CHAPTER 2

The Hirsch Index and Related Impact Measures

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Introduction

The Hirsch index (or h-index), introduced in 2005 by Jorge Hirsch, is one of the most popular indicators in information science and in informetrics. Hundreds of articles have been written on the h-index or related h-type indices. To quote Ball (2007, p. 737), "the h-index does seem to be able to identify good scientists, and is becoming widely used informally, for example to rank applicants for research posts." A review of this important topic is thus warranted.

In the first section we define the h-index and lay out its advantages and disadvantages. To deal with some of the limitations of the h-index, several other "h-type" indices (also called impact measures) have been introduced. These are discussed in the next section. We then discuss the application of h-type indices to entities other than authors (e.g., journals or topics) and provide an overview of case studies. In the fourth section we consider the intrinsic properties impact factors have or should have. It is not possible to define exactly what a good impact measure is, but we can give axiomatic characterizations of some impact measures and look at the influence of production (i.e., number of articles and number of citations to these articles) on some impact measures.

In the fifth section we discuss some informetric models for these h-type measures (e.g., their dependence on the total number of articles and on the total number of citations). We also describe dynamical aspects of these indices—the influence of transformations on h-type indices—and study distributions of h-type indices, noting that these are of a different nature from the distributions of the impact factor. In the next section we study h-type indices as a function of time (e.g., the career of an author) and examine the possible shapes of these functions as well as case studies. We close with some conclusions and challenges.

This review does not describe many case studies (although an attempt is made to present a complete reference list up to the end of 2008) but focuses on the "philosophy" behind h-type indices as impact or performance measures and their potentialities as new informetric indicators.

The Hirsch Index: Definition, Advantages, and Disadvantages

We start with an historical note, communicated to this author by R. Rousseau and reported by Edwards (2005). It appears that the Hirsch index (of course not with this name) was defined some 35 years earlier by the astrophysicist Sir Arthur Stanley Eddington as follows (in a communication to the geophysicist Harold Jeffreys). In order to record his cycling prowess, Eddington records n, being the highest number of days on which he had cycled n or more miles. As will become clear, this is nothing other than Hirsch's index (but in cycling terminology).

Hirsch's (2005b) definition is: A scientist has index h if h of his/her N_p papers have at least h citations each, and the other (N_p- h) papers have no more than h citations each. It must be remarked, however, that the first formulation by Hirsch (2005a) in the arXiv paper of August 17, 2005 had the word "fewer" instead of "no more" in this definition. This was corrected in the arXiv paper on September 29, 2005 and subsequently published as Hirsch (2005b). This correction was indeed necessary because, in the formulation of August 17, the h-index does not always exist. Consider the following simple example: Say an author has 5 papers, 4 of them with 3 citations and the fifth with 1. Here (h = 3) but when, in this definition, "no more" is replaced by "fewer," the h-index does not exist (cf. remarks on this by Glänzel [2006a] and Rousseau [2006b]). Note that Hirsch's (2005b) correction was not mentioned by Anderson, Hankin, and Killworth (2008), by Lehman, Jackson, and Lautrup (2008), or by Sidiropoulos and Katsaros (2008).

Another, equivalent, definition is as follows: If we rank an author's papers in decreasing order by the number of citations they receive, then this author's h-index is the highest rank $r = h$ such that the papers at ranks 1, 2, …, h, each have h or more citations.

Although we do not like the term, the papers at ranks 1, …, h constitute the so-called h-core (introduced by Rousseau, 2006b). Note that this h-core is not really a core set of papers, although they are the h most visible papers.

An example is presented based on this author's own publication and citation record (compiled October 22, 2008, using Web of Science [WoS]). Table 2.1 presents papers ranked in decreasing order of received citations. From this it is clear that h = 17 (this is clear from the table up to r = 18; for later use, we include authors with ranks up to 25).

What does the h-index measure? In itself the h-index does not give a value to an informetric unity. This is in contrast with other informetric indicators such as the impact factor, which, essentially, is an average number of citations per paper. The h-index is not an average,

Table 2.1 Citation data for L. Egghe (Web of Science, October 22, 2008)

Rank	Number of citations
1	58
2	52
3	45
4	37
5	37
6	33
7	27
8	25
9	22
10	19
11	18
12	18
13	18
14	18
15	17
16	17
17	17
18	17
19	16
20	15
21	14
22	14
23	13
24	13
25	12

not a percentile, not a fraction: It is a totally new way of measuring performance, impact, visibility, quality, … (see also the fourth section of this chapter) for instance, of the career of a scientist (for other applications, see the third section). It is a simple measure without any threshold (e.g., for the Garfield-Sher impact factor one limits the citation period to two years, which is, as is well-known, not optimal in some fields). This should explain the popularity of the h-index. The h-index is so popular that Scopus and Web of Science, less than two years after its introduction, have decided to incorporate it as an indicator.

The h-index is a single, simple measure that combines papers (an indicator of quantity) and citations (an indicator of quality or impact). It is a robust measure in two ways: It is not influenced by a set of infrequently cited papers or by the (even severe) increase of citations to already highly cited papers. The first aspect is certainly an advantage of the h-index: A researcher should not be "punished" for writing some low-cited papers, as long as this researcher writes highly cited papers as well. The fact that the h-index does not count the actual citations to papers is, in our view, a disadvantage of the h-index: Once a paper belongs to the h-core, it does not matter how many more citations it will receive. We feel that very highly cited papers should contribute more to an impact measure than less cited papers. Other measures have, therefore, been introduced (see the next section). To illustrate the insensitivity of the h-index we compared in 2006 the citation data of E. Garfield and F. Narin (see Egghe, 2006c). The data can be found in Table 2.2.

As one readily sees, both scientists had h-index values of 27. But Garfield has no less than 14 papers with more than 100 citations (the top one has 625 citations) while Narin has only one paper with more than 100 citations. This clearly illustrates the insensitivity of the h-index.

Of course, like any citation indicator, the h-index is field dependent and thus h-indices of authors in different fields cannot be compared. Table 2.3 presents the top researchers in the fields of physics, chemistry, and computer science (Ball, 2005).

The differences are clear: The h-indices of the physicists are all lower than those of the chemists. The h-indices of the computer scientists are, in turn, low in comparison with those of the physicists. For more on this field-dependence, see the next section.

New researchers have a clear disadvantage because of the short length of their careers and they should not be compared with researchers with long careers. On the other hand, established researchers may rest on their laurels because the number of citations received may increase even if no new papers are published. In other words: The h-index can never decrease. In the next section we review measures that take career length into account.

In citation analysis there is the problem of how to treat self-citations (Egghe & Rousseau, 1990; Smith, 1981; Vinkler, 1986). It is no different for the h-index—see Schreiber (2007b) and Zhivotovsky and Krutovsky (2008). Another problem is how to deal with multi-authored papers (citing as well as cited). Several papers dealing with this issue are discussed in the next section. Burrell (2007e) also discusses both problems (self-citations and multi-authorship). Finally, one should also be aware that a researcher's h-index depends on the citation database that is used. One can use WoS, Scopus, or Google Scholar—see Bar-Ilan (2008c), Jacsó (2008a, 2008b, 2008c, 2008d, 2008e), Meho and Yang (2007), and Meho and Rogers (2008). Meho and Yang (2007) show that not only h-index values can change when one goes from one database to another but that, when comparing (ranking) h-indices for a set of researchers, the ranks can also change significantly. Bar-Ilan (2008c) and Jacsó (2008a, 2008e) compare the performance of WoS, Scopus, and Google Scholar in various respects. Meho and Rogers (2008) conducted a similar study showing the better coverage provided by Scopus. Note also that the website www.harzing.com/pop.htm offers software that retrieves and analyzes citation data based on Google Scholar: The h-index and several variants

Table 2.2 Citation data and h-index for E. Garfield and F. Narin in 2006

E. Garfield		F. Narin	
Total citations	Rank of paper	Total citations	Rank of paper
625	1	112	1
149	2	95	2
138	3	86	3
132	4	82	4
132	5	73	5
129	6	71	6
127	7	70	7
111	8	63	8
109	9	59	9
108	10	55	10
107	11	55	11
105	12	53	12
104	13	52	13
101	14	52	14
96	15	44	15
91	16	41	16
89	17	38	17
88	18	37	18
87	19	35	19
85	20	33	20
80	21	33	21
67	22	29	22
63	23	28	23
41	24	28	24
29	25	28	25
28	26	27	26
27	27	27	27
26	28	26	28
.	.	.	.
.	.	.	.
.	.	.	.

(to be introduced in the next section) can be calculated. McKercher (2008) used this software.

As with any indicator, the h-index is a single number, thereby reducing a scientist's career to a one-dimensional measurement. In the last section we add a time dimension to the h-index (and other h-type indices).

The h-index is shown to be robust with regard to errors in citation lists (Vanclay, 2007) or missing publications (Rousseau, 2007b). Hirsch (2007) demonstrates that his h-index is a good indicator—better than total number of citations or papers or citations per paper (impact factor)—of future achievement.

More discussion of the (dis)advantages of the h-index can be found in works by Antonakis and Lalive (2008); Bar-Ilan (2006); Bornmann and Daniel (2005, 2007b); Braun, Glänzel, and Schubert (2005, 2006); Costas and Bordons (2007b); Egghe (2006a, 2006b, 2006c); Glänzel (2006b);

Table 2.3 Top researchers in physics, chemistry, and computer science, by h-indices

Physics		
1.	Ed Witten, Institute for Advanced Study, Princeton	$h = 110$
2.	Marvin Cohen, University of California, Berkeley	$h = 94$
3.	Philip Anderson, Princeton University	$h = 91$
4.	Manuel Cardona, Max Planck Institute for Solid State Research, Stuttgart, Germany	$h = 86$
5.	Frank Wilczek, Massachusetts Institute of Technology	$h = 68$
Chemistry		
1.	George Whitesides, Harvard University	$h = 135$
2.	Elias James Corey, Harvard University	$h = 132$
3.	Martin Karplus, Harvard University	$h = 129$
4.	Alan Heeger, University of California, Santa Barbara	$h = 114$
5.	Kurt Wüthrich, Swiss Federal Institute of Biology, Zurich	$h = 113$
Computer Science		
1.	Hector Garcia-Molina, Stanford University	$h = 70$
2.	Deborah Estrin, University of California, Los Angeles	$h = 68$
3.	Ian Foster, Argonne National Laboratory, Illinois	$h = 67$
4.	Scott Shenker, International Computer Science Institute, Berkeley	$h = 65$
4.	Don Towsley, University of Massachusetts, Amherst	$h = 65$
4.	Jeffrey D. Ullman, Stanford University	$h = 65$

Glänzel and Persson (2005); Jin, Liang, Rousseau, and Egghe (2007); Kelly and Jennions (2007); Liu and Rousseau (2007); Popov (2005); Rousseau (2006b, 2008a); Sidiropoulos and Katsaros (2008); Thelwall (2008); van Leeuwen (2008); van Raan (2006); and Wendl (2007).

