

Cyclotomic Units

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May 5, 2024

Chapter 1

Introduction

This is my first experimental mini-project to formalize the theory of cyclotomic units in Lean.

Chapter 2

The Index of Cyclotomic Units

2.1 Cyclotomic units

Definition 2.1.1. BLAH.

2.2 Index theorem

Theorem 2.2.1. *Let $n \in \mathbb{N}$ and let $K = \mathbb{Q}(\zeta_n)$ where ζ_n is a primitive n -th root of unit. Let O_K be the ring of integers of K . Let C_K be the group of cyclotomic units in K according to Definition 2.1.1. The index of the group of cyclotomic units $\mu_n(K)$ in the group of units $U(K)$ is given by*

$$[O_K^\times : C_K] = h_{K^+},$$

where h_{K^+} is the class number of the maximal real subfield K^+ of K .

Proof.

□