INFERENTIAL STATISTICS AND HYPOTHESIS TESTING ASSIGNMENT

(Submitted By- Darshna)

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not. Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

Answer a.) :

The type of probability distribution that would accurately portray the above scenario is **Binomial Probability Distribution**. In the above scenario number of sample is fixed (10 drugs), has only two possible outcomes (a drug is produce a satisfactory result or not satisfactory result) and probability of success is same in all trails.

Three conditions binomial probability distribution followed:-

i. Total number of trials is fixed at n.

ii. Each trial is binary, i.e., has only two possible outcomes - success or failure.

iii. Probability of success is same in all trials, denoted by p.

b.) Calculate the required probability.

Answer b.) :

Let S and NS be the events of choosing the satisfactory result and not satisfactory result, respectively. Now, the probabilities of the occurrence of S is P(S) and NS is P(NS) need to be calculated from the given information.

Let the probability of the occurrence of NS be x. Similarly, for S, the probability of its occurrence would be 4*x.

From the equation,

$$x + 4x = 1$$

$$\Rightarrow 5x = 1$$

$$\Rightarrow x = \frac{1}{5}$$

$$P(NS) = \frac{1}{5}$$

$$P(S) = \frac{4}{5}$$

The theoretical probability that at most, 3 drugs are not able to do a satisfactory job are given below:-

The probability of the occurrence of not satisfactory result is P(NS) = 1/5 = 0.2

We have a binomial distribution where n=10, r=3 and p=0.2 and we need to calculate P(X<=3) = P(X=0) + P(X=1) + P(X=2) + P(X=3).

Binomial distribution:-

$$P(X = r) = {}^{n}C_{r}(p)^{r}(1-p)^{n-r}$$

Combination formula:-

$$C(n,r) = \frac{n!}{(r!(n-r)!)} = ?$$

Where ***n*** is the number of things to choose from, and we choose ***r*** of them, no repetition, order doesn't matter.

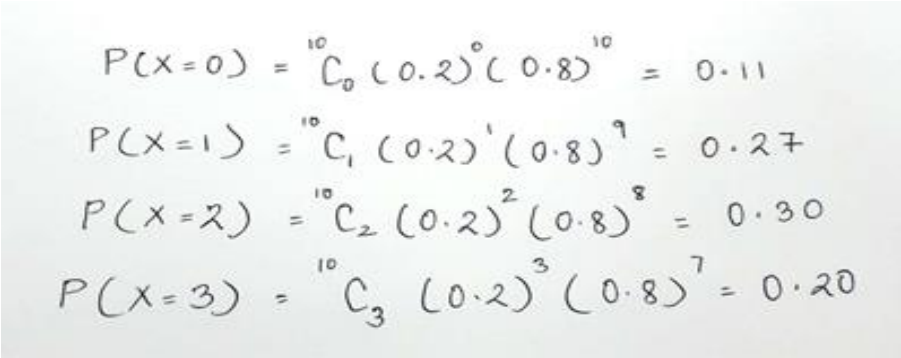
$${}^{10}C_0 = 1$$

$${}^{10}C_1 = 10$$

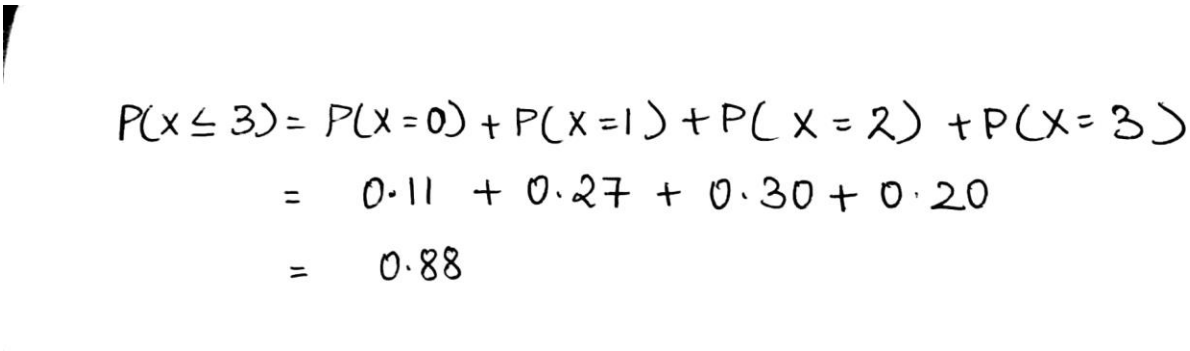
$${}^{10}C_2 = 45$$

$${}^{10}C_3 = 120$$

Now,



So, P(X<=3)



Required probability is 0.88

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.)Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words