Variants of the h-Index

As mentioned already, the h-index has some clear disadvantages such as its insensitivity to the actual number of citations to the articles in the h-core or the advantage that researchers with longer careers have (but it must be emphasized that it was Hirsch's explicit purpose to give an index that measures impact of "long" careers). Multiple authorship, self-citations and field-dependence are also issues that must be dealt with in connection with the h-index.

In fact, Hirsch himself introduced in the first defining article (Hirsch 2005b) a variant of the h-index: The m-quotient, which is simply the h-index divided by career length (time since the publication of the first paper). Burrell (2007a) gives a similar definition (in a time-dependent context—see also the last section), which he calls the h-rate. A variant is to divide the h-index not by time but by the total number of papers published by an author (called the normalized h-index). Sidiropoulos and Katsaros (2008) do this. Rousseau (personal communication) feels that this is a bad idea but thinks it is useful for journals (instead of authors) (see the next section for an application of the h-index to journals): In this way different years with different numbers of published articles can be compared, as Rousseau (2006a) has shown.

Iglesias and Pecharromán (2007) claim that dividing the h-index by the average number of citations per paper produces an index that can be used for inter-area comparisons. We are convinced that this index is more field independent than the h-index, but we do not think it is useable as a completely field-independent index. Batista, Campiteli, Kinouchi, and Martinez (2006) and Campiteli, Batista, and Martinez (2007) seem to prove that dividing the h-index by the total number of authors in the set of h papers yields a relatively field-independent indicator. We are skeptical about the claim by Valentinuzzi, Laciar, and Atrio (2007) that they have found two discipline-independent h-type indices.

Radicchi, Fortunato, and Castellano (2008) also try to construct a field-independent h-index. They do not divide the h-index by the average number of citations per paper in the field (Iglesias & Pecharromán, 2007) but rather divide the actual number of citations to each of a researcher's papers by this average number. In addition, they divide the number of a researcher's publications by the average number of publications in the field. Using these "rescaled" numbers, they apply the definition of the h-index, thereby obtaining the "generalized h-index."

Qiu, Ma, and Cheng (2008) also mention the field-dependence of the h-index as well as the fact that the h-index is not very well suited to evaluating the outputs of young researchers. They introduce the Paper Quality Index (PQI), a relative index based on the impact factor of a field and the total number of citations in that field. This index is based on the impact factor and is, therefore, not of h-type.

Molinari and Molinari (2008a, 2008b) make a log-log plot of the h-index versus number of papers, yielding a regression line of the form $\ln h = A - \beta \ln N$ (N = number of papers). Then h_m is defined as

$$h_m = e^A = \frac{h}{N^\beta}$$

which is defined as the impact (or quality) index and which is claimed to be a correction for size. The name is not well chosen because all indices discussed here are impact indices (see also the fourth section).

Several papers deal with modifications of the h-index in connection with co-authors. Campiteli and colleagues (2007) and Batista and colleagues (2006) use the h-index, divided by the average number of authors per paper in the h-core. This index is denoted h_I and equals $h / \bar{T}$ where $\bar{T} = T / h$, with T the total number of authors in the h-core, hence $h_I = h^2 / T$.

Independently, both Egghe (2008f) and Schreiber (2008c, 2008d) introduce the same measure h_m (Schreiber) and h_F (Egghe) using fractional paper counts (thereby replacing entire ranks by fractional ranks being 1 divided by the number of co-authors of the paper). Egghe (2008f) also suggests the measure h_f whereby one uses fractional citation counts instead of fractional paper counts. He presents a mathematical theory of h_f and h_F, noting that, in practice, the corrections h_f or h_F on h are not very large. Wan, Hua, and Rousseau (2007) and Chai, Hua, Rousseau, and Wan (2008) go even further and define the so-called "pure h-index" where one can even take into account a scientist's relative position in the byline of an article. In this connection see also Kim and Seo (2007).

Kosmulski (2006) introduces the $h^{(2)}$-index which is an h-type index that is easier to calculate than the h-index because one needs a shorter list of papers in decreasing order of number of citations: An author has Kosmulski's index $h^{(2)}$ if $r = h^{(2)}$ is the highest rank such that all papers at ranks 1, …, $h^{(2)}$ have at least $(h^{(2)})^2$ citations. In Table 2.1, this would mean $h^{(2)} = 5$ for this author. The numbers $h^{(2)}$ are hence much smaller than the h-indices and we feel that, for this reason, $h^{(2)}$ does not discriminate very well among authors. In addition, the calculation of the h-index is not very time consuming and, as has been mentioned, can be calculated in both Web of Science and Scopus. Notwithstanding this criticism of the $h^{(2)}$-index, the Applications section of this chapter reviews an interesting application of the $h^{(2)}$-index to downloads.

Levitt and Thelwall (2007) give a generalization of the Kosmulski index. They define the Hirsch k-frequency $f(k)$ as the number of documents that are cited at least kh times. Note that $f(h) = h^{(2)}$ is the Kosmulski index. The variable k enables one to illustrate the distribution of the highly cited documents. They also introduce a "normalized" h-index:

$$h_{\mathrm{norm}} = 100\frac{h^2}{T}$$

where T is equal to the number of documents of the set.

Hu and Chen (2009) introduce the so-called Major Contribution Index (MCI): An author's Hirsch index, based on his/her papers in which this author plays a major role (e.g., first or corresponding author). We do not see the value of this h-index variant: It is the number of citations that counts and this for any published paper of an author!

None of the variants of the h-index deals with the insensitivity of the h-index to the number of citations to the highly cited papers. This will be discussed now. Note that—in our opinion—we do not need variants of the h-index dealing with the lowly cited papers. Hence we think that the v-index, introduced by Riikonen and Vihinen (2008) (being h divided by the total number of papers), does not make much sense as an impact measure.

First we introduce the g-index, as given by Egghe (2006a, 2006b, 2006c). Note that the actual number of citations to the first 17 papers of this author (Table 2.1) is in fact of no importance as long as this number is at least 17. The insensitivity of the h-index is also clear from Table 2.2 in the comparison of the careers of Garfield and Narin. Egghe notes that the h-index satisfies the property that the first h papers, together, have at least $h^{(2)}$ citations (indeed, in the h-core we have h papers each having at least h citations). Then Egghe defines the g-index as the highest rank with this property. In other words, with the same ranking of papers in decreasing order of number of citations received, we define the g-index as the highest rank such that the first g papers have at least g^2 citations together. In other words: The first g papers have at least g citations, on average. Note that, per definition, $g \geq h$.

In Table 2.1, the total number of citations to the first 24 papers equals $580 > 24^2$ and the total number of citations to the first 25 papers equals $592 < 25^2$ so that $g = 24$ for this author (as of October 22, 2008). Egghe (2006c) performs the same calculation on the (extended) citation data for Garfield and Narin (Table 2.2). Although their h-indices (in 2006) are equal ($h = 27$), Egghe finds that Garfield's g-index is 59 and Narin's is 40, demonstrating the g-index's greater discriminatory power. This is confirmed by Schreiber (2008a, 2008b) and Tol (2008). Costas and Bordons (2008) also confirm this finding, noting that for prolific scientists the ranks based on the g-index are lower than those based on the h-index, and their g-index/h-index ratios are, on average, higher. Burrell (2007c) compares the g-index with the h-index and finds that they are both proportional to career length (see also the section on time-dependent h-type sequences). Rosenstreich and Wooliscroft (2009) apply the g-index to the ranking of management journals.

Jin and colleagues (2007) introduce the R-index, another attempt to improve the sensitivity of the h-index to the number of citations to highly cited papers. It is defined as

$$R = \sqrt{\sum_{i=1}^{h} c_i} \tag{1}$$

where documents, as usual, are ranked in decreasing order of the number of received citations, c_i is the number of citations to the i^{th} paper, and h is the h-index. So, as is also the case with the g-index, this measure takes into account the actual c_i-values in the h-core. It is an improvement of the A-index introduced by Jin (2006) (the name was suggested by Rousseau [2006b]; see also Jin et al. [2007]), which defined as the average number of citations to papers in the h-core:

$$A = \frac{1}{h}\sum_{i=1}^{h} c_i \tag{2}$$

This measure has the undesirable property that an increase in the number of citations might lead to a decrease in the A-index (see the section on desirable properties of impact measures). Consider a situation where four papers have 6, 5, 4, and 3 citations respectively: $h = 3$ and $A = \frac{1}{3}(6 + 5 + 4) = 5$. Add one citation to the fourth paper: 6, 5, 4, 4; now $h = 4$ and $A = \frac{1}{4}(6 + 5 + 4 + 4) = \frac{19}{4} < 5$, which is not desirable: More citations should not lead to a strict decrease in an impact measure. Note that the g- and R-indices do not suffer from this deficiency. The square root in (1) is necessary to conform with the h-index in case all $c_i = h$, $i = 1, \ldots, h$. Then $R = h$, as is readily seen.

Glänzel (2008c, p. 66) compares the h-index and A-index with so-called characteristic scores and scales (CSS), which are mean citation rates of subsets of an author's papers. One example of a CSS is the average number of citations to those papers that have at least a number of citations larger than or equal to the overall average number of citations.

