b0u0.png', dpi=300)

# ax[1].plot(res[:,0], sexp, '-', label=r'$(b_{\mathrm{exp}} - b_{\mathrm{mod}})/\sigma_
# ax[1].plot((thexp - theta) / sig_thexp, sexp, '-', label=r'$\theta_{\mathrm{exp}} - \t
# ax[1].legend(loc=1, fancybox=True, framealpha=.8)
# ax[1].grid()
# ax[1].set_ylabel('Distance along plume axis, s/[cm]')

# ax[1].set_title('Objective fn: %.3f' % objFn[bi][ui])
```

```
/usr/share/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:31: RuntimeWarning: divid
```



(*Left*) Visualisation (logarithmic scale) of the probability density of the parameter space. (*Right*) Image of experimental plume with calculated and estimated trajectories. #(*Right*) Difference between experimental (guessed) and model solutions as a function of the distance along the plume axis.

1 To do:

- Identify which cases produce minima in the objective function

In [8]:
```
# Uncomment the following line to transform this notebook into a latex file
!jupyter nbconvert --to latex plumeAnalyser.ipynb
```

```
[NbConvertApp] Converting notebook plumeAnalyser.ipynb to latex
[NbConvertApp] Support files will be in plumeAnalyser_files/
[NbConvertApp] Making directory plumeAnalyser_files
[NbConvertApp] Writing 37194 bytes to plumeAnalyser.tex
```

In [9]:
```
# Now run pdflatex plumeAnalyser from the command line
```

In [11]:
```
CovV.shape
```

Out[11]:
```
(143, 143)
```