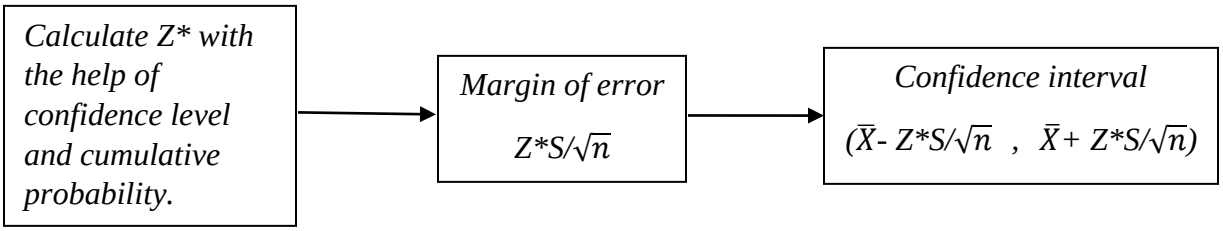
Answer a.) :

For finding the range in which the population mean might lie Central Limit Theorem is used because CLT helps to assume the sample mean would be normally distributed, with mean and standard deviation. Using this assumption it is easy to find margin error, confidence interval and so many things.

Margin of error- A margin of error tells you how many points your results will differ from the real population value.

Confidence interval- A confidence interval gives an estimated range of values which is include an unknown population parameter.

Cumulative probability- Cumulative probability is the probability that a random variable is less than or equal to the specified value.



All the properties of the required method:-

I. Sampling distribution's mean ($\mu_{\bar{x}}$) = Population mean (μ)

II. Sampling distribution's standard deviation (Standard error) = $\sigma/\sqrt{n}$, where σ is the population's standard deviation and n is the sample size

III. For n>30, sampling distribution becomes a normal distribution.

b.) Find the required range.

Answer b.) :

Let's say sample size n**,** mean $\bar{X}$ and standard deviation S. So the **y% confidence interval** (i.e. confidence interval corresponding to y% confidence level) for μ will be given by the range:

$$(\bar{X} - \frac{Z^*S}{\sqrt{n}}, \bar{X} + \frac{Z^*S}{\sqrt{n}})$$

Where, **Z*** is associated with a **y% confidence level**.

Here $\bar{X}$ = 207 seconds

S = 65 seconds

n = 100

Confidence level = 95%

Step – 1

Cumulative probability

$$y + \left(\frac{100 - y}{2}\right)$$

y is the confidence level =95

So, Cumulative probability

$$= 95 + \left(\frac{100 - 95}{2}\right)$$
$$= 95 + \left(\frac{5}{2}\right)$$
$$= 95 + 2.5$$
$$= 97.5\ \%$$

Cumulative probability = 97.5 %

Now, check the z-table for Z-value corresponding to 0.975. Thus 1.96 is the Z* for the original confidence level.

Z* = 1.96

Step – 2

Next calculate Margin of error:-

$$\text{margin of error} = \frac{z^* s}{\sqrt{n}}$$
$$= \frac{1.96 \times 65}{\sqrt{100}}$$
$$= \frac{127.4}{10}$$
$$= 12.74$$

Margin of error = 12.74

Step - 3

The confidence interval or range in which population mean lie is given by-

$$\text{Confidence interval} = \left(\bar{X} - \frac{z^* s}{\sqrt{n}}, \bar{X} + \frac{z^* s}{\sqrt{n}}\right)$$

$$= (207 - 12.74, 207 + 12.74)$$

$$= (194.26, 219.74)$$

Confidence interval or range = (194.26, 219.74)

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

Answer a.) :

The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Here, the claim contains ≤ sign (i.e. the at most sign), so the null hypothesis is the original claim and alternate hypothesis is > sign.

Null hypothesis- Ho: $\mu \leq 200$

Alternate hypothesis- Hi: $\mu > 200$

Sample size n = 100

Mean ($\bar{X}$) = 207 seconds

Standard deviation (σ) = 65 seconds

Significance level (α) = 5 % = 0.05

Sampling distribution for the given sample = $\sigma/\sqrt{n}$

For making decision to either reject or fail to reject the null hypothesis 2 hypothesis methods are used.

i. Critical Value Method

ii. p – value Method

Critical value Method

Step – 1

Here, value of α is 0.05 (of 5%), so the area of the rejection region would be 0.05 and the area of the acceptance region would be 0.95. Since this is an upper-tailed test, the critical region is only on the right-hand side of the distribution, and the area of the critical region would be 0.05.

The area of the critical region in this case would be 0.05, which would be the area beyond the UCV (upper critical value) point in the distribution. So, the area till the UCV point would be **1 - 0.05, i.e. 0.95**. This would be the cumulative probability of that point.