Sezer and Gokceoglu (2009) define the so-called "fuzzy academic performance index," which is built around three indicators—total number of publications, total number of citations, and academic life span—and one notes a high correlation of this index with the h-index. It can be seen from Table 2.1 that the R-index of this author is 21.86, the square root of the sum of the number of citations to the $h = 17$ most highly cited articles. In order to normalize for the age of an article (which is more advantageous for attracting more citations), Jin and colleagues (2007) introduce the AR-index:

$$AR = \sqrt{\sum_{i=1}^{h} \frac{c_i}{a_i}} \tag{3}$$

where h and c_i are as given and a_i is the age of article i. The AR-index is an approach to define an h-type index that can actually decrease. In this way authors cannot "rest on their laurels." Rousseau and Jin (2008), Jin (2007), and Jin and Rousseau (2007) discuss the AR-index further; Glänzel (2007a) provides comments on the R- and AR-indices. Järvelin and Persson (2008) criticize the AR approach and propose another way of taking age into account (their proposal, however, is not an h-type index).

Rousseau and Jin (2008) present an alternative for (3). Instead of the square of (3):

$$\sum_{i=1}^{h} \frac{c_i}{a_i} \tag{4}$$

one could also define

$$AR_1^2 = \frac{\sum_{i=1}^{h} c_i}{\sum_{i=1}^{h} a_i} \tag{5}$$

similar in form to the meta-journal average impact factor and the meta-journal global impact factor, respectively. Egghe and Rousseau (1996a, 1996b) discuss their difference—and hence, indirectly, the difference between (4) and (5). However, Rousseau and Jin (2008) indicate some deficiencies with the index defined in (5).

Another way to take the citation scores of the (highly cited) papers into account is to weight the articles according to their citations: This is the weighted h-index denoted h_w (Egghe & Rousseau, 2008). The discrete h_w-index suffers from the same problem as the A-index but Egghe and Rousseau show that the continuous model does not suffer from this aberration.

Anderson and colleagues (2008) present another attempt to solve the problem of the insensitivity of the h-index to the number of citations to the highest cited articles. Although it is not a practical solution (they have to take into account all citations to all papers), there is some theoretical interest in their proposal. Their inspiration comes from a theory of partitions, described by Andrews (1998). A table (called a Ferrers graph) is constructed as follows: Papers are vertically ranked in decreasing order of citations received. Instead of writing the number of received citations, dots (equal to this number) are shown horizontally, for each paper. From the upper left corner, squares of side length 1, 2, 3, … are made. First the square of side-length 1 (consisting of the first citation dot to the first paper) is made and this point receives a weight of 1. The square of side-length 2 is made by adding to this square of side-length 1 the second citation (dot) to the first paper and the first two citations (dots) to the second paper. So we add three citations each receiving a weight of ⅓. The next square with side-length 3 consists of the square of length 2, extended with five citations (each receiving a weight of ⅕): The third citation (dot) to the first paper and to the second paper and the first three citations (dots) to the third paper, and so on. We can continue like this until all citations to all papers are covered. Figure 2.1 provides an example.

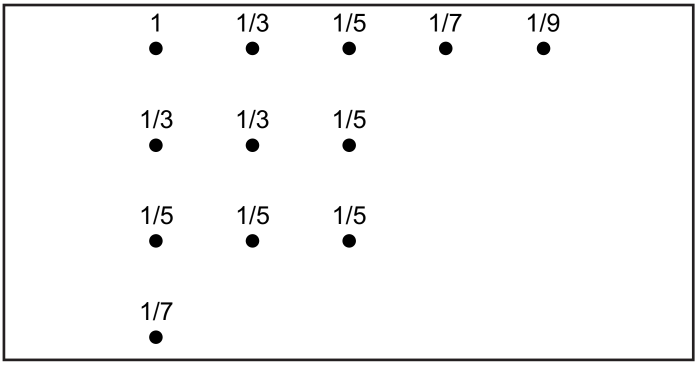

Figure 2.1 Ferrers graph of (5,3,3,1): four papers with respectively 5, 3, 3 and 1 citation(s)

The sum of all the weights of given citations is the so-called tapered h-index; it is denoted h_T and is a (theoretical) natural extension of the h-index h. Indeed, the h-index h equals the side-length of the largest completed (filled in) square of points, being also equal to the sum of all the scores in this square (called the Durfee square) (in Figure 2.1, $h = 3$). Hence, clearly h £ h_T (in Figure 2.1, $h_T = 3 + \frac{2}{7} + \frac{1}{9}$) but h_T takes into account ("in an h-type way") all citations to all papers (as said, this is not practical and hence a disadvantage of the tapered h-index h_T).

Egghe (2008a) shows how these Ferrers graphs can be transformed to yield the g-index in a natural (theoretical) way. He considers the so-called conjugate sequence of a partition (decreasing sequence of coordinates of a vector) and shows that its h-index is the same as the h-index of the original partition. Egghe (2005) studies Lorenz curves for partitions, and their conjugates are studied in both discrete and continuous (introduced by Egghe [2008a]) ways.

Van Eck and Waltman's (2008) study of generalizations of the h-index and the g-index involves an extra parameter α. The generalized h-index h_α is the highest rank such that all the papers at this rank (and lower ranks) have at least αh citations. Note that $h_1 = h$. Similarly, the generalized g-index g_α is defined as the highest rank such that the first g_α papers have, together, at least αg^2 citations. Note again that $g_1 = g$. The authors demonstrate properties for varying α. Independently, Levitt and Thelwall (2007) also define h_α, but in another notation: They define the Hirsch-frequency $f(k)$ as the number of documents in the collection that are cited at least kh times. Hence $f(k) = h_\alpha$ for $\alpha = k$.

Deineko and Woeginger (in press) and Woeginger (2008d) even generalize this by replacing αh and αg^2 by general functions $s(k)$, $k = 1, 2, \ldots$. Subsequent sections of this chapter discuss axiomatic characterizations of these indices (including the h-index and g-index) and their informetric properties.

Bornmann, Mutz, and Daniel (2008) compare nine of these indices (h, m quotient of Hirsch, g, $h^{(2)}$, A, R, AR, h_w, and a new m-index). The m-index is the median number of citations received by papers in the h-core. They classify (using factor analysis) indices as:

- Describing the most productive core of a scientist's output (such as h, m quotient, g, and $h^{(2)}$)
- Depicting the impact of the papers in the core (such as A, m-index, R, AR, and h_w)

(See also Bornmann, Mutz, Daniel, Wallon, & Ledin, 2008.)

Furthermore, Bornmann, Mutz, and Daniel (2009) argue, the classical bibliometric indicators "number of publications" and "total citation counts" load so high on these two factors that, in this (statistical) vision, h-type indices are not necessarily required. Several papers discussed in the next section report similar conclusions.

Bornmann, Mutz, and Daniel (2007) introduce the b-index of a scientist as the number of his papers that belong to the top 10 percent of papers in a field. Because this index uses a threshold value, we consider this as a non-h-type measure.

Leydesdorff (in press) compares the h-index statistically (using principal component analysis) with non-h-type indices (such as PageRank, impact factor, Scimago Journal Ranking, and network centrality measures). He finds that the h-index combines the dimensions of size and impact.

Ruane and Tol (2008) introduce rational interpolations of the h-index (i.e., between h and $h + 1$); see also Tol (2008) and Guns and Rousseau (2009a) (who also study the g-index). Rousseau (2008d) introduces real-valued h-indices and g-indices, which receive further attention from Guns and Rousseau (2009a). These real-valued indices are based on the piecewise linearly interpolated citation curve. Researchers who have investigated other h-type indices include Sidiropoulos and Katsaros (2008); Rousseau, Guns, and Liu (2008); Kosmulski (2007); Boell and Wilson (2008); and Ruane and Tol (2008).

Applications

It is clear from Hirsch's (2005b) original definition that the h-index (and hence also the other h-type indices) has been defined in order to have a simple informetric indicator of the impact of a researcher's career. Worldwide the h-index (and some of its variants) is used for this purpose and we have mentioned that the h-index is available in WoS and Scopus. Researchers in the same field (and with equally long careers) can be compared using the h-index. The h-index can also be used to predict future achievements and for deciding on tenure positions or grants (Hirsch, 2005b).

Soon after the introduction of the h-index, Braun and colleagues (2005, 2006) noticed the important fact that the h-index can also be applied to journals: The same definition applies to a ranked set of articles (in decreasing order of the number of citations they received) of a journal (e.g., for a certain period or for all articles of the journal). The same goes, of course, for the other h-type indices. This leads to the constatation that the journal impact factor (IF) has a serious competitor, especially because the IF is dependent on the defined citing and publication period: This arbitrariness in the definition of IF does not apply to the h-index and derivatives. Miller (2006) claims the superiority of the h-index over the IF for physics and Vanclay (2007) expresses a similar preference, although Vanclay (2008b) subsequently reports high correlation between journal rankings based on IF and rankings based on the h-index. Harzing and van der Wal (2009) find the h-index for journals, based on Google Scholar, to be a better measure of journal impact than the IF itself. Vanclay (2006) comments on the work by Braun and colleagues (2005).

In general, one finds comparisons of the h-index with other bibliometric indicators such as total number of publications, total number of citations, average number of citations per publication (IF), and even peer review for authors, journals, or groups of authors in papers by Bornmann and Daniel (2007a); Bornmann, Wallon, and Ledin (2008); Costas and Bordons (2007b); and van Raan (2006). Wan, Hua, Rousseau, and Sun (2009) compare the h-index for journals with a new indicator, the "download immediacy index" (DII) defined as the number of downloads of a journal's articles within one publication year, divided by the number of articles published in that journal during that year (this measure was introduced by Wan, Hua, Du, & Song, 2007). The conclusions are, in general, the same: All these rankings correlate strongly. From this point of view, research evaluation does not need the h-index, although all authors agree that the h-index is a valuable, simple tool that, due to the previously mentioned correlation, can to a certain extent, replace the other indicators. Bornmann, Marx, and Schier (2009) reach a similar conclusion: h-type indices correlate highly with IF and hence, from a statistical point of view, are empirically redundant. Coincidentally Reedijk and Moed (2008) come to the conclusion that the "impact" of impact factors is decreasing, which indirectly leads to a higher importance of the h-index for journals. However, Honekopp and Kleber (2008) reach an opposite conclusion as far as the prediction of future success of an article is concerned.

In addition to applications to authors, groups of authors (e.g., institutes), and journals there are possible applications to all source-item relations in an information production process (IPP) (cf. Egghe, 2005). An IPP is a set of sources (e.g., authors, journals) that produce items (e.g., articles) where one indicates which sources produce which items (see Egghe [2005] or Egghe & Rousseau [1990] for many more examples). These sources can be ranked in decreasing order of the number of items and hence the h-index (or any h-type measure) of this IPP can be calculated—see also Egghe (in press c).

