Supporting Information

"Laboratory Experiments of Forced Plumes in a Density-Stratified Crossflow and Implications for Volcanic Plumes"

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Figure S1 2

Figure S2 2

Table S1 3

Figure S3 4

Figure S4 4

Table S2 5

Figure S5 6

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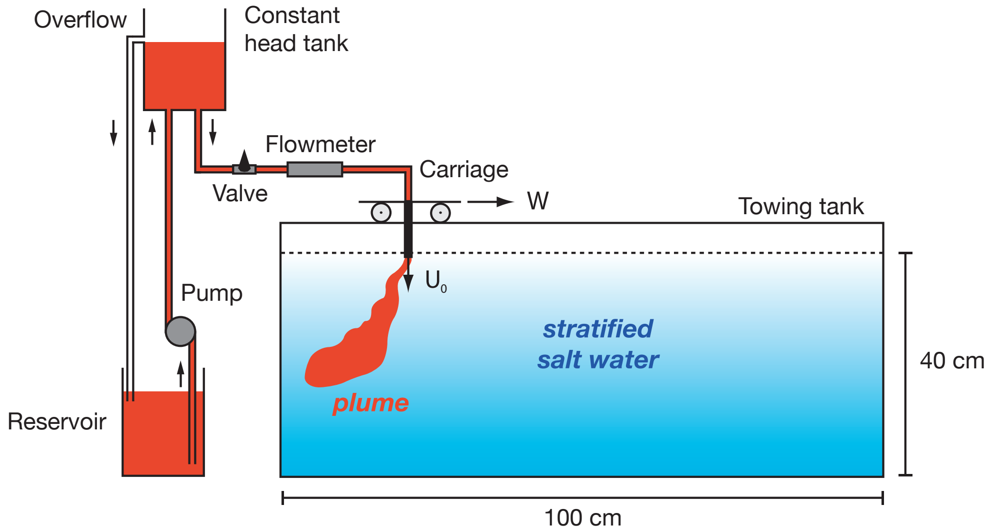

Figure S1: Schematic diagram of the experimental apparatus.

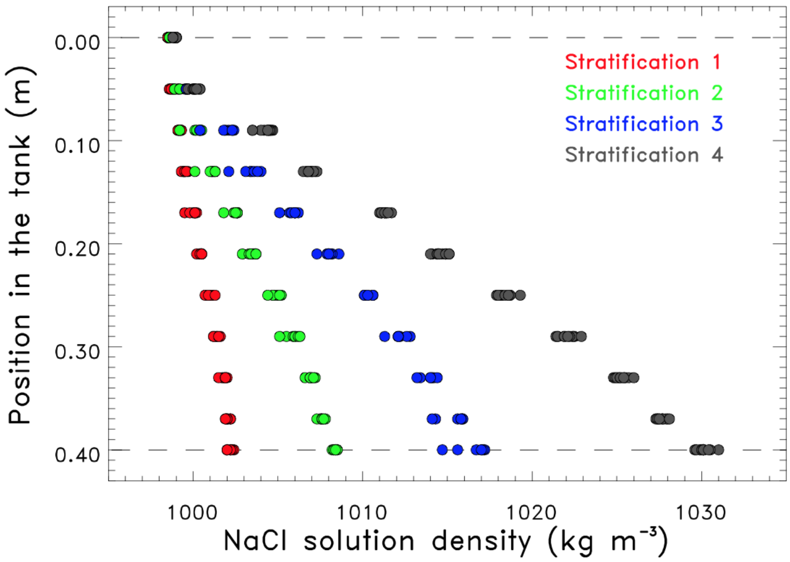

Figure S2: Density profiles used in our experiments. Red, green, blue, and black circles correspond to Brunt-Väisälä frequencies of approximately 0.305 ± 0.003, 0.488 ± 0.006, 0.657 ± 0.011, and $0.870 \pm 0.005\ s^{-1}$, respectively.