Cumulative probability = 0.95

Step – 2

The z-critical score for the upper-tailed test at 5% significance level. The z-score for 0.950 is not there in the z-table. So, look for the numbers nearest to 0.950. See that the z-score for 0.9495 is 1.64 (1.6 on the horizontal bar and 0.04 on the vertical bar), and the z-score for 0.9505 is 1.65. So, taking the average of these two, the z-score for 0.9500 is 1.645.

z - score = 1.645

Step – 3

$$\text{Critical value} = \mu + Z_c\left(\sigma/\sqrt{N}\right)$$
$$= 200 + \left(1.645 \times \left(\frac{65}{\sqrt{100}}\right)\right)$$
$$= 200 + (1.645 \times 6.5)$$
$$= 200 + (10.6925)$$
$$= 210.6925$$

Critical value = 210.6925

Since 207 ($\bar{x}$) is less than 210.6925, $\bar{x}$ lies in the acceptance region and fail to reject the null hypothesis.

p – value Method

Step – 1

z - score for sample mean

$$\text{z-score} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{207 - 200}{65/10}$$

$$= \frac{70}{65}$$

$$= 1.076$$

$$= 1.08$$

z – score = 1.08

Step – 2

See that the cumulative probability of z-score 1.08 is 0.8599 (1.0 on the horizontal bar and 0.08 on the vertical bar) in z – table.

Cumulative probability = 0.8599

Step – 3

For one-tailed test

$$\text{P-value} = 1 - 0.8599$$

$$= 0.1401$$

$$= 14\%$$

Since the p-value is greater than the significance level (0.1401 > 0.05). Failed to reject the null hypothesis.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current hypothesis test conditions (sample size, mean, and standard deviation), the value of α and β come out to 0.05 and 0.45 respectively.

Now, a different sampling procedure is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other.

Answer b.)

Two type of error in the above case

Type-I error: - The painkiller drug needs to have a time of effect of at most 200 seconds (time of effect <= 200) to be considered as having done a satisfactory job but rejected.

Type-II error: - The painkiller drug do not have a time of effect of at most 200 seconds (time of effect >200) to be considered as having done a satisfactory job but accepted.

Below are the two conditions where either method be more preferred than the other:-

First condition- Patient have severe headache.

Consequences of type-I error is the time of effect is less than equal to 200 seconds for satisfactory result but this is rejected.

Consequences of type-II error is the time of effect is greater than 200 seconds which is harmful for patient.

So, for the safety reason type-I error is preferred here instead of type-II error.

Second condition- Patient have mild headache

Consequences of type-I error is reject the time of effect is less than equal to 200 seconds for satisfactory result but for mild headache not necessary less than equal to 200 seconds.

Consequences of type-II error is accept the time of effect greater than 200 seconds because for mild pain we can wait for more time.

So, here type-II error is preferred instead of type-I.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign for its existing subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer 4:

A/B testing basically is a method which compare two versions of webpages or apps against each other in order to decide which one performs better. By displaying two variations of campaign, team can see which one attracts more interaction and conversions from customers.

A/B testing used because of the following reason:-

i. **Improved Content Engagement**- Marketing team consider and create variables for their A/B tests, they also create lists of potential improvements. Consequently, the simple act of running A/B tests invariably makes their final versions better for their customers.

ii. **Reduced Bounce Rate**-When visitors just "bounce" from the online ad campaign without spending any time reviewing the content or viewing other pages is 'Bounce Rate'. A/B testing will helps to find a best combination of elements that keeps visitors on the online ad long enough to provide them with value from the content which may ultimately lead to a sale.

iii. **Increased Conversion Rates**- A/B testing is most effective way of creating content that converts more visitors into buyers.

iv. **Ease of Analysis**- When analysing data from an A/B test, we can easily determine a 'winner' and a 'loser' based on simple, straightforward metrics.

v. **Reduced Risks**- A/B testing is help to examine the customer behaviour and examine visitors on the campaign in order to committing major decision which helps to increase the chance of success.

A/B testing process-

Before A/B testing

1. Pick one variable for test- We can test more than one variable for a single online ad campaign, but testing them one at a time.

2. Identify Goals-Main conversion goals are the metrics that we are using to determine whether or not the modified version is more successful than the real version. Goals can be anything from page size, text font and text colour of the taglines.

3. Create a 'control' and a 'challenger'- The real version of taglines is 'control' and modified version of taglines against the 'control' is 'challenger'.

4. Split sample groups equally and randomly.

5. Decide how significant results need to be.

During A/B testing

1. Use an A/B testing tools.

2. Test both variations simultaneously- When we run A/B tests, we will need to run the two variations at the same time, otherwise we may be left second-guessing our results.

3. Give the A/B test enough time to produce useful data- Test run long enough in order to obtain a substantial sample size. Otherwise, it will be difficult to tell whether there was a statistically difference between the two variations.

4. Ask for feedback from real users.

After the A/B testing

1. Focus on goal metric- Focus on the primary goal metric, when do the analysis.

2. Measure the significance of results using A/B testing calculator.

3. Take action based on results.