In this sense the paper by Banks (2006) is very interesting because it defines the h-index for (scientific) topics and compounds. Thus "hot" topics can be detected in diverse research areas. Bar-Ilan (2007, 2008a) applies this idea to the h-index of the topic "h-index." Egghe and Rao (2008a), Jin and Rousseau (2008), Minasny, Hartemink, and McBratney (2007), Rousseau (2007a), and The STIMULATE6 Group (2007) undertake similar investigations.

Csajbók, Berhidi, Vasas, and Schubert (2007) study the h-index for countries (by various science fields). Liu and Rousseau (2007, 2009) calculate the h-index for book classifications and the number of times the books in these classifications are borrowed, an example of how the h-index can be used in library management. Guan and Gao (2009) apply the h-index to citations to patents.

Because the h-index can be applied in any IPP, Egghe and Rao (2008b) studied different h-indices for groups of authors. If we look at a group of authors (e.g., in an institute or field) we can arrange these authors in decreasing order of number of publications. The corresponding h-index is denoted h_p. If we arrange these authors in decreasing order of the total number of citations received, we arrive at the h-index h_C. Rousseau and Rons (2008) propose a similar h-index, but using the total number of different citing papers (instead of all received citations). If we consider the group of authors as one meta-author and if we arrange the papers of this meta-author in decreasing order of the number of citations received we have the global h-index h_G, also studied by van Raan (2006) and mentioned by Prathap (2006). Egghe and Rao (2008b) prove interrelations among these h-indices; they also examine the successive h-index, h_2. This successive h-index was introduced, independently, by Prathap (2006) and Schubert (2007) and we define it now.

Suppose we have a group of authors, for example, at an institute, and that each author has an h-index. If we arrange these authors in decreasing order of their h-indices then we can define the h-index of this list (denoted h_2) as the highest rank such that the first h_2 authors have an h-index of at least h_2. This is called the successive h-index because it was derived from the h-indices of the authors. Schubert (2007) offers a slightly different example, replacing authors by journals (e.g., of a publisher); with this approach the h-indices of these journals determine the successive h-index, h_2, of this publisher (see also Braun et al., 2005, 2006). One can even go to a higher aggregation level, using the h_2-indices to determine the successive h-index, h_3, of a country. Egghe and Rao (2008b) also study the successive h-index h_2 in comparison with the previously defined h_p, h_C, and h_G for a group of authors. Egghe (2008g) provides a description of the modeling of successive h-indices; see also the section on informetric models.

Ruane and Tol (2008) apply successive h-indices to the field of economics in the Republic of Ireland; Tol (2008) introduces the successive g-index and applies it to the same topic. As has been mentioned, Tol (2008, p. 153) finds that "the successive g-index has greater discriminatory power than the successive h-index." Arencibia-Jorge and Rousseau (in press) apply successive h-indices to Cuban institutions. Arencibia-Jorge, Barrios-Almaguer, Fernández-Hernández, and Carvajal-Espino (2008) undertake a similar study. Rousseau and colleagues (2008) investigate the (successive) h-index in the generalized framework of a conglomerate (which is a slight extension of an IPP). Da Silva, Palazzo de Oliveira, Valdeni de Lima, Warpechowski, Hannel, Medianeira Denardi, and colleagues (2009) use successive h-indices in the construction of a relative h-index for groups (among a set of groups). They combine this relative h-index for groups with the Gini index (Gini, 1909) of the group (based on its members' h-indices) to yield the α-index. This index measures the quality of a group and is independent of the size of the group. Jacsó (2007) addresses the problem of recognizing author names for h-index calculations.

H-indices have numerous applications. They can be used not only with citations but also with download data (O'Leary, 2008). Hua, Rousseau, Sun, and Wan (2009) find that, in general, downloads yield higher numbers than citations and, therefore, one does not use the h-index but the Kosmulski $h^{(2)}$-index. We consider this an interesting use and advantage of the $h^{(2)}$-index.

One could determine h-indices of websites (based on their in- or out-links). We even feel (although there are apparently no references yet) that the h-index (and h-type indices) could be used in econometrics and other -metrics fields as a new assessment indicator, for example, of wealth or income.

We talk nowadays of a "3-tier" evaluation system (peer review, citations, downloads) in a "2-tier" communication system (pre-publication in a repository [e-print server], publication in a [e-]journal). The British government has announced that, after 2008, it will base funding assessments for universities on h-type indicators (Ball, 2007).

We have mentioned applications of the h-index to researchers, journals, institutes, and topics. Additional application areas are mentioned here, grouped by broad topic.

- **Authors, Career Assessment**: Aguayo-Albasini and Campillo-Soto (2008); Bar-Ilan (2006); Batista and colleagues (2006); Bornmann, Wallon, and Ledin (2008); Brahler and Decker (2005); Costas and Bordons (2007a); Cronin and Meho (2006); de Araujo (2008); Dodson (2008); Dorta-Contreras, Arencibia-Jorge, Marti-Lahera, and Araujo-Ruiz (2008); Engqvist and Frommen (2008); Ferrand (2007); Frangopol (2005); Glänzel and Persson (2005); Harnad (2007); Heaney (2007); Hermes-Lima, Alencastro, Santos, Navas, and Beleboni (2007); Humberto Romero, Garcia, and Kiwi (2009); Imperial and Rodriguez-Navarro (2007); Jones (2008); Kellner (2008); Kellner and Ponciano (2008); Kelly and Jennions (2006, 2007); Levitt and Thelwall (2009); Lovegrove and

Johnson (2008); Lufrano and Staiti (2008); Machacek and Kolcunova (2008); McKercher (2008); Minasny and colleagues (2007); Mugnaini, Packer, and Meneghini (2008); Oppenheim (2007); Purvis (2006—a 20-line note); Rao (2007); Rau (2007); Rodriguez, Janssens, Debackere, and De Moor (2007, 2008); Romero, Cortés, Escudero, Lopez, and Moreno (2007); Saad (2006); Salgado and Paez (2007); Sanderson (2008); Sangam and Girji (2008); Schreiber (2007a); Sebire (2008); Sidiropoulos and Katsaros (2008); Sidiropoulos, Katsaros, and Manolopoulos (2007); Timofieiev, Snásel, and Dvorský (2008); Torro-Alves, Herculano, Tercariol, Filho, and Graeff (2007); Ursprung and Zimmer (2007); van Haselen (2007); van Leeuwen (2008); Vanclay (2007); Vinkler (2007).

- **Journals**: Barendse (2007); Gracza and Somoskovi (2008); Harzing and van der Wal (2009); Jeang (2007); Krauskopf and Krauskopf (2008); Levitt and Thelwall (2008); Liu, Rao, and Rousseau (in press); Olden (2007); Orbay and Karamustafao⇔lu (2008); Rousseau (2006a); Rousseau and Small (2005); Saad (2006); Satyanarayana and Sharma (2008); Sebire (2008); Sidiropoulos, Katsaros, and Manolopoulos (2007); Vanclay (2007, 2008a, 2008b).
- **Research Groups and Institutions**: de Araujo (2008); Grothkopf and Stevens-Rayburn (2007); Kinney (2007); Molinari and Molinari (2008a, 2008b); Mugnaini and colleagues (2008); Pires da Luz, Marques-Portella, Mendlowicz, Gleiser, Silva Freire Coutinho, and Figueira (2008); Torro-Alves and colleagues (2007); van Raan (2006).
- **Countries**: Lufrano and Staiti (2008); Mahbuba and Rousseau (2008), Pilc (2008).
- **Topics**: Marx and Barth (2008); Minasny and colleagues (2007); Rousseau (2007a); The STIMULATE6 Group (2007).
- **Library Management**: Liu and Rousseau (2007, 2009).

It is clear that the majority of case studies on h-type indices focuses on authors (researchers) and/or career assessment, which is in line with the original purpose of the h-index as defined by Hirsch (2005a, 2005b).

We end this section with an intriguing question: If we consider "all" publications of all authors as if they were written by one meta-author M, what is the h-index of M? This h-index could then be considered as the h-index of "humanity." We know that, in essence, the answer to this problem has no real value because we mix all scientific disciplines, which is not allowed when dealing with citation analysis (and by extension when dealing with the h-index), so the question should be considered as merely an intriguing challenge.

Of course one has to define what "all" publications means. We have limited ourselves to WoS. Henry Small (personal communication, May 2007) reported (and this information was published by Egghe and Rao [2008a] with Small's permission) that, depending on the time period used (up to 1955 or up to 1972), the h-index of WoS (standing for "all" publications) is between 1,500 and 2,000. Note that in May 2007, WoS (up to 1972) contained a little more than 33.10^6 documents of which about 80 percent have received at least one citation (Small, personal communication, May 2007).

Theories of Impact and Performance Measurement

Glänzel (1996), supported by Rousseau (2002), explains the need for standards in informetric research. This is certainly an issue when introducing a new topic like h-type indices.

In this section we examine theoretical properties of h-type indices so that we can learn to "understand" their true nature. But first, what do we want? In other words what do we expect from an impact or performance measure (the terms should be standardized, so we suggest, from now on, using the term "impact measure")?

Let us compare this with concentration measures (i.e., those capable of measuring concentration or inequality) (Egghe, 2005, chapter 4), which were introduced in econometrics, 100 or so years ago. Concentration measures have little to do with impact measures but act, like impact measures, on a decreasing sequence $x_1, x_2, \ldots, x_N$ of positive numbers. We will show that the g-index has a concentration property. Here we introduce briefly the notion of a concentration measure and show that we can define, in an exact way, what we expect from a concentration measure. Then we go back to impact measures and investigate whether such an exact "wish list" can also be given for impact measures.

Lorenz (1905) invented concentration theory in econometrics in 1905. It has also found its way into informetrics because, as in econometrics, many phenomena in informetrics are very concentrated in the sense that few sources have many items and many sources have few items (e.g., few papers receive many citations and many papers receive few [or no] citations [Egghe, 2005]).