Run	N (s^{-1})	Q_0 ($m^3\,s^{-1}$)	M_0 ($m^4\,s^{-2}$)	F_0 ($m^4\,s^{-3}$)	U_0 ($m\,s^{-1}$)	W ($m\,s^{-1}$)	$\Delta\rho/\rho_a$	W^*	Ri_0	Re_0	Group
1	0.31	2.6×10^{-5}	7.0×10^{-6}	4.3×10^{-7}	0.272	0.012	0.00170	0.044	1.2×10^{-3}	1495	B
2	0.31	5.0×10^{-5}	2.6×10^{-5}	8.4×10^{-7}	0.526	0.024	0.00170	0.046	3.3×10^{-4}	2894	B
3	0.30	2.5×10^{-5}	6.6×10^{-6}	4.2×10^{-7}	0.263	0.048	0.00170	0.182	1.3×10^{-3}	1447	A
4	0.30	2.5×10^{-5}	6.6×10^{-6}	4.2×10^{-7}	0.263	0.024	0.00170	0.091	1.3×10^{-3}	1447	A
5	0.31	5.2×10^{-5}	2.8×10^{-5}	8.6×10^{-7}	0.544	0.012	0.00170	0.022	3.1×10^{-4}	2990	B
6	0.30	5.0×10^{-5}	2.6×10^{-5}	8.4×10^{-7}	0.526	0.048	0.00170	0.091	3.3×10^{-4}	2894	A
7	0.31	1.0×10^{-4}	1.1×10^{-4}	1.7×10^{-6}	1.052	0.048	0.00170	0.046	8.3×10^{-5}	5788	A
8	0.48	2.5×10^{-5}	6.6×10^{-6}	1.1×10^{-6}	0.263	0.012	0.00451	0.046	3.5×10^{-3}	1447	B
9	0.49	2.4×10^{-5}	6.1×10^{-6}	1.1×10^{-6}	0.254	0.012	0.00451	0.047	3.8×10^{-3}	1399	B
10	0.49	2.5×10^{-5}	6.6×10^{-6}	1.1×10^{-6}	0.263	0.024	0.00451	0.091	3.5×10^{-3}	1447	B
11	0.49	2.3×10^{-5}	5.7×10^{-6}	1.0×10^{-6}	0.246	0.048	0.00451	0.195	4.0×10^{-3}	1350	A
12	0.49	5.0×10^{-5}	2.6×10^{-5}	2.2×10^{-6}	0.526	0.048	0.00451	0.091	8.8×10^{-4}	2894	A
13	0.48	5.1×10^{-5}	2.7×10^{-5}	2.2×10^{-6}	0.535	0.024	0.00451	0.045	8.5×10^{-4}	2942	B
14	0.49	9.8×10^{-5}	1.0×10^{-4}	4.3×10^{-6}	1.035	0.048	0.00451	0.046	2.3×10^{-4}	5691	A
15	0.67	2.5×10^{-5}	6.6×10^{-6}	2.1×10^{-6}	0.263	0.012	0.00841	0.046	6.6×10^{-3}	1447	B