Lorenz concentration theory goes as follows. Let $X = (x_1,x_2,\ldots,x_N)$ be a decreasing vector with positive coordinates x_i, $i = 1, \ldots, N$. The Lorenz curve $L(X)$ of X is the polygonal curve connecting (0,0) with the points

$$\left(\frac{i}{N}, \sum_{j=1}^{i} a_j\right), \quad i = 1, \ldots, N$$

where

$$a_i = \frac{x_i}{\sum_{j=1}^{N} x_j} \tag{6}$$

Note that for $i = N$ we have (1,1) as end point of $L(X)$. Let X and $Y = (y_1, y_2, \ldots, y_N)$ be two such vectors. We say that X is more concentrated than Y if $L(X) > L(Y)$, that is, if the Lorenz curve of X is strictly above that of Y (except, of course, in the common points (0,0) and (1,1)). We also say that the coordinates of X are more unequal than those of Y.

That Lorenz curves are the right tool to measure concentration (or inequality) is seen in Muirhead's (1903) results, stating[1] $L(X) > L(Y)$ if and only if X is constructed, starting from Y, by a finite number of applications of elementary transfers. An elementary transfer (e.g., on $Y = (y_1, \ldots, y_N)$) changes Y into the vector

$$\left(y_1, \ldots, y_i + a, \ldots, y_j - a, \ldots, y_N\right) \tag{7}$$

where $1 \leq i < j \leq N$ and $a > 0$. Because Y is decreasing, this means, in econometric terms that "we take away ($a > 0$) from the poor (j) and give it to the rich (i)," producing a more unequal (concentrated) situation, which is applied repeatedly to yield X out of Y.

Now we define a function f, acting on N-dimensional vectors $X = (x_1, \ldots, x_N)$ with values $f(X) = f(x_1, \ldots, x_N) \geq 0$ to be a good concentration measure if

$$L(X) > L(Y) \text{ implies } f(X) > f(Y) \tag{8}$$

Well-known, good concentration measures are the variation coefficient (i.e., the standard deviation of X divided by its average) and the Gini index (Gini, 1909; or see Egghe, 2005, Chapter 4).

So we were able to define in a mathematically exact and unique way what we expect from a concentration measure. We are, at this time, unable to do the same for impact measures. Indeed, each of the different impact measures we have defined so far has different impact properties, and it can be proved (see later) that there does not exist a measure that satisfies all of these impact properties. Let us start with the work of Woeginger (2008b), who defines an impact measure (therein called a scientific impact index) as a function f, acting on vectors (we assume that the vectors X, Y, … are decreasing, i.e., their coordinates are in decreasing order) $X = (x_1, \ldots, x_N)$ and $Y = (y_1, \ldots, y_N)$ (note that now M can be different from N) such that

(i) $f(0, \ldots, 0) = 0$ (in any dimension, including 0)

(ii) $X \leq Y$ implies $f(X) \leq f(Y)$. Here $X \leq Y$ is defined as follows: $X \leq Y$ if $N \leq M$ and $x_i \leq y_i$ for $i = 1, \ldots, N$.

It is clear that (i) and (ii) are necessary conditions for an impact measure but that they are far from sufficient: A zero vector has no impact. In other words, if an author has no papers with citations or has no papers at all, the impact of this author is zero and if on every common coordinate (say rank) of two authors X and Y the paper (at that rank) of Y has more ≥ citations than the paper (at that rank) of X, then Y has more impact than X. These requirements for an impact measure are non-controversial but are not enough to call f a good impact measure; in fact it might have been better had Woeginger chosen another name for f satisfying (i) and (ii) (cf. the requested standardization in informetrics mentioned previously) but we will keep the name in order not to confuse the reader.

Woeginger (2008b, p. 227) then continues his search for "desired elementary properties," other than (i) and (ii), for impact measures. He formulates five "fairly natural axioms" (p. 227) that are desired for impact measures f (a so-called wish-list, as Cater and Kraft [1989] suggested in the context of information retrieval). Again the vectors X, Y are decreasing.

A1. If the $(N + 1)$-dimensional vector Y results from the N-dimensional vector X by adding a new article with $f(X)$ citations, then $f(Y) \leq f(X)$.
A2. If the $(N + 1)$-dimensional vector Y results from the N-dimensional vector X by adding a new article with $f(X) + 1$ citations, then $f(Y) > f(X)$.
B. If the N-dimensional vector Y results from the N-dimensional vector X by increasing the number of citations of a single article, then $f(Y) \leq f(X) + 1$.
C. If the N-dimensional vector Y results from the N-dimensional vector X by increasing the number of citations of every article by at most 1, then $f(Y) \leq f(X) + 1$.
D. If the $(N + 1)$-dimensional vector Y results from the N-dimensional vector X by first adding an article with $f(X)$ citations and then increasing the number of citations of every article by at least 1, then $f(Y) > f(X)$.

Although somewhat artificial, these are all "desirable" properties of a good impact measure. Only A1 looks controversial but this axiom shows the robustness required of an impact measure. It is better understood as: Adding an article with $f(X)$ citations will not increase $f(X)$. In fact A1 and (ii) together imply $f(X) = f(Y)$. Also, axioms B and C are not completely natural: Why should $f(Y)$ be maximally one unit larger than $f(X)$? These axioms clearly are meant to describe the h-index and certainly not other h-type indices such as the g-index or R-index.

Woeginger (2008b) shows A2 => D but also that A1 and A2 => not B, so that there is no impact measure that can satisfy all these axioms. He goes on to prove that an impact measure *f* is the h-index if and only if *f* satisfies A1, B, and D. This reveals much about the true nature of the h-index as an impact measure. Other, less important measures are also characterized using some of the axioms. Quesada (2009) gives a similar characterization of the h-index (but now for the real-valued version of it, i.e., with values in the positive real numbers).

Woeginger (2008d) provides another characterization of the h-index. First we introduce a new axiom.

> S. For every decreasing vector X, we have $f(X) = f(R(X))$ where $R(X)$ is the mirrored vector $X = (x_1,\dots,x_N)$ (depicted in the plane with the indices [papers] as abscissa and the x_i [citations] as ordinates) over the first bissectrix (the 45° line).

Then Woeginger proves that an impact measure *f* is the h-index if and only if *f* satisfies A1, S, and D.

In all these characterizations it is easy to show that h satisfies these axioms; the difficult part is the reverse (if) part. Axiom S is logical as regards the definition of the h-index.

Characterizing the g-index is an even more difficult task that was also performed by Woeginger (2008a). He adds to (i) and (ii) the trivial requirement that an impact measure should also satisfy: If $X = (x_1,\dots,x_N)$ and $Y = (x_1,\dots,x_N, 0)$, then $f(X) = f(Y)$. This is indeed logical: Adding a new paper without citations cannot increase (or decrease) the impact. It is slightly strange that Woeginger keeps the same name for a measure *f* satisfying these three axioms as one that satisfies only (i) and (ii) (the standardization problem again). Woeginger needs other "desirable" properties to characterize the g-index than those for the h-index, showing the different nature of the g-index. The three axioms needed are (for X, Y decreasing)

> E. If the $(N + 1)$-dimensional vector Y results from the N-dimensional vector X by first adding an article with $f(X)$ or $f(X) + 1$ citations and then increasing the number of citations of every article by one, then $f(Y) = f(X) + 1$.

Axiom E is of the same nature as axiom D. The difference is needed for technical reasons. Of a completely different nature are the following two axioms (again for decreasing vectors X and Y).

> T_1. Let $X = (x_1, \dots, x_N)$ and let $1 \leq i < j \leq N$. If the vector Y results from X by setting $x_k = y_k$ for all $k = 1, \dots, N$ but $k \neq i$, $k \neq j$, $y_i = x_i + 1$, and $y_j = x_j - 1$, then $f(Y) \geq f(X)$.

T_2. Let $X = (x_1,\ldots,x_N)$ and let $1 \leq i < j \leq f(X)$. If the vector Y results from X by setting $x_k = y_k$ for all $k = 1, \ldots, N$ but $k \neq i$, $k \neq j$, $y_i = x_i - 1$, and $y_j = x_j + 1$, then $f(Y) = f(X)$.

Next Woeginger proves that an impact measure f is the g-index if and only if f satisfies E, T_1, and T_2. Axiom T_2 is merely a technical item in order to obtain the characterization of the g-index. Axiom T_1 is very interesting: In view of Muirhead's theorem, we see that T_1 (called the transfer property) is nothing else than property (8), except for the inequality sign. This is strict in (8) (>) but in T_1 it is not (≥). Due to the requirement that impact measures should be robust with respect to a change of citation quantities, we think that the non-strict inequality is a good property in this context. So the g-index (and essentially only the g-index) satisfies this econometric concentration property. What does this mean from the point of view of impact measures? Essentially it says that, if we have two situations (e.g., two authors) with the same number of papers and the same number of citations in total, then the author for whom the citations are more concentrated over the papers has the higher impact. In other words, it is better to write one paper with 100 citations (and nine papers without a citation) than to write 10 papers with 10 citations each. That is a good property of the g-index. Lehmann and colleagues (2008) and Leydesdorff (in press) also recognize a desired property for good impact (performance) measures. Woeginger's theorem, characterizing the g-index, is not trivial but it is trivial to see that the g-index satisfies axiom T_1.

Note also that the h-index does not satisfy T_1: For $x = (3,3,3)$ we have $h(X) = 3$ but for $Y = (4,3,2)$ we have $h(Y) = 2 < h(X)$.

Egghe (2008b) discusses the "econometric" aspects of the g-index and also defines the Kosmulski variant $h^{(2)}$ of the h-index for the g-index. He shows that this variant satisfies T_1. Egghe further discusses the case that $L(X) = L(Y)$: Then we require that

$$\sum_{i=1}^{N} x_i > \sum_{i=1}^{N} y_i$$

should imply that X has the highest impact, which is proved to be true for most impact measures.

Gagolewski and Grzegorzewski's (2009) work is similar to Woeginger's. Several h-type indices such as the h-index, Woeginger's w-index, and Kosmulski's MAXPROD index are characterized by the maximal generalized circles or ellipses that are still dominated by the citation function, that is the step function on unit intervals having the decreasing citation scores as ordinates.

Deineko and Woeginger (in press) study a new family of impact measures that generalize the Kosmulski index and provide axiomatic characterizations of these measures. Woeginger (2008c) does the same for a family of generalizations of the g-index. Van Eck and Waltman (2008) study similar generalizations of the h-index and the g-index. Rousseau (2008d) checks some of Woeginger's axioms on the g-index, the $h^{(2)}$-index, and the R-index.

Marchant (2009a) characterizes some impact measures (impact in a broad sense): total number of publications, total number of citations, maximal number of citations, number of papers with at least α citations (α = a threshold), and the h-index. All these measures are ranking devices (e.g., of authors) and Marchant characterizes them according to their ranking properties. Most ranking properties are technical and sometimes cumbersome (as Marchant himself indicates). It takes no fewer than six ranking properties to characterize the h-index; Rousseau (2008a) remarks that one other important ranking property defined by Marchant (2009a) is not a property of the h-index: The weak independence property. This ranking property is defined as follows. Consider two scientists A and B (represented by their total publications and all the citations received by these publications) and suppose we have a ranking device that gives a lower rank to B than to A (i.e., B is preferred to A or states otherwise, according to a certain impact measure, B has more impact than A). Assume that one adds the same publications (each with their citations) to the publication lists of scientists A and B. Then B should still be preferred over A: The impact measure of B should still be larger than A's. This apparently logical requirement is, strangely enough, not satisfied by the impact measure h. Indeed, as Rousseau (2008a) argues, let A be represented by the vector (5,5) and B by the vector (5,3,3): A has two publications with each receiving 5 citations and B has three publications with 5, 3, and 3 citations. Hence A has h-index 2 and B has h-index 3. Suppose we add to both authors two publications with 4 citations each. Then A and B are now represented as: A = (5,5,4,4), B = (5,4,4,3,3). Now A's h-index is 4 (up from 2 to 4) and B's h-index still is 3, now smaller than A's.

Neither Rousseau nor Marchant argues that ranking according to the h-index is bad, but they underline that, whatever method one uses, one must know the properties of the ranking method. Marchant (2009a) presents a good inventory.

We underscore Rousseau's comments that the g-index and the R-index also lack this property. For the R-index we can take the example (which is a slight modification of Rousseau's examples so that it can also be used for the R-index). In the first case we have

$$R_A = \sqrt{10} < R_B = \sqrt{11}$$

and in the second case we have

$$R_A = \sqrt{18} > R_B = \sqrt{13}$$

For the g-index we offer an example. Let first $A = (6,2)$ and $B = (5,2,2)$. We have $g_A = 2$ (even adding a zero to A does not yield a g_A-value of 3) and $g_B = 3$. Now add two articles with 4 citations each to A and B: $A = (6,4,4,2)$, $B = (5,4,4,2,2)$. Now $g_A = 4$ and $g_B = 3$.

Impact measures are examples of scoring rules. Marchant (2009b) characterizes general scoring rules.

This section has made it clear that it is not easy to define exactly what we mean by a "good" impact measure. Furthermore there is the intriguing problem of the influence of productivity on impact measures. Productivity can be defined as an author's total number of publications, but what is its influence on impact measures? Egghe (in press c) has studied impact measures of the h-type (h, g, R, h_w) and non-h-type (average number of citations per paper) [i.e., general IF], median citation, or fraction of the papers with at least a certain number of citations). Relations, such as an increasing relation between an impact measure and productivity, cannot be proved in general although, intuitively, we feel that such a relationship is plausible. Egghe, Goovaerts, and Kretschmer (2007) indicate this in connection with productivity versus collaboration (in the sense of "fraction of co-authored papers"), at least in fields in the exact sciences.

The study of the general relation between productivity and impact measures has not yet been undertaken and, hence, is flagged here as a research challenge.

Lehman and colleagues (2008) (see also Lehmann, Jackson, & Lautrup, 2005) also study impact measures of the h- and non-h-types (similar to those Egghe [in press c] studied) and conclude, for instance that the average number of citations per paper is a superior indicator of scientific quality (based on statistical arguments). This is, however, disproved by Hirsch (2007).

Informetric Models for h-Type Indices

Most models for impact measures follow the simple framework of Lotka's law. This law is used in two ways. We assume that the number of authors $f(n)$ with $n = 1, 2, 3 \ldots$ publications equals

$$f(n) = \frac{C}{n^{\alpha}} \tag{9}$$

where $C > 0$, $\alpha > 1$. This is the classical Lotka's law (Lotka, 1926); it is not controversial. Theoretically it applies in every IPP where sources have (or produce) items: The number of sources with n items is given by formula 9. A second application used in this text is the paper-citation relation (say for an author): The number of papers with n citations (received) is given by formula 9. Of course, each application requires its dedicated C and α and can be obtained via a fitting procedure (see Rousseau & Rousseau, 2000; or Egghe, 2005, appendix).

Applying formula 9 to the paper-citation relation is somewhat controversial because we can see from van Raan (2001a, 2001b) as well as Brantle and Fallah (2007), Lehmann and colleagues (2008), Radicchi and colleagues (2008), and Molinari and Molinari (2008a using the rank-frequency version of formula 9—see formula 10) that formula 9 does not completely apply. For formula 9 to apply one needs a linearly decreasing data set in a log-log, scale which is not completely true: In practice we see a concavely decreasing graph in a log-log scale, modeled by van Raan (2001a, 2001b) by Bessel functions—see also earlier work of Glänzel, Burrell, and Sichel and the recent article in which Burrell (2008a) extends Lotkaian informetrics using a Pareto type II distribution. All these functions are difficult to use in calculations and using such a model would prevent us from creating informetric models for impact measures.

Using Lotka's law in the past (e.g., Egghe, 2005) we have obtained good approximations of reality and this turns out to be the case with impact measures.

Of course, for calculatory reasons we take $n \geq 1$ as a continuous variable. In this framework it is well known (Egghe, 2005) that Lotka's law (9) is equivalent to Zipf's law: In the first application, if we rank the authors in decreasing order of their number of publications, then the number of publications $g(r)$ of the author on rank r equals

$$g(r) = \frac{D}{r^{\beta}} \tag{10}$$

$$\beta = \frac{1}{\alpha - 1} \tag{11}$$

$D > 0$ and where β, is given by

The notation r $g(r)$ is classical and used by Egghe (2005). We use this notation here but note that one should not confuse this function $g(.)$ with the g-index.

In the second application we rank the papers (of an author) in decreasing order of the number of citations received. Then the number $g(r)$ of citations to the paper at rank r is given by (10), where (11) is always valid.

Having the rank-frequency function $g(r)$ at our disposition (in the papers-citations context as has been described), we can give an elegant variant of the definition of the h-index: The h-index is simply the side of the square determined by the function $g(r)$ and its intersection with the first bissectrix. In other words, if we intersect the curve r $g(r)$ in

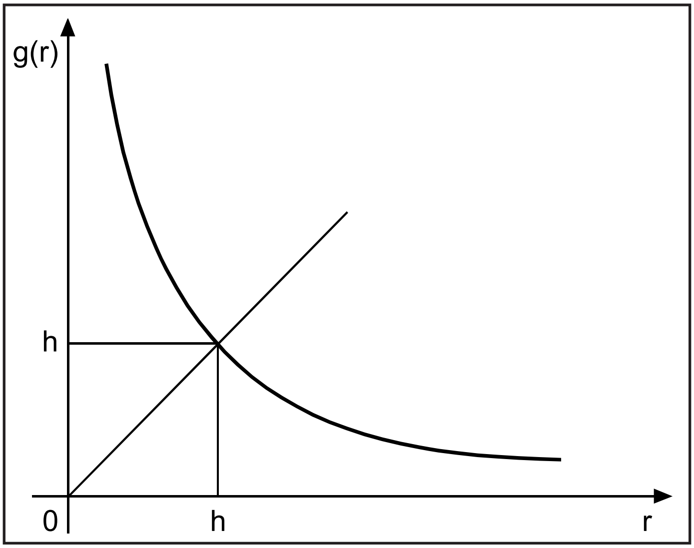

Figure 2.2 Geometric illustration of the determination of the h-index

the plane with the first bissectrix, then the point of intersection has coordinates (h,h), thereby determining the Hirsch index h (hence also the equation $g(h) = h$ defines the h-index). Figure 2.2 provides a graphical illustration. Hirsch (2005a, 2005b) presents this definition, which also appears in papers by Miller (2006), Valentinuzzi and colleagues (2007), Woeginger (2008d), and, indirectly, Woeginger (2008b).

The exponent α is called Lotka's exponent; it is very important in informetrics and will play a similarly important role in the modeling of impact measures. A classical value is $\alpha = 2$; this value is a turning point of informetric properties.

So much for this short introduction to Lotkaian informetrics; Egghe (2005) provides more detail. Egghe and Rousseau (2006) have obtained the simplest model for the impact for the h-index. Because the result is basic and its proof simple we present it here. Suppose (9) is valid for the

$$h = T^{\frac{1}{\alpha}} \tag{12}$$

$$T = \int_1^{\infty} f(n)dn = \frac{C}{\alpha - 1} \tag{13}$$

paper-citation relation (of an author or of a journal). Let there be *T* cited papers in total. Then the h-index of this author or journal is given by

$$\int_{n}^{\infty} f(n')\,dn' = \frac{C}{\alpha - 1} n^{1-\alpha} \tag{14}$$

Proof: By definition of *f* is the total number of papers given by because $\alpha > 1$. The total number of papers with *n* or more citations is given by

$$Th^{1-\alpha} = h \tag{14a}$$

So (13) and (14) imply that the total number of papers with *n* or more citations is given by $Tn^{1-\alpha}$. For $n = h$ (the h-index) we need, by definition of *h*

whence (12).

Result (12) shows that, because $\alpha > 1$, the h-index is a concavely increasing function of *T*, the total number of papers. We will use (12) in the next section (in a time-dependent context) to model h-index sequences. Here we restrict ourselves to fixed time.

Glänzel (2006a) obtained an approximation of (12) via an argument with discrete values of *n* (see also Glänzel, 2007b, 2008b). Egghe and Rousseau (2006) also proved the Zipf variant of (12) and it is confirmed by Molinari and Molinari (2008a) (reference to Egghe and Rousseau 2006 is given but, surprisingly, not mentioned in the text); see also Kinney (2007). Kinney is apparently not aware of formula (12) but it seems—experimentally—that (12) is valid for $1/\alpha \approx 0.4$ (there called the "master-curve"). Given the prevalent view that citation analysis is highly dependent on the field under study, we doubt that these experimental findings are universal and in fact they are not confirmed by Molinari and Molinari (2008a). Formula (12) was experimentally veri-

$$A = \int_{1}^{\infty} nf(n)\,dn = \frac{C}{\alpha - 2} \tag{15}$$

$$A = T\frac{\alpha - 1}{\alpha - 2} \tag{15a}$$

$$h = \left(\frac{\alpha - 2}{\alpha - 1} A \right)^{\frac{1}{\alpha}} \quad \textbf{(16)}$$

fied by Egghe and Rao (2008a) on papers containing *N*-grams of variable length. Egghe and Rousseau (2006) also found a relation between h and A = the total number of citations: If $\alpha > 2$ we have, by definition of f that Hence (13) and (15) yield
and hence, by (12):
, also a concavely increasing function of A.

Both formulae (12) and (16) are confirmed in the experimental work of van Raan (2006): On a log-log scale, (12) and (16) are linear functions of T and A respectively, meaning log h is a linear function of log T and log A. Van Raan (2006) finds a linearly correlated cloud of points. Similar correlations are studied in Ball and Ruch (2008); Barendse (2007); Bornmann and Daniel (2007a); Bornmann, Wallon, and Ledin (2008); Costas and Bordons (2007b); Iglesias and Pecharromán (2007); Saad (2006); and Schubert and Glänzel (2007). Minasny and colleagues (2007) also report the concave relation (16).

Schubert and Glänzel (2007) and Glänzel (2008b; see also 2007a) study statistical relations between the h-index and so-called composite indicators (combining publication output and mean citation rate). They report strong linear statistical relations. It must, however, be noted that these composite indicators have one factor equaling (in our notation) $T^{1/\alpha}$,

$$g = \left(\frac{\alpha - 1}{\alpha - 2} \right)^{\frac{\alpha - 1}{\alpha}} T^{\frac{1}{\alpha}} \quad \textbf{(17)}$$

$$g = \left(\frac{\alpha - 1}{\alpha - 2} \right)^{\frac{\alpha - 1}{\alpha}} h \quad \textbf{(18)}$$

$$R = \sqrt{\frac{\alpha - 1}{\alpha - 2}} \; T^{\frac{1}{\alpha}} \quad \textbf{(19)}$$

$$R = \sqrt{\frac{\alpha - 1}{\alpha - 2}} \; h \quad \textbf{(20)}$$

which is h itself according to (12), the other factor being a power of the mean citation rate. Glänzel (2008b) proposes using z-statistics to analyze the tail of citation distributions in light of the h-index.

Other h-type indices can, likewise, be modeled (if $\alpha > 2$). For the g-index (Egghe, 2006c)
by (12); for the R-index
by (12) (see Jin et al., 2007) and similarly for the weighted h-index h_w (Egghe & Rousseau, 2008). One might conclude, based on (18) and (20), that g and R are linear functions of h. This is not really true: If h varies,

$$\frac{1}{\alpha - 1}$$

then, by (12) and (13), it is likely that α varies and hence not only h varies in (18) and (20). Only when α is constant and when C varies in (13) we find that h varies (by (12)) and that g and R are linear in h (by (18) and (20)).

Beirlant, Glänzel, Carbonez, and Leemans (2007) propose an alternative for T-dependent indices:

$$\psi(r) = Ar^{b} \tag{21}$$

$$\varphi(j) = Bj^{c} \tag{22}$$

(incidentally equal to Zipf's exponent β,—see formula 11) is called the extreme value index. Rousseau (2008c) calculates (and compares) the

$$h^{*} = B^{\frac{\delta-1}{\delta}} A^{\frac{1}{\delta}} T^{\frac{b}{\delta}} \tag{23}$$

h-index and the g-index in some simple distribution models.

$$\delta = 1 + \frac{b(\alpha - 1)}{c} \tag{24}$$

Let us further study the simple formula (12) for the h-index: $h = T^{1/\alpha}$. This allows us to see how h will change if the publication-citation rela-

$$h^{*} = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} h \tag{25}$$

tion changes. Egghe (2008c) does this using results obtained previously (Egghe 2007c, 2008e). If we apply a power law transformation to the paper rank r: ψ (r) and to the citation density n: φ (n) of the form

$$h^* = 2^{\frac{\alpha-1}{\alpha}} h \quad (26)$$

$$h < h^* < 2h \quad (27)$$

(A,B,b,c > 0) then we prove that the h-index of the transformed system, denoted h^*, is given by
where T is the total number of papers before the transformation and where

$$h^* = 2^{\frac{1}{\alpha}} h \quad (28)$$

$$h < h^* < 2h \quad (29)$$

the Lotka exponent of the transformed system. Egghe (2008c) considers the simple case, $b = c = 1$ which gives, by (23) and (24):

This has the following simple application:

$$h^* = 2h \quad (30)$$

(i) ψ (r) = r, φ (j) = 2j: Papers remain the same but they double their number of citations. Then
which yields
: Doubling of citations does not multiply h by 2, a logical fact.

(ii) ψ (r) = 2r, φ (j) = j: Double the number of papers with the same number of citations. Now we have
and again we have
, again a logical fact.

$$h^* = 2^{\frac{2}{\alpha}-1} h \quad (31)$$

(iii) $\psi(r) = 2r$, $\varphi(j) = 2j$: If we double papers and citations then we have
which is logical.

(iv) $\psi(r) = r$, $\varphi(j) = j/2$. Double the number of papers but divide their number of citations by 2. This principle occurs (approximately) in uncontrolled lists where the same paper occurs twice and where the citations to this paper are divided over the two occurrences of the paper. This also occurs (approximately) when an author has "publicities," i.e., where an author publishes "the least publishable unit" and where,

$$\varphi(T) = \frac{C^*}{T^{\alpha^*}}$$

therefore, fewer citations per paper are obtained. Now we have

$$h = T^{\frac{1}{\alpha}}$$

$$\varphi(h) = \frac{C^*}{h^{\alpha\alpha^*}} \tag{32}$$

Does this practice raise the h-index? We have from (31) that $h^* > h$ if and only if $\alpha < 2$, so for low values of α. By (12) this means high values of h, that is, for prolific authors this practice pays off.

$$h_2 = S^{\frac{1}{\alpha\alpha^*}} \tag{33}$$

The next problem to be addressed is that of the distribution of the h-index (given a group of authors). First we deal with the size-frequency function $\varphi(h)$, that is, the number of authors with h-index h. As discussed in the beginning of this section, we assume that both the paper-citation relation and the author-paper relation conform with Lotka's law. Let $f(n) = C/n^{\alpha}$ be the number of papers with n citations and

the number of authors with T papers. If there are T papers and because $f(n) = C/n^{\alpha}$ we find that this author has h-index
(see formula 12) hence $T = h^{\alpha}$. If we put this in the formula for φ we have that

Hence the h-index is distributed according to Lotka's law with exponent $\alpha\alpha^*$, which Egghe (2007a) has proved. A similar result was proved for the g-index. Rao (2007) gives empirical evidence for the high degree of skewness of (32) (because the Lotka exponent is $\alpha\alpha^*$ with $\alpha > 1$ and $\alpha^* > 1$).

We have two applications of this result. First we can model successive h-indices. If there are S authors it follows from (32) and (12) that
is the h-index of the group of authors (e.g., an institute). Egghe (2008g) obtained this result. Further successive h-indices can be modeled in like fashion.

Egghe (2008h) gives the second application. Because we have result (32) we find that the rank-frequency distribution $h(r)$ is Zipf's law, as

$$h_1 = T_1^{\frac{1}{\alpha}}$$

$$h_2 = T_2^{\frac{1}{\alpha}}$$

$$h = \left(T_1 + T_2\right)^{\frac{1}{\alpha}} = \left(h_1^{\alpha} + h_2^{\alpha}\right)^{\frac{1}{\alpha}}$$

remarked in the beginning of this section, hence is convexly decreasing. Because Zipf's is a power law on a log-log scale, the function $h(r)$ is a decreasing straight line. This result was also experimentally confirmed based on the h-indices of the mathematics journals in WoS. Pyykkö (2008) provides confirmation where most $h(r)$-curves in a log-log scale are linear. This also shows that $h(r)$ has a totally different shape from IF(r), the impact factor rank distribution. Based on the Central Limit Theorem, Egghe (in press b) proved that IF(r) has an S-shape, first convexly decreasing and then concavely decreasing, hence completely different from the shape of $h(r)$—although Beirlant and Einmahl (2008) show the asymptotic normality of the h-index (as T ∞).

Several researchers have studied merging aspects of the h-index. Glänzel (2008a) discusses the merging of disjoint sets. The stochastic proof essentially boils down to the following observation. Let us have two

sets of papers consisting of T_1 and T_2 papers, respectively. If both paper-citation systems satisfy Lotka's law (9) with the same α then their h-indices are (see formula 12):
and
, respectively. Hence $T_1 = h_1^{\alpha}$ and $T_2 = h_2^{\alpha}$. So $T_1 + T_2 = h_1^{\alpha} + h_2^{\alpha}$, hence is the h-index of the merged system. A similar result can be found in work by Molinari and Molinari (2008a). Egghe (2008d) discusses more general merging models, where the merged sets are not necessarily disjoint. Only inequalities between the h-indices of the sets and the h-index of the merged set can be given. Two merging devices are given: In the merging one can add the citation scores or one can take the maximum of the citation scores. Examples of both scoring systems are given. Vanclay (2007) provides an example of merging, using the max-device.

Rousseau (2007b) and Vanclay (2007) discuss the robustness of the h-index. Rousseau shows its robustness with respect to missing publications and Vanclay with respect to citation errors.

We conclude with some references on different topics. Basu (2007) compares the Random Hierarchical Model (see Basu, 1992) with a formula derived by Hirsch (2005b) under unexplained assumptions on the rank-order citation distribution of papers. We refer to Egghe (2008f) for the mathematical derivation of some inequalities between the h-index h (where authorship is counted in the total way) and the fractional h-indices h_f and h_F (described previously) and to Egghe (in press c) for some mathematical results on the influence of productivity on some impact measures. Finally Torra and Narukawa (2008) interpret the h-index and the total number of citations as fuzzy integrals.

Time Series of h-Type Indices

All h-type indices are single numbers—as is the case for any indicator. We can add (literally) an extra dimension by looking at h-type indices over time t. In case of, for example, an author, we can let t vary over the career (or any other time span). This yields a much stronger evaluation tool than just one number and can also indicate (by extrapolation) what the value of this index will be in the (near) future.

Much of this section is devoted to h-index sequences, but most theoretical results are also valid for other impact measures (because they are the h-index multiplied by a number dependent only on Lotka's exponent α; see formulae 17–20, for example).

The h-type measures can be calculated only when we know the publication period and the citation period. So it is clear that different h-index sequences are possible. Liu and Rousseau (2008) defined not less than 10 different h-index sequences, using dedicated tables indicating which publication and citation periods are used. Egghe (in press a) studies four different h-index sequences, including the "real career h-index sequence" which, we think, is the most important h-index sequence: For every year $t = 1, 2, \ldots, t_m$ (t_m is, e.g., the present [or last] year of the career) we calculate the h-index $h(t)$ based on the papers published in the years 1, 2, … , t and on the number of citations that these papers received in the same period. Such an h-index sequence is, obviously, increasing. Further in this section we will indicate when such a sequence increases concavely, linearly, or convexly. Hirsch (2005a, 2005b) originated the real career h-index sequence, but see also Burrell (2007b).

It is best that $h(t)$ in this real career h-index sequence is calculated at every present year t (i.e., calculate $h(t)$ from year to year, say each time at the end of the year). In this way one does not have to truncate for citation years in the past. Such truncations are very tedious to execute, for

$$h^*(t) = \left(T(t_m) - h(t_m - t)^{\alpha}\right)^{\frac{1}{\alpha}} \quad \textbf{(34)}$$

example, in WoS where citation data are presented up to now. Of course, because the h-index was defined in 2005, this is not possible for most authors, whose careers began (long) before that date. So, for example, in the case of this author (whose publication career started in 1978) we truncated citation from that year onward (Egghe, 2009a), which produces a linearly increasing h-index sequence.

This problem led Liang (2006) to study another h-index sequence, one going backward in time: So the h-index at time t, denoted $h^*(t)$, uses papers and citations to these papers in the years t_m - t, … , t_m. Note that for this h-index sequence one always has a citation period up to now, so that truncation in WoS is not necessary. Liang was the first to study h-index sequences; Burrell (2007a) noticed that Liang's h-index sequence used time going backward. He remarks that the real career h-index sequence, as introduced previously, is the more natural, which is true. Burrell further remarks that, for a theoretical model, the difference between the two h-index sequences will not be large. This is, however, not true as the mathematical theory, as Egghe's (2009a)

$$h(t) = T(t)^{\frac{1}{\alpha}} \quad \textbf{(35)}$$

$$h(t) = b^{\frac{1}{\alpha}} t^{\frac{1}{\alpha}} \quad \textbf{(36)}$$

development shows. It is proved there that, for every t in the interval $[0, t_m]$ (continuous time model),
in the Lotkaian model (12) and where $T(t_m)$ is the total number of (cited) publications at time t_m (e.g., now). Based on (34) one can show that the sequences $h(t)$ and $h^*(t)$ are very different. For example, if $h(t)$ is convex (including the linear case) then $h^*(t)$ is strictly concave. Within the

$$h(t) = ct^{\frac{\beta+1}{\alpha}} \tag{37}$$

Lotkaian model it is shown that both sequences are equal if and only if the researcher has a constant production of publications per time unit.

Another h-index sequence, where time goes forward (the normal case) but where one does not have to execute tedious citation truncation actions is the so-called total career h-index sequence. Here, as in the real career h-index sequence, the h-index at time t is calculated based on publications of years 1, …, t but one considers, for every t, the total citation period 1, …, t_m. Ye and Rousseau (2008) perform several fitting exercises for this total career h-index sequence (for authors, journals, universities, and companies).

Let us go back to the real career h-index sequence $h(t)$, which is the most important. Egghe (2009a) gives a simple mathematical model for $h(t)$, based on (12). Denote by $T(t)$ the total number of cited publications at time t. Then (12), applied to $T(t)$ gives
Assume that the author has a constant number b of publications per year. Hence at time t there are $T(t) = bt$ publications yielding, by (35)
which is a concavely increasing sequence (because $\alpha > 1$). As has been mentioned, this author's real career h-index sequence is increasing linearly. This can be modeled by assuming that the author produces more papers as t increases: Assume that the author produces at year t, bt^β papers ($\beta > 0$; the case $\beta = 0$ corresponds to the case of constant production). Egghe (2009a) then finds that
where c is a constant. We now have that $h(t)$ is strictly concave if $\beta + 1 < \alpha$, is linear if $\beta + 1 = \alpha$, and is convex if $\beta + 1 > \alpha$·. Because this author's $h(t)$ is linear we can conclude that, for this author, $\beta + 1 \approx \alpha$ (e.g., for $\alpha \approx 2$ we have $\beta \approx 1$, yielding also a linear increase in the number of publications for this author, which is confirmed in practice).

Burrell (2007b, 2007d) studies the time dependence of the h-index using a different model for the publication and citation process: In each case a Poisson process is assumed (of course, with different rates). In this model Burrell (2007b) proves that the h-index is approximately linear in career length and in the logarithm of the publication rate and citation rate. Burrell (2007d) shows that Jin's A-index (formula (2)) is approximately a linear function of time and of h; and the total number of citations to the papers in the h-core has, approximately, a square-law relationship with time and hence also with Jin's A and the h-index. Burrell (2007a, 2007c) also confirmed the linear dependence of the h-index on career length (see also Burrell [2008b] for a case study). Hirsch (2005b) presents a simple model. Burrell (2007c) also notes that the g-index is proportional to career length and hence to the h-index. This follows from (18) proved by Egghe (2006c). Da Fontoura Costa (2006) finds linear as well as concave h-index sequences for authors.

Anderson and colleagues (2008) compare authors' h-index sequences and h_T-index sequences (h_T is the tapered h-index, described previously). They show that there is an essentially linear relationship between h and h_T. This requires an explanation. The result, however, shows that the "trouble" of calculating the more difficult-to-obtain h_T-index is not really worth the effort (but, as has been mentioned, there is some theoretical interest in the tapered h-index, h_T).

Egghe (2009b) proposes a general Lotkaian growth model for sources and for items. Based on this model, he derives a time-dependent growth model for the h-index $h(t)$ and the g-index $g(t)$. He also has presented special cases of this result (Egghe, 2007b, 2007d).

Rousseau and Ye (2008) define a dynamic h-type index based on the R-index at time t multiplied by $h'(t)$, the derivative of the h-index sequence. This is a difficult calculation to execute. Using computer simulations, Guns and Rousseau (2009b) generate h-index sequences. For these simulations they use peak models (fast linear increase followed by a slow linear decrease) for the distribution of the number of received citations. They find that many simulations yield a linear h-index sequence, although convex as well as concave cases occurred.

Finally we mention the case studies of journal h-index sequences by Liu, Rao, and Rousseau (in press) and Rousseau (2006a) and of h-index sequences of countries by Mahbuba and Rousseau (2008).

Conclusions and Challenges

As is clear from this review, the h-index and its variants are extensively studied in informetrics, experimentally and theoretically, although its introduction was by Hirsch (2005b), a physicist. Outside informetrics, h-type indices are mostly used in case studies.

We have described advantages and disadvantages of the h-index and variants of the h-index (such as the g-index and R-index) that try to avoid some of the h-index's limitations.

The h-index, originally conceived of as a way to measure the citation impact of a researcher and to predict future performance, was soon applied to other sources. An important application of the h-index (or its variants) is to journals, as proposed by Braun and colleagues (2005, 2006). One is then immediately interested in comparing the performance of the h-index for a journal with the (classical) impact factor (IF). Although there is some disagreement on the matter, we are inclined to say that the h-index will challenge the impact factor. One reason is that the IF has time thresholds for both the publication period (2 years) and the citation period (1 year), which is not optimal in several fields; the h-index does not impose such time limitations. Also, both Scopus and WoS allow one to calculate the h-index for any set of articles, hence not only for the publication set of an author but also of a journal (for a particular year or all years).

As mentioned, the application of h-type indices to topics and compounds (Banks, 2006) is a potentially very interesting (determining "hot" topics) area for research. The study of h-type indices related to downloads (of articles in an e-journal or a repository) or even outside our field (e.g., in econometrics) is another challenging area.

Further study is needed on "the real nature" of impact measures. Not all given "desirable properties" for impact measures, presented in the fourth section (mainly by Marchant and Woeginger) are so natural, because they are—simply—needed to characterize some h-type indices (such as the h-index and g-index) and hence were not formulated based on a philosophy on "desired properties" for impact measures. More standardized properties are needed as is the case, for example, in concentration theory using Lorenz curves.

More studies are needed on informetric models for h-type indices. We would recommend extending informetric models to some non-Lotkaian cases of the paper-citation relation.

Finally, time series of h-type indices are very important. They add, literally, an extra dimension to this assessment tool, describing the evolution of a researcher's career (to take one example). We have noted that many different time series can be studied (different publication and citation periods), but they are also dependent on the source used (e.g., Scopus, WoS, Google Scholar).

The shapes of these time series still have to be determined in many research areas, not only for authors but also for institutes and topics.

More research-funding bodies can be expected to use h-type indices (and their sequences) for assessment purposes and consequent funding (cf. Ball, 2007). More e-literature sources will add such indicators to their information products (Scopus, WoS, but also institutional repositories), and thereby become readily available to governments, universities, and research centers.

We also note that the long-standing controversy surrounding citation analysis is diminishing thanks to the creation of objective indicators such as the h-index. This is certainly the case in the exact, applied, and medical sciences (every researcher wants to know his/her h-index). Just as it is a challenge for citation analysis in general, it is a challenge to apply h-type indices to the human and social sciences in an appropriate way.

Endnote

1. Muirhead's (1903) theorem was published two years before Lorenz (1905) introduced the Lorenz curve. Therefore, Muirhead did not use the Lorenz terminology but a combina-

torial variant of it. Here we present the Lorenz variant of Muirhead's theorem. Hardy, Littlewood, and Pólya (1952) and Egghe and Rousseau (1991) also discuss Muirhead's theorem.

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