16	0.64	2.5×10^{-5}	6.6×10^{-6}	2.1×10^{-6}	0.263	0.024	0.00841	0.091	6.6×10^{-3}	1447	B
17	0.64	2.5×10^{-5}	6.6×10^{-6}	2.1×10^{-6}	0.263	0.048	0.00841	0.182	6.6×10^{-3}	1447	A
18	0.67	5.1×10^{-5}	2.7×10^{-5}	4.2×10^{-6}	0.535	0.048	0.00841	0.090	1.6×10^{-3}	2942	A
19	0.66	5.2×10^{-5}	2.8×10^{-5}	4.3×10^{-6}	0.544	0.024	0.00841	0.044	1.5×10^{-3}	2990	B
20	0.66	5.1×10^{-5}	2.7×10^{-5}	4.2×10^{-6}	0.535	0.012	0.00841	0.022	1.6×10^{-3}	2942	C
21	0.66	1.0×10^{-4}	1.1×10^{-4}	8.3×10^{-6}	1.061	0.048	0.00841	0.045	4.0×10^{-4}	5836	B
22	0.87	1.4×10^{-5}	2.1×10^{-6}	1.2×10^{-6}	0.149	0.06	0.00871	0.402	2.1×10^{-2}	820	A
23	0.86	1.3×10^{-5}	1.9×10^{-6}	1.9×10^{-6}	0.140	0.06	0.01471	0.428	4.0×10^{-2}	772	A
24	0.87	1.4×10^{-5}	2.1×10^{-6}	6.8×10^{-6}	0.149	0.06	0.04905	0.402	1.2×10^{-1}	820	A
25	0.87	1.5×10^{-5}	2.4×10^{-6}	6.9×10^{-6}	0.158	0.012	0.04705	0.076	1.0×10^{-1}	868	C
26	0.86	1.6×10^{-5}	2.6×10^{-6}	7.3×10^{-6}	0.167	0.003	0.04705	0.018	9.1×10^{-2}	916	C
27	0.87	1.6×10^{-5}	2.6×10^{-6}	7.3×10^{-6}	0.167	0.006	0.04705	0.036	9.1×10^{-2}	916	C
28	0.87	1.5×10^{-5}	2.4×10^{-6}	6.9×10^{-6}	0.158	0.027	0.04705	0.171	1.0×10^{-1}	868	B
29	0.87	2.0×10^{-5}	4.2×10^{-6}	4.5×10^{-6}	0.210	0.0048	0.02313	0.023	2.8×10^{-2}	1158	C
30	0.87	2.0×10^{-5}	4.2×10^{-6}	4.5×10^{-6}	0.210	0.0012	0.02313	0.006	2.8×10^{-2}	1158	C
31	0.87	2.0×10^{-5}	4.2×10^{-6}	4.5×10^{-6}	0.210	0.036	0.02313	0.171	2.8×10^{-2}	1158	B
32	0.88	2.0×10^{-5}	4.2×10^{-6}	4.6×10^{-6}	0.210	0.012	0.02364	0.057	2.9×10^{-2}	1158	C

Table S1: Experimental conditions. $N^2 = -g/\rho_a \times \mathrm{d}\rho/\mathrm{d}z$: stratification frequency; $Q_0 = \pi U_0 R_0^2$: volume flux; $M_0 = U_0 Q_0$: momentum flux; $F_0 = g'_0 Q_0$: buoyancy flux; $g'_0 = g(\Delta\rho/\rho_a)$: reduced gravity; $W^* = W/U_0$: wind velocity ratio; $\mathrm{Ri}_0 = g'_0 R_0/U_0^2$: source Richardson number; $\mathrm{Re}_0 = U_0 R_0/\nu$: source Reynolds number; g: acceleration of gravity; R_0: vent radius; U_0: exit velocity; W: ambient velocity; $\Delta\rho = \rho_a - \rho_0$: density gradient; ρ_0: jet density; ρ_a: ambient density; ν: kinematic viscosity. The vent radius is $R_0 = 0.0055$ m for all the experiments. Group A = weak plume; Group B = distorted plume; Group C = strong plume.

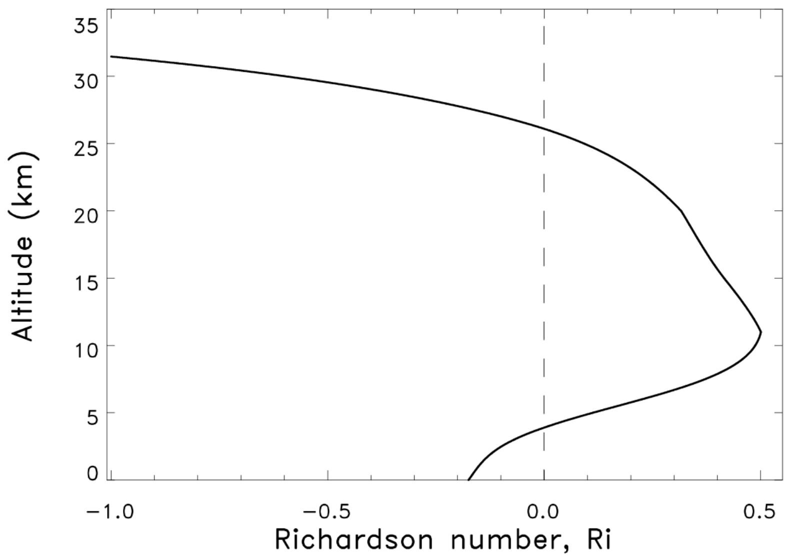

Figure S3: Typical profile of the Richardson number in a volcanic plume. Calculations have been made using the model of *Carazzo et al.* (2008).

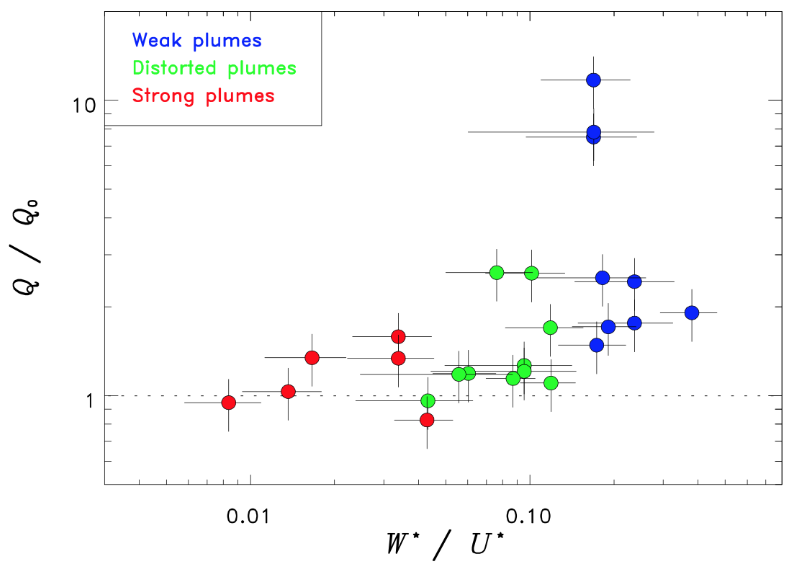

Figure S4: Dimensionless volumetric flow rate Q/Q_0 as a function of the dimensionless number $W^{\star}/U^{\star}$ in our experiments.

Name	Eruption	$MDR^\dagger$ [kg s^{-1}]	$W^\dagger$ [m s^{-1}]	$N^\dagger$ [s^{-1}]	H_0 [km]	$W^\star$ [−]	$U^\star$ [−]	Group	Source
CN1	Cerro Negro, April 1992	3×10^5	7.5	0.0114	3.0	0.039	0.184	B	1
CN2	Cerro Negro, Nov.-Dec. 1995	9.4×10^3	8.4	0.0121	1.2	0.044	0.078	A	2
Et	Etna, 19-24 July 2001	6×10^3	15	0.0124	1.1	0.079	0.071	A	3
He	Hekla, August 1980	2×10^6	9.3	0.0162	3.7	0.049	0.322	B	4
Mi	Miyakejima, August 2000	1.2×10^6	4.5	0.0125	4.0	0.024	0.266	C	5
MSH	Mt St Helens, 18 May 1980	2×10^7	16.3	0.0151	7.0	0.085	0.563	B	6
Pi1	Pinatubo, 12 June 1991	6×10^6	3.3	0.0148	5.3	0.017	0.417	C	7
Pi2	Pinatubo, 15 June 1991	3×10^8	12.4	0.0177	12.2	0.065	1.153	C	8
Re	Reventador, 3 November 2002	1×10^7	6	0.0164	5.5	0.031	0.477	C	9
Ru	Ruapehu, 17 June 1996	2×10^5	23.2	0.0101	3.0	0.122	0.228	A	10

Table S2: Source and environmental conditions for the independent data-set of historical explosive volcanic eruptions. MDR: mass discharge rate in kg s^{-1}; W: average wind speed in m s^{-1}; N: average stratification frequency in s^{-1}; H_0 natural length scale for a pure plume rising in a calm stratified environment in km (*Morton et al.*, 1956); $W^\star$: wind velocity ratio; $U^\star$: plume velocity ratio. Group A = weak plume; Group B = distorted plume; Group C = strong plume. The length scale H_0 is calculated using MDR and N. $W^\star$ and $U^\star$ are calculated assuming a choked jet at the vent according to which U_0 is given by *Woods and Bower* (1995): $U_0 = 1.8\sqrt{x_{g_0} R_{g_0} T_0}$, where x_{g_0} is the amount of gas at the base of the volcanic column, R_{g_0} is the volcanic gas constant, and T_0 is the magma temperature. We consider a range of gas contents between 1 and 5 wt%, a range of temperature between 1000 K and 1400 K, and we take $R_{g_0} = 461$ J kg^{-1} K^{-1}. These values give $U_0 = 122 - 323$ m s^{-1} whose uncertainties are included in the error bars for $W^\star$ and $U^\star$ (see Figure 4 of the paper).
Sources : †: Data from *Mastin* (2014).
1 : see http://www.volcano.si.edu/volcanoes/region14/nicarag/cerroneg/14neg06f.png
2 : see http://www.volcano.si.edu/Photos/full/041047.jpg
3 : see http://www.volcano.si.edu/Photos/full/101098.jpg
4 : see Figs. 8, 9, 10 & 11 of *Gronvold et al.* (1983)
5 : see http://ayay.co.uk/backgrounds/nature/mountains/mount-miyakejima-eruption-2000.jpg
6 : see Fig. 10b of *Carey and Sparks* (1986)
7 : see http://pubs.usgs.gov/pinatubo/hoblitt2/fig11.jpg
8 : see http://www.crystalinks.com/june15pinatubovolcano.jpg
9 : see Fig. 1 of *Chakraborty et al.* (2006)
10 : see http://info.geonet.org.nz/download/attachments/950553/Ruapehu-Erupts-Jun-96-lge.jpg

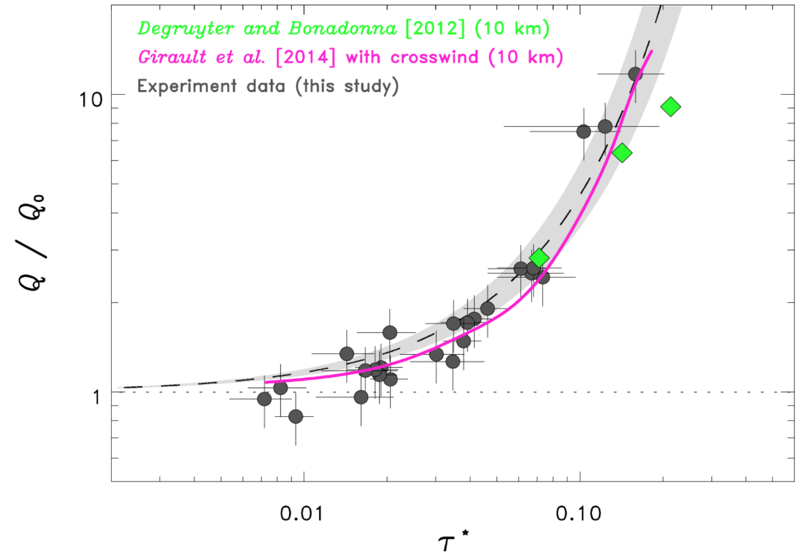

Figure S5: Comparison of our experimentally-determined curve with 1D models of a volcanic plume.

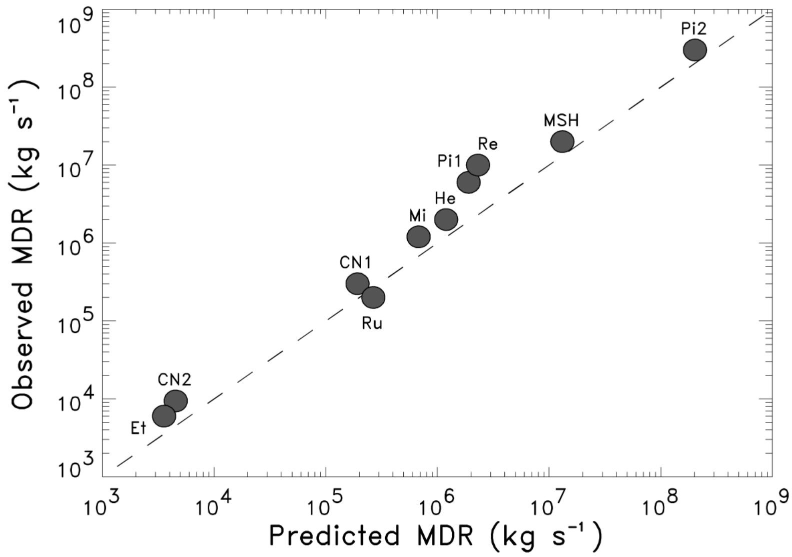

Figure S6: Comparison between the predicted (using Equations 7 to 10 of the paper) and observed mass discharge rate of historical explosive volcanic eruptions (Table S2).

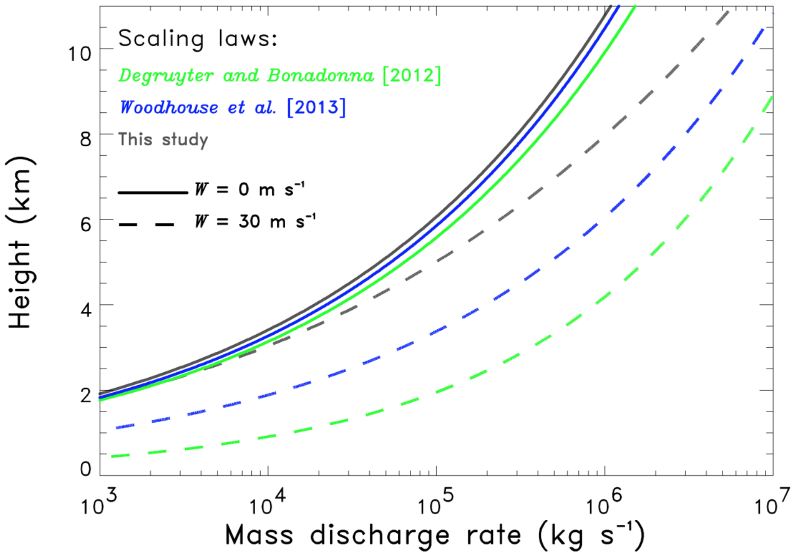

Figure S7: Maximum column height as a function of the source mass discharge rate and wind velocity at the tropopause calculated using the scaling laws presented in this study, and those of *Degruyter and Bonadonna* (2012) and *Woodhouse et al.* (2013). Solid and dashed lines give the predictions in a mid-latitude atmosphere for no wind ($W = 0\,\mathrm{m\,s^{-1}}$) and windy ($W = 30\,\mathrm{m\,s^{-1}}$) conditions, respectively. Green lines are calculated using Eq.6 of *Degruyter and Bonadonna* (2012). Red lines correspond to the functional approximation given in Figure 5 caption of *Woodhouse et al.* (2013). Blue lines give our predictions assuming an andesitic magma with an initial temperature of 1200 K and a density of $2400\,\mathrm{kg\,m^{-3}}$. The specific heat of the volcanic gas and solid particles are $2000\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and $1050\,\mathrm{J\,kg^{-1}\,K^{-1}}$ (*Carazzo et al.*, 2008). Scaling laws in the paper are determined assuming that $H_0 = 0.55H_{obs}$ (*Carazzo et al.*, 2008) and taking $U_0 = 150\,\mathrm{m\,s^{-1}}$ as an average velocity